Group Member: Eleanor Ting A24CS0247 Wong Zi Ning A24CS0313

1. $a_0 = 1$ $\quad a_n = 3a_{n-1} + 2$

$a_0 = 1$

$a_1 = 3(1) + 2 = 5$

$a_2 = 3(5) + 2 = 17$

$a_3 = 3(17) + 2 = 53$

$a_4 = 3(53) + 2 = 161$ $\quad \therefore$ First five terms = 1, 5, 17, 53, 161

2. $b_n = b_{n-1} + n^2$ $\quad b_1 = 1$

$b_2 = 1 + 2^2$

$= 5$

$b_3 = 5 + 3^2$

$= 14$

$b_4 = 14 + 4^2$

$= 30$

3. $a_n = 2a_{n-1} + 5$ $\quad a_0 = 3$

$a_1 = 2(3) + 5 = 11$

$a_2 = 2(11) + 5 = 27$

$a_3 = 2(27) + 5 = 59$

$a_4 = 2(59) + 5 = 123$

4. $T_n = 2T_{n-1} + 1$ $\quad T_1 = 1$

$T_1 = 1$

$T_2 = 2(1) + 1 = 3$

$T_3 = 2(3) + 1 = 7$

$T_4 = 2(7) + 1 = 15$

$T_1 = 2^1 - 1 = 1$

$T_2 = 2^2 - 1 = 3$

$T_3 = 2^3 - 1 = 7$

$T_n = 2^n - 1$

Assume : $T_n = 2^n - 1$

$T_{n+1} = 2T_n + 1$

$T_{n+1} = 2(2^n - 1) + 1$

$T_{n+1} = 2^{n+1} - 2 + 1$

$T_{n+1} = 2^{n+1} - 1$

Counting Methods & Probability

1. a. $\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6}$

$10 \times 9 \times 8 \times 7 \times 6 = 30240$

b. $\underline{5} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6}$

0 8
2
4
6

$5 \times 9 \times 8 \times 7 \times 6 = 15120$

2. a. $10! = 3628800$

b. $9! \times 2! = 725760$

3. a. Possible outcomes : (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

Total = 6 outcomes

Probability = $\frac{6}{36} = \frac{1}{6}$

b. All outcomes = $6 \times 6 = 36$

Outcomes without 6 = $5 \times 5 = 25$

Probability = $\frac{36-25}{36} = \frac{11}{36}$ #

At least one 6 :

(1,6), (2,6), (3,6), (4,6), (5,6), (6,6)

(6,1), (6,2), (6,3), (6,4), (6,5)

c. Two dice are equal = (1,1), (2,2), (3,3), (4,4), (5,5), (6,6)

$= \frac{6}{36} = \frac{1}{6}$

4. a. $^{15}C_4 = C(15,4) = \frac{15!}{4!(15-4)!} = 1365$

b. $^{13}C_2 = C(13,2) = \frac{13!}{2!(13-2)!} = 78$

5. a. STATISTICS

S(3) T(3) I(2) A(1) C(1)

Total = $\frac{9!}{2! \times 3! \times 2!} = 15120$

b. TTT | S A I S I C S

S(3) I(2)

Total = $\frac{8!}{3!\,2!} = 3360$

Permutations and Combinations

1. a. $^{12}P_8 = \frac{12!}{(12-8)!} = 19958400$

   b. $\underline{4}\ \underline{10}\ \underline{9}\ \underline{8}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{4} = 4 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 = 2419200$

   A B C D   1 2 3 4

2a. ENGINEERING

   E(3) N(3) G(2) I(2) R(1)

   without Restrictions $= \frac{11!}{3! \times 3! \times 2! \times 2! \times 1!} = 277200$

b. $\underbrace{\text{EEE} | \text{NGINRING}}_{9!}$

   E's together $= \frac{9!}{3! \times 2! \times 2! \times 1!} = 15120$

3.a. Distinct arrangement $= (8-1)! = 7! = 5040$

   b. Indistinguishable when flipped $= \frac{(8-1)!}{2} = 2520$

4. a. $^{18}C_6 = 18564$

   b. $^{10}C_4 \times {}^8C_2 = 5880$

   $^{10}C_3 \times {}^8C_3 = 6720$

   $^{10}C_2 \times {}^8C_4 = 3150$

   $^{10}C_1 \times {}^8C_5 = 560$

   $^{10}C_0 \times {}^8C_6 = 28$

   Total $= 16338$

5.a. $\frac{30!}{10! \times 12! \times 8!} = 3.7848 \times 10^{12}$

   b. $\frac{29!}{9! \times 12! \times 8!} = 1.2616 \times 10^{12}$

6. a. 0 ways. If at least one of each type must be chosen, customer need to buy at least 10 cakes.

   // $^{10}C_5 = 252$ ways

b. $C(n+r-1, r) = \frac{(n+r-1)!}{r!(n-1)!}$

n: number of types (10 cakes)

r: number being chosen (5 cakes) ∴ repetition allowed, order doesn't matter

$C(10+5-1, 5) = \frac{14!}{5!(9!)}$

$= 2002$

Pigeonhole Principle

1. $\{1, 2, 3 \ldots\ldots 99\} = \{(1,2), (3,4), (5,6) \ldots (97, 98), (99)\}$

There are 50 groups.

- 49 groups containing two consecutive integers
- 1 group contains a single integer 99.

By Pigeonhole principle, since we are putting 50 numbers into 49 complete pairs, at least two of our chosen numbers must fall in the same pair.

If two number fall into the same pair, they must be consecutive. Hence, at least two numbers must be consecutive

2. When divided by 8, any integer gives a remainder of 0, 1, 2, 3, 4, 5, 6 or 7

Pigeonholes : 8 possible remainders

Pigeons: 9 positive integers

$\lceil \frac{9}{8} \rceil = \lceil 1.125 \rceil = 2$

At least two integers will share the same remainder when divided by 8

Let $x \equiv r \pmod 8$, where $r \in \{0, 1, 2, 3, \ldots 7\}$

$x \equiv r \pmod 8$ and $y \equiv r \pmod 8$

$x - y \equiv 0 \pmod 8$

∴ The difference $x - y$ is divisible by 8

Example : $95 \div 8 = 11$ remainder 7	$31 \div 8 = 3$ remainder 7
$23 \div 8 = 2$ remainder 7	$15 \div 8 = 1$ remainder 7
$95 - 23 = 72$ ($72 = 8 \times 9$)	$31 - 15 = 16$ ($16 = 2 \times 8$)

3. Pigeonholes = 7 days in a week

Pigeons = 30 students

$\lceil \frac{30}{7} \rceil = \lceil 4.286 \rceil = 5$

$\therefore$ There are at least 5 students born in the same day of a week.

since $5 > 2$, there are at least 2 students born in the same day of a week

4. Pigeons = 9 socks

Pigeonholes = 3 colors (red, blue, green)

$\lceil \frac{9}{3} \rceil = \lceil 3 \rceil = 3$

$\therefore$ At least one colour must have at least 3 socks

Let X be the 9 socks , $Y = \{$red, blue, green$\}$

Define a function $f: X \to Y$ by $f(s)$ = red , if the color of sock is red ,

$f(s)$ = blue, if the color of sock is blue , $f(s)$ = green, if the color of sock is green .