Name : _____

Student ID : _____ Section : _____

Date : _____

Assignment 1 – Set Theory and Logic, Function

Group of 3, Due date: 17 November 2024

1. Let A={ 1, 3, 5, 7 }, B={ 3, 4, 5, 6 }, and C={ 5, 6, 7, 8 }.
   a) Prove that (A ∩ B) ∪ (B ∩ C) ⊆ B.
   b) Prove or disprove: (A ∪ C) – B = (A – B) ∪ (C – B).
   c) Find A ⊕ (B ∩ C) and determine whether this result is equal to (A ⊕ B) ⊕ C.

2. Define the following sets:
   - P={ x ∈ Z | x is a prime number less than 20 }
   - E={ x ∈ Z | x is an even number less than 20 }
   - D={ x ∈ Z | x is a divisor of 36 and less than 20 }

   Using these sets:
   a) Translate the statement: "All elements of PPP that are not in DDD are also in EEE" into a formal logic expression using quantifiers.
   b) Prove or disprove the above statement in (a) by checking each element.
   c) Prove that (P∩E)∪D=D using set theory laws.

3. Let U be the universal set of all integers from 1 to 30. Define the following subsets of U:
   - A={ x ∈ U | x is a multiple of 2}
   - B={ x ∈ U | x is a multiple of 3}
   - C={ x ∈ U | x is a multiple of 5}
   a) Construct a Venn diagram that represents U, A, B, and C.
   b) Identify the number of elements in each region of the Venn diagram (e.g., A ∩ B ∩ C , A−B−C, etc.).

4. Let X={ 0, 1, 2 } and Y={ a, b }.
   a) Find the Cartesian product X×Y and list each ordered pair.
   b) Define a relation R ⊆ X × X such that (x, y) ∈ R (x, y) if and only if x + y is even. List all pairs in R.
   c) Determine if R is reflexive, symmetric, or transitive, and provide proof for each property.

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5. Consider the following statements:
   a) Prove the contrapositive of the following statement: "If $x \in Z$ is even and x^2 is also even, then x is a multiple of 4."
   b) Determine whether the following statement is true or false. If true, prove it. If false, provide a counterexample:
      - "For all integers *m* and *n*, if *mn* is even, then *m* is even or *n* is even."
   c) Using a proof by contradiction, show that the square root of 3 is irrational.

6. Let $f: Z \rightarrow Z$ be defined by $f(x) = 3x + 2$.
   a) Prove or disprove that f is one-to-one.
   b) Prove or disprove that f is onto.
   c) Find $f^{-1}(y)$, if it exists, for $y \in Z$, and determine the conditions under which f has an inverse function.

7. Let $g: R \rightarrow R$ be defined by $g(x) = x^2$ and $h(x) = x + 1$.
   a) Compute $(g \circ h)(x)$ and $(h \circ g)(x)$.
   b) Determine whether $g \circ hg$ and $h \circ gh$ are one-to-one, onto, or neither.
   c) If possible, find the inverse of $(h \circ g)(x)$.