

# Asymptotic analysis of piezoelectric energy harvester

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## 1 Summary of the interested equations

Here we are interested in the classical model of a piezoelectric cantilever beam energy harvester, whose model is described using the following set of equations:

$$u'''' - \lambda^2 u = 0, \quad (1)$$

and the accompanying boundary conditions:

$$\begin{cases} u(0) = 0 \\ u'(0) = 0 \\ u''(1) + \frac{j\lambda\beta\alpha^2}{j\lambda\beta + 1} u'(1) = 0 \\ u'''(1) = 0 \end{cases}, \quad (2)$$

where  $\lambda$  is the eigenvalues for the problem,  $u$  denotes the displace function of the cantilever beam,  $\beta$  is the dimensionless externally connected resistance, and  $\alpha$  is the dimensionless piezoelectric coefficient. They can be expressed as follows

$$\lambda = \omega \sqrt{\frac{m_p l_p^4}{B_p}}, \quad \beta = R_l C_p \sqrt{\frac{B_p}{m_p l_p^4}}, \quad \alpha = e_p \sqrt{\frac{l_p}{C_p B_p}}, \quad (3)$$

where  $\omega$  is angular frequency,  $m_p$  is line mass density,  $l_p$  is the length of the cantilever beam,  $B_p$  is the bending stiffness,  $C_p$  is the inherent capacitance of the piezoelectric layer,  $e_p$  is the charge accumulation number,  $R_l$  is the externally connected resistance.

In practical applications, dielectric property of piezoelectric materials indicate that the parameter  $\beta$  is