Investigation upon the performance of piezoelectric energy harvester with flexible extensions

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1. Model Description

We seek to investigate the influence of a flexible extension upon the overall performance of a classic piezoelectric cantilever beam energy harvester. In our problem, the energy harvester is comprised of two parts: the primary beam part and the beam extension part, as shown in Figure 1.

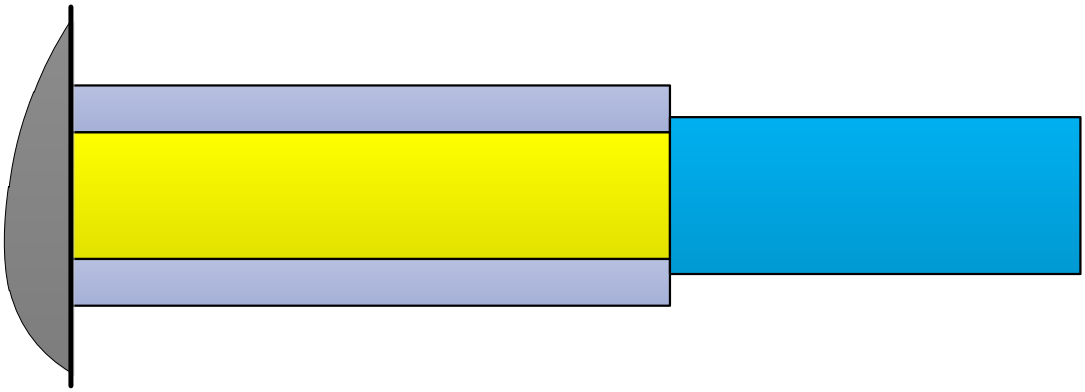

Figure 1: Schematic configuration of the piezoelectric energy harvester with flexible extension.

Following the classical analyzing process of piezoelectric bimorph cantilever beams [1, 2], we can simply list the following dimensional equations for the piezoelectric primary beam part as:

$$\begin{cases} M_p(x_1,t) = B_p \dfrac{\partial^2 w_1(x_1,t)}{\partial x_1^2} - e_p V_p(t) \\ q_p(x_1,t) = e_p \dfrac{\partial^2 w_1(x_1,t)}{\partial x_1^2} + \epsilon_p V_p(t), \end{cases} \tag{1}$$

where where $M_p(x_1,t)$ is the moment at cross section of x_1 and $q_p(x_1,t)$ is the corresponding line charge density on the electrode. $w_1(x_1,t)$ is the displacement function of the primary beam part with $0 \le x_1 \le l_p$ and $V_p(t)$ is the voltage across the electrodes. The corresponding coefficients B_p, e_p, and ϵ_p are defined as

$$B_p = \frac{2}{3} b \left\{ E_s h_s^3 + c_{11}^E \left[(h_s + h_p)^3 - h_s^3 \right] \right\}, \quad e_p = b e_{31} \left(h_s + \frac{1}{2} h_p \right), \quad \epsilon_p = \frac{b \epsilon_{33}^S}{2 h_p} \tag{2}$$

in which c_{11}^E and E_s are the elastic constants of the piezoelectric layer and the structure layer, respectively, e_{31} is the piezoelectric charge constant of the piezoelectic layer, ϵ_{33}^S is the dielectric constant of the piezoelectric layer, h_s and h_p are the half structure layer thickness and piezoelectric layer thickness, respectively, l_p is the length of the primary beam part, and b is the width of the primary beam part.

In terms of the mechanical balance, the equation of a piezoelectric beam can be established using the Euler-Bernoulli assumptions as follows

$$B_p \frac{\partial^4 w_1(x_1,t)}{\partial x_1^4} + m_p \frac{\partial^2 w_1(x_1,t)}{\partial t^2} = 0 \tag{3}$$

where $m_p = 2b(\rho_s h_s + \rho_p h_p)$ is the line mass density of the primary beam part with ρ_s and ρ_p being the volumetric density of the structure layer and the piezoelectric layer, respectively. In turn, principally the piezoelectric energy harvester can be regarded as a current source. So we need to know the charge accumulated on the electrode $Q_p(t)$, which is calculated as

$$Q_p(t) = \int_0^{l_p} q_p(t)\ dx_1 \ = \ e_p \left[\frac{\partial w_1(x_1,t)}{\partial x_1}\right]\Bigg|_0^{l_p} + C_p V_p(t) \tag{4}$$

where $C_p = \epsilon_p l_p$ is the inherent capacitance of the piezoelectric layer. According to the Kirchhoff's law, the electric equilibrium equation is

$$\frac{dQ_p(t)}{dt} + \frac{V_p(t)}{R_l} = 0 \tag{5}$$

where R_l is the externally connected resistive load.

When it comes to the beam extension part ($0 \le x_2 \le l_e$), the governing equations are

$$B_e \frac{\partial^4 w_2(x_2,t)}{\partial x_2^4} + m_e \frac{\partial^2 w_2(x_2,t)}{\partial t^2} = 0 \tag{6}$$

where $w_2(x_2,t)$ is the displacement of the extension beam at position $0 \le x_2 \le l_e$, $B_e = \frac{2}{3} b h_e^3$ is the equivalent bending stiffness of the extension beam, $m_e = \rho_e h_e$ is the line mass density of the extension beam, ρ_e is the volumetric mass density of the extension beam, h_e is the half thickness of the extension beam, and l_e is the length of the extension beam. As a result, the defining relations for the cross section moment $M_e(x_2,t)$ at the position x_2 is

$$M_e(x_2,t) = B_e \frac{\partial^2 w_2(x_2,t)}{\partial x_2^2}. \tag{7}$$

The related boundary conditions are listed as follows. When $x_1 = 0$ at the fixed end of the primary beam,

$$w_1(0,t) = w_b(t), \quad w_1'(0,t) = 0, \tag{8}$$

where $w_b(t)$ is the base excitation displacement function. Usually we use a harmonic vibration in the experiment where $w_b(t) = Re\left\{\xi_b e^{j\sigma t}\right\}$ with σ being the angular frequency of the base excitation signal and $j = \sqrt{-1}$ being the imaginary unit. To be more accurate, the amplitude ξ_b is generally set to be a real constant designated by the controller. At the connection point of the primary beam and the beam extension where $x_1 = l_p$ and $x_2 = 0$,

$$\begin{cases} w_1(l_p,t) = w_2(0,t) \\ \dfrac{\partial w_1(l_p,t)}{\partial x_1} = \dfrac{\partial w_2(0,t)}{\partial x_2} \\ B_p \dfrac{\partial^2 w_1(l_p,t)}{\partial x_1^2} - e_p V_p(t) = B_e \dfrac{\partial^2 w_2(0,t)}{\partial x_2^2} \\ B_p \dfrac{\partial^3 w_1(l_p,t)}{\partial x_1^3} = B_e \dfrac{\partial^3 w_2(0,t)}{\partial x_2^3} \end{cases}, \tag{9}$$

and at the free end of the beam extension where $x_2 = l_e$, we have

$$\frac{\partial^2 w_2(l_e,t)}{\partial x_2^2} = 0, \quad \frac{\partial^3 w_2(l_e,t)}{\partial x_2^3} = 0 \tag{10}$$

1.1. Harmonic Balance Analysis

Generally in the literature [1, 2], mode decomposition method or finite element method are used to solve the above described equations. Here in this contribution, as we are interested in the steady state response of the piezoelectric energy harvester, and the above described system are linear, harmonic balance method is used. Hence, as a result of the base excitation $w_b(t) = Re\left\{\xi_b e^{j\sigma t}\right\}$, we can set the steady state response of the displacements $w_1(x_1,t)$ and $w_2(x_2,t)$ of the primary beam and the beam extension respectively as

$$w_1(x_1,t) = \tilde{w}_1(x_1)e^{j\sigma t}, \quad w_2(x_2,t) = \tilde{w}_2(x_2)e^{j\sigma t}, \tag{11}$$

the steady state voltage response $V_p(t)$ and charge accumulation $Q_p(t)$ as

$$V_p(t) = \tilde{V}_p e^{j\sigma t}, \quad Q_p(t) = \tilde{Q}_p e^{j\sigma t}, \tag{12}$$

and the cross section moment $M_p(x_1,t)$ and $M_e(x_2,t)$ described as

$$M_p(x_1,t) = \tilde{M}_p(x_1)e^{j\sigma t}, \quad M_e(x_2,t) = \tilde{M}_e(x_2)e^{j\sigma t}. \tag{13}$$

As a result, the system of equations for the piezoelectric energy harvester can be summarized as

$$\begin{cases} B_p \dfrac{\partial^4 \tilde{w}_1(x_1)}{\partial x_1^4} - m_p\sigma^2\tilde{w}_1(x_1) = 0 \\ B_e \dfrac{\partial^4 \tilde{w}_2(x_2)}{\partial x_2^4} - m_e\sigma^2\tilde{w}_2(x_2) = 0 \\ j\sigma\tilde{Q}_p + \dfrac{\tilde{V}_p}{R_l} = 0 \end{cases}, \tag{14}$$

$$\begin{cases} \tilde{M}_p(x_1) = B_p \dfrac{\partial^2 \tilde{w}_1(x_1)}{\partial x_1^2} - e_p\tilde{V}_p \\ \tilde{Q}_p = e_p\left[\dfrac{\partial \tilde{w}_1(x_1)}{\partial x_1}\right]\Big|_0^{l_p} + C_p\tilde{V}_p \\ \tilde{M}_e(x_2) = B_e \dfrac{\partial^2 \tilde{w}_2(x_2)}{\partial x_2^2} \end{cases}, \tag{15}$$

and the boundary conditions become

$$\begin{cases} \tilde{w}_1(0) = \xi_b, & \dfrac{\partial \tilde{w}_1}{\partial x_1}(0) = 0 \\ w_1(l_p,t) = w_2(0,t), & \dfrac{\partial \tilde{w}_1(l_p)}{\partial x_1} = \dfrac{\partial \tilde{w}_2(0)}{\partial x_2} \\ B_p \dfrac{\partial^2 \tilde{w}_1(l_p)}{\partial x_1^2} - e_p\tilde{V}_p = B_e \dfrac{\partial^2 \tilde{w}_2(0)}{\partial x_2^2}, & B_p \dfrac{\partial^3 \tilde{w}_1(l_p)}{\partial x_1^3} = B_e \dfrac{\partial^3 \tilde{w}_2(0)}{\partial x_2^3} \\ \dfrac{\partial^2 \tilde{w}_2(l_e)}{\partial x_2^2} = 0, & \dfrac{\partial^3 \tilde{w}_2(l_e)}{\partial x_2^3} = 0 \end{cases}. \tag{16}$$

From the equations (14), (15), and (16), we can eliminate the electrical quantities $\tilde{Q}_p$ and $\tilde{V}_p$ by incorporating them into the boundary conditions. Actually, from equations (14) and (15), we have

$$\tilde{V}_p = \frac{j\sigma R_l e_p}{j\sigma R_l C_p + 1} \left[\frac{\partial \tilde{w}_1(x_1)}{\partial x_1}\right]\Bigg|_0^{l_p} \tag{17}$$

which can actually be used to eliminate the term $\tilde{V}_p$ in the boundary conditions (16). In the end, we can simplify the problem as a combination of the governing equations

$$\begin{cases} B_p \dfrac{\partial^4 \tilde{w}_1(x_1)}{\partial x_1^4} - m_p \sigma^2 \tilde{w}_1(x_1) = 0 \\ B_e \dfrac{\partial^4 \tilde{w}_2(x_2)}{\partial x_2^4} - m_e \sigma^2 \tilde{w}_2(x_2) = 0 \end{cases} \tag{18}$$

and the boundary conditions

$$\begin{cases} \tilde{w}_1(0) = \xi_b, \quad \dfrac{\partial \tilde{w}_1}{\partial x_1}(0) = 0 \\ \tilde{w}_1(l_p) = \tilde{w}_2(0), \quad \dfrac{\partial \tilde{w}_1(l_p)}{\partial x_1} = \dfrac{\partial \tilde{w}_2(0)}{\partial x_2} \\ B_p \dfrac{\partial^2 \tilde{w}_1(l_p)}{\partial x_1^2} + \dfrac{j\sigma R_l e_p^2}{j\sigma R_l C_p + 1} \dfrac{\partial \tilde{w}_1(l_p)}{\partial x_1} = B_e \dfrac{\partial^2 \tilde{w}_2(0)}{\partial x_2^2}, \quad B_p \dfrac{\partial^3 \tilde{w}_1(l_p)}{\partial x_1^3} = B_e \dfrac{\partial^3 \tilde{w}_2(0)}{\partial x_2^3} \\ \dfrac{\partial^2 \tilde{w}_2(l_e)}{\partial x_2^2} = 0, \quad \dfrac{\partial^3 \tilde{w}_2(l_e)}{\partial x_2^3} = 0 \end{cases} . \tag{19}$$

which actually manifests as a boundary value problem.

2. Dimensionless Problem

Using the following dimensionless group

$$\tilde{w}_1, \tilde{w}_2 \sim \xi_b, \quad \tilde{x}_1 \sim l_p, \quad \tilde{x}_2 \sim l_e \tag{20}$$

we can nondimensionalize the above formulated boundary value problem with respect to the following variables:

$$\tilde{w}_1 = \xi_b u_1, \quad \tilde{w}_2 = \xi_b u_2, \quad \tilde{x}_1 = l_p x, \quad \tilde{x}_2 = l_e x. \tag{21}$$

Note that here we use one independent space variable x to nondimensionalize two previously used variables x_1 and x_2. This comes from the fact that the variables x_1 and x_2 are not coupled with each other in the sense that the primary beam and the extension beam do not overlap each other except for their joint point where $x_1 = l_p$ and $x_2 = 0$. Thus the two variables do not occur in the equations simultaneously except for the boundary conditions. As for the boundary conditions, the change of variables does not affect the values of the equations. Therefore, the two parts of the piezoelectric energy harvester beam are in fact independent of each other except for the joining point. In one word, the equation (21) does not change the problem in essence.

Hence, the above boundary value problem is further changed into the combination of the governing equations

$$\begin{cases} \dfrac{B_p}{l_p^4} u_1'''' - m_p \sigma^2 u_1 = 0 \\ \dfrac{B_e}{l_e^4} u_2'''' - m_e \sigma^2 u_2 = 0 \end{cases} \tag{22}$$

and the boundary conditions

$$\begin{cases} u_1(0)=1, \quad u_1'(0)=0 \\ u_1(1)=u_2(0), \quad \dfrac{1}{l_p}u_1'(1)=\dfrac{1}{l_e}u_2'(0) \\ \dfrac{B_p}{l_p^2}u_1''(1)+\dfrac{j\sigma R_l e_p^2}{j\sigma R_l C_p+1}\dfrac{1}{l_p}u_1'(1)=\dfrac{B_e}{l_e^2}u_2''(0), \quad \dfrac{B_p}{l_p^3}u_1'''(1)=\dfrac{B_e}{l_e^3}u_2'''(0) \\ u_2''(1)=0, \quad u_2'''(1)=0 \end{cases}. \tag{23}$$

in which the prime means the derivative with respect to x. The equations can again be organized in a more compact form

$$\begin{cases} u_1''''-\nu^2 u_1=0 \\ u_2''''-\nu^2\lambda_m\lambda_l^4/\lambda_B u_2=0 \end{cases} \tag{24}$$

and the boundary conditions

$$\begin{cases} u_1(0)=1, \quad u_1'(0)=0 \\ u_1(1)=u_2(0), \quad \lambda_l u_1'(1)=u_2'(0) \\ u_1''(1)+\dfrac{j\nu\beta}{j\nu\beta+1}\alpha^2 u_1'(1)=\lambda_B/\lambda_l^2 u_2''(0), \quad u_1'''(1)=\lambda_B/\lambda_l^3 u_2'''(0) \\ u_2''(1)=0, \quad u_2'''(1)=0 \end{cases}. \tag{25}$$

where

$$\nu=\sigma\sqrt{\frac{m_p l_p^4}{B_p}}, \quad \lambda_B=\frac{B_e}{B_p}, \quad \lambda_m=\frac{m_e}{m_p}, \quad \lambda_l=\frac{l_e}{l_p} \tag{26}$$

$$\beta=R_l C_p\sqrt{\frac{B_p}{m_p l_p^4}}, \quad \alpha=e_p\sqrt{\frac{l_p}{C_p B_p}} \tag{27}$$

The system (24) and (25) is a two-point boundary value problem. The problem can readily be solved by a Chebyschev collocation method using the MATLAB package *Chebfun* [3].

3. Influence of extension part upon energy harvester performance

The basic geometry and material properties of the materials used in the proposed piezoelectric energy harvester with flexible extensions are summarized in Table 1. Note that some of the parameters, like length l_e, Young's modulus Y_e, and volumetric density ρ_e of the beam extension, are actually changing across different simulations. In the simulation, base excitation frequency fr and external load resistance R_l, which change the dimensionless values of ν and β, respectively, are of critical importance in the sense that these two parameters reflect the influence of vibration source frequency spectrum and external load circuit. For every set of parameter values, we set the base excitation frequency fr to change from 1 Hz to 100 Hz, which covers the usual frequency range of natural vibration sources, and set the load resistance R_l to change from 1 Ω to 10 $M\Omega$, which is inspired by the Ref. [1] and takes into account the dielectric property of piezoelectric materials. Actually, when the load resistance $R_l = 1\ \Omega$, the piezoelectric energy harvester is said to be in a short-circuit condition as the general equivalent resistance of the structure is much larger than R_l. On the other hand, when $R_l = 10\ M\Omega$, the system is close to an open-circuit condition where no external load is connected to the output electrodes.

In the following, we will investigate the influences of length l_e, Young's modulus Y_e, and volumetric density ρ_e of the beam extension upon the performance of the piezoelectric energy harvester with flexible extensions separately.

Table 1: Geometric, material, and electromechanical parameters of the simulation for piezoelectric energy harvester with flexible extension

Parameter item	**Parameter value**
Length of the primary beam, l_p (mm)	100
Width of the whole energy harvester, b (mm)	20
Half thickness of the structure, h_s (mm)	0.25
Thickness of the piezoelectric layer, h_p (mm)	0.2
Young's modulus of the structure, Y_s (Gpa)	100
Young's modulus of the piezoelectric layer, Y_p (Gpa)	66
Mass density of the substructure, ρ_s (kg/m^3)	7165
Mass density of the piezoelectric layer, ρ_p (kg/m^3)	7800
piezoelectric constant, d_{31} (pm/V)	-190
Permittivity, ϵ_{33}^S (nF/m)	15.93
Length of the beam extension, l_e (mm)	30
Young's modulus of the beam extension, Y_e (Gpa)	2.3
Mass density of the beam extension, ρ_e (kg/m^3)	1.38
Half thickness of the structure, h_e (mm)	0.25

3.1. beam extension length l_e or length ratio λ_l

The presence of the beam extension is firstly controlled by the beam extension length l_e or length ratio λ_l, equivalently. When $\lambda_l = 0.0$, no beam extension is attached the resultant energy harvester reduces to the classical piezoelectric energy harvesting bimorph. [1] In our research, this case is referred to as a reference. In the considered range of length ratio $0 \leq \lambda_l \leq 1$, the base excitation problem is solved with respect to different base excitation frequency fr and external load resistance R_l.

In the first place, we plot the resulting amplitude of output voltage V_p with respect to the length ratio of λ_l = 0, 0.2, 0.4, 0.6, 0.8, 1.0 in Figure 2 and the corresponding amplitude of output power P_p in Figure 3.

According to Figure 2 and Figure 3, for given λ_B and λ_m, frequency response of the piezoelectric energy harvester in the interested frequency range $(1 - 100\ Hz)$ changes in accordance with the extension length l_e and thus the length ratio λ_l. When λ_l is relatively small, say $\lambda_l = 0.1$ and $\lambda_l = 0.2$ (as shown in Figure 2(b)) in our case, the vibration of the whole beam is almost the same as that of the case where $\lambda_l = 0.0$. Only slight change of the resonant frequency of eigen mode (We call this mode $L1$) is observed. Therefore the beam extension part is playing a negligible role in the motion of the primary beam part. As a result, energy harvesting performances of the proposed piezoelectric energy harvesters with flexible extensions are similar to that of a classic cantilever beam based piezoelectric energy harvester.

With the increase of the length ratio λ_l, say $\lambda_l = 0.4$, 0.6, or 0.8, there begins to exist an extra resonant and anti-resonant mode in the considered frequency range, compared with that of the case where $\lambda_l = 0.0$. What's more, for the cases of $\lambda_l = 0.4$ and 0.6, the shift of the resonant frequency of mode $L1$ is significant compared with the case of small λ_l. Meanwhile, when $\lambda_l = 0.8$, the frequency shift is again negligible.

With further increase of the length ratio to $\lambda_l = 1.0$, two extra resonant and anti-resonant modes occur in the considered frequency range and the frequency shift begins to be important.

In this view, the attachment of the flexible extension part may introduces extra vibration modes in the considered frequency range. Noting that typical piezoelectric energy harvesters rely on resonant modes to work, this will actually expands the bandwidth of the proposed piezoelectric energy harvester. On the other hand, the flexible extension also provides a way to tune the bandwidth of a piezoelectric energy harvester. The point to be noticed is the existence of anti-resonant modes. They correspond to low output voltage and output power, and to some degree narrows the bandwidth of the energy harvester. However, it is still not worse than an otherwise non-resonant vibration, as in both cases the

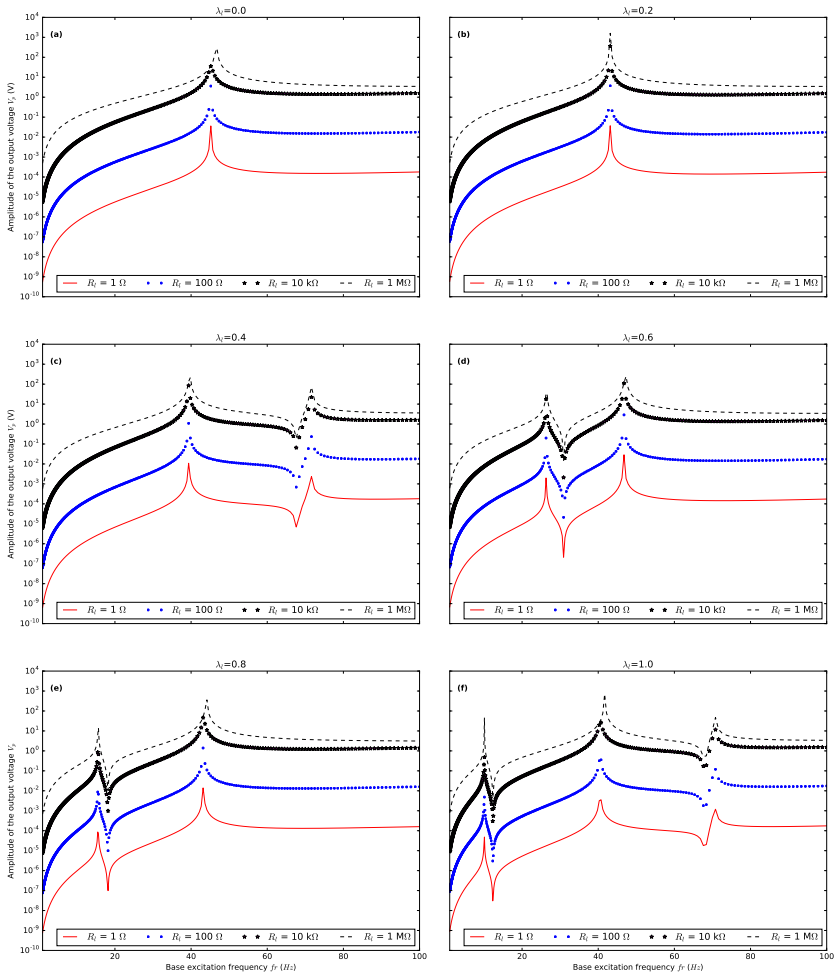

Figure 2: Output voltage V_p (amplitude) of the piezoelectric energy harvester with flexible extension versus length ratio λ_l at different frequency f and load resistance R_l.

electrical output of the piezoelectric energy harvester is unusable.

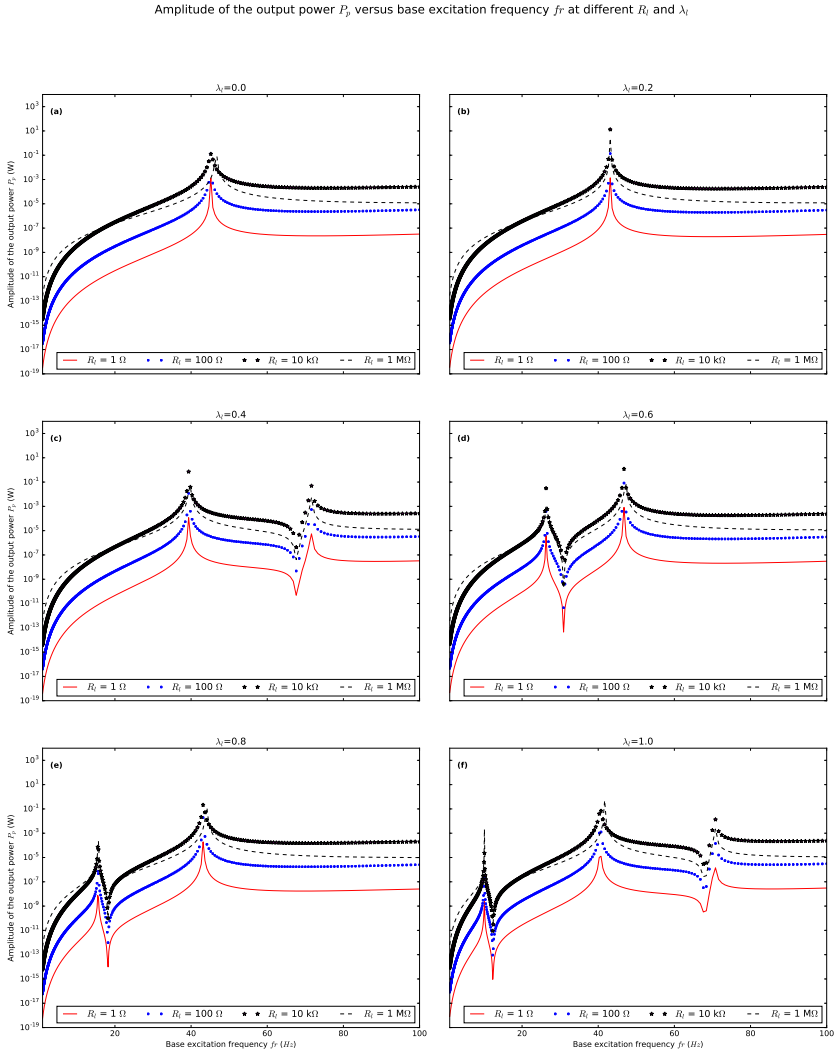

Figure 3: Output power P_p (amplitude) of the piezoelectric energy harvester with flexible extension versus length ratio λ_l at different frequency f and load resistance R_l.

In the second place, we compare the case where $\lambda = 0.3$ and 0.4 with the reference case where $\lambda = 0.0$ in terms of different externally connected resistance R_l. The resulting amplitude of output voltage V_p and output power P_p are shown in Figure 4. It is obvious that in the range of low R_l, a power law exists between the amplitudes of output voltage and power and the connected resistance R_l. For all the chosen frequency values fr, the increase of R_l leads to an increase in V_p, and finally it is found that V_p approaches asymptotically to a limit value V_p^{lim}. This value actually corresponds to the open-circuit output voltage of the piezoelectric energy harvester. At the same time, the value of P_p exhibits an

obvious maximum at some values of R_l between $10k\Omega$ and $1M\Omega$. And away from the maximum value, we see also a power law between the value of R_l and P_p. This indicates that an asymptotic analysis may help us to simplify the analysis of performance of piezoelectric energy harvesters. But this is out of the scope of our current contribution and will be covered in the furture research.

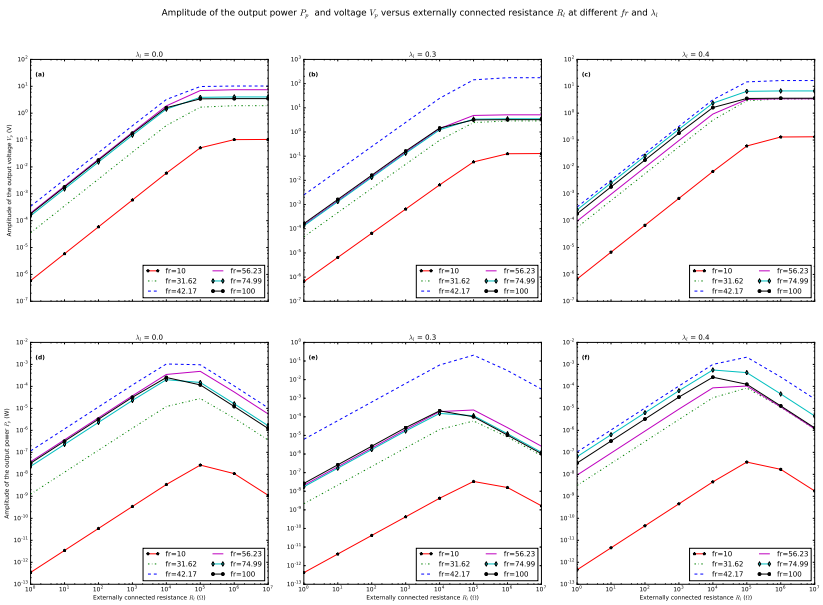

Figure 4: Output voltage V_p (amplitude) and output power P_p (amplitude) of the piezoelectric energy harvester with flexible extension versus bending stiffness ratio λ_l at different frequency f and load resistance R_l.

In the third place, at different values of fr and R_l, we directly plot the amplitudes of V_p and P_p with respect to λ_l, which are shown in Figure 5 and Figure 5. It is shown from the figures that at given values of R_l and fr, output performances of the proposed piezoelectric energy harvesters can be tuned by the parameter λ_l. For example, when $fr = 31.62\ Hz$ and $R_l = 10\ k\Omega$, a maximum peak around $\lambda_l = 0.5$ is shown if we increase the values of λ_l. Compared with the case of no extension ($\lambda_l = 0$), the amplitude of output voltage V_p is about 3 times larger, while the amplitude of output power P_p is about 3 times larger. This indicates that the addition of the flexible extension can substantially increase the output performance of a piezoelectric energy harvester. It should be noted that in case of large values of fr, like $fr = 100Hz$, the tuning performance of the flexible extension is no longer siginificant.

3.2. beam extension bending stiffness or bending stiffness ratio λ_B

It is easily seen from the diagram that the change of extension length l_e have a great influence on the frequency response of the piezoelectric energy harvester. At the given values of λ_B and λ_m, the contained vibration modes in the frequency range considered does change with respect to the length ratio λ_l. When λ_l is relatively small, which is below 0.3 in our case, no extra vibration modes can be found in the frequency range of $1 - 100\ Hz$. Hence the energy harvesting performances of the proposed energy harvesters are similar to that of a pure cantilever beam piezoelectric energy harvester. (note: it will be better if I can compare the resonant energy harvesting performance in this case) With further increase of the length ratio λ_l, there begins to exist an extra resonant and anti-resonant mode in the considered frequency range. In this view, the change of length ratio actually expands the bandwidth of the energy harvesting performances. Even when

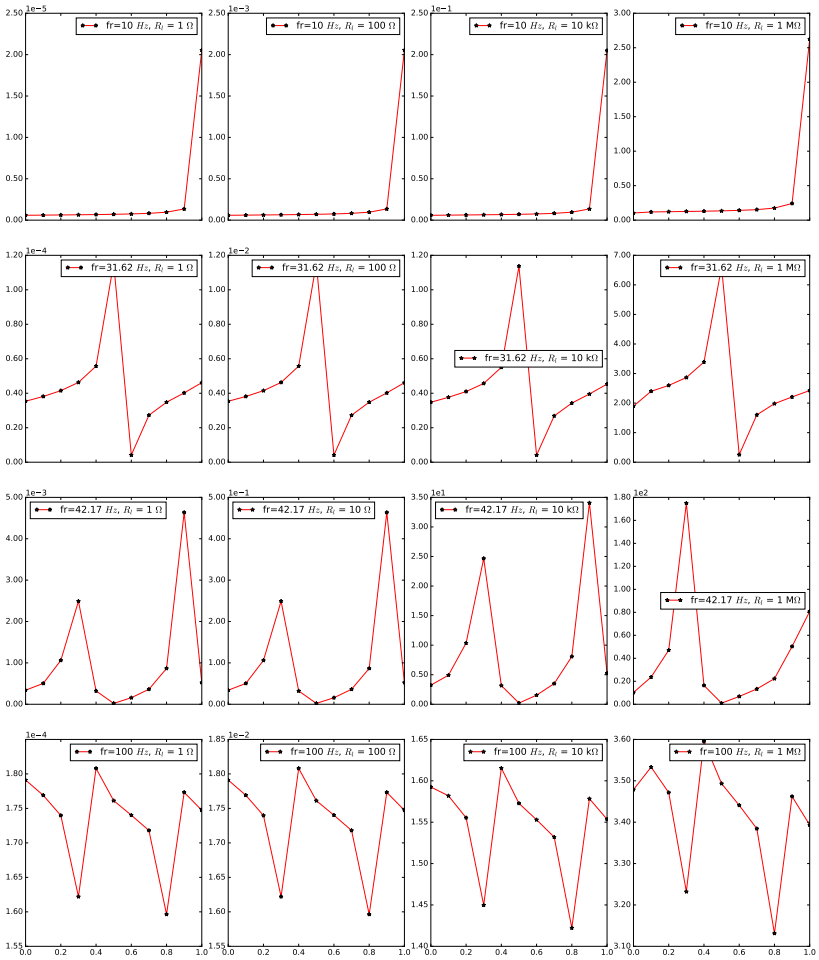

Figure 5: Output voltage V_p (amplitude) and output power P_p (amplitude) of the piezoelectric energy harvester with flexible extension versus bending stiffness ratio λ_l at different frequency f and load resistance R_l.

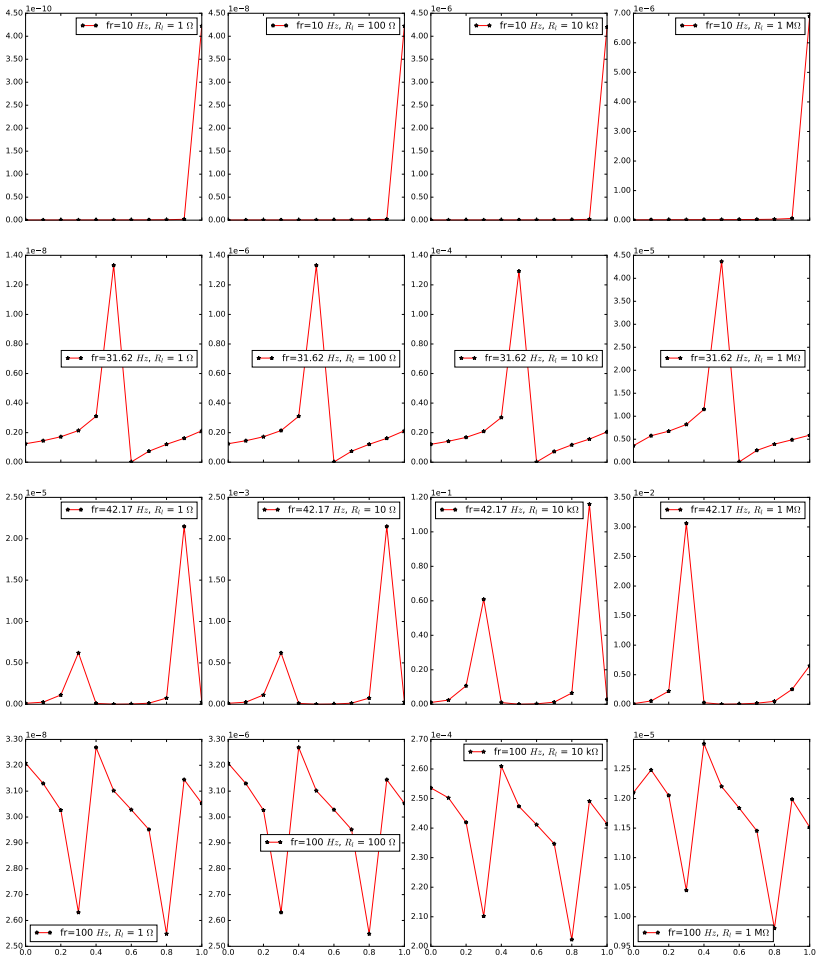

Figure 6: Output voltage V_p (amplitude) and output power P_p (amplitude) of the piezoelectric energy harvester with flexible extension versus bending stiffness ratio λ_l at different frequency f and load resistance R_l.

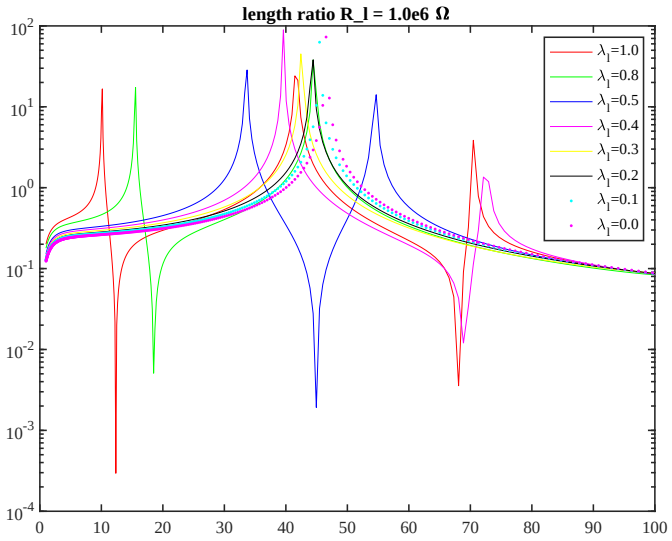

Figure 7: Output voltage V_p (amplitude) and output power P_p (amplitude) of the piezoelectric energy harvester with flexible extension versus bending stiffness ratio λ_l at different frequency f and load resistance R_l.

the length ratio $\lambda_l = 1.0$, three resonant modes are found in the frequency range. The point to be noticed is the existence of anti-resonant mode, which largely narrows the bandwidth of the energy harvester. This accompanying characteristic is to be further investigated in the following research.

3.3. beam extension line density or line density ratio λ_m

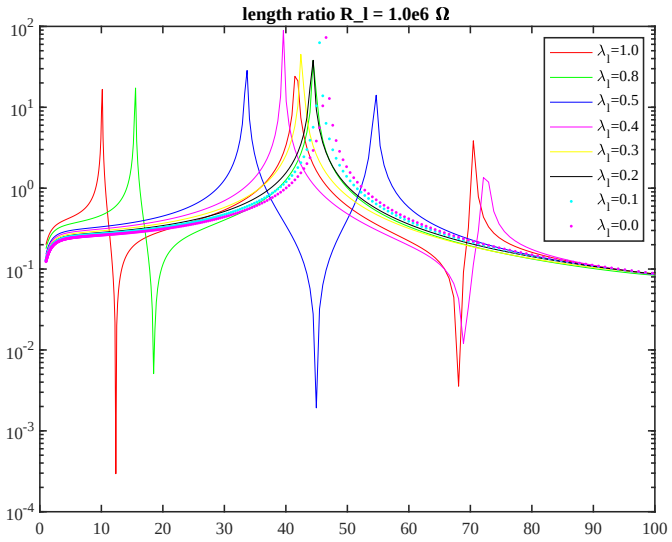

Figure 8: Output voltage V_p (amplitude) and output power P_p (amplitude) of the piezoelectric energy harvester with flexible extension versus line mass density ratio λ_m at different frequency f and load resistance R_l.

It is easily seen from the diagram that the change of extension length l_e have a great influence on the frequency response of the piezoelectric energy harvester. At the given values of λ_B and λ_m, the contained vibration modes in the frequency range considered does change with respect to the length ratio λ_l. When λ_l is relatively small, which is below 0.3 in our case, no extra vibration modes can be found in the frequency range of $1 - 100\ Hz$. Hence the energy harvesting performances of the proposed energy harvesters are similar to that of a pure cantilever beam piezoelectric energy harvester. (note: it will be better if I can compare the resonant energy harvesting performance in this case) With further increase of the length ratio λ_l, there begins to exist an extra resonant and anti-resonant mode in the considered frequency range. In this view, the change of length ratio actually expands the bandwidth of the energy harvesting performances. Even when the length ratio $\lambda_l = 1.0$, three resonant modes are found in the frequency range. The point to be noticed is the existence of anti-resonant mode, which largely narrows the bandwidth of the energy harvester. This accompanying characteristic is to be further investigated in the following research.

4. Discussion

5. Conclusion

Here in this contribution, we investigate the method of flexible extension to tune the energy harvesting performance of piezoelectric cantilever energy harvester.

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