Dear Editor,

We would like to thank all the reviewers for their careful and thorough reading of this manuscript and for their thoughtful comments and constructive suggestions. They help improve the quality of our manuscript. We list our response to the reviewer’s comments point by point below (the reviewer’s comments are in ***bold italics and colored blue*** while the authors’ reply is in black):

**Comments from Reviewer #1**:

***This paper is concerned with the modeling and analysis of piezoelectric vibration energy harvesting, assuming beam-like structures. It provides helpful analytical solutions in the context of a geometrically- and physically-linear formulation. It further offers asymptotic expansion analysis as a means to obtain approximations that allow much easier interpretation, e.g. regarding the interplay of key parameters on the output measures of central interest. The paper is generally well-written and of strong scientific interest and content. It also clearly fits the scope of the Archive of Applied Mechanics.***

***Editorial issues:***

***i. Quite a few minor suggestions that the reviewer proposes regarding wording, formatting etc., can be found in the appended commented version of the manuscript.***

Response: Thanks very much for the comments. We have gone through and revised the manuscript thoroughly according to the comments.

***Scientific content issues:***

***ii. My biggest concern is that the approach seems so straightforward and builds on rather simple (linear) constitutive and structural models that I am strongly suspecting something very similar must have been done before — although I have not conducted a comprehensive literature study to fully support this claim. Perhaps a more thorough literature overview could be added in the introduction, with a clear discussion on the short-comings of the state-of-the-art and how this contribution fills these gaps. So far, this is really only put in context of paper [17], by the Inman & Ertürk group(s).***

Response: Thanks for the valuable comments. Actually, when the idea presented in this manuscript first came to the authors’ mind, we were also surprised that no background research was done on this simple and straightforward idea. Later, we took a literature search and found that asymptotic analysis of piezoelectric composite structures usually based on geometrical small parameters and the resulting simplified stress and strain relations. The results for these asymptotic analyses are usually the simplified or reduced-order model for piezoelectric composite structures like beams and plates, which are actually the starting point of the present contribution. Another kind of related asymptotic analysis for this problem is the limiting behavior of piezoelectric energy harvester under open-circuit and closed-circuit conditions, which are actually covered in the equations (12) in this contribution and can readily be recovered by setting $\sigma = 0$ and $\sigma = \infty$. As a comparison, our scheme of asymptotic analysis is focused on the smallness of a physical parameter (the so-called $\delta$ in the contribution). This kind of smallness is actually specific to problem of piezoelectric energy harvesting. Though by no means our literature search is complete, a mild conclusion can be made that our idea is not covered previously in the literature.

***iii. It would also be interesting to discuss the necessity of increasing the complexity of the underlying (linear) constitutive and structural models and the possible merit of the asymptotic expansion analysis in these situations.***

Response: Thanks very much for the reviewer’s comments. We added some discussions in the conclusion part. Actually, the smallness of parameter $\delta$ is guaranteed by structural and material properties used in the piezoelectric energy harvesting devices and thus also holds with complex constitutive relations and structural nonlinearity. In this sense, the recorded asymptotic analysis in this contribution can be readily generalized to more complex models.

***Recommendation: The reviewer suggests that minor modifications according to the comments above be made before the manuscript can be accepted.***

Response: Thanks very much for the reviewer’s comments. We have gone through the manuscript again end made revisions according to the comments.

**Comments from Reviewer #2**:

***The current paper develops an exact theoretical solution to analyze the classical piezoelectric energy harvesters. Simplified asymptotic expansions of the solutions are derived to provide relatable insight on the mechanics of these types of energy harvesters. This paper provides a novel analytical framework which could be useful in understanding the output performance measures of such systems which in turn can be used as potential design criteria. Except for a few paragraphs, some of which are mentioned below, the manuscript is well written and clear. I only have minor comments which are as follows:***

***Minor comments:***

***1- Page 2 line 19-26 which discusses Rayleigh-Ritz method and claims that this method cannot be used for modal analysis of a system should be clarified. There are multiple works in the literature, which use this method to capture the modes of different linear and nonlinear systems.***

Response: Thanks for the comments. The reviewers are right that the Rayleigh-Ritz method is extremely useful and successful in the analysis of piezoelectric structures. Here we revise the manuscript to make it clear that Rayleigh-Ritz method provides a general way to do the analysis but usually requires more resources and time to do the calculation and is thus inconvenient for preliminary structural design.

***2- Page 4, line 45: It should be explained how equation 9d is developed. (Better referencing to the equations used could be helpful).***

Response: We thank the reviewer for pointing out this issue. The derivation process for equation 9 is essentially the same as those presented in reference [17]. We add the reference here for clarity.

***3- Fig. 2 and page 6 line 23-36: In the Erturk and Inman's model ([17] in the manuscript), with increasing resistance, the natural frequency also increases (closed circuit natural frequency is smaller than the open circuit one).***

***This happens consistently across the first three modes. Is this true in the developed method in the manuscript (shown in Fig. 2)? For the 2nd and 3rd mode it seems that it is not following the same trend. Could you please explain, if this is not following the same trend, why we do not observe the same trend as in [17]?***

Response: Thanks very much for the reviewer’s comments. Actually in our method, the results for natural frequency at different externally connected resistance are exactly the same as those found by Erturk and Inman, which is shown in Fig. 2. Open-circuit and closed-circuit conditions can be easily represented as $\beta = 0$ and $\beta = \infty$ which goes into equation (12) in different ways.

***4- Fig 3. phase plot: Would it not be better to increase the range of the Y axis so that the other plots that are mentioned in the legend also become visible? (The plots are not visible for 3 cases of electromechanical coupling factors).***

Response: Thanks very much for the comments. The cases for $\delta = 0$ and $\delta = \infty$ correspond to the solution with zero phase shift in the displacement function u. We have changed the y axis limit to make them visible.

***5- Fig (3)-(10): The utilized parameters for the simulations should either be clearly defined, or proper referencing be made. (Parameters that define the non-dimensional ones, for instance what stiffness, mass and … was used?).***

Response: Thanks very much for the reviewer’s comments. To make comparisons with the results obtained by Erturk and Inman, we have actually adopted the same physical parameter values unless otherwise assigned explicitly. Based on the definitions of the dimensionless parameters in equation (12), the physical parameters $f\_b$ and $R\_l$ can be totally determined by the three dimensionless parameters $\delta$, $\beta$, and $\sigma$. But in some of the figure legends, both dimensionless parameters and physical parameters are given for clarity.

***6- Fig. 4 and page 6, line 1-2: Please discuss the physics of: why the phase reaches a minimum value at a certain electromechanical coupling factor.***

Response: Thanks very much for the reviewer’s comments. We have gone through the manuscript again end made revisions according to the comments.

***7- Fig. 6 and page 12 and 13: Did you check whether expansions of higher orders will give more accurate results for the plot near the resonance frequency? Also, why are only 2 expansions shown here, considering that it could not capture the behavior of the system at the resonance frequency, which is essential for understanding the piezoelectric energy harvesters? Also, did you check whether these approximations were able to capture the higher harmonics? (Perhaps two sets of figures for the behavior of the system at higher harmonics would be interesting to see in the appendix).***

Response: Thanks very much for the reviewer’s comments. We have gone through the manuscript again end made revisions according to the comments.

***8- Fig. 7: This figure at its current format is difficult to understand. Would it not be better to also prepare a surface plot (similar to Fig. 9), and prepare a dashed line that shows the \delta\_p^c over the surface plot?***

Response: Thanks very much for the reviewer’s comments. We have gone through the manuscript again end made revisions according to the comments.

***9- Figure 10 and 11: The results of these figures should be compared with the existing ones in the literature and thoroughly discussed as to why we observe such trends.***

Response: Thanks very much for the reviewer’s comments. We have gone through the manuscript again end made revisions according to the comments.

***Textual comments:  
1- Page 4, line 15: "is be decomposed as" should be changed to "is to be decomposed". "to" should be added.***

Response: Thanks very much for the reviewer’s comments. We have gone through the manuscript again end made revisions according to the comments.

***2- Page 5, line 3: Punctuation should be fixed: "Therefore the method" needs "," after "Therefore".***

Response: Thanks very much for the reviewer’s comments. We have gone through the manuscript again end made revisions according to the comments.

***3- Page 6, line 40: Punctuation should be fixed: "parameters, σ, β and δ. which …". "which" should be capitalized if the sentence is finished.***

Response: Thanks very much for the reviewer’s comments. We have gone through the manuscript again end made revisions according to the comments.

***4- Page 6, line 27: Punctuation should be fixed: "However the" needs"," after "However".***

Response: Thanks very much for the reviewer’s comments. We have gone through the manuscript again end made revisions according to the comments.

***5- Fig 3 caption: "phase (left panel) of", I believe this was meant to be "right panel"?***

Response: Thanks very much for the reviewer’s comments. We have gone through the manuscript again end made revisions according to the comments.

***References and citations:  
1- Needs to add references for equations (1), (2), (13)***

***2- Line 36: Needs to cite to the equations that were used to develop equation (9).***

Response: Thanks very much for the reviewer’s comments. We have gone through the manuscript again end made revisions according to the comments.