Reviewer #1:

This paper is concerned with the modeling and analysis of piezoelectric vibration energy harvesting, assuming beam-like structures. It provides helpful analytical solutions in the context of a geometrically- and physically-linear formulation. It further offers asymptotic expansion analysis as a means to obtain approximations that allow much easier interpretation, e.g. regarding the interplay of key parameters on the output measures of central interest. The paper is generally well-written and of strong scientific interest and content. It also clearly fits the scope of the Archive of Applied Mechanics.

Editorial issues:

i. Quite a few minor suggestions that the reviewer proposes regarding wording, formatting etc., can be found in the appended commented version of the manuscript.

Scientific content issues:

ii. My biggest concern is that the approach seems so straightforward and builds on rather simple (linear) constitutive and structural models that I am strongly suspecting something very similar must have been done before — although I have not conducted a comprehensive literature study to fully support this claim. Perhaps a more thorough literature overview could be added in the introduction, with a clear discussion on the short-comings of the state-of-the-art and how this contribution fills these gaps. So far, this is really only put in context of paper [17], by the Inman & Ertürk group(s).

iii. It would also be interesting to discuss the necessity of increasing the complexity of the underlying (linear) constitutive and structural models and the possible merit of the asymptotic expansion analysis in theses situations.

Recommendation: The reviewer suggests that minor modifications according to the comments above be made before the manuscript can be accepted.

Reviewer #2:

The current paper develops an exact theoretical solution to analyze the classical piezoelectric energy harvesters. Simplified asymptotic expansions of the solutions are derived to provide relatable insight on the mechanics of these types of energy harvesters. This paper provides a novel analytical framework which could be useful in understanding the output performance measures of such systems which in turn can be used as potential design criteria. Except for a few paragraphs, some of which are mentioned below, the manuscript is well written and clear. I only have minor comments which are as follows:

Minor comments:

1- Page 2 line 19-26 which discusses Rayleigh-Ritz method and claims that this method cannot be used for modal analysis of a system should be clarified. There are multiple works in the literature, which use this method to capture the modes of different linear and nonlinear systems.

2- Page 4, line 45: It should be explained how equation 9d is developed. (Better referencing to the equations used could be helpful).

3- Fig. 2 and page 6 line 23-36: In the Erturk and Inman's model ([17] in the manuscript), with increasing resistance, the natural frequency also increases (closed circuit natural frequency is smaller than the open circuit one).

This happens consistently across the first three modes. Is this true in the developed method in the manuscript (shown in Fig. 2)? For the 2nd and 3rd mode it seems that it is not following the same trend. Could you please explain, if this is not following the same trend, why we do not observe the same trend as in [17]?

4- Fig 3. phase plot: Would it not be better to increase the range of the Y axis so that the other plots that are mentioned in the legend also become visible? (The plots are not visible for 3 cases of electromechanical coupling factors).

5- Fig (3)-(10): The utilized parameters for the simulations should either be clearly defined, or proper referencing be made. (Parameters that define the non-dimensional ones, for instance what stiffness, mass and … was used?).

6- Fig. 4 and page 6, line 1-2: Please discuss the physics of: why the phase reaches a minimum value at a certain electromechanical coupling factor.

7- Fig. 6 and page 12 and 13: Did you check whether expansions of higher orders will give more accurate results for the plot near the resonance frequency? Also, why are only 2 expansions shown here, considering that it could not capture the behavior of the system at the resonance frequency, which is essential for understanding the piezoelectric energy harvesters? Also, did you check whether these approximations were able to capture the higher harmonics? (Perhaps two sets of figures for the behavior of the system at higher harmonics would be interesting to see in the appendix).

8- Fig. 7: This figure at its current format is difficult to understand. Would it not be better to also prepare a surface plot (similar to Fig. 9), and prepare a dashed line that shows the \delta\_p^c over the surface plot?  
9- Figure 10 and 11: The results of these figures should be compared with the existing ones in the literature and thoroughly discussed as to why we observe such trends.

Textual comments:  
1- Page 4, line 15: "is be decomposed as" should be changed to "is to be decomposed". "to" should be added.

2- Page 5, line 3: Punctuation should be fixed: "Therefore the method" needs "," after "Therefore".

3- Page 6, line 40: Punctuation should be fixed: "parameters, σ, β and δ. which …". "which" should be capitalized if the sentence is finished.

4- Page 6, line 27: Punctuation should be fixed: "However the" needs"," after "However".

5- Fig 3 caption: "phase (left panel) of", I believe this was meant to be "right panel"?

References and citations:  
1- Needs to add references for equations (1), (2), (13)

2- Line 36: Needs to cite to the equations that were used to develop equation (9).