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Cite as: AIP Conference Proceedings **2116**, 300002 (2019); https://doi.org/10.1063/1.5114302
Published Online: 24 July 2019

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Optimal Homotopy Asymptotic Method in the Study of Energy Harvesting Problems

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Abstract. An investigation of the nonlinear behavior of a bistable energy harvesting device is proposed by using a powerful analytical technique, namely the Optimal Homotopy Asymptotic Method (OHAM). This techni que proves its applicability in this kind of problems demonstrating a very fast convergence, due to the presence of the convergence-control parameters, whose optimal values, which are determined using rigorous procedure, ensure the convergence of the solutions after the first iteration.

INTRODUCTION

Vibration energy harvesting is one of the most challenging tasks for engineers nowadays and represents a major field of research, taking into account the actual need to exploit renewable energy, especially as an alternative for micropowering for small electronics. Basically, the mechanism which governs the conversion of vibration energy to electricity may use piezoelectric, electromagnetic, electrostatic, or magnetostrictive transductions [1]. A remarkable solution from technological point of view is represented by bistable energy harvesting devices, exhibiting nonlinear characteristics and proving some unique features. A review of the recent research on vibration energy harvesting via bistable systems is developed by Harne and Wang [2], emphasizing the most important achievements in this field in recent years. In the early stage of research, the nonlinear behavior of bistable energy harvesting devices was numerically investigated [3], but after that, analytical approaches have been preferred, since analytical techniques provide a deep inside and a more rigorous assessment of the nonlinear behavior of such devices. In this respect, some analytical techniques were successfully employed, such as the harmonic balance method [4], the Melnikov theory [5], the method of multiple scales [6], and so on.

This study presents an alternative to the investigation of the nonlinear behavior of bistable energy harvesting devices, offering an explicit analytical solution by means of the Optimal Homotopy Asymptotic Method (OHAM). This method is able to deal with strongly nonlinear dynamic problems such as those specific to this kind of devices, since its application does not depend on the presence of small parameters in the governing equations.

APLICATION OF OHAM FOR THE BISTABLE ENERGY HARVESTING DEVICE

We will apply in what follows the Optimal Homotopy Asymptotic Method (OHAM) [7-10] to investigate the model of the bistable energy harvesting device [4]

$$\begin{aligned} &\ddot{x}+\mu_1\dot{x}+x+x^3-\varepsilon\theta v=\Gamma\cos\omega t \\ &\dot{v}+\mu_2 v+\theta\dot{x}=0 \end{aligned} \tag{1}$$

International Conference of Numerical Analysis and Applied Mathematics (ICNAAM 2018)
AIP Conf. Proc. 2116, 300002-1–300002-4; https://doi.org/10.1063/1.5114302
Published by AIP Publishing. 978-0-7354-1854-7/$30.00

with the initial conditions

$$x(0)=A,\quad \dot{x}(0)=0 \\ v(0)=B \tag{2}$$

As usual in this kind of developments, we switch the variable under the transformation $\tau=\Omega t$ so that the governing equations may be written as

$$x''+\frac{\mu_1}{\Omega}x'+x+x\left(\frac{1}{\Omega^2}-1\right)+\frac{x^3}{\Omega^2}-\frac{\varepsilon\theta}{\Omega^2}v-\frac{\Gamma}{\Omega^2}\cos\frac{\omega}{\Omega}\tau=0 \tag{3}$$

$$v'+\frac{\mu_2}{\Omega}v+\theta x'=0 \tag{4}$$

Following the procedure described in [7-10], we could identify the linear operators for the governing equations under the form

$$Lx=x''+x \tag{5}$$
$$Lv=v' \tag{6}$$

Therefore, the initial approximation will be determined from the linear differential equation

$$Lx_0=0,\quad x_0(0)=A, x_0'(0)=0 \tag{7}$$

having the solution

$$x_0(\tau)=A\cos\tau \tag{8}$$

The corresponding linear differential equation which offers the initial approximation for v is

$$Lv_0+g(v)=0,\quad v_0'+B\sin\tau=0,\quad v_0(0)=B \tag{9}$$

and consequently, the initial approximation for v will be

$$v_0(\tau)=B\cos\tau \tag{10}$$

The nonlinear operator for the first governing equation is

$$N(x)=\frac{\mu_1}{\Omega}x'+x\left(\frac{1}{\Omega^2}-1\right)+\frac{x^3}{\Omega^2}-\frac{\varepsilon\theta}{\Omega^2}v-\frac{\Gamma}{\Omega^2}\cos\frac{\omega}{\Omega}\tau \tag{11}$$

and using the initial approximations, we obtain

$$N(x_0)=\alpha\sin\tau+\beta\cos\tau+\frac{A^3}{4}\cos 3\tau \tag{12}$$

where

$$\alpha=-\frac{\mu_1 A}{\Omega},\quad \beta=A\left(\frac{1}{\Omega^2}-1\right)+\frac{3}{4\Omega^2}A^3-\frac{\varepsilon\theta}{\Omega^2}B-\Gamma \tag{13}$$

Similarly, for the second governing equation the nonlinear operator is

$$N(v)=\frac{\mu_2}{\Omega}v+\theta x'-B\sin\tau \tag{14}$$

and taking into account the obtained initial approximations, we have

$$N(v_0) = \gamma \cos \tau + \delta \sin \tau \tag{15}$$

where

$$\gamma = \frac{\mu_2}{\Omega} B, \quad \delta = -(B + \theta A) \tag{16}$$

At this point, following the procedure, we can write the linear differential equations which will offer the first-order approximate solutions. For the first-order approximate solution x_1 we have

$$x_1'' + x_1 = (C_1 + 2C_2 \cos 2\tau + 2C_3 \sin 2\tau + 2C_4 \cos 4\tau + 2C_5 \sin 4\tau)(\alpha \sin \tau + \beta \cos \tau + \frac{A^3}{4} \cos 3\tau) \tag{17}$$

with the initial conditions

$$x_1(0) = x_1'(0) = 0 \tag{18}$$

while for the first-order approximate solution v_1 we have

$$v_1' = (C_6 + 2C_7 \cos 2\tau + 2C_8 \sin 2\tau + 2C_9 \cos 4\tau + 2C_{10} \sin 4\tau)(\gamma \cos \tau + \delta \sin \tau) \tag{19}$$

with the initial conditions

$$v_1(0) = 0 \tag{20}$$

Taking into account also the necessary conditions to avoid secular terms, after simple developments, taking into account that OHAM solutions are obtained by adding initial and first-order approximations

$$\begin{aligned} \tilde{x}(t) &= x_0(t) + x_1(t, C_i), \quad i = 1,2,\ldots,p \\ \tilde{v}(t) &= v_0(t) + v_1(t, C_j), \quad j = 1,2,\ldots,s \end{aligned} \tag{21}$$

we obtain for the problem under study the following approximate solutions:

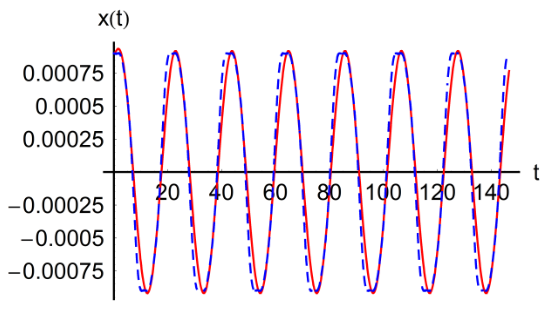
$$\begin{aligned} \tilde{x} &= A \cos \tau + \frac{\alpha(C_2 - C_4) + \beta(C_3 + C_5)}{8}(3 \sin \tau - \sin 3\tau) + \frac{\alpha(C_5 - C_3) + \beta(C_2 + C_4)}{8}(\cos \tau - \cos 3\tau) + \\ & \frac{\alpha C_4 + \beta C_5 + \frac{A^3 C_3}{4}}{24}(5 \sin \tau - \sin 5\tau) + \frac{\beta C_4 - \alpha C_5 + \frac{A^3 C_3}{4}}{24}(\cos \tau - \cos 5\tau) + \frac{A^3 C_5}{192}(7 \sin \tau - \sin 7\tau) + \\ & + \frac{A^3 C_4}{192}(\cos \tau - \cos 7\tau) \end{aligned} \tag{22}$$

$$\begin{aligned} \tilde{v} &= B \cos \tau + \frac{\delta(C_7 - C_9) + \gamma(C_8 + C_{10})}{8}(3 \sin \tau - \sin 3\tau) + \frac{\delta(C_{10} - C_8) + \gamma(C_7 + C_9)}{8}(\cos \tau - \cos 3\tau) + \\ & \frac{\delta C_9 + \beta C_{10}}{24}(5 \sin \tau - \sin 5\tau) + \frac{\gamma C_9 - \delta C_{10}}{24}(\cos \tau - \cos 5\tau) \end{aligned} \tag{23}$$

The optimal values of the convergence-control parameters are obtained following the procedure described in [7], after simple computations implemented using Mathematica software. The procedure is very easy to implement since it always lead to solving a system of algebraic equations, which is an easy task for a regular computer. Obviously, the number of algebraic equations equals the number of convergence-control parameters involved in the analytical solutions, which could be smaller or larger, depending on the requested accuracy of the solutions.

$$C_1 = -0.123891491137, C_2 = 0.032258789096, C_3 = -0.119585826329, C_4 = 0.271318654200,$$
$$C_5 = 0.143967330110, C_6 = 0.691132430852, C_7 = -0.647752766861, C_8 = -0.240998734434, \quad (24)$$
$$C_9 = 0.275185158088, C_{10} = 0.035255406368$$

The numerical and analytical solutions of Eqs. (1) and (2) are presented in Fig. 1 for a set of specific physical parameters.



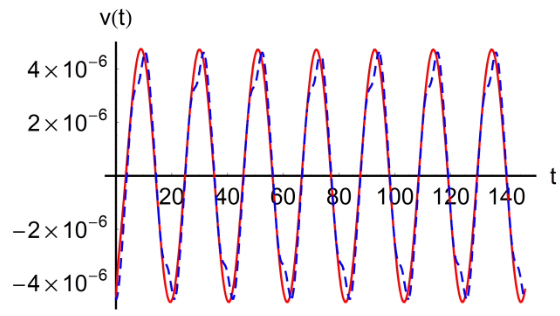


FIGURE 1. Analytical and numerical solutions for (a) x and respectively (b) v, for μ_1=2, μ_2=0.5, ω=0.3, Γ=0.001, θ=0.01
______ numerical results; _ _ _ _ _ analytical results (22), (23)

The Fig.1 presents a comparison between the analytical and numerical results for a specific set of physical parameters involved in the governing equations.

CONCLUSIONS

The nonlinear behavior of a bistable energy harvesting device is investigated in this paper. The principal goal of our work has been to construct an approximation of the solution on large interval by means of a powerful analytical technique, namely the Optimal Homotopy Asymptotic Method (OHAM) in order to test its applicability in this kind of nonlinear problems. Our procedure is based on a new construction of the solution and especially on the involvement of the so-called convergence-control parameters C_1, …, C_{10}. These parameters lead to agood accuracy of our approximate analytical solution comparing to numerical results. It is to remark that if a better accuracy is needed, then we just simply increase the number of convergence-control parameters involved in the analytical solutions. It was proved that OHAM is an explicit analytical method, very effective, rapidly convergent to an accurate solution after the first iteration.

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