1. $A = \{1, 3, 5, 7\}$

$B = \{3, 4, 5, 6\}$

$C = \{5, 6, 7, 8\}$

a) $(A \cap B) \cup (B \cap C) \subseteq B$

$(A \cap B) = \{3, 5\}$

$(B \cap C) = \{5, 6\}$

$(A \cap B) \cup (B \cap C) = \{3, 5, 6\}$

(b) $(A \cup C) - B = (A - B) \cup (C - B)$

$(A \cup C) = \{1, 3, 5, 6, 7, 8\}$

$(A \cup C) - B = \{1, 7, 8\}$

$(A - B) = \{1, 7\}$

$(C - B) = \{7, 8\}$

$(A - B) \cup (C - B) = \{1, 7, 8\}$

$(A \cup C) - B = (A - B) \cup (C - B)$

$\{1, 7, 8\} = \{1, 7, 8\}$ # prove

(c) $A \oplus (B \cap C)$

$(A - (B \cap C)) \cup ((B \cap C) - A)$

$(B \cap C) = \{5, 6\}$

$A - (B \cap C) = \{1, 3, 7\}$

$(B \cap C) - A = \{6\}$

$(A - (B \cap C)) \cup ((B \cap C) - A) = \{1, 3, 6, 7\}$

$(A \oplus B) \oplus C$

$A \oplus B = (A - B) \cup (B - A)$

$(A - B) = \{1, 7\}$

$(B - A) = \{4, 6\}$

$(A - B) \cup (B - A) = \{1, 4, 6, 7\}$

$((A - B) \cup (B - A)) - C) \cup (C - ((A - B) \cup (B - A)))$

$= \{1, 4\}$ $= \{5, 8\}$

$= \{1, 4, 5, 8\}$

$A \oplus (B \cap C) \neq (A \oplus B) \oplus C$

2. $P = \{x \in \mathbb{Z} \mid x$ is a prime number less than $20\}$

$E: \{x \in \mathbb{Z} \mid x$ is an even number less than $20\}$

$D = \{x \in \mathbb{Z} \mid x$ is a divisor of 36 and less than $20\}$

a) $\forall x: \{(P(x) \wedge \neg D(x) \rightarrow E(x))$

b) $P = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$E = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

$D = \{1, 2, 3, 4, 6, 9, 12, 18\}$

$P - D = \{5, 7, 11, 13, 17, 19\}$

$(P - D) \neq E$, statement is false

c) $(P \cap E) \cup D = D$

$(P \cap E) = \{2\}$

$(P \cap E) \cup D = \{1, 2, 3, 4, 6, 9, 12, 18\}$

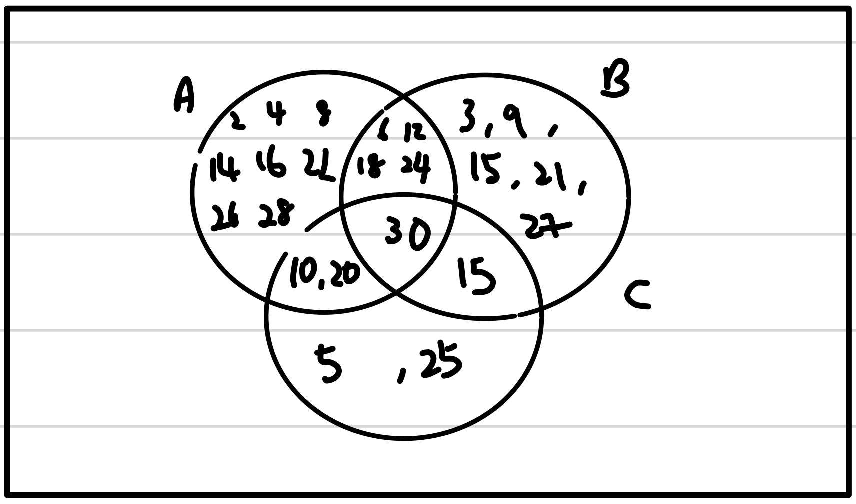
$(P \cap E) \cup D$

$(D \cup P) \cap (D \cup E)$

$\{2\}$ is a subset of D

Hence, $(P \cap E) \cup D = D$, prove!

3.



b) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10,$
$11, 12, 13, 14, 15, 16, 17, 18, 19, 20.$
$21. 22, 23, 24, 25, 26, 27, 28. 29, 30\}$

$A = \{x \in U \mid x$ is a multiple of $2\}$

$B = \{x \in U \mid x$ is a multiple of $3\}$

$C = \{x \in U \mid x$ is a multiple of $5\}$

$A = \{2, 4, 6, 8. 10, 12. 14, 16, 18, 20, 22, 24, 26, 28, 30\}$ 15 elements

$B = \{3, 6, 9, 12, 15, 18, 21, 24. 27, 30\}$ 10 elements

$C = \{5, 10, 15, 20, 25, 30\}$ 6 elements

$A \cap B = \{6, 12, 18, 24, 30\}$ 5 elements

$A \cap C = \{10, 20, 30\}$ 3 elements

$B \cap C = \{15, 30\}$ 2 elements

$A \cap B \cap C : \{30\}$ 1 elements

4 a) $X = \{0, 1, 2\}$ $Y = \{a, b\}$

$|X \times Y| = \{(0,a), (0,b), (1,a), (1,b), (1,c), (2,a), (2,b)\}$

b) $R \subseteq X \times X$ $(x, y) \in R(x, y)$ $x+y$ = even

$R = \{(0,0), (0,2), (1,1), (2,0), (2,2)\}$

(c)

$$M_R: \begin{array}{c|c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array} \times \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \overset{M_{R^2}}{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}$$

Reflexive, $(0,0), (1,1), (2,2) \in R$

Symmetric, $(0,2), (2,0) \in R$

Transitive, $(0,0), (0,2) \in R$, $(0,2) \in R$

$(0,2), (2,0) \in R$, $(0,0) \in R$

$(0,2), (2,2) \in R$, $(0,2) \in R$

$(2,0), (0,0) \in R$, $(2,0) \in R$

$(2,2), (2,0) \in R$, $(2,0) \in R$

5a) If $x \in \mathbb{Z}$ is even, x^2 also even. x is a multiple of 4.

If P then Q, if not Q then not P

contra: If not Q, then not P

P: x is even and x^2 is even.

Q: x is a multiple of 4

contra: If x is not a multiple of 4, then x is not even and x^2 is not even.

$\neg Q(x)$ is true

Suppose x is not a multiple of 4, x could be odd ($x = 2k+1$, for some integer k) 1,3,5

or even but not divisible by 4 (2, 6, 10)

1. If x is odd, then x^2 is odd (If $x=3$, then $x^2 = 9 \Rightarrow$ odd)
2. If x is even but not a multiple of 4, $x = 4k+2$ (2, 6, 10)

- If $x = 2k$, where k is odd, then $x^2 = (2k)^2 = 4k^2$
- $4k^2$ is divisible by 4, x^2 is even, but x is not divisible by 4.

$\Rightarrow$ If x is not a multiple of 4, it doesn't mean x has to be odd. It could be an even number that is not divisible by 4.

If x is even, $x = 2m$

If x is not divisible by 4 → means not a multiple of 4

↳ so x is even (a multiple of 2), does not have a factor 4

If x is a multiple of 4, $x = 4k$

If x is even but not a multiple of 4, $x = 4k+2$

cuz can divide by 2

remainder 2 after divide by 4

↳ can write like this

$x = 2(\underbrace{2k+1}_{\text{odd integer}})$

5b) For all integers m and n, if mn is even, them m is even or n is even.

The statement is true

Proof:

- An integer is even if and only if it's divisible by 2. Hence, an integer is odd if it is not divisible by 2.
- Suppose mn is even, then mn is divisible by 2. Thus, at least m or n is even and divisible by 2.

∴ We can conclude that the statement is true.

c). Suppose that the conclusion is false. Then $\sqrt{3}$ is rational.

- $\sqrt{3}$ can expressed as $\frac{p}{q}$. (p and q have no common factor other than 1, $q \neq 0$)

$$\sqrt{3} = \frac{p}{q}$$

$$3 = \frac{p^2}{q^2}$$

$$3q^2 = p^2$$

Let $p = 3n$

$$3q^2 = (3n)^2 = 9n^2$$

$$3q^2 = 9n^2$$

$$q^2 = 3n^2$$

∴ This shows that q^2 is divisible by 3. Hence, q must also divisible by 3.

Contradiction: Both p and q are divisible by 3 which contradicts our assumption. Hence. our assumption is false, $\sqrt{3}$ is not rational.

so the statement is true.

6a) $f: \mathbb{Z} \to \mathbb{Z}$ $\quad f(x) = 3x + 2$

for all integer. n_1 and n_2 if $f(x_1) = f(x_2)$, then $n_1 = n_2$

Let $f(n_1) = f(n_2)$

$$3x_1 + 2 = 3x_2 + 2$$

$$3x_1 = 3x_2$$

$$x_1 = x_2$$

$\therefore$ This shows that f is one-to-one

b) $f^{-1}(y) = x$

$$y = 3x + 2$$

$$x = \frac{y-2}{3}$$

$$f^{-1}(y) = \frac{y-2}{3}$$

$\therefore$ f is not onto, not every $y \in \mathbb{Z}$ satisfies $x \in \mathbb{Z}$. $(y-2)$ must divisible by 3

E.g. $x = 7$ $\quad y = 7-2/3 = 1.17$

(c) $y = 3x + 2$

$x = \frac{y-2}{3}$

condition: $\quad y \in \mathbb{Z}$ and $y-2$ must divisible by 3 $(2, 5, 8, \dots)$

$y = 2$, $2-2=0$, divisible by 3 $\quad f^{-1}(2) = \frac{2-2}{3} = 0$

$y = 5$, $5-2 = 3$, divisible by 3 $\quad f^{-1}(5) = \frac{5-2}{3} = 1$

$\therefore$ For $f^{-1}(y)$ to exist, f must be one-to-one and onto. It is proven that f is one-to-one but not onto. f does not have inverse function for all $y \in \mathbb{Z}$.

7. $g: R \to R \quad g(x) = x^2 \quad h(x) = x+1$

$g \circ h(x) = g(h(x))$

$= (x+1)^2$

$h \circ g(x) = h(g(x))$

$= x^2+1$

(b) $g \circ h(x) = (x+1)^2$

$= x^2 + 2x + 1$

Let $g \circ h(x_1) = g \circ h(x_2)$

$(x_1+1)^2 = (x_2+1)^2$

$x_1 + 1 = \pm(x_2+1)$

$x_1 = x_2$ or $x_1 = -x_2 - 2$

$\therefore$ Not one-to-one, because for each $y \in Z$, there is more than one $x \in X$.

$g \circ h(x) = (x+1)^2$ onto ?

$g \circ h(x) = (x+1)^2$

$y = (x+1)^2$

$\sqrt{y} = x+1$

$x = \sqrt{y} - 1$

If $y = -1$, $x = \infty$

$\therefore$ Since $g \circ h(x) = (x+1)^2$, $(y \geq 0)$. not all real numbers can be reached.

Therefore, $g \circ h(x)$ is not onto

(c) $h \circ g(x) = x^2+1$, one to one?

Let $h \circ g(x_1) = h \circ g(x_2)$

$x_1^2 + 1 = x_2^2 + 1$

$x_1^2 = x_2^2$

$x_1 = \pm x_2$

$\therefore$ Not one-to-one

(Horizontal line test have two intersection points)

$h \circ g(x) = x^2+1$, onto ?

$y = x^2+1$

$x = \sqrt{y-1}$

$y \notin R$, If $y = 0$, $x = \infty$

$\therefore h \circ g(x)$ is not onto.