3.4 解:

(1) $I_1 = -\log 0.6 = \log\frac{5}{3} = 0.737$ bit

$I_2 = -\log 0.4 = \log\frac{5}{2} = 1.322$ bit

(2)

X \ Y	b_1	b_2	
a_1	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{3}{5}$
a_2	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{2}{5}$
	$\frac{4}{5}$	$\frac{1}{5}$	

$I(a_1;b_1) = I(b_1) - I(b_1|a_1)$

$= -\log\frac{4}{5} + \log\frac{5}{6} = \log\frac{25}{24} = 0.0589$ bit

$I(a_2;b_1) = I(b_1) - I(b_1|a_2)$

$= -\log\frac{4}{5} + \log\frac{3}{4} = \log\frac{15}{16} = -0.0931$ bit

$I(a_1;b_2) = -\log\frac{1}{5} + \log\frac{1}{6} = \log\frac{5}{6} = -0.263$ bit

$I(a_2;b_2) = -\log\frac{1}{5} + \log\frac{1}{4} = \log\frac{5}{4} = 0.322$ bit

(3) $H(X) = H(0.6, 0.4) = 0.971$ bit

$H(Y) = H(\frac{4}{5}, \frac{1}{5}) = 0.722$ bit

$H(XY) = E[-\log P(X,Y)]$

$= -\frac{1}{2}\log\frac{1}{2} - \frac{1}{10}\log\frac{1}{10} - \frac{3}{10}\log\frac{3}{10} - \frac{1}{10}\log\frac{1}{10} = 1.6855$ bit

(4)

$H(X|Y) = H(XY) - H(Y)$

$$= 1.6855 - 0.722 = 0.9635 \text{ bit}$$

$$= -\frac{1}{2}\log\frac{5}{8} - \frac{1}{10}\log\frac{1}{2} - \frac{3}{10}\log\frac{3}{8} - \frac{1}{10}\log\frac{1}{2}$$

(5)

$$H(Y|X) = H(XY) - H(X)$$

$$= 1.6855 - 0.971 = 0.7145 \text{ bit}$$

(6)

$$I(X;Y) = H(X) - H(X|Y)$$

$$= 0.971 - 0.9635 = 0.0075 \text{ bit}$$

3.10 解：

二元对称离散信道为均匀信道，则

$$C = 1 - H\left(\frac{1}{100}\right) = 1 + \frac{1}{100}\log\frac{1}{100} + \frac{99}{100}\log\frac{99}{100}$$

$$= 0.9192 \text{ bit/符号}$$

$$C' = 0.9192 \times 1000 = 919.2 \text{ bit/s}$$

3.11 解：

$$C = \log 4 - H(0.5, 0.5) = 1 \text{ bit/符号}$$

3.12 解：

该信道为准对称信道，

对称子矩阵为：

$$\begin{bmatrix} 1-p-q & p \\ p & 1-p-q \end{bmatrix} \qquad \begin{bmatrix} q \\ q \end{bmatrix}$$

则

$$C=\log 2-[(1-q)\log(1-q)+q\cdot\log(2q)]$$

$$-H(p,q,1-p-q)=1-q-$$

$$(1-q)\log(1-q)+p\log p+(1-p-q)\log(1-p-q)$$

3.15 解

$$C=\log 2-H(0.8,0.2)$$

$$=1-(-0.8\log 0.8-0.2\log 0.2)=0.278$$

$$C_3=3+2.4\log 0.8+0.6\log 0.2=0.8342\ \text{bit/符号}$$

信道矩阵

$$[P]=\begin{array}{c|cccccccc} & 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \\ \hline 000 & 0.512 & 0.128 & 0.128 & 0.032 & 0.128 & 0.032 & 0.032 & 0.008 \\ 001 & 0.128 & 0.512 & 0.032 & 0.128 & 0.032 & 0.128 & 0.008 & 0.032 \\ 010 & 0.128 & 0.128 & 0.512 & 0.128 & 0.128 & 0.008 & 0.128 & 0.032 \\ 011 & 0.032 & 0.128 & 0.128 & 0.512 & 0.008 & 0.032 & 0.032 & 0.128 \\ 100 & 0.128 & 0.032 & 0.032 & 0.008 & 0.512 & 0.128 & 0.128 & 0.032 \\ 101 & 0.032 & 0.128 & 0.008 & 0.032 & 0.128 & 0.512 & 0.128 & 0.128 \\ 110 & 0.032 & 0.008 & 0.128 & 0.032 & 0.128 & 0.032 & 0.512 & 0.128 \\ 111 & 0.008 & 0.032 & 0.032 & 0.128 & 0.032 & 0.128 & 0.128 & 0.512 \end{array}$$

3.21 证明：

5.10

$$P=\begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ 0 & \frac{1}{2} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & \frac{2}{3} & \frac{1}{3}\\ 0 & \frac{1}{3} & \frac{2}{3}\end{bmatrix}=\begin{bmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ 0 & \frac{1}{2} & \frac{1}{2}\end{bmatrix}$$

设$\begin{bmatrix}X\\ p(x)\end{bmatrix}=\begin{bmatrix}a_1 & a_2\\ q & 1-q\end{bmatrix}$

$$p(x=a_1,z=c_1)=p(x=a_1,y=b_1)=\frac{q}{3}$$

$$p(x=a_1,z=c_2)=p(x=a_1,y=b_2)=\frac{q}{3}$$

$$p(x=a_1,z=c_3)=p(x=a_1,y=b_3)=\frac{q}{3}$$

$$p(x=a_2,z=c_1)=p(x=a_2,y=b_1)=0$$

$$p(x=a_2,z=c_2)=p(x=a_2,y=b_2)=\frac{1-q}{2}$$

$$p(x=a_2,z=c_3)=p(x=a_2,y=b_3)=\frac{1-q}{2}$$

$$\therefore p(z=c_1)=p(y=b_1)=\frac{q}{3},p(z=c_2)=p(y=b_2)=\frac{3-q}{6},p(z=c_3)=p(y=b_3)=\frac{3-q}{6}$$

$$\therefore H(Z)=H(Y)=H\left(\frac{q}{3},\frac{3-q}{6},\frac{3-q}{6}\right)=-\frac{q}{3}\log\frac{q}{3}-\frac{3-q}{3}\log\frac{3-q}{6}$$

$$H(Z\mid X)=H(Y\mid X)=qH\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)+(1-q)H\left(0,\frac{1}{2},\frac{1}{2}\right)=1-q+q\log 3$$

$$\begin{aligned}\therefore I(X;Z)&=H(Z)-H(Z\mid X)\\ &=H(Y)-H(Y\mid X)\\ &=I(X;Y)\end{aligned}$$

3.22

证明:

当 $n=1$ 时，错误概率 $p=\frac{1}{2}(1-(1-2p))$ 成立;

假设 $n=k$ 成立，即 k 个串接信道的错误概率为 $\frac{1}{2}[1-(1-2p)^k]$;

当 $n=k+1$ 时，其错误概率为:

$$
\begin{aligned}
&\overline{p}\frac{1}{2}[1-(1-2p)^k]+p\left(1-\frac{1}{2}[1-(1-2p)^k]\right)\\
&=\frac{\overline{p}}{2}-\frac{\overline{p}}{2}(1-2p)^k+p-\frac{p}{2}[1-(1-2p)^k]\\
&=\frac{1}{2}-\frac{\overline{p}}{2}(1-2p)^k+\frac{p}{2}(1-2p)^k\\
&=\frac{1}{2}-\frac{1}{2}(1-2p)^k+p(1-2p)^k\\
&=\frac{1}{2}[1-(1-2p)^{k+1}]
\end{aligned}
$$

当 $n\to\infty$ 时，错误概率近似为 $\frac{1}{2}$，总信道矩阵为 $\begin{bmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$，此时不论输入为何分布，输出均为等概率分布。其互信息为:

$$\lim_{n\to\infty} I(X_0;X_n)=H(X_n)-H(X_n\mid X_0)=1-H(X_n\mid X_0)=0\text{比特/符号}$$