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Asymptotic derivation of a curved piezoelectric interface model and homogenization of piezoelectric composites

S.-T. GU[1,a], Q.-C. HE[1,b], V. PENSEE[1,c] and J.-Z. ZHOU[2,d]

[1]Université Paris-Est, Laboratoire de Modélisation et Simulation Multi Echelle, FRE3160, CNRS, 5 Boulevard Descartes, 77454 Marne-la-Vallée Cedex 2, France

[2]Institute of Civil Engineering and Architecture, Hubei University of Technology, Wuhan, China

[a]gu@univ-paris-est.fr, [b]he@univ-paris-est.fr, [c]Vincent.Pensee@univ-paris-est.fr, [d]hgzhongzhou@sohu.com

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Abstract. In this work, we derive a general piezoelectric interface model by using a coordinate-free asymptotic approach. Next, this interface model is applied to the homogenization of fibrous piezoelectric composites. The overall piezoelectric properties are calculated and compared to the ones obtained by using the three-phase model.

Introduction

The study of interfaces is a lastingly standing issue in mechanics of materials and structures. This is because the overall mechanical behaviour of a heterogeneous material or a structure is strongly affected by the interface between the material constituents or the structural elements. The objective of the present work is twofold. First, it aims to derive a general piezoelectric interface model by using a coordinate-free asymptotic approach. Second, it has the purpose of applying the established general interface model to the homogenization of fibrous piezoelectric composites. The approach we develop in solving this problem is inspired from those proposed by Bövik [2], Hashin [4] and Benveniste [1]. However, our work has two original salient features: (i) it deals with a coupled phenomenon instead of a non-coupled one in all the previous relevant works; (ii) it carries out the derivation and gives the results in a coordinate-free way. The last feature is particularly important for the interface model to be conveniently applied to the cases where the interface has a complex geometrical form. Indeed, we need not to determine the principal curvature coordinates involved in the works [2, 4, 1]. Furthermore, we rediscover the coordinate-free forms of their elastic and conduction interface models by setting the coupling terms in our model to be zero. In this sense, the present work can be viewed as a coordinate-free piezoelectric extension of those works [2, 4, 1].
The general piezoelectric interface model can be applied to the homogenization of a transversely isotropic composite consisting of transversely isotropic piezoelectric fibres surrounded by transversely isotropic piezoelectric layers and embedded in a transversely isotropic piezoelectric matrix. First, the effective piezoelectric moduli of this composite are estimated by considering the latter as a three-phase composite and by using a generalized self-consistent method. Next, the effective piezoelectric moduli of the composite are calculated by taking the latter to be a two-phase composite with the phase interfaces described by our interface model and by using the concept of equivalent inclusions. Finally, the effective piezoelectric moduli of the composite obtained with the interphases and with the interfaces are compared so as to verify the usefulness and pertinence of our general interface model for solving the problem of homogenizing piezoelectric composites.

Governing Equations

Consider a curved piezoelectric interphase of small constant thickness h between two piezoelectric media denoted media 1 and media 2 (Fig. 1(a)). The sub-domains occupied by the interphase and by media 1 and 2 are respectively denoted by $\Omega^{(0)}$, $\Omega^{(1)}$ and $\Omega^{(2)}$. The interfaces S_1 between $\Omega^{(1)}$ and $\Omega^{(0)}$ and S_2 between $\Omega^{(2)}$ and $\Omega^{(0)}$ are supposed to be perfect. The main objective is to replace the interphase by an imperfect interface S_0 which correspond to the middle surface parallel to S_1 and S_2. We denote $\mathbf{n}$ the unit normal vector on S_i (i=0, 1, 2).

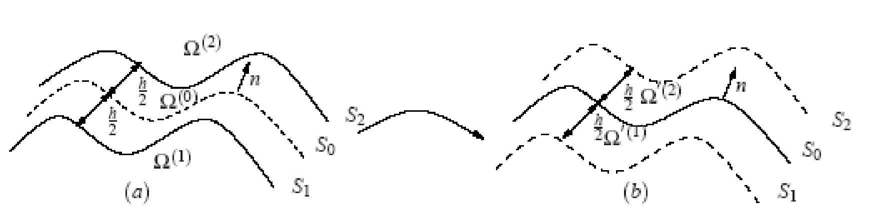

Figure 1. Replacement of the three-phase configuration (a) of Medium(1)/Interphase(0)/Medium(2) by the two-phase configuration (b) of Medium(1)/Interface/Medium(2).

The material constituting each sub-domain $\Omega^{(i)}$ (i=0,1,2) is assumed to be piezoelectric and is characterized by the two constitutive relations.

$$\Sigma_{ij} = K_{ijkl}E_{kl} - \Pi_{kij}e_k \quad \text{and} \quad d_i = \Pi_{ijk}E_{jk} + M_{ki}e_k \tag{1}$$

where $\mathbf{\Sigma}$ is the cauchy stress tensor, $\mathbf{E}$ is the infinitesimal strain tensor, $\mathbf{e}$ is the electric vector and $\mathbf{d}$ is the electric displacement vector. In these equations, K is the fourth order elastic stiffness tensor, $\mathbf{\Pi}$ the third order piezoelectric tensor and M the second order dielectric tensor.

In addition, the interfaces S_1 and S_2 are supposed to be perfect, it follows that the electric potential φ, the normal component d_n of the electric displacement vector, the traction vector $\mathbf{t} = \mathbf{\Sigma}.\mathbf{n}$ and the displacement vector u are continuous across them. These conditions reads as

$$\varphi^{(0)}|_{S_i} = \varphi^{(i)}|_{S_i}\,,\;\; d_n^{(0)}|_{S_i} = d_n^{(i)}|_{S_i}\,,\;\; u^{(0)}|_{S_i} = u^{(i)}|_{S_i}\,,\;\; t^{(0)}|_{S_i} = t^{(i)}|_{S_i} \tag{2}$$

where i=1or 2.

Imperfect Interface Model of a Thin Piezoelectric Interphase

The modelling of the thin piezoelectric interphase as an imperfect interface consists of the characterization of jump relations to be satisfied by the electric potential φ, the normal component of the electric displacement vector d_n, the traction vector $\mathbf{t}$ and the displacement vector $\mathbf{u}$ .These relations are established following the framework proposed in [1, 2].

First, we consider the electric potential φ. In the configuration of Fig1. (a), we express the electric potential $\varphi^{(0)}$ at the mid-surface S_0 in terms of the electric potential at the interface S_1 and S_2 by means of a Taylor expansion:

$$\varphi^{(0)}|S_0 = \varphi^{(0)}|S_1 + \frac{h}{2}\nabla\varphi^{(0)}.\mathbf{n}|S_1 + 0(h^2) \tag{3}$$

$$\varphi^{(0)}|S_0 = \varphi^{(0)}|S_2 - \frac{h}{2}\nabla\varphi^{(0)}.\mathbf{n}|S_2 + 0(h^2) \tag{4}$$

Subtracting (3) from (4) and making use of the conditions (2) leads to

$$\varphi^{(2)}|S_2 - \varphi^{(2)}|S_1 = \frac{h}{2}(\nabla\varphi^{(0)}.\mathbf{n}|S_1 + \nabla\varphi^{(0)}.\mathbf{n}|S_2) + 0(h^2) \tag{5}$$

We introduce the surface operator for expressing $\mathsf{B}^{(i)}(\mathbf{t},\mathbf{u},\varphi,d_n)$, i=0,1 or 2 according to the medium considered, which is an expression of $\nabla\varphi.\mathbf{n}$ as function of continuous quantities across S_1 and S_2. Denoting $\mathbf{P} = \boldsymbol{\delta} - \mathbf{n}\otimes\mathbf{n}$ where $\boldsymbol{\delta}$ is the second order identity tensor, after some calculations, we obtain

$$\begin{aligned}\mathsf{B}^{(i)}(\mathbf{t},\mathbf{u},\varphi,d_n) = &-\frac{d_n}{\mathbf{n}.\mathbf{M}^{(i)}.\mathbf{n}} + \frac{[\mathbf{\Pi}^{(i)} : (\nabla\mathbf{u}.\mathbf{P})].\mathbf{n}}{\mathbf{n}.\mathbf{M}^{(i)}.\mathbf{n}} \\ &+ \frac{\mathbf{n}.\mathbf{\Pi}^{(i)}.\mathbf{n}}{\mathbf{n}.\mathbf{M}^{(i)}.\mathbf{n}}.\mathsf{A}^{(i)}(\mathbf{t},\mathbf{u},\varphi,d_n) - \frac{[\mathbf{M}^{(i)}.(\mathbf{P}.\nabla\varphi)].\mathbf{n}}{\mathbf{n}.\mathbf{M}^{(i)}.\mathbf{n}}\end{aligned} \tag{6}$$

where

$$\begin{aligned}\mathsf{A}^{(i)}(\mathbf{t},\mathbf{u},\varphi,d_n) = \mathbf{B}^{(i)}.(\mathbf{t} + \frac{\mathbf{n}.\mathbf{\Pi}^{(i)}.\mathbf{n}}{\mathbf{n}.\mathbf{M}^{(i)}.\mathbf{n}}d_n - [\mathsf{K}^{(i)} : (\nabla\mathbf{u}.\mathbf{P})].\mathbf{n} - [\mathbf{\Pi}^{(i)T}.\mathbf{P}.\nabla\varphi].\mathbf{n} \\ - \frac{(\mathbf{n}.\mathbf{\Pi}^{(i)T}.\mathbf{n})[\mathbf{\Pi}^{(i)} : (\nabla\mathbf{u}.\mathbf{P})].\mathbf{n}}{\mathbf{n}.\mathbf{M}^{(i)}.\mathbf{n}} + \frac{(\mathbf{n}.\mathbf{\Pi}^{(i)T}.\mathbf{n})[\mathbf{M}^{(i)}.(\mathbf{P}.\nabla\varphi)].\mathbf{n}}{\mathbf{n}.\mathbf{M}^{(i)}.\mathbf{n}})\end{aligned} \tag{7}$$

with

$$\mathbf{B}^{(i)} = [\mathbf{n}.\mathsf{K}^{(i)}.\mathbf{n} + \frac{(\mathbf{n}.\mathbf{\Pi}^{(i)T}.\mathbf{n}) \otimes (\mathbf{n}.\mathbf{\Pi}^{(i)}.\mathbf{n})}{(\mathbf{n}.\mathbf{M}^{(i)}.\mathbf{n})}]^{-1}$$

Introducing (6) in (5), we have

$$\varphi^{(2)}|S_2 - \varphi^{(2)}|S_1 = \tag{8}$$
$$\frac{h}{2}(\mathsf{B}^{(0)}(\mathbf{t}^{(1)},\mathbf{u}^{(1)},\varphi^{(1)},d_n^{(1)})|S_1 + \mathsf{B}^{(0)}(\mathbf{t}^{(2)},\mathbf{u}^{(2)},\varphi^{(2)},d_n^{(2)})|S_2) + 0(h^2)$$

In the configuration of Fig1. (b), the interphase has been eliminated and replaced by an interface positioned at the location of S_0 with a jump between the inner and outer face. So it is necessary to express all fields at the locations S_1 and S_2 by the field in the field of two sides of S_0 denoted by (+) and (-), respectively. After some calculation, the jump on φ across the imperfect interface S_0 is given by

$$[\varphi] = \frac{h}{2}(\mathsf{B}^{(0)}(\mathbf{t}^{(+)},\mathbf{u}^{(+)},\varphi^{(+)},d_n^{(+)}) + \mathsf{B}^{(0)}(\mathbf{t}^{(-)},\mathbf{u}^{(-)},\varphi^{(-)},d_n^{(-)})) \tag{9}$$
$$-\frac{h}{2}(\mathsf{B}^{(2)}(\mathbf{t}^{(+)},\mathbf{u}^{(+)},\varphi^{(+)},d_n^{(+)}) + \mathsf{B}^{(1)}(\mathbf{t}^{(-)},\mathbf{u}^{(-)},\varphi^{(-)},d_n^{(-)})) + 0(h^2)$$

Following the same steps, the jumps, across the imperfect interface, in displacement vector, traction vector and electric displacement vector are obtained and are given through the replacement of the operator B in the formulation by the operators A, C, and D, respectively, for example:

$$[\mathbf{u}] = \frac{h}{2}(\mathsf{A}^{(0)}(\mathbf{t}^{(+)},\mathbf{u}^{(+)},\varphi^{(+)},d_n^{(+)}) + \mathsf{A}^{(0)}(\mathbf{t}^{(-)},\mathbf{u}^{(-)},\varphi^{(-)},d_n^{(-)})) \tag{10}$$
$$-\frac{h}{2}(\mathsf{A}^{(2)}(\mathbf{t}^{(+)},\mathbf{u}^{(+)},\varphi^{(+)},d_n^{(+)}) + \mathsf{A}^{(1)}(\mathbf{t}^{(-)},\mathbf{u}^{(-)},\varphi^{(-)},d_n^{(-)})) + 0(h^2)$$

Note that the following operators have been used

$$\mathsf{C}^{(i)}(\mathbf{t},\mathbf{u},\varphi,d_n) = -\nabla[\mathbf{\Pi}^{(i)}:(\nabla\mathbf{u}.\mathbf{P})]:\mathbf{P} - \nabla[(\mathbf{\Pi}^{(i)}.\mathbf{n}).\mathsf{A}^{(i)}]:\mathbf{P} \tag{11}$$
$$+\nabla[\mathbf{M}^{(i)}:(\mathbf{P}.\nabla\varphi)]:\mathbf{P} + \nabla[(\mathbf{M}^{(i)}.\mathbf{n}).\mathsf{B}^{(i)}]:\mathbf{P}$$

$$\mathsf{D}^{(i)}(\mathbf{t},\mathbf{u},\varphi,d_n) = -\nabla[\mathsf{K}^{(i)}:(\nabla\mathbf{u}.\mathbf{P})]:\mathbf{P} - \nabla[(\mathsf{K}^{(i)}.\mathbf{n}).\mathsf{A}^{(i)}]:\mathbf{P} \tag{12}$$
$$+\nabla[\mathbf{\Pi}^{(i)}:(\mathbf{P}.\nabla\varphi)]:\mathbf{P} + \nabla[(\mathbf{\Pi}^{(i)}.\mathbf{n}).\mathsf{B}^{(i)}]:\mathbf{P}$$

Effective Behaviour of Transverse Piezoelectric Composites under Anti-plane Shear

In the section, we study the special case of three coaxial cylindrical phases which is composed by an isotropic transverse and homogeneous interphase between two isotropic transverse and homogeneous media. Let us assume that the three media have the same principal direction $\mathbf{m}$ that is perpendicular to the unit normal $\mathbf{n}$ of the interface.

The overall properties are calculated using the equivalent inclusion method proposed in elasticity by [3] which has been extended here to the piezoelectricity. Due to the limitation of pages, details are omitted and only anti-plane shear boundary conditions are considered. These conditions allow the determination of the overall elastic moduli K_{44}^*, the overall piezoelectric modulus Π_{44}^* and the overall dielectric modulus M_{11}^*.

The model is illustrated considering the following material properties with the rapport between the rayon of the matrix and the rayon of the inclusion $b/a = 1.1$:

$$K_{44}^{(1)} = 35.3Gpa,\quad K_{44}^{(0)} = \frac{K_{44}^{(1)}}{2},\quad K_{44}^{(2)} = \frac{K_{44}^{(1)}}{10},\quad \Pi_{15}^{(1)} = 10C/m^2,$$
$$\Pi_{15}^{(2)} = \frac{\Pi_{15}^{(1)}}{10},\quad M_{11}^{(1)} = 15.1nC^2/Nm^2,\quad M_{11}^{(0)} = \frac{M_{11}^{(1)}}{2},\quad M_{11}^{(2)} = \frac{M_{11}^{(1)}}{10}.$$

The evolution of overall moduli K_{44}^{*}, Π_{44}^{*} and M_{11}^{*} with the piezoelectric modulus $\Pi_{11}^{(0)}$ are presented on Figures 2, 3 and 4. For each, the interphase thickness is considered. The predictions of the model are compared to the one given by the well known three phase model. Figures 2, 3 and 4 show a good agreement between the predictions of these two models.

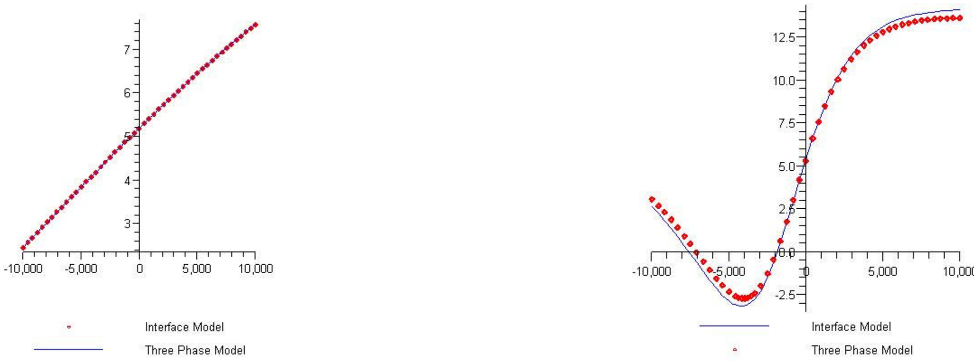

Figure 2. Evolutions of the elastic modulus Π_{15}^{*} for two ratio $h/a = 0.001$ (left) and $h/a = 0.01$ (right).

Figure 3. Evolutions of the elastic modulus K_{15}^{*} for two ratio $h/a = 0.001$ (left) and $h/a = 0.01$ (right).

Figure 4. Evolutions of the elastic modulus M_{15}^{*} *for two ratio* $h/a = 0.001$ *(left) and* $h/a = 0.01$ *(right).*

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