

# Investigation upon the performance of piezoelectric energy harvester with flexible extensions

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## 1. Model Description

We seek to investigate the influence of a flexible extension upon the overall performance of a classic piezoelectric cantilever beam energy harvester. In our problem, the energy harvester is comprised of two parts: the primary beam part and the beam extension part, as shown in Figure 1.

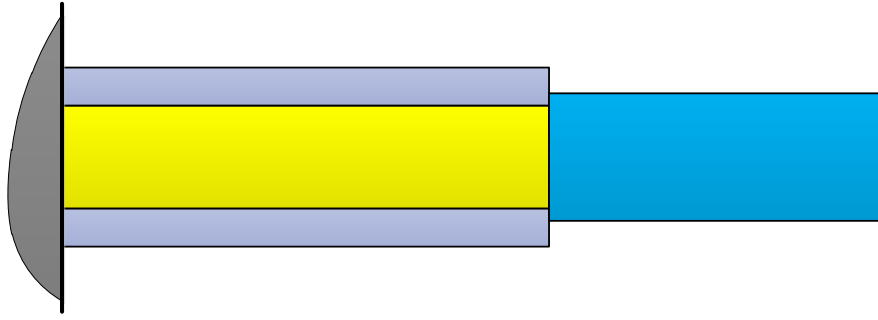


Figure 1: Schematic configuration of the piezoelectric energy harvester with flexible extension.

Following the classical analyzing process of piezoelectric bimorph cantilever beams [1, 2], we can simply list the following dimensional equations for the piezoelectric primary beam part as:

$$\begin{cases} M_p(x_1, t) = B_p \frac{\partial^2 w_1(x_1, t)}{\partial x_1^2} - e_p V_p(t) \\ q_p(x_1, t) = e_p \frac{\partial^2 w_1(x_1, t)}{\partial x_1^2} + \epsilon_p V_p(t), \end{cases} \quad (1)$$

where where  $M_p(x_1, t)$  is the moment at cross section of  $x_1$  and  $q_p(x_1, t)$  is the corresponding line charge density on the electrode.  $w_1(x_1, t)$  is the displacement function of the primary beam part with  $0 \leq x_1 \leq l_p$  and  $V_p(t)$  is the voltage across the electrodes. The corresponding coefficients  $B_p$ ,  $e_p$ , and  $\epsilon_p$  are defined as

$$B_p = \frac{2}{3}b \{ E_s h_s^3 + c_{11}^E [(h_s + h_p)^3 - h_s^3] \}, \quad e_p = b e_{31} \left( h_s + \frac{1}{2} h_p \right), \quad \epsilon_p = \frac{b \epsilon_{33}^S}{2 h_p} \quad (2)$$

in which  $c_{11}^E$  and  $E_s$  are the elastic constants of the piezoelectric layer and the structure layer, respectively,  $e_{31}$  is the piezoelectric charge constant of the piezoelectric layer,  $\epsilon_{33}^S$  is the dielectric constant of the piezoelectric layer,  $h_s$  and  $h_p$  are the half structure layer thickness and piezoelectric layer thickness, respectively,  $l_p$  is the length of the primary beam part, and  $b$  is the width of the primary beam part.

In terms of the mechanical balance, the equation of a piezoelectric beam can be established using the Euler-Bernoulli assumptions as follows

$$B_p \frac{\partial^4 w_1(x_1, t)}{\partial x_1^4} + m_p \frac{\partial^2 w_1(x_1, t)}{\partial t^2} = 0 \quad (3)$$

where  $m_p = 2b(\rho_s h_s + \rho_p h_p)$  is the line mass density of the primary beam part with  $\rho_s$  and  $\rho_p$  being the volumetric density of the structure layer and the piezoelectric layer, respectively. In turn,

principally the piezoelectric energy harvester can be regarded as a current source. So we need to know the charge accumulated on the electrode  $Q_p(t)$ , which is calculated as

$$Q_p(t) = \int_0^{l_p} q_p(t) dx_1 = e_p \left[ \frac{\partial w_1(x_1, t)}{\partial x_1} \right] \Big|_0^{l_p} + C_p V_p(t) \quad (4)$$

where  $C_p = \epsilon_p l_p$  is the inherent capacitance of the piezoelectric layer. According to the Kirchhoff's law, the electric equilibrium equation is

$$\frac{dQ_p(t)}{dt} + \frac{V_p(t)}{R_l} = 0 \quad (5)$$

where  $R_l$  is the externally connected resistive load.

When it comes to the beam extension part ( $0 \leq x_2 \leq l_e$ ), the governing equations are

$$B_e \frac{\partial^4 w_2(x_2, t)}{\partial x_2^4} + m_e \frac{\partial^2 w_2(x_2, t)}{\partial t^2} = 0 \quad (6)$$

where  $w_2(x_2, t)$  is the displacement of the extension beam at position  $0 \leq x_2 \leq l_e$ ,  $B_e = \frac{2}{3} b h_e^3$  is the equivalent bending stiffness of the extension beam,  $m_e = \rho_e h_e$  is the line mass density of the extension beam,  $\rho_e$  is the volumetric mass density of the extension beam,  $h_e$  is the half thickness of the extension beam, and  $l_e$  is the length of the extension beam. As a result, the defining relations for the cross section moment  $M_e(x_2, t)$  at the position  $x_2$  is

$$M_e(x_2, t) = B_e \frac{\partial^2 w_2(x_2, t)}{\partial x_2^2}. \quad (7)$$

The related boundary conditions are listed as follows. When  $x_1 = 0$  at the fixed end of the primary beam,

$$w_1(0, t) = w_b(t), \quad w_1'(0, t) = 0, \quad (8)$$

where  $w_b(t)$  is the base excitation displacement function. Usually we use a harmonic vibration in the experiment where  $w_b(t) = \text{Re} \{ \xi_b e^{j\sigma t} \}$  with  $\sigma$  being the angular frequency of the base excitation signal and  $j = \sqrt{-1}$  being the imaginary unit. To be more accurate, the amplitude  $\xi_b$  is generally set to be a real constant designated by the controller. At the connection point of the primary beam and the beam extension where  $x_1 = l_p$  and  $x_2 = 0$ ,

$$\left\{ \begin{array}{l} w_1(l_p, t) = w_2(0, t) \\ \frac{\partial w_1(l_p, t)}{\partial x_1} = \frac{\partial w_2(0, t)}{\partial x_2} \\ B_p \frac{\partial^2 w_1(l_p, t)}{\partial x_1^2} - e_p V_p(t) = B_e \frac{\partial^2 w_2(0, t)}{\partial x_2^2} \\ B_p \frac{\partial^3 w_1(l_p, t)}{\partial x_1^3} = B_e \frac{\partial^3 w_2(0, t)}{\partial x_2^3} \end{array} \right. \quad (9)$$

and at the free end of the beam extension where  $x_2 = l_e$ , we have

$$\frac{\partial^2 w_2(l_e, t)}{\partial x_2^2} = 0, \quad \frac{\partial^3 w_2(l_e, t)}{\partial x_2^3} = 0 \quad (10)$$

### 1.1. Harmonic Balance Analysis

Generally in the literature [1, 2], mode decomposition method or finite element method are used to solve the above described equations. Here in this contribution, as we are interested in the steady state response of the piezoelectric energy harvester, and the above described system are linear, harmonic balance method is used. Hence, as a result of the base excitation  $w_b(t) = \text{Re} \{ \xi_b e^{j\sigma t} \}$ , we can set the steady state response of the displacements  $w_1(x_1, t)$  and  $w_2(x_2, t)$  of the primary beam and the beam extension respectively as

$$w_1(x_1, t) = \tilde{w}_1(x_1) e^{j\sigma t}, \quad w_2(x_2, t) = \tilde{w}_2(x_2) e^{j\sigma t}, \quad (11)$$

the steady state voltage response  $V_p(t)$  and charge accumulation  $Q_p(t)$  as

$$V_p(t) = \tilde{V}_p e^{j\sigma t}, \quad Q_p(t) = \tilde{Q}_p e^{j\sigma t}, \quad (12)$$

and the cross section moment  $M_p(x_1, t)$  and  $M_e(x_2, t)$  described as

$$M_p(x_1, t) = \tilde{M}_p(x_1)e^{j\sigma t}, \quad M_e(x_2, t) = \tilde{M}_e(x_2)e^{j\sigma t}. \quad (13)$$

As a result, the system of equations for the piezoelectric energy harvester can be summarized as

$$\begin{cases} B_p \frac{\partial^4 \tilde{w}_1(x_1)}{\partial x_1^4} - m_p \sigma^2 \tilde{w}_1(x_1) = 0 \\ B_e \frac{\partial^4 \tilde{w}_2(x_2)}{\partial x_2^4} - m_e \sigma^2 \tilde{w}_2(x_2) = 0, \\ j\sigma \tilde{Q}_p + \frac{\tilde{V}_p}{R_l} = 0 \end{cases} \quad (14)$$

$$\begin{cases} \tilde{M}_p(x_1) = B_p \frac{\partial^2 \tilde{w}_1(x_1)}{\partial x_1^2} - e_p \tilde{V}_p \\ \tilde{Q}_p = e_p \left[ \frac{\partial \tilde{w}_1(x_1)}{\partial x_1} \right] \Big|_0^{l_p} + C_p \tilde{V}_p, \\ \tilde{M}_e(x_2) = B_e \frac{\partial^2 \tilde{w}_2(x_2)}{\partial x_2^2} \end{cases} \quad (15)$$

and the boundary conditions become

$$\begin{cases} \tilde{w}_1(0) = \xi_b, \quad \frac{\partial \tilde{w}_1}{\partial x_1}(0) = 0 \\ w_1(l_p, t) = w_2(0, t), \quad \frac{\partial \tilde{w}_1(l_p)}{\partial x_1} = \frac{\partial \tilde{w}_2(0)}{\partial x_2} \\ B_p \frac{\partial^2 \tilde{w}_1(l_p)}{\partial x_1^2} - e_p \tilde{V}_p = B_e \frac{\partial^2 \tilde{w}_2(0)}{\partial x_2^2}, \quad B_p \frac{\partial^3 \tilde{w}_1(l_p)}{\partial x_1^3} = B_e \frac{\partial^3 \tilde{w}_2(0)}{\partial x_2^3} \\ \frac{\partial^2 \tilde{w}_2(l_e)}{\partial x_2^2} = 0, \quad \frac{\partial^3 \tilde{w}_2(l_e)}{\partial x_2^3} = 0 \end{cases} \quad (16)$$

From the equations (14), (15), and (16), we can eliminate the electrical quantities  $\tilde{Q}_p$  and  $\tilde{V}_p$  by incorporating them into the boundary conditions. Actually, from equations (14) and (15), we have

$$\tilde{V}_p = \frac{j\sigma R_l e_p}{j\sigma R_l C_p + 1} \left[ \frac{\partial \tilde{w}_1(x_1)}{\partial x_1} \right] \Big|_0^{l_p} \quad (17)$$

which can actually be used to eliminate the term  $\tilde{V}_p$  in the boundary conditions (16). In the end, we can simplify the problem as a combination of the governing equations

$$\begin{cases} B_p \frac{\partial^4 \tilde{w}_1(x_1)}{\partial x_1^4} - m_p \sigma^2 \tilde{w}_1(x_1) = 0 \\ B_e \frac{\partial^4 \tilde{w}_2(x_2)}{\partial x_2^4} - m_e \sigma^2 \tilde{w}_2(x_2) = 0 \end{cases} \quad (18)$$

and the boundary conditions

$$\begin{cases} \tilde{w}_1(0) = \xi_b, \quad \frac{\partial \tilde{w}_1}{\partial x_1}(0) = 0 \\ \tilde{w}_1(l_p) = \tilde{w}_2(0), \quad \frac{\partial \tilde{w}_1(l_p)}{\partial x_1} = \frac{\partial \tilde{w}_2(0)}{\partial x_2} \\ B_p \frac{\partial^2 \tilde{w}_1(l_p)}{\partial x_1^2} + \frac{j\sigma R_l e_p^2}{j\sigma R_l C_p + 1} \frac{\partial \tilde{w}_1(l_p)}{\partial x_1} = B_e \frac{\partial^2 \tilde{w}_2(0)}{\partial x_2^2}, \quad B_p \frac{\partial^3 \tilde{w}_1(l_p)}{\partial x_1^3} = B_e \frac{\partial^3 \tilde{w}_2(0)}{\partial x_2^3} \\ \frac{\partial^2 \tilde{w}_2(l_e)}{\partial x_2^2} = 0, \quad \frac{\partial^3 \tilde{w}_2(l_e)}{\partial x_2^3} = 0 \end{cases} \quad (19)$$

which actually manifests as a boundary value problem.

## 2. Dimensionless Problem

Using the following dimensionless group

$$\tilde{w}_1, \tilde{w}_2 \sim \xi_b, \quad \tilde{x}_1 \sim l_p, \quad \tilde{x}_2 \sim l_e \quad (20)$$

we can nondimensionalize the above formulated boundary value problem with respect to the following variables:

$$\tilde{w}_1 = \xi_b u_1, \quad \tilde{w}_2 = \xi_b u_2, \quad \tilde{x}_1 = l_p x, \quad \tilde{x}_2 = l_e x. \quad (21)$$

Note that here we use one independent space variable  $x$  to nondimensionalize two previously used variables  $x_1$  and  $x_2$ . This comes from the fact that the variables  $x_1$  and  $x_2$  are not coupled with each other in the sense that the primary beam and the extension beam do not overlap each other except for their joint point where  $x_1 = l_p$  and  $x_2 = 0$ . Thus the two variables do not occur in the equations simultaneously except for the boundary conditions. As for the boundary conditions, the change of variables does not affect the values of the equations. Therefore, the two parts of the piezoelectric energy harvester beam are in fact independent of each other except for the joining point. In one word, the equation (21) does not change the problem in essence.

Hence, the above boundary value problem is further changed into the combination of the governing equations

$$\begin{cases} \frac{B_p}{l_p^4} u_1'''' - m_p \sigma^2 u_1 = 0 \\ \frac{B_e}{l_e^4} u_2'''' - m_e \sigma^2 u_2 = 0 \end{cases} \quad (22)$$

and the boundary conditions

$$\begin{cases} u_1(0) = 1, \quad u_1'(0) = 0 \\ u_1(1) = u_2(0), \quad \frac{1}{l_p} u_1'(1) = \frac{1}{l_e} u_2'(0) \\ \frac{B_p}{l_p^2} u_1''(1) + \frac{j\sigma R_l e_p^2}{j\sigma R_l C_p + 1} \frac{1}{l_p} u_1'(1) = \frac{B_e}{l_e^2} u_2''(0), \quad \frac{B_p}{l_p^3} u_1'''(1) = \frac{B_e}{l_e^3} u_2'''(0) \\ u_2''(1) = 0, \quad u_2'''(1) = 0 \end{cases} \quad (23)$$

in which the prime means the derivative with respect to  $x$ . The equations can again be organized in a more compact form

$$\begin{cases} u_1'''' - \nu^2 u_1 = 0 \\ u_2'''' - \nu^2 \lambda_m \lambda_l^4 / \lambda_B u_2 = 0 \end{cases} \quad (24)$$

and the boundary conditions

$$\begin{cases} u_1(0) = 1, \quad u_1'(0) = 0 \\ u_1(1) = u_2(0), \quad \lambda_l u_1'(1) = u_2'(0) \\ u_1''(1) + \frac{j\nu\beta}{j\nu\beta + 1} \alpha^2 u_1'(1) = \lambda_B / \lambda_l^2 u_2''(0), \quad u_1'''(1) = \lambda_B / \lambda_l^3 u_2'''(0) \\ u_2''(1) = 0, \quad u_2'''(1) = 0 \end{cases} \quad (25)$$

where

$$\nu = \sigma \sqrt{\frac{m_p l_p^4}{B_p}}, \quad \lambda_B = \frac{B_e}{B_p}, \quad \lambda_m = \frac{m_e}{m_p}, \quad \lambda_l = \frac{l_e}{l_p} \quad (26)$$

$$\beta = R_l C_p \sqrt{\frac{B_p}{m_p l_p^4}}, \quad \alpha = e_p \sqrt{\frac{l_p}{C_p B_p}} \quad (27)$$

The system (24) and (25) is a two-point boundary value problem. The problem can readily be solved by a Chebyshev collocation method using the MATLAB package *Chebfun* [3].

Table 1: Geometric, material, and electromechanical parameters of the simulation for piezoelectric energy harvester with flexible extension

Parameter item	Parameter value
Length of the primary beam, $l_p$ (mm)	100
Width of the whole energy harvester, $b$ (mm)	20
Half thickness of the structure, $h_s$ (mm)	0.25
Thickness of the piezoelectric layer, $h_p$ (mm)	0.2
Young's modulus of the structure, $Y_s$ (Gpa)	100
Young's modulus of the piezoelectric layer, $Y_p$ (Gpa)	66
Mass density of the substructure, $\rho_s$ (kg/m <sup>3</sup> )	7165
Mass density of the piezoelectric layer, $\rho_p$ (kg/m <sup>3</sup> )	7800
piezoelectric constant, $d_{31}$ (pm/V)	-190
Permittivity, $\epsilon_{33}^S$ (nF/m)	15.93
Length of the beam extension, $l_e$ (mm)	30
Young's modulus of the beam extension, $Y_e$ (Gpa)	2.3
Mass density of the beam extension, $\rho_e$ (kg/m <sup>3</sup> )	1.38
Half thickness of the structure, $h_e$ (mm)	0.25

### 3. Influence of extension part upon energy harvester performance

The basic geometry and material properties of the materials used in the proposed piezoelectric energy harvester with flexible extensions are summarized in Table 1. Note that some of the parameters, like length  $l_e$ , Young's modulus  $Y_e$ , and volumetric density  $\rho_e$  of the beam extension, are actually changing across different simulations. In the simulation, base excitation frequency  $fr$  and external load resistance  $R_l$ , which change the dimensionless values of  $\nu$  and  $\beta$ , respectively, are of critical importance in the sense that these two parameters reflect the influence of vibration source frequency spectrum and external load circuit. For every set of parameter values, we set the base excitation frequency  $fr$  to change from 1 Hz to 100 Hz, which covers the usual frequency range of natural vibration sources, and set the load resistance  $R_l$  to change from 1  $\Omega$  to 10 M $\Omega$ , which is inspired by the Ref. [1] and takes into account the dielectric property of piezoelectric materials. Actually, when the load resistance  $R_l = 1 \Omega$ , the piezoelectric energy harvester is said to be in a short-circuit condition as the general equivalent resistance of the structure is much larger than  $R_l$ . On the other hand, when  $R_l = 10 \text{ M}\Omega$ , the system is close to an open-circuit condition where no external load is connected to the output electrodes.

In the following, we will investigate the influences of length  $l_e$ , Young's modulus  $Y_e$ , and volumetric density  $\rho_e$  of the beam extension upon the performance of the piezoelectric energy harvester with flexible extensions separately.

#### 3.1. Beam extension length $l_e$ or length ratio $\lambda_l$

The presence of the beam extension is primarily indicated by the beam extension length  $l_e$ , or equivalently the length ratio  $\lambda_l$ . When  $\lambda_l = 0.0$ , no beam extension is attached to the primary beam and the resultant piezoelectric energy harvester reduces to the classic piezoelectric energy harvesting bimorph [1], which is referred to as a reference for comparison. In this contribution, the range of length ratio  $\lambda_l$  to be considered is set to be  $0.0 \leq \lambda_l \leq 1.0$ . For each value of length ratio  $\lambda_l$ , the base excitation problem is solved with respect to different base excitation frequency  $fr$  and load resistance  $R_l$ . In this process, the Young's modulus of the beam extension part is set to be  $Y_e = 2.3 \text{ GPa}$  while its volumetric density is set to be  $\rho_e = 1.38 \times 10^3 \text{ kg/m}^3$ .

In the first place, we would like to explore the influence of base excitation frequency  $fr$  upon performance of the piezoelectric energy harvester at different values of load resistance  $R_l$  and length ratio  $\lambda_l$ . Let the frequency range of interest be 1 – 100 Hz. The length ratio  $\lambda_l$  is chosen to be 0.0, 0.2, 0.4, 0.6, 0.8, or 1.0, and load resistance  $R_l$  is chosen to be  $10^0 \Omega$ ,  $10^2 \Omega$ ,  $10^4 \Omega$ , or  $10^6 \Omega$ . The resulting amplitudes of output voltage  $V_p$  and output power  $P_p$  are plotted against base excitation frequency  $fr$  in Figure 2 and Figure 3, respectively. In this process, the values of beam extension part density  $\rho_e$  and Young's modulus  $Y_e$  are kept unchanged, so do the values of  $\lambda_m$  and  $\lambda_B$ .

According to Figure 2 and Figure 3, for the given values  $\lambda_B$  and  $\lambda_m$ , frequency response of the piezoelectric energy harvester in the interested frequency range changes in accordance with the extension length  $l_e$  and thus the length ratio  $\lambda_l$ .

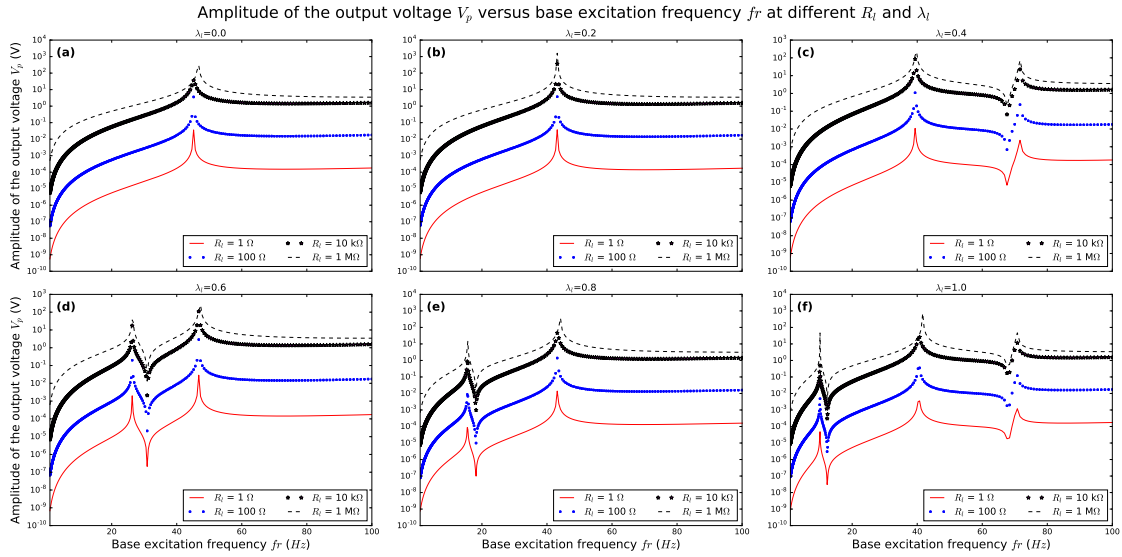


Figure 2: Output voltage  $V_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus base excitation frequency  $f_r$  at different length ratio  $\lambda_l$  and load resistance  $R_l$ .

When  $\lambda_l$  is relatively small, say  $\lambda_l = 0.2$ , as shown in Figure 2(b), frequency response of the *PEHFE* piezoelectric energy harvester with flexible extension is almost the same as that of the *CPEH* case where  $\lambda_l = 0.0$ . That is to say, there is only resonant peak in the considered frequency range and the corresponding resonant frequency is around  $42 \text{ Hz}$  for different values of load resistance  $R_l$ . Therefore we can conclude that in this situation, the beam extension part is playing a negligible role in the motion of the primary beam part. As a result, energy harvesting performances of the proposed piezoelectric energy harvesters with flexible extensions are similar to that of a classic piezoelectric energy harvester.

With the increase of the length ratio  $\lambda_l$ , say  $\lambda_l = 0.4, 0.6$ , or  $0.8$ , as shown in Figure 2(b), (c), and (d), respectively, there begins to exist an extra resonant and anti-resonant mode in the considered frequency range, compared with that of the case of no flexible extension where  $\lambda_l = 0.0$ . An increase of the value of  $\lambda_l$  causes the decrease of the resonant and anti-resonant frequencies of the newly occurring mode. What's more, for the cases of  $\lambda_l = 0.4$  and  $0.6$ , the resonant frequency of the previously existing mode, which corresponds to the first resonant mode of a classic piezoelectric energy harvester, is shifted to an obvious degree. This can be explained as follows. The motion of the proposed piezoelectric energy harvester with flexible extension is controlled by the primary beam part and the beam extension part. With the increase of  $\lambda_l$ , resonant frequency of the beam extension part approaches that of the primary beam. Thus the two parts interact with each other at a high level. resonant frequency of the whole piezoelectric energy harvester with flexible extension is therefore tuned by the beam extension part. When the resonant frequency of the beam extension is far small than that of the primary beam, which is the case for  $\lambda = 0.8$ , as shown in Figure 2(d), the two parts have little effect upon each other.

If we further increase the length ratio to  $\lambda_l = 1.0$ , more resonant and anti-resonant modes occur in the considered frequency range. It can be found from Figure 2(d) and Figure 3(d) that, amplitude of the output voltage  $V_p$  and output power  $P_p$  at the newly occurring resonant mode is comparable in order of magnitude to that of the classic piezoelectric energy harvester.

In this view, attachment of the flexible extension part actually increases the working frequency range of the piezoelectric energy harvester, as typical piezoelectric energy harvesters rely on resonant modes to work. On the other hand, the flexible extension also provides a way to tune the bandwidth of a piezoelectric energy harvester. An obvious way to do so is to choose the parameters of the beam extension part, so that the beam extension part and the primary beam part have similar resonant frequency and interact strongly with each other. The point to be noticed is the existence of anti-resonant modes. They correspond to low output voltage and output power, and to some degree narrows the bandwidth of the energy harvester. However, it is still not worse than an otherwise non-resonant vibration, as in both cases the electrical output of the piezoelectric energy harvester is unusable.

In the second place, we investigate the influence of load resistance  $R_l$  upon performance of the

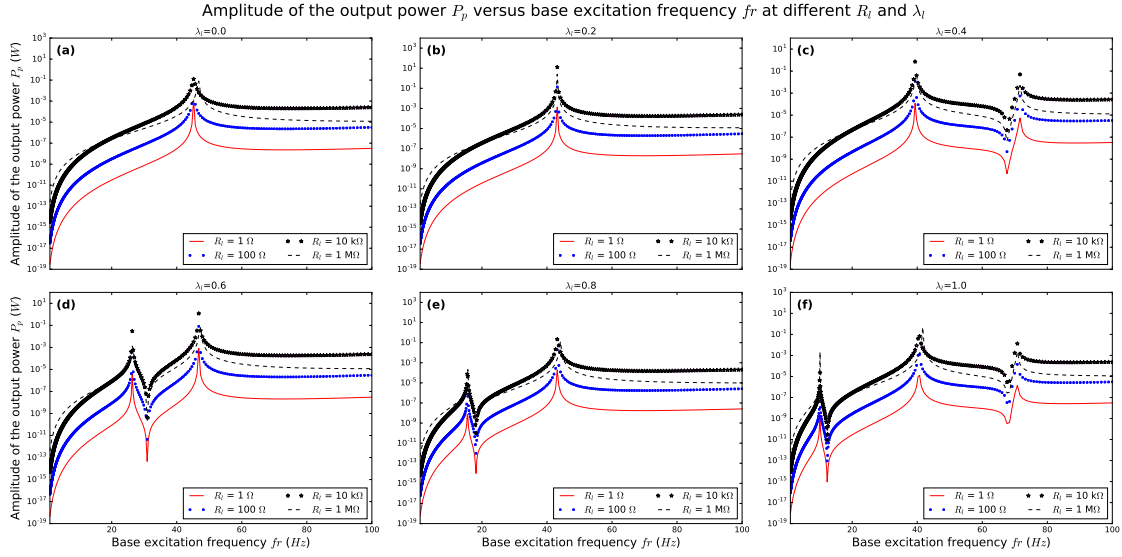


Figure 3: Output power  $P_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus base excitation frequency  $f_r$  at different length ratio  $\lambda_l$  and load resistance  $R_l$ .

piezoelectric energy harvester at different values of base excitation frequency  $f_r$  and length ratio  $\lambda_l$ . The resulting amplitudes of output voltage  $V_p$  and output power  $P_p$  are shown in Figure 4 and Figure 5, respectively. It is obvious that in the range of low  $R_l$ , a power law exists between the amplitudes of output voltage  $V_p$  and output power  $P_p$  and load resistance  $R_l$ . For all the chosen values of base excitation frequency  $f_r$ , the increase of  $R_l$  leads to an increase in  $V_p$ . Ultimately,  $V_p$  approaches asymptotically to a limit value  $V_p^{lim}$ . This value actually corresponds to the amplitude of open-circuit output voltage of the piezoelectric energy harvester. At the same time, the value of  $P_p$  exhibits an obvious maximum at some values of  $R_l$  between 10 k $\Omega$  and 1 M $\Omega$ . And on both sides away from the maximum value, we see also a power law between the value of  $R_l$  and  $P_p$ , as shown in Figure 5. This indicates that an asymptotic analysis may help us to simplify the analysis of performance of piezoelectric energy harvesters. But this is out of the scope of our current contribution and will be covered in the future research.

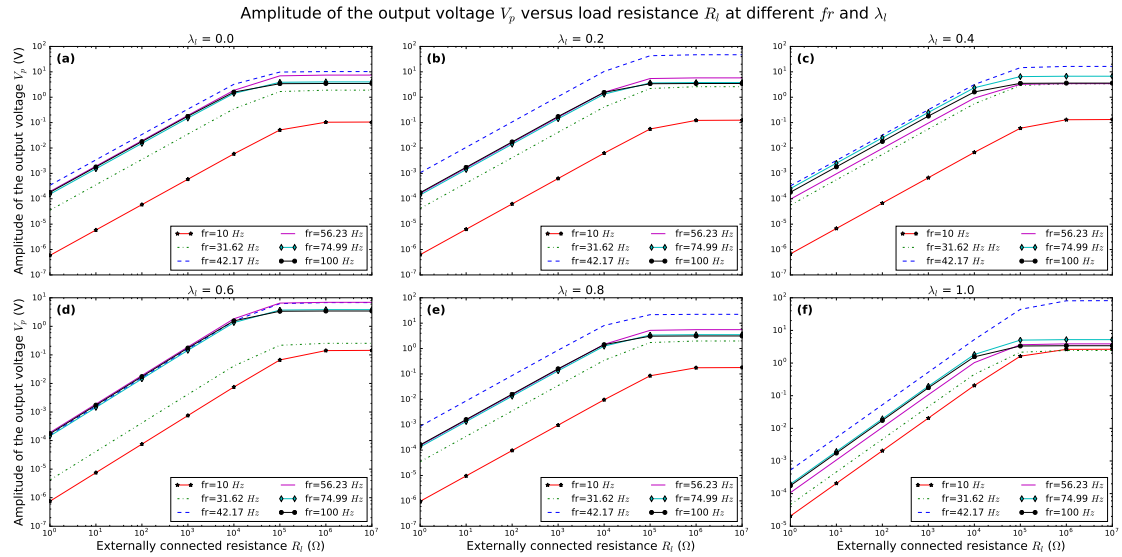


Figure 4: Output voltage  $V_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus load resistance  $R_l$  at different frequency  $f_r$  and length ratio  $\lambda_l$ .

In the third place, at different values of  $f_r$  and  $R_l$ , we directly plot the amplitudes of  $V_p$  and  $P_p$  with respect to  $\lambda_l$ , which are shown in Figure 6 and Figure 6, respectively. It is shown that at given values of  $R_l$  and  $f_r$ , output performances of the proposed piezoelectric energy harvesters can

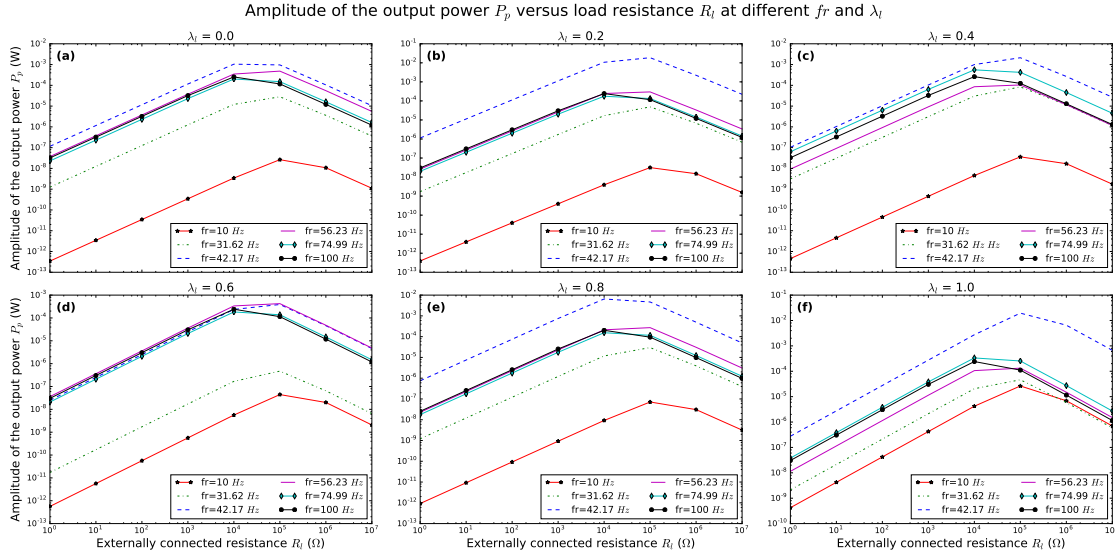


Figure 5: Output power  $P_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus load resistance  $R_l$  at different frequency  $fr$  and length ratio  $\lambda_l$ .

be tuned by the parameter  $\lambda_l$ . For example, when  $fr = 31.62 \text{ Hz}$  and  $R_l = 10 \text{ k}\Omega$ , a maximum peak of  $V_p$  and  $P_p$  is found around  $\lambda_l = 0.5$ . Compared with the case of no flexible extension ( $\lambda_l = 0$ ), the amplitude of output voltage  $V_p$  is about 13 times larger, while the amplitude of output power  $P_p$  is about 3 times larger. Similar phenomena can be found when  $fr = 42.17 \text{ Hz}$ . This indicates that the addition of the flexible extension can substantially increase the output performance of a piezoelectric energy harvester. It should be noted that in case of large values of  $fr$ , like  $fr = 100 \text{ Hz}$ , the tuning performance of the flexible extension is no longer significant.

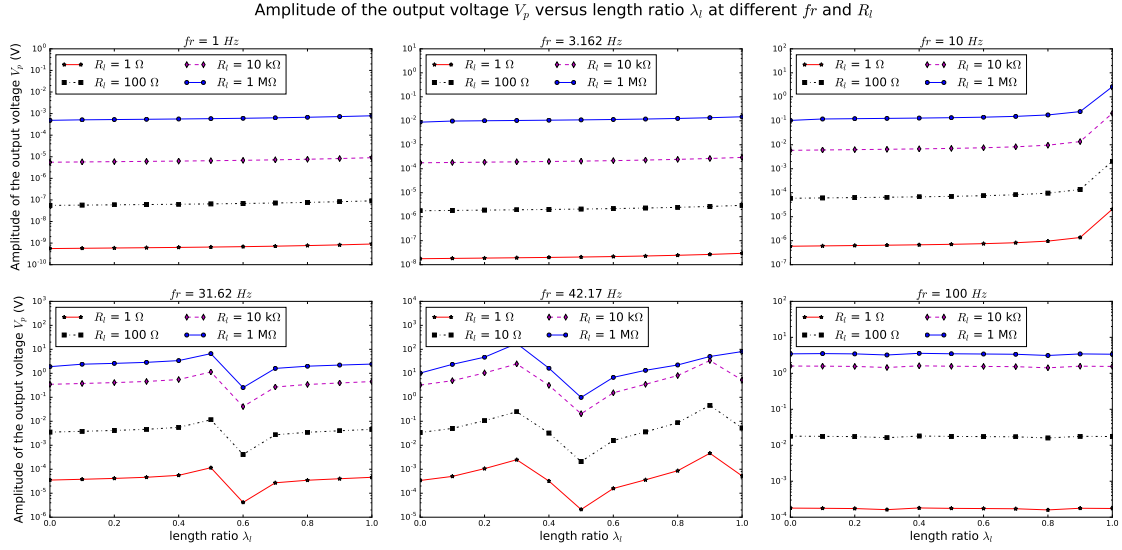


Figure 6: Output voltage  $V_p$  (amplitude) and output power  $P_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus length ratio  $\lambda_l$  at different frequency  $f$  and load resistance  $R_l$ . **to be revised in the legend, add label (a), (b)**

### 3.2. Beam extension Young's modulus $Y_e$ or bending stiffness ratio $\lambda_B$

To investigate the influence of bending stiffness ratio  $\lambda_B$  upon the performance of the piezoelectric energy harvester with flexible extension, we set different values of Young's modulus  $Y_e$  of the beam extension part to change the values of  $\lambda_B$ . Considering the properties of commonly used engineering materials [4], the values of  $Y_e$  is set to be in the range of  $0.01 \text{ GPa} \leq Y_e \leq 400 \text{ GPa}$ . For each value of bending stiffness ratio  $\lambda_B$ , the base excitation problem is solved with respect to



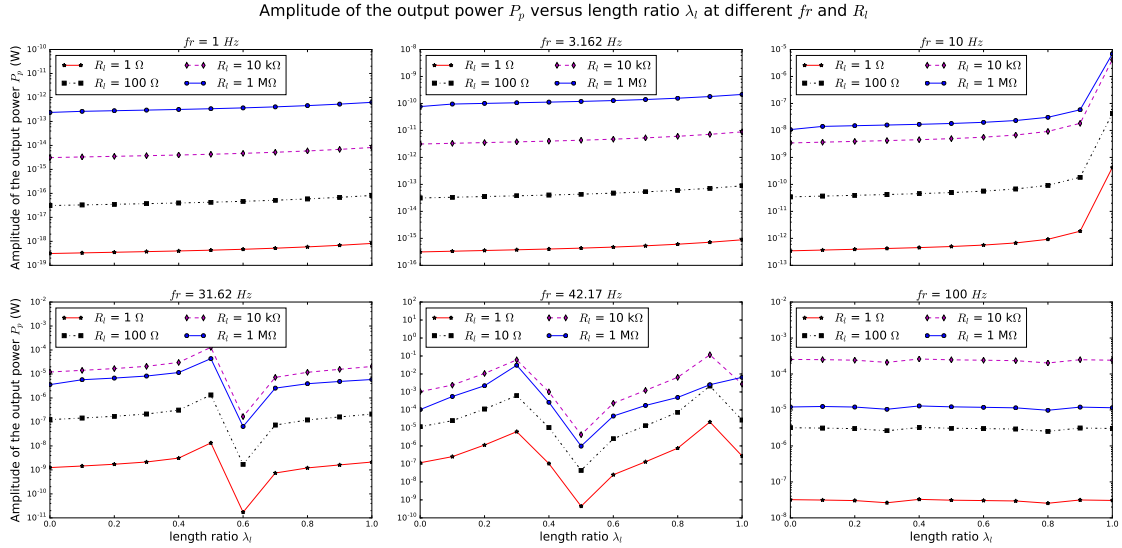


Figure 7: Output voltage  $V_p$  (amplitude) and output power  $P_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus length ratio  $\lambda_l$  at different frequency  $f$  and load resistance  $R_l$ . to be revised in the legend, add label (a), (b)

different base excitation frequency  $fr$  and load resistance  $R_l$ . In this process, the length ratio is fixed to be  $\lambda_l = 0.3$  while its volumetric density is set to be  $\rho_e = 1.38 \times 10^3 \text{ kg/m}^3$ .

Firstly, the influence of base excitation frequency  $fr$  upon performance of the piezoelectric energy harvester is shown Figure 8 and Figure 9 at different values of load resistance  $R_l$  and bending stiffness ratio  $\lambda_B$ . To have a clear clue about the value of  $\lambda_B$ , it is found that for a Young's modulus of  $Y_e = 0.01 \text{ GPa}$ , the corresponding value of bending stiffness ratio is  $\lambda_B = 2.39 \times 10^{-5}$ , while for a Young's modulus of  $Y_e = 200 \text{ GPa}$ , the corresponding value of bending stiffness ratio is  $\lambda_B = 0.478$ .

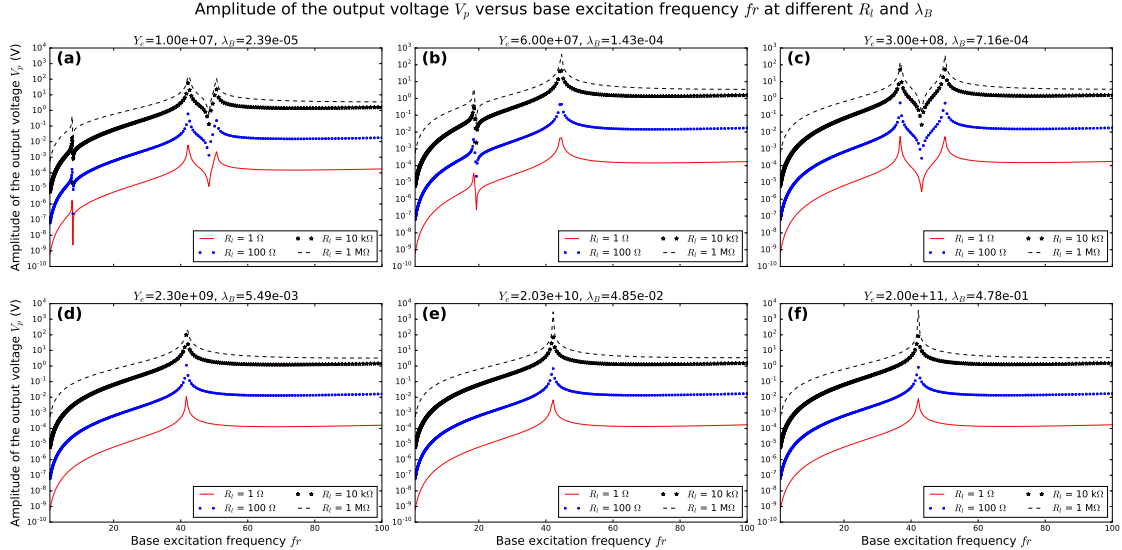


Figure 8: Output voltage  $V_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus base excitation frequency  $fr$  at different bending stiffness ratio  $\lambda_B$  and load resistance  $R_l$ . to be revised in the legend, change figure title

It is easily seen from Figure 8 and Figure 9 that bending stiffness ratio  $\lambda_B$  also affects frequency response of the proposed piezoelectric energy harvester with flexible extension, but in a different manner from length ratio  $\lambda_l$ .

When the bending stiffness ratio  $\lambda_B$  is small, say  $Y_e = 0.01 \text{ GPa}$ , as shown in Figure 8(a) and Figure 9(a), there exist multiple resonant modes in the considered frequency range. And some of

the resonant modes are potential to be used for energy harvesting.

At the given values of  $\lambda_l$  and  $\lambda_m$ ,

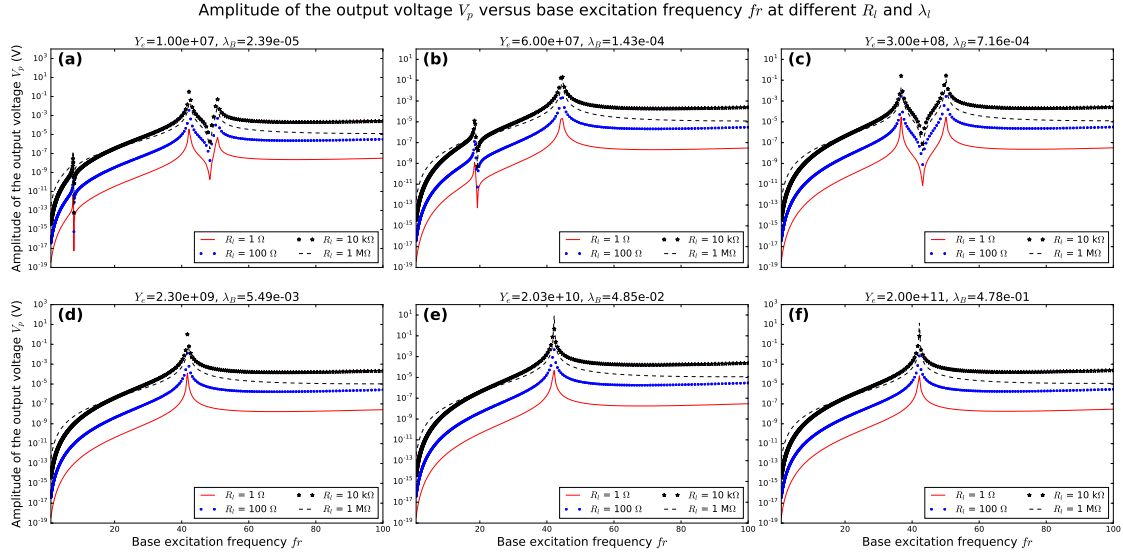


Figure 9: Output power  $P_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus base excitation frequency  $f_r$  at different bending stiffness ratio  $\lambda_B$  and load resistance  $R_l$ . to be revised in the legend.  
change figure title

It is easily seen from the diagram that the change of extension length  $l_e$  have a great influence on the frequency response of the piezoelectric energy harvester. At the given values of  $\lambda_B$  and  $\lambda_m$ , the contained vibration modes in the frequency range considered does change with respect to the length ratio  $\lambda_l$ . When  $\lambda_l$  is relatively small, which is below 0.3 in our case, no extra vibration modes can be found in the frequency range of 1 – 100 Hz. Hence the energy harvesting performances of the proposed energy harvesters are similar to that of a pure cantilever beam piezoelectric energy harvester. (note: it will be better if I can compare the resonant energy harvesting performance in this case) With further increase of the length ratio  $\lambda_l$ , there begins to exist an extra resonant and anti-resonant mode in the considered frequency range. In this view, the change of length ratio actually expands the bandwidth of the energy harvesting performances. Even when the length ratio  $\lambda_l = 1.0$ , three resonant modes are found in the frequency range. The point to be noticed is the existence of anti-resonant mode, which largely narrows the bandwidth of the energy harvester. This accompanying characteristic is to be further investigated in the following research.

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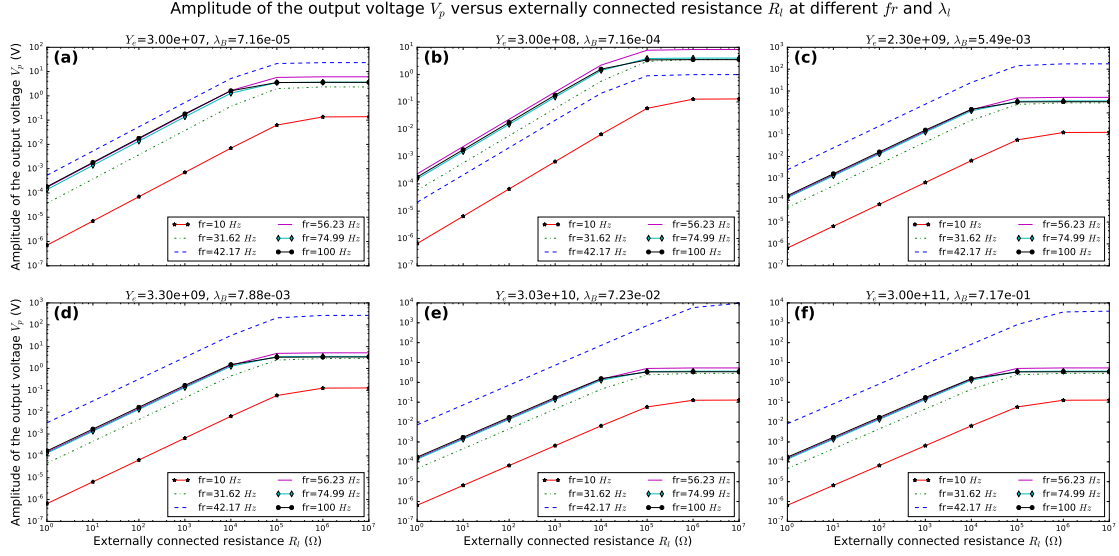


Figure 10: Output voltage  $V_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus bending stiffness ratio  $\lambda_B$  at different frequency  $fr$  and load resistance  $R_l$ .

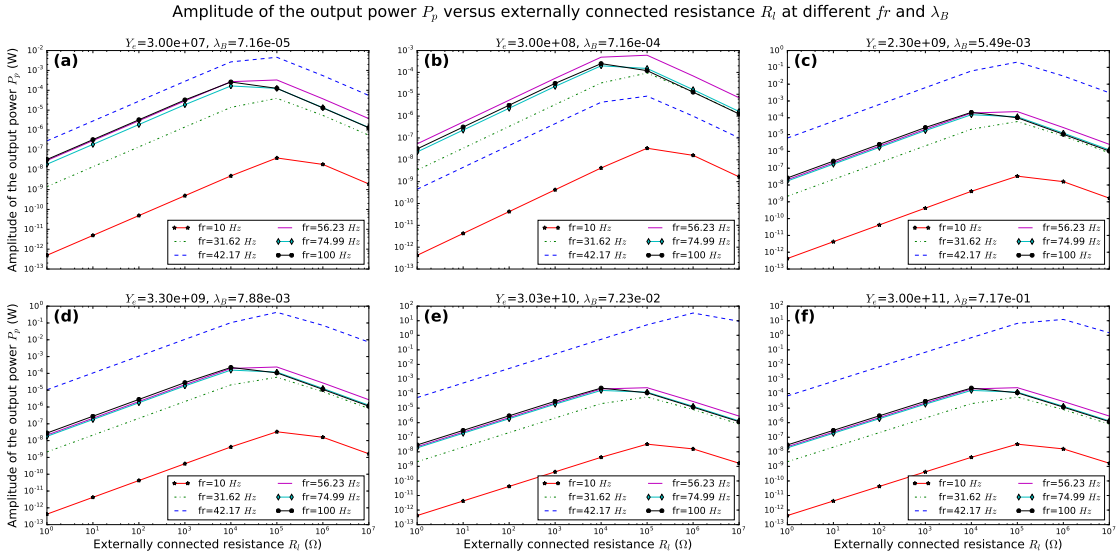


Figure 11: Output power  $P_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus bending stiffness ratio  $\lambda_B$  at different frequency  $fr$  and load resistance  $R_l$ .

and anti-resonant mode in the considered frequency range. In this view, the change of length ratio actually expands the bandwidth of the energy harvesting performances. Even when the length ratio  $\lambda_l = 1.0$ , three resonant modes are found in the frequency range. The point to be noticed is the existence of anti-resonant mode, which largely narrows the bandwidth of the energy harvester. This accompanying characteristic is to be further investigated in the following research.

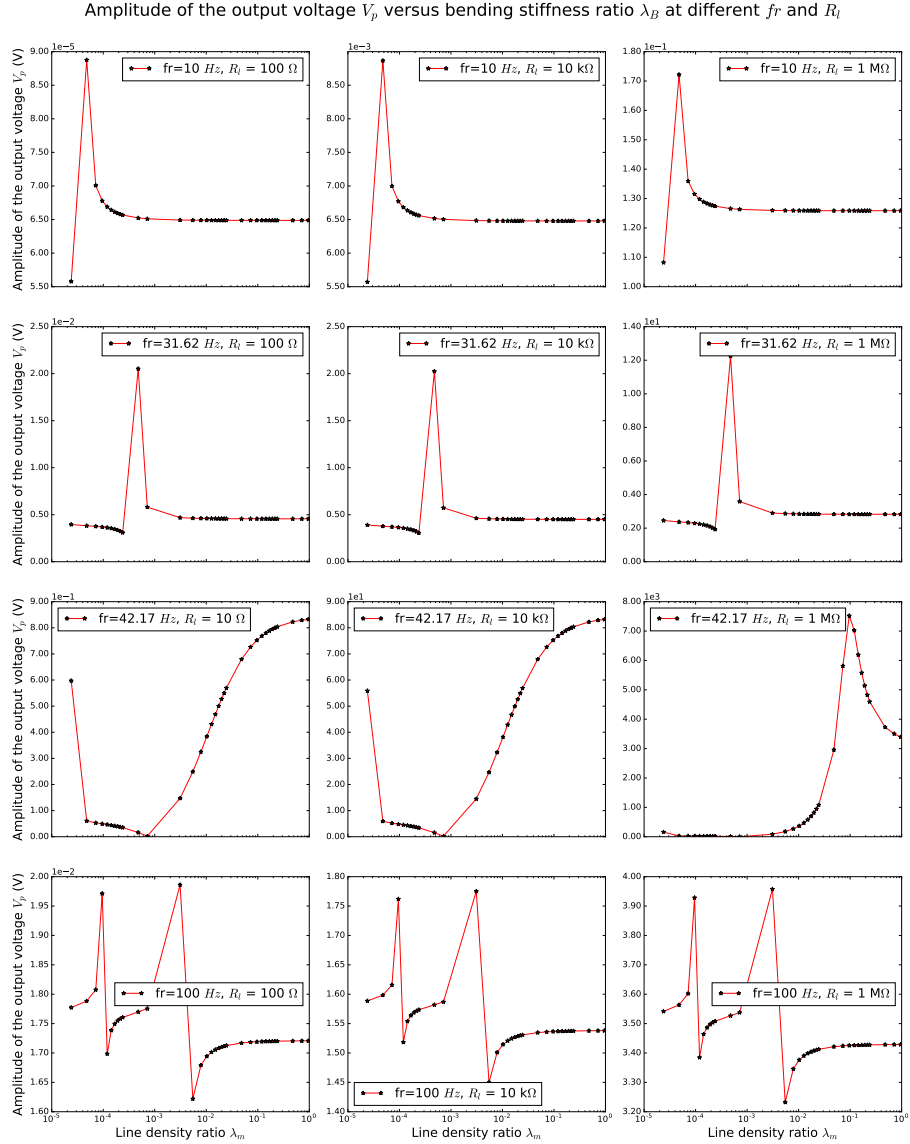


Figure 12: Output voltage  $V_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus bending stiffness ratio  $\lambda_B$  at different frequency  $fr$  and load resistance  $R_l$ .

It is easily seen from the diagram that the change of extension length  $l_e$  have a great influence on the frequency response of the piezoelectric energy harvester. At the given values of  $\lambda_B$  and  $\lambda_m$ , the contained vibration modes in the frequency range considered does change with respect to the length ratio  $\lambda_l$ . When  $\lambda_l$  is relatively small, which is below 0.3 in our case, no extra vibration modes can be found in the frequency range of 1 – 100 Hz. Hence the energy harvesting performances of the proposed energy harvesters are similar to that of a pure cantilever beam piezoelectric energy harvester. (note: it will be better if I can compare the resonant energy harvesting performance in this case) With further increase of the length ratio  $\lambda_l$ , there begins to exist an extra resonant and anti-resonant mode in the considered frequency range. In this view, the change of length ratio actually expands the bandwidth of the energy harvesting performances. Even when the length ratio  $\lambda_l = 1.0$ , three resonant modes are found in the frequency range. The point to be noticed is the

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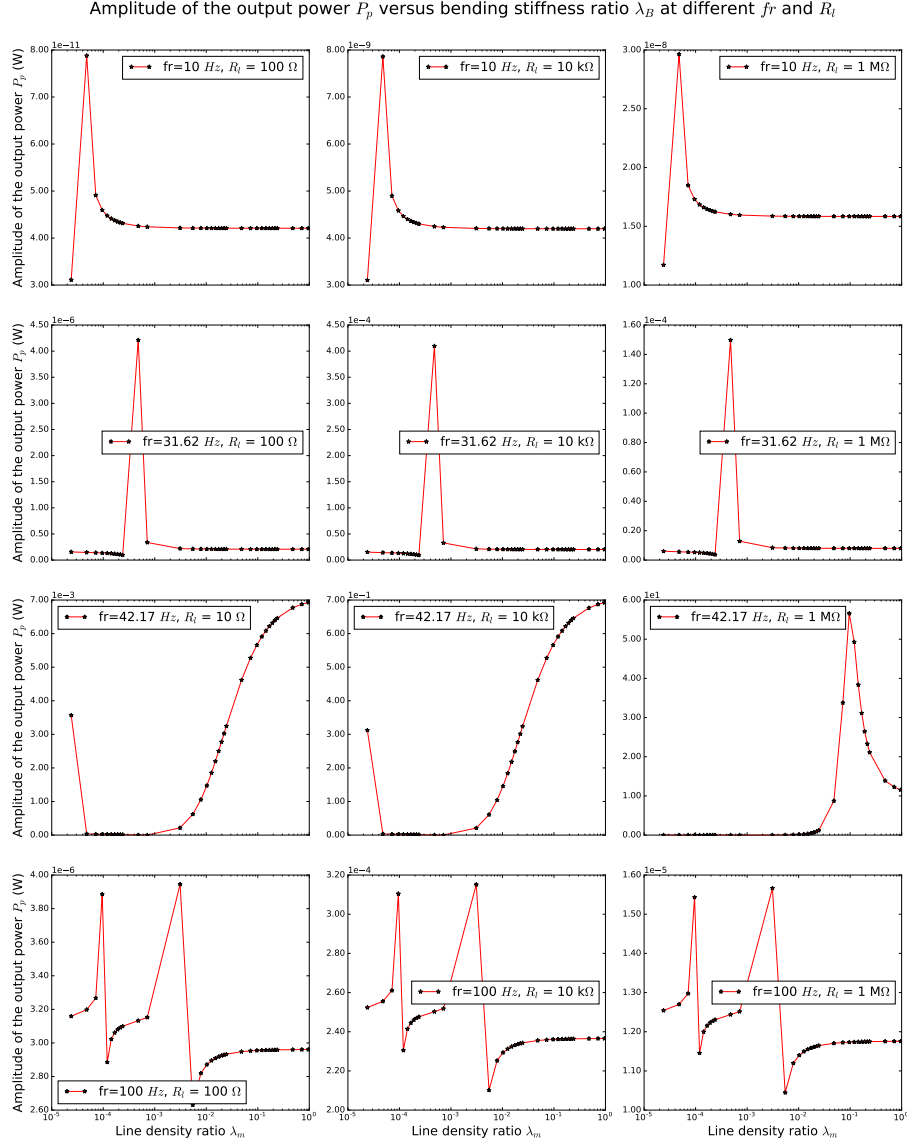


Figure 13: Output power  $P_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus bending stiffness ratio  $\lambda_B$  at different frequency  $fr$  and load resistance  $R_l$ .

It is easily seen from the diagram that the change of extension length  $l_e$  have a great influence on the frequency response of the piezoelectric energy harvester. At the given values of  $\lambda_B$  and  $\lambda_m$ , the contained vibration modes in the frequency range considered does change with respect to the length ratio  $\lambda_l$ . When  $\lambda_l$  is relatively small, which is below 0.3 in our case, no extra vibration modes can be found in the frequency range of 1 – 100 Hz. Hence the energy harvesting performances of the proposed energy harvesters are similar to that of a pure cantilever beam piezoelectric energy harvester. (note: it will be better if I can compare the resonant energy harvesting performance in this case) With further increase of the length ratio  $\lambda_l$ , there begins to exist an extra resonant and anti-resonant mode in the considered frequency range. In this view, the change of length ratio actually expands the bandwidth of the energy harvesting performances. Even when the length ratio  $\lambda_l = 1.0$ , three resonant modes are found in the frequency range. The point to be noticed is the existence of anti-resonant mode, which largely narrows the bandwidth of the energy harvester. This accompanying characteristic is to be further investigated in the following research.

### 3.3. beam extension line density or line density ratio $\lambda_m$

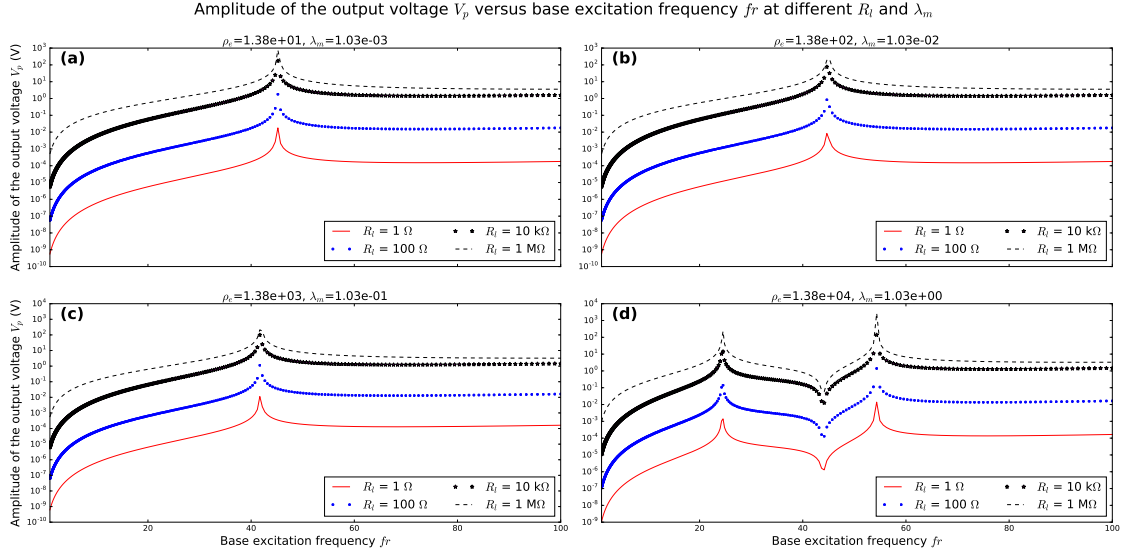


Figure 14: Output voltage  $V_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus base excitation frequency  $f_r$  at different line density ratio  $\lambda_m$  and load resistance  $R_l$ .

It is easily seen from the diagram that the change of extension length  $l_e$  have a great influence on the frequency response of the piezoelectric energy harvester. At the given values of  $\lambda_B$  and  $\lambda_m$ , the contained vibration modes in the frequency range considered does change with respect to the length ratio  $\lambda_l$ . When  $\lambda_l$  is relatively small, which is below 0.3 in our case, no extra vibration modes can be found in the frequency range of 1 – 100 Hz. Hence the energy harvesting performances of the proposed energy harvesters are similar to that of a pure cantilever beam piezoelectric energy harvester. (note: it will be better if I can compare the resonant energy harvesting performance in this case) With further increase of the length ratio  $\lambda_l$ , there begins to exist an extra resonant and anti-resonant mode in the considered frequency range. In this view, the change of length ratio actually expands the bandwidth of the energy harvesting performances. Even when the length ratio  $\lambda_l = 1.0$ , three resonant modes are found in the frequency range. The point to be noticed is the existence of anti-resonant mode, which largely narrows the bandwidth of the energy harvester. This accompanying characteristic is to be further investigated in the following research.

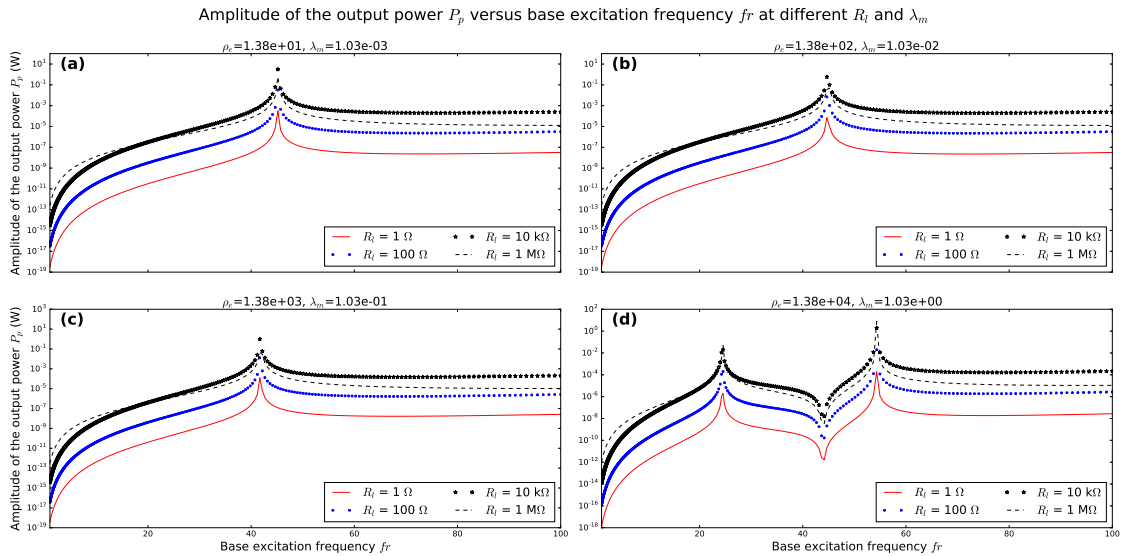


Figure 15: Output power  $P_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus base excitation frequency  $f_r$  at different line density ratio  $\lambda_m$  and load resistance  $R_l$ .

It is easily seen from the diagram that the change of extension length  $l_e$  have a great influence on the frequency response of the piezoelectric energy harvester. At the given values of  $\lambda_B$  and  $\lambda_m$ , the contained vibration modes in the frequency range considered does change with respect to the length ratio  $\lambda_l$ . When  $\lambda_l$  is relatively small, which is below 0.3 in our case, no extra vibration modes can be found in the frequency range of 1 – 100  $Hz$ . Hence the energy harvesting performances of the proposed energy harvesters are similar to that of a pure cantilever beam piezoelectric energy harvester. (note: it will be better if I can compare the resonant energy harvesting performance in this case) With further increase of the length ratio  $\lambda_l$ , there begins to exist an extra resonant and anti-resonant mode in the considered frequency range. In this view, the change of length ratio actually expands the bandwidth of the energy harvesting performances. Even when the length ratio  $\lambda_l = 1.0$ , three resonant modes are found in the frequency range. The point to be noticed is the existence of anti-resonant mode, which largely narrows the bandwidth of the energy harvester. This accompanying characteristic is to be further investigated in the following research.

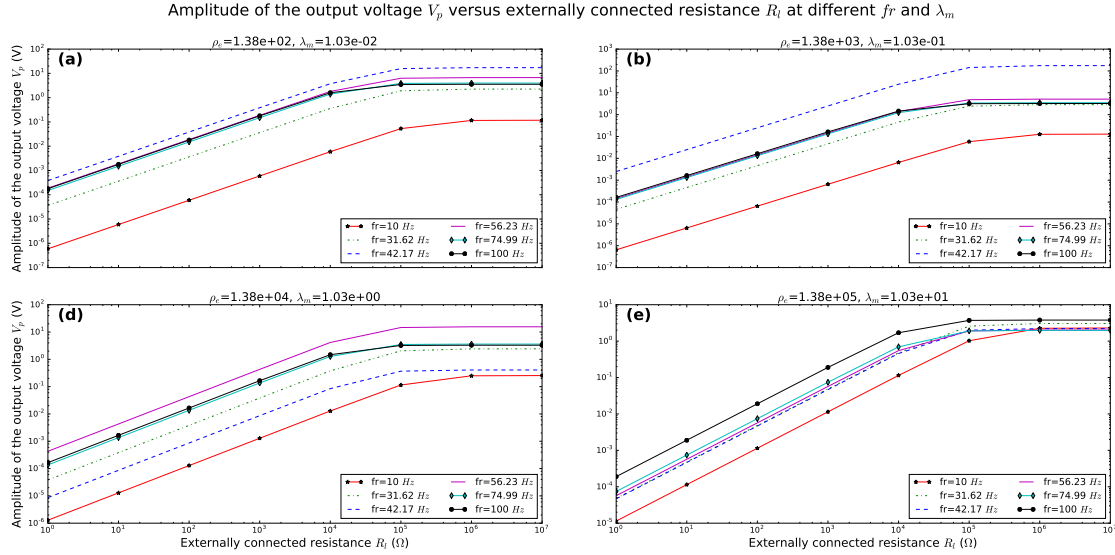


Figure 16: Output voltage  $V_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus line density ratio  $\lambda_m$  at different frequency  $fr$  and load resistance  $R_l$ .

It is easily seen from the diagram that the change of extension length  $l_e$  have a great influence on the frequency response of the piezoelectric energy harvester. At the given values of  $\lambda_B$  and  $\lambda_m$ , the contained vibration modes in the frequency range considered does change with respect to the length ratio  $\lambda_l$ . When  $\lambda_l$  is relatively small, which is below 0.3 in our case, no extra vibration modes can be found in the frequency range of 1 – 100  $Hz$ . Hence the energy harvesting performances of the proposed energy harvesters are similar to that of a pure cantilever beam piezoelectric energy harvester. (note: it will be better if I can compare the resonant energy harvesting performance in this case) With further increase of the length ratio  $\lambda_l$ , there begins to exist an extra resonant and anti-resonant mode in the considered frequency range. In this view, the change of length ratio actually expands the bandwidth of the energy harvesting performances. Even when the length ratio  $\lambda_l = 1.0$ , three resonant modes are found in the frequency range. The point to be noticed is the existence of anti-resonant mode, which largely narrows the bandwidth of the energy harvester. This accompanying characteristic is to be further investigated in the following research.

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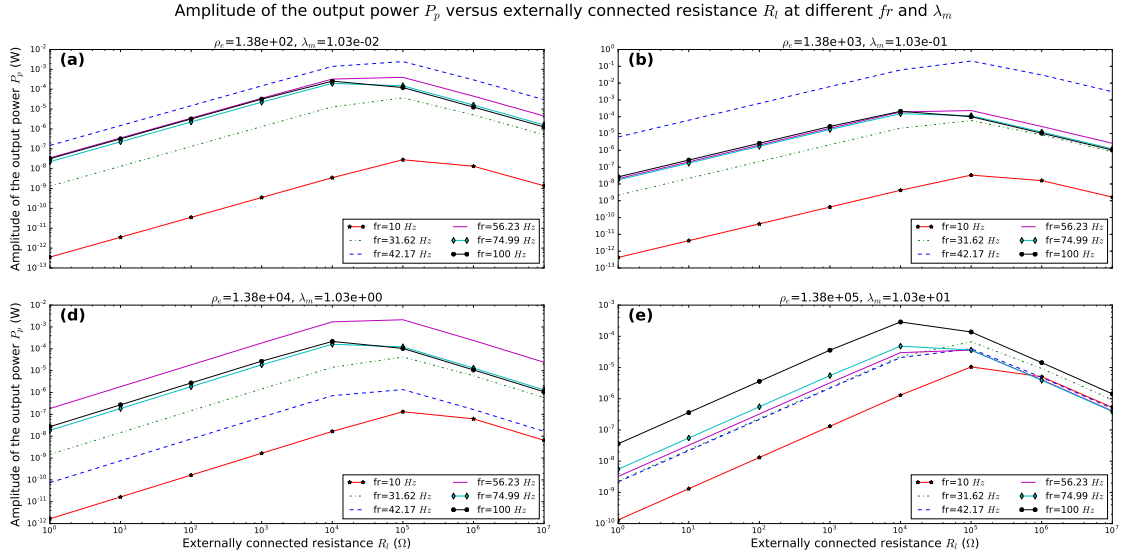


Figure 17: Output power  $P_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus line density ratio  $\lambda_m$  at different frequency  $fr$  and load resistance  $R_l$ .

existence of anti-resonant mode, which largely narrows the bandwidth of the energy harvester. This accompanying characteristic is to be further investigated in the following research.

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#### 4. Discussion

#### 5. Conclusion

Here in this contribution, we investigate the method of flexible extension to tune the energy harvesting performance of piezoelectric cantilever energy harvester.



Amplitude of the output voltage  $V_p$  versus line density ratio  $\lambda_m$  at different  $fr$  and  $R_l$

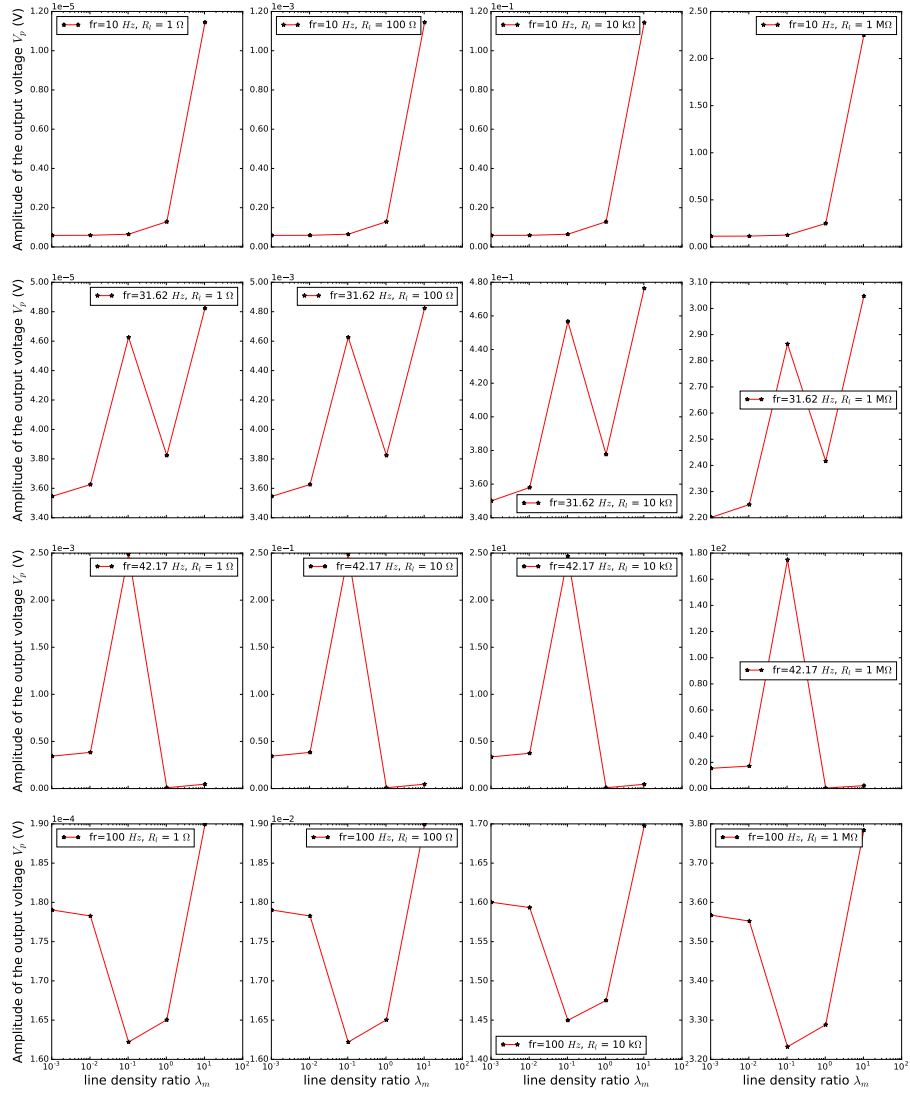


Figure 18: Output voltage  $V_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus line density ratio  $\lambda_m$  at different frequency  $fr$  and load resistance  $R_l$ .

Amplitude of the output power  $P_p$  versus line density ratio  $\lambda_m$  at different  $fr$  and  $R_l$

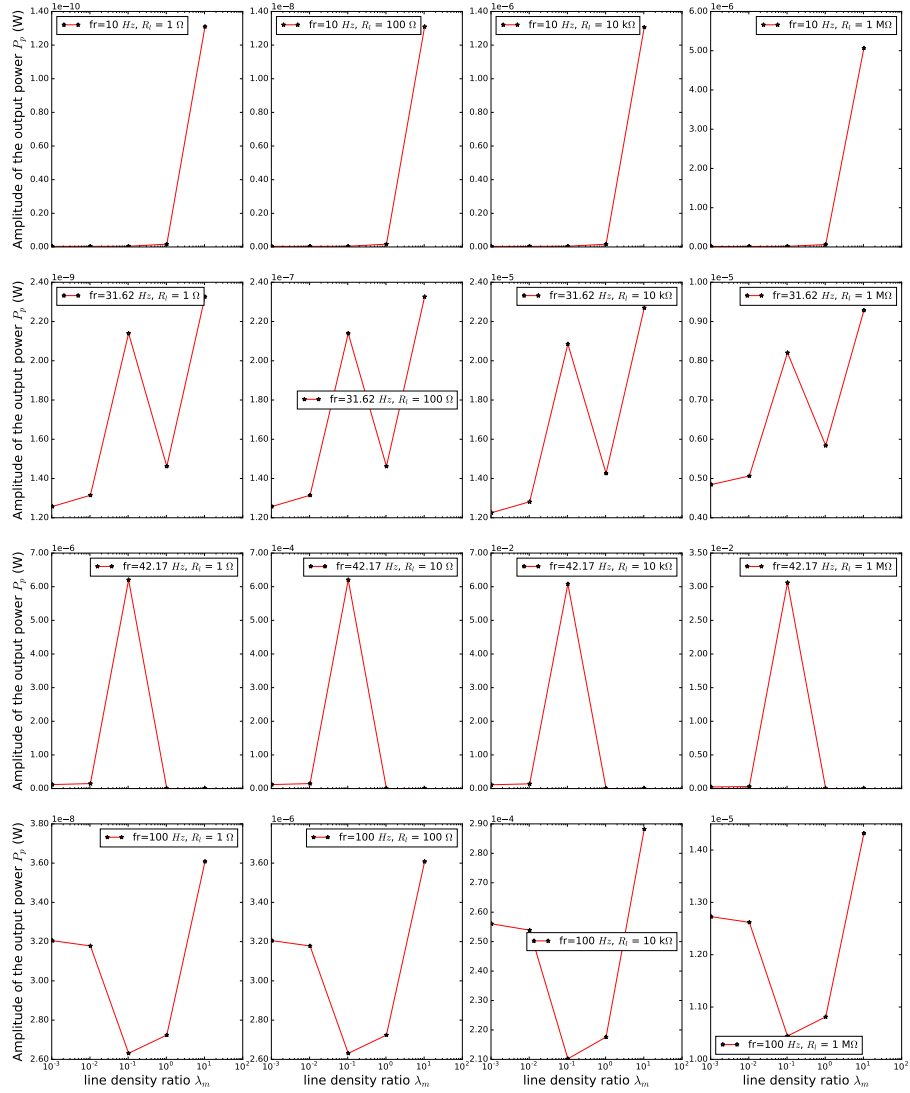


Figure 19: Output power  $P_p$  (amplitude) of the piezoelectric energy harvester with flexible extension versus line density ratio  $\lambda_m$  at different frequency  $fr$  and load resistance  $R_l$ .

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