

4.1

$$(1) \bar{L} = \sum_i P(s_i) L_i$$

$$= 3.11 \text{ 码元 / 符号}$$

$$(2) H(S) = H(0.2, 0.19, 0.18, 0.17, 0.15, 0.10, 0.01) = 2.609 \text{ bit / 符号}$$

$$R = \frac{H(S)}{\bar{L}} = 0.8389 \text{ bit / 码元}$$

$$(3) R' = \bar{L} \log 2 = 3.11 \text{ bit / 符号}$$

$$(4) \eta = \frac{H(S)}{\bar{L} \log 2} = 0.8389$$

4.4

$$(1) H(S) = H(0.125, 0.125, 0.25, 0.5) = 1.75 \text{ bit / 符号}$$

$$(2) \bar{L} = \sum_i P(S_i) L_i = 1.75 \text{ 码元 / 符号}$$

$$(3) \text{ 由于 } \eta = \frac{R}{R_{\max}} = \frac{H(S)}{\bar{L} \log r} = 1$$

\therefore 码元0和1是独立等概分布

$$\therefore P(0) = P(1) = 0.5$$

$$H(X) = H(0.5, 0.5) = 1 \text{ bit / 符号}$$

$$P(1|0) = P(1|1) = P(1) = 0.5$$

$$P(0|1) = P(0|0) = P(0) = 0.5$$

5.7 解:

$$N=1, H(S) = H(0.9, 0.1) = 0.469 \text{ bit/符号}$$

$$\begin{pmatrix} S \\ C \end{pmatrix} = \begin{pmatrix} s_1 & s_2 \\ 0 & 1 \end{pmatrix}, \eta_1 = \frac{H(S)}{C} = 0.881$$

$$N=2, \begin{pmatrix} S^2 \\ P(S) \\ C \end{pmatrix} = \begin{pmatrix} s_1 s_1 & s_1 s_2 & s_2 s_1 & s_2 s_2 \\ 0.81 & 0.09 & 0.09 & 0.01 \\ 0 & 10 & 110 & 111 \end{pmatrix}$$

$$\therefore \bar{L}_2 = 0.81 + 0.09 \cdot 2 + 0.09 \cdot 3 + 0.01 \cdot 3 \\ = 1.29 \text{ bit/符号}^2$$

$$\bar{C} = \frac{\bar{L}_2}{2} = 0.645 \text{ bit/符号元}$$

$$\eta_2 = \frac{H(S)}{\bar{C}} = 0.727$$

同理求 η_3

$N \rightarrow \infty$, 由香农第一定理, 必存在
唯一可译码, 使

$$\lim_{N \rightarrow \infty} \frac{\bar{L}_N}{N} = H_r(S)$$

$$\therefore \lim_{N \rightarrow \infty} \eta_N = 1$$

5.9

霍夫曼编码：

$$\begin{bmatrix} s_i \\ P(s_i) \\ L_i \\ W_i \end{bmatrix} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & s_5 \\ 0.25 & 0.2 & 0.2 & 0.2 & 0.15 \\ 2 & 2 & 2 & 3 & 3 \\ 01 & 10 & 11 & 000 & 001 \end{bmatrix}$$

$$\therefore \bar{L} = \sum_i P(s_i) L_i = 2.35 \text{ 码元 / 符号}$$

$$\eta = \frac{H(S)}{\bar{L}} = 0.979 \approx R$$

$$H(S) = H(0.25, 0.2, 0.2, 0.2, 0.15) \\ = 2.3037 \text{ bit/符号}$$

构造码： $\begin{pmatrix} s_i \\ C \end{pmatrix} = \begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 \\ 00 & 01 & 10 & 110 & 111 \end{pmatrix}$

$$\bar{L} = 2 \times 0.25 + 2 \times 0.2 + 2 \times 0.2 + 3 \times 0.2 + 3 \times 0.15 \\ = 2.35$$

$$\therefore \eta = R = 0.979$$

5.10 解： D 维， r 片叶 +

空叶数： $R_{D-1}((r-D)(D-2))$

$$R_3((8-4)(4-2)) = R_3(8) = 2$$

$$P_e = 1 - (0.125 + 0.15 + 0.35) = 0.375$$

$$(2) F(b_1) = a_1, F(b_2) = a_2, F(b_3) = a_3$$

$$P_e = 0.375$$