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The 'Indian wire trick' via parametric excitation: a comparison between theory and experiment

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A seemingly paradoxical experiment is described whereby a length of wire is stabilized upside down by vertical periodic oscillation of its support. The experimental results reveal an upper and a lower bound on the excitation frequency for stability. The results of recent theories are presented and used to explain the essential details of the observations. The theory relies on a novel phenomenon of so-called resonance–tongue interaction. The result is verified via asymptotic calculations based on a one-dimensional rod model and numerical results on a spatially discretized system of links. This gravity-defying effect has potential application to the stabilization of other spatially extended systems via parametric excitation.

Keywords: rod mechanics; experimental dynamics; parametric excitation

1. Introduction

There has long been a fascination with phenomena that defy gravity, such as the legendary Indian rope trick and various more reputable scientific phenomena (Dullin & Easton 1999; Mahadevan *et al.* 1998; Thomsen & Tcherniak 2001). For example, it has been known since Stephenson (1908) that rapid, small-amplitude, parametric (i.e. vertical) harmonic excitation can balance a single pendulum in an upside-down position. Several authors (see, for example, Otterbein 1982) have wondered whether the Indian rope trick may be performed this way. Indeed it was demonstrated (Acheson & Mullin 1993) and proved mathematically (Acheson 1993) that a chain of N linked pendulums may be stabilized upside down by high enough frequencies. Unfortunately, in the limit of a rope, i.e. $N \to \infty$, the required frequency tends to infinity. It was reported (Acheson & Mullin 1998) that a rope with bending stiffness, namely a wire, can be stabilized at finite frequency, but no theory of why it works nor any experimental results have been published. Here we present quantitative details and provide an explanation for observations on the stabilization of upright wire which would otherwise fall over under its own weight. In particular, we show that there are upper and lower bounds on the excitation frequency required to perform this gravity-defying 'Indian wire trick'.

2. Experimental results

The experimental set-up was as follows. A length of intrinsically straight domestic 'curtain wire' (which consists of a tightly wound steel spring *ca.* 3 mm in diameter clad with a 0.5 mm plastic coating) was held upright by a clamp. This was driven vertically using a sliding crank device to supply a close approximation to sinusoidal excitation. The amplitude of this oscillation was held fixed at $\Delta = 2.2$ cm peak to peak. The length of wire beyond the clamp, l, was allowed to vary, as was the frequency of excitation ω within a range 0–35 Hz. It was found that the longest length of wire which was able to support its own weight was $l = l_c = 55.3$ cm. Longer lengths resulted in catastrophic collapse to a buckled state. This bifurcation is subcritical as the buckled states are of large amplitude to the extent that the tip of of the wire is beneath the support, as shown in figure 1*a* (cf. the experiments by Benjamin reported in Iooss & Joseph (1990)).

Taking a fixed length l a little greater than l_c, initially held loosely upright, and switching on the parametric excitation, it was found that the vertical position was made stable for a range $\omega_1 < \omega < \omega_2$ (see figure 1*b*). This stability region in the (l, ω)-plane is depicted in figure 2*a* as the area between the two curves $\omega_1(l)$ and $\omega_2(l)$ for $l > l_c$. These results were obtained by taking a fixed l and making a sweep in frequency with increments of 0.5 Hz, allowing, at each ω value, time for transient effects to decay. The results represent, for each l value, the average over several such sweeps.

The lower stability boundary $\omega_1(l)$ is characterized by the wire falling over upon reduction of ω, rather like the static buckle of the unforced problem. This falling is typically preceded, upon decreasing ω, by a definite lean (figure 1*c*).

The upper instability at $\omega_2(l)$ was observed to be more dramatic. As the frequency is increased, the wire suddenly becomes unstable to large-amplitude planar oscillations which quite often cause permanent plastic deformation of the helical coil. At the onset of the instability, the oscillations were observed to be at approximately the same frequency as the vertical drive and to be predominantly in the third vibration mode (figure 1*d*). The fact that the oscillations were at the same frequency as the drive and not (say) at half the drive frequency was checked by subjecting the experiment to strobe lighting with varying frequency, and repeating 20 times.

Indeed, using standard rod theory assuming linear elasticity, equation (3.1) below, one can calculate the natural vibration frequencies of the wire at the critical length $l = l_c$. The bending stiffness of the wire was calculated from the value of l_c using the third relation in equation (3.2) below and the knowledge that buckling occurs in theory at $b = b_c = 0.127\,59$. By this method, the first three vibration frequencies are calculated to be 4.9, 14.4 and 28.5 Hz. The corresponding modes are the second, third and fourth vibration modes, respectively, since the first mode by definition has a natural frequency of zero at the critical length $l = l_c$. It might therefore seem surprising at this stage that the dynamic instability at forcing frequency ω_2, which is *ca.* 25–30 Hz, did not have a bigger component of the fourth mode of vibration. We shall offer an explanation for this in the theoretical analysis to follow.

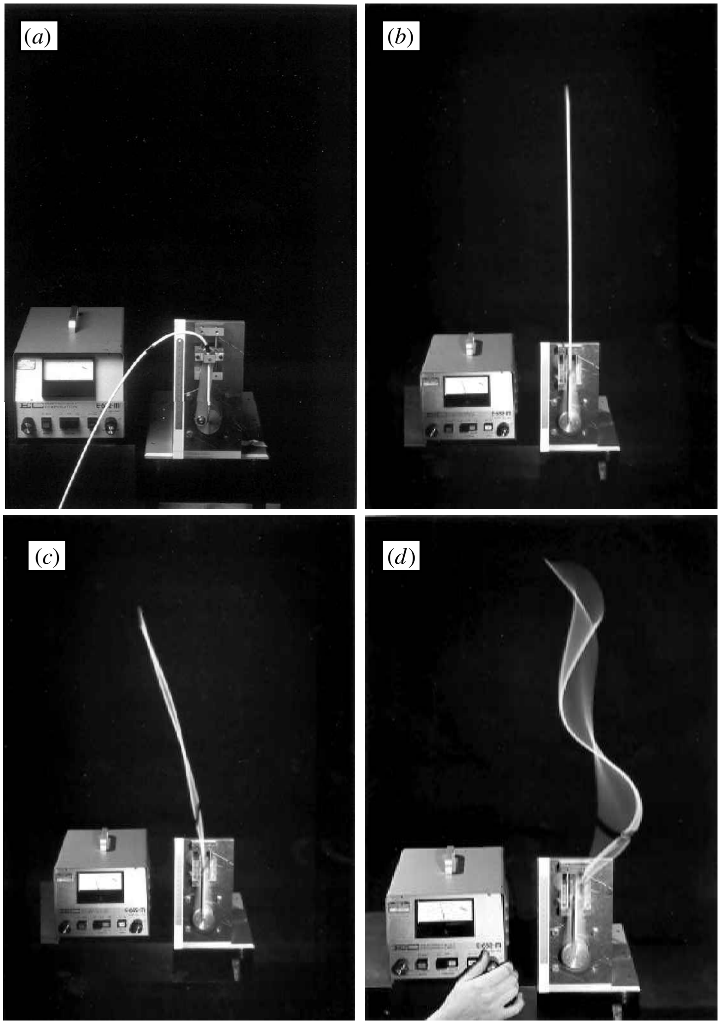

Figure 1. The experimental demonstration: (a) the sagging buckled wire; (b) the stabilized vertical wire; (c) leaning motion near the lower-frequency 'falling-over' instability; and (d) motion near the higher-frequency dynamic instability.

Despite care being taken to use new specimens of wire whenever there was evidence of permanent plastic deformation, the experiments revealed that transient effects can play a large role. In particular, evidence was found of additional resonant instabilities within the region of figure 2a that we have referred to as dynamically 'stable'. That is, lateral oscillations were sometimes excited which could involve a mixture of different spatial modes and which could cause a loss of stability well before ω_2 upon slow increase in ω. For the majority of the 'dynamically stable' region though, the wire was found to be stable such that the tip could be given a slight push and the resulting lateral motion would decay. A repeatable exception to this was for ω just above ω_1, where a small-amplitude circular motion of the tip was often excited. This motion is an order of magnitude slower than the vertical excitation and had a tendency for its frequency to decrease to zero as ω was decreased to ω_1.

The same phenomenon of stabilization beyond the critical length for buckling was also found to be possible using other materials, such as a 2 m length of niobium wire. However, we focus on the curtain-wire results since they were robust and the lengths of wire involved were practical. We now turn to a theoretical explanation of these results.

3. Theoretical explanation

A simple theory (Acheson 1993) predicts the stabilization upside down of a system of N jointed pendulums without bending stiffness (verified experimentally in Acheson & Mullin (1993)). These results do not directly apply here since taking $N \to \infty$ results in a string with no bending stiffness and a prediction that the required frequency of excitation would be infinite.

Instead, it is useful to consider a continuum model in which the wire is modelled as a linearly elastic rod with bending stiffness B (Champneys & Fraser 2000). In the absence of damping, and ignoring geometric nonlinearity, the lateral displacement u at arclength s along the wire ($s = 0$ being the clamped end) and time t is governed by the dimensionless equation

$$\eta \ddot{u} + (1 - \eta\varepsilon \cos t)[(1 - s)u']' + bu'''' = 0. \tag{3.1}$$

Here a prime denotes differentiation with respect to s and a dot denotes differentiation with respect to t. The dimensionless parameters are

$$\eta = \frac{4\pi^2\omega^2 l}{g}, \qquad \varepsilon = \frac{\Delta}{l}, \qquad b = \frac{B}{mgl^3}, \tag{3.2}$$

η and ε being the dimensionless frequency and amplitude of the excitation and b the scaled bending stiffness.

By analysing the fundamental instability in this model using asymptotic methods, a simple dimensionless formula was derived in Champneys & Fraser (2000) for a lower bound on the excitation frequency for stability of the upright wire:

$$\omega^2 > 1.27 \frac{lg}{4\pi^2\Delta^2}\left(1 - \frac{l_c^3}{l^3}\right). \tag{3.3}$$

This lower bound is plotted for the experimental values of l_c and Δ as a dashed line in figure 2*a*. The bound essentially arises from a one-degree-of-freedom analysis of the first mode of vibration near its buckling instability. It is in an equivalent form to that which one gets for a rigid pendulum, namely that the product of the excitation frequency ω and amplitude Δ should be greater than a prescribed value (Acheson 1993; Levi 1999). Such an inequality can also be explained physically using the same arguments as for the rigid pendulum. By averaging over the high frequency of excitation, in the limit of a large product $\omega\Delta$, simple geometry reveals an effect that lateral motion causes an effective restoring force (see Levi (1999) for the details).

However, this physical argument does not explain why the formula (3.3) is such a gross underestimate of the experimental boundary ω_1 (see figure 2*a*), nor does it predict the upper instability boundary ω_2. In order to explain these features, we shall present here the results of a new double-scale asymptotic analysis of equation (3.1), the detailed methodology of which appears elsewhere (Fraser & Champneys 2002).

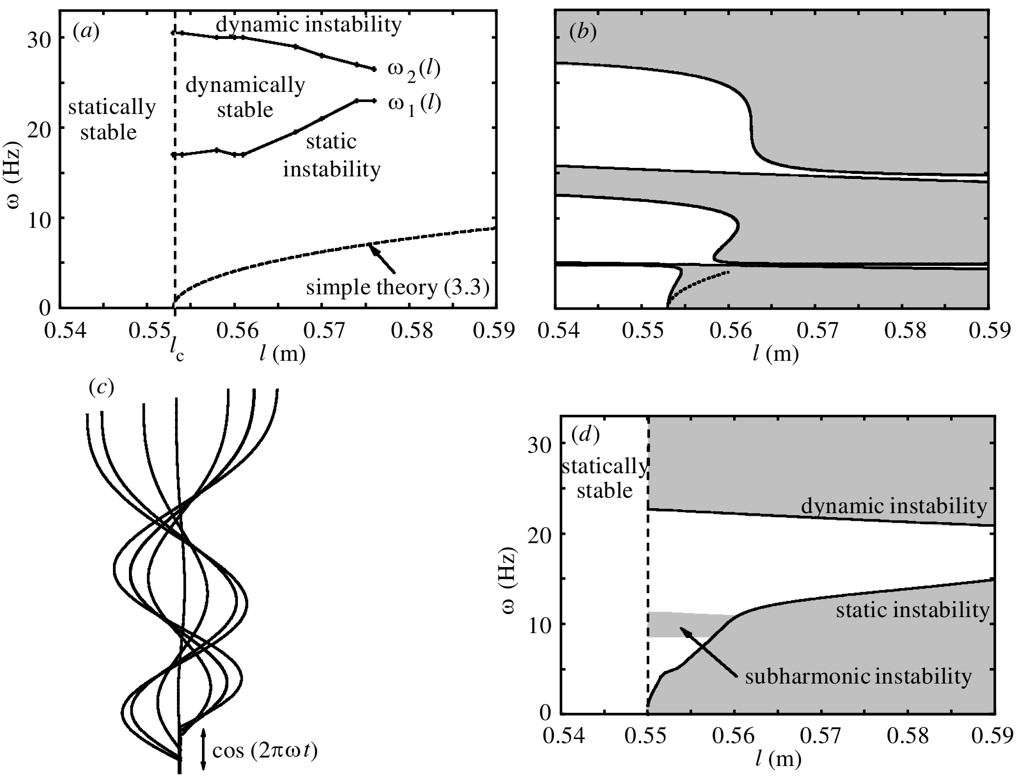

Figure 2. Quantitative details of the experimental results and various theories. See text for detailed explanation. (*a*) Experimental data and the 'lower bound' (3.3) for stability. (*b*) Resonance–tongue interaction asymptotic results. Shaded regions represent instability. (*c*) The shape of the harmonic resonance of the third spatial mode. (*d*) Numerical results on the discrete link model for $N = 16$.

These results reveal a hidden subtlety in this problem which arises from the interaction between harmonic resonances of certain vibration modes and the pure falling-over instability of the first mode. The analysis is valid in the asymptotic limit $\varepsilon \to 0$ around certain special values η_c of the dimensionless frequency η, for which two instabilities occur at the same value of the third dimensionless parameter b. The most dramatic case is when one of the instabilities is the harmonic (at the same frequency of the drive) resonance of any of the vibration modes, and the other is the pure falling-over mode. The asymptotic description of both resonance–tongue boundaries in the (b, ε)-plane is such that the coefficient of the quadratic-in-ε terms becomes infinite. This in turn causes an increase in the parameter range leading to stability for η values just less than the critical η_c, and the vanishing of the stability region for $\eta > \eta_c$.

Just such a critical interaction is found to occur for the fourth spatial mode at $\eta = 1813.5$ (corresponding to $\omega = 28.5$ Hz in the experimental coordinates) and for the third spatial mode at $\eta = 460.7$ (corresponding to $\omega = 14.4$ Hz). The harmonic response of the third spatial mode of the wire according to the theory is depicted in figure 2*c*. Note its similarity with the experimentally observed dynamic instability (figure 1*d*).

The full results of the asymptotic analysis for the experimental values of l_c and Δ are plotted in figure 2*b*. The shaded parameter regions correspond to where this analysis predicts instability. Note the large 'dolphin's nose' shaped stability region which bears a close resemblance to the experimental stability region in figure 2*a*. This stability region has two horizontal asymptotes, at approximately $\omega = 14.8$ and 28.5 Hz, as ℓ decreases. As ℓ increases though, the upper boundary of the stability region approaches the lower boundary. The asymptotic value that both of these boundaries are approaching there is $\omega \approx 14.8$ Hz, the resonant frequency of the third vibration mode. Thus it is the unfolding—at finite amplitude $\Delta = 0.022$, which implies $\varepsilon \approx 0.02$—of the interaction between the harmonic instability of the *third* spatial mode and the pure buckling instability that is causing the shape of the instability boundary. This reasoning accords with the experimental observation that the dynamic instability was predominantly in the third spectral mode.

Note that the results of figure 2*b* show other, smaller windows of stability at lower frequency. The lowest of these is quadratically tangent as $l \to l_c$ to the curve (3.3) depicted in figure 2*a* (the lower portion of the dashed line in reproduced in figure 2*b* for comparison). Other zones of instability within the stability regions given by the theory leading to figure 2*b* are also predicted using similar asymptotic theory (results not depicted). They correspond to tongues of subharmonic resonance of various vibration modes. These regions of instability are significantly narrower than the dolphin's nose shaped ones, and may well account for the observation of extra instabilities that were sometimes excited by transient effects in the experiments. It is also worthwhile noting that the asymptotic methods can be extended to deal with the effects of geometric nonlinearity (Fraser & Champneys 2002). Such an analysis predicts circular oscillations whose frequency tends to zero as the falling-over instability is approached ($\omega \to \omega_1$), just as observed in the experiments.

Finally, in order to test the theoretical results numerically and also to include material damping, which is evidently present in the experiment, we have derived a model consisting of N identical rigid links coupled by stiff, damped linear springs. We may consider this is a simple finite-element discretization. By correctly scaling the mass and length of each pendulum, we checked that in the limit $N \to \infty$ such a model approaches the continuum model (3.1) with an additional damping term $\Gamma \dot{u}''''$ on the left-hand side. Here Γ is a dimensionless parameter representing internal material damping. By using numerical bifurcation techniques (Doedel *et al.* 1997) we have been able to plot out boundaries of stability in parameter space for N up to 32. A striking convergence is found with an increase in N. The results for $N = 16$ and $\Gamma = 0.004$ and other parameters as for the experiment are plotted in figure 2*d*, with shading again representing instability. The same characteristic shape of the stability boundary is recovered, with a small region of subharmonic instability of the second vibration mode. Approximate quantitative agreement is found with the experiment apart from a down-shift in the frequencies, which might be due to an overly simplistic model of material damping, or failure to catch the true bending stiffness of the wire. This down-shift did not disappear as N increased.

4. Conclusion

We have described for the first time the experimental details of a counterintuitive mechanical phenomenon. The experiment throws up several interesting questions.

(i) Why does the wire stay up at all?

(ii) Why does the simple criterion (3.3) (the dashed line in figure 2*a*) fail to be quantitatively accurate?

(iii) Why is the upper instability boundary found to be a harmonic resonance rather than a subharmonic one as is more common in structural dynamics?

The answer to (i) is, as we remarked earlier, more or less the same as the physical explanation of what keeps the simple pendulum up, as explained in Levi (1999). The answers to (ii) and (iii) we have shown to be related and are to do with the new 'resonance–tongue interaction' that we have identified. This theory, which is only strictly valid for an asymptotically small amplitude of excitation (i.e. far less than $\varepsilon = 0.02$ used in the experiment), has been supported by numerical bifurcation analysis of a carefully discretized model which is valid for arbitrary excitation amplitudes. Note, however, that neither form of modelling takes into account the particular (nonlinear) material properties of the curtain wire used in the experiment, which may account for some of the discrepancy between the three sets of data (cf. parts (*a*),(*b*) and (*d*) of figure 2). Nevertheless, strong evidence has been presented that it is this new resonance–tongue interaction that qualitatively explains the shape of the stability region for this experiment.

The concept that instability mechanisms corresponding to different spatial modes can interact so strongly has potential significance in other multi-degree-of-freedom parametrically excited problems. Examples of such problems include the claim by Wolf (1969, 1970) to stabilize heavy viscous oil over water. Other possibilities include auto-parametrically resonant mechanical systems (Nayfey 2000; Tondl *et al.* 2000), such as pendulum dampers, and the dynamics of cable-stayed bridges (Lilien & Pinto da Costa 1994), where subharmonic resonance is typically thought to be the only important dynamic resonance that must be considered by designers. Our results show that harmonic resonances can also lead to practically significant effects, under conditions when they interact with a static bifurcation. Moreover, the presence of harmonic resonance can render a single-degree-of-freedom static bifurcation analysis wildly inaccurate.

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