Assignment 4

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a) ii and iii

b) ii because it requires less parameters (is the simplest one)

c) $P(M_1 | N)$, where $N \in \{1, 2, 3\}$ and $M_1 \in \{0, 1, 2, 3, 4\}$

$M_1 \backslash N$	1	2	3
0	$f + e(1-f)$	f	f
1	$(1-f)(1-2e)$	$e(1-f)$	$e(1-f)$
2	$e(1-f)$	$(1-2e)(1-f)$	$e(1-f)$
3	0	$e(1-f)$	$(1-2e)(1-f)$
4	0	0	$e(1-f)$

d) F_1 true F_2 true : $N > 5$

F_1 true F_2 false : $N = 4$

F_1 false F_1 true : not reasonable

F_1 false F_2 false : $N = 2$

$N = 2$, or $N = 4$ or $N > 5$

e) It is given the $e > f$, so the most probable or most likely number is $N = 2$ because it's probability $e^2(1-f)^2$

$N = 4$ probability is $e(1-f) \cdot f$

$N > 5$ probability is f^2

Considering that $e > f$, we can assume

$e^2(1-f)^2 > e(1-f) \cdot f$ ~~thus~~ thus $N = 2$ is more likely occur than $N = 4$

The probability for $N > 5$ is so small (negligible) (f^2), thus it is smaller than $N = 2$

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Question 2

$$P(B|j,m) \propto P(B,j,m) = \sum_{e,a} P(B,j,m,e,a)$$

$$= \sum_{e,a} P(B)\,P(e)\cdot P(a|B,e)\cdot P(j|a)\cdot P(M|a)$$

$$= \sum_{e} P(B)\,P(e)\cdot \sum_{a} P(a|B,e)\,P(j|a)\,P(M|a)$$

$$= \sum_{e} P(B)\cdot P(e)\cdot f_1(j,m|B,e)$$

$$= P(B) \sum_{e} P(e)\,f_1(j,m|B,e)$$

$$= P(B)\,f_2(j\ m|B)$$

$$P(B|+j,+m) = a\,P(B) \sum_{e} P(e)\cdot \sum_{a} P(a|B,e)\cdot P(+j|a)\cdot P(+m|a)$$

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a) $P(B \mid +j, +m) = \alpha P(B) \sum_a P(a \mid B, e) P(+m \mid a) P(+j \mid a) \times$

$$\times \sum_e P(e)$$

$$= \alpha P(B) \times \sum_e P(e) \left[0.7 \times 0.9 \times \begin{pmatrix} 0.95 & 0.29 \\ 0.94 & 0.001 \end{pmatrix} + \right.$$

$$\left. + 0.5 \times 0.001 \times \begin{pmatrix} 0.05 & 0.71 \\ 0.06 & 0.999 \end{pmatrix} \right]$$

$$= \alpha P(B) \times \sum_e P(e) \left[0.63 \times \begin{pmatrix} 0.95 & 0.29 \\ 0.94 & 0.001 \end{pmatrix} + 0.5 \times 10^{-4} \times \begin{pmatrix} 0.05 & 0.71 \\ 0.06 & 0.99 \end{pmatrix} \right.$$

$$= \alpha P(B) \times \sum_e P(e) \begin{bmatrix} 0.598525 & 0.183055 \\ 0.59223 & 0.0012295 \end{bmatrix}$$

put the values of e:

$$= \alpha P(B) \times \left[0.998 \times \begin{pmatrix} 0.59223 \\ 0.0011295 \end{pmatrix} + 0.002 \times \begin{pmatrix} 0.598525 \\ 0.59223 \end{pmatrix} \right]$$

$$= \alpha P(B) \times \left[\begin{pmatrix} 0.001197 \\ 0.000366 \end{pmatrix} + \begin{pmatrix} 0.591045 \\ 0.001127 \end{pmatrix} \right]$$

$$= \alpha P(B) \times \begin{pmatrix} 0.592242 \\ 0.001492 \end{pmatrix}$$

put the values of P(B)

$$= \alpha \begin{pmatrix} 0.001 \\ 0.999 \end{pmatrix} \times \begin{pmatrix} 0.592242 \\ 0.001492 \end{pmatrix}$$

$$= \boxed{\langle 0.284, 0.716 \rangle}$$

# of addition = 7
multiplication = 16
division = 2

total number of arithmetic operations is 23, whereas enumeration algorithm is 25.

Question 3

a) When eliminating D we generate a new factor f_2;

$$f_2(A,+c,E,F) = \sum_d P(F|d) \cdot P(E|d) \cdot \cancel{f_1} (A,+c,d)$$

This leaves us with factors:

$$P(+c), P(G|F,+c), P(A), f_2(A,+c,E,F)$$

When eliminating G we generate a new factor f_3:

$$f_3(+c,F) = \sum_g P(g|+c,F)$$

This leaves us with the factors:

$$P(+c), P(A), f_2(A,+c,E,F), f_3(+c,F)$$

When eliminating F we generate a new factor f_4:

$$f_4(A,+c,E) = \sum_f f_2(A,+c,E,f) f_3(+c,f)$$

This leaves us with the factors:

$$P(+c), P(A), f_4(A,+c,E)$$

b) $$P(A,E|+c) = \frac{P(+c)P(A)f_4(A,+c,E)}{\sum_{a,e} P(+c)P(a)f_4(a,+c,e)}$$

c) The largest factor generated is $f_2(A,+c,E,F)$
It has 8 entries (2^3)

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d) Variable Eliminated | Factor Generated

Variable Eliminated	Factor Generated
B	$f_1(A, +c, D)$
G	$f_2(+c, F)$
F	$f_3(+c, D)$
D	$f_4(A, +c, E)$

Question 4

a)(i) $P(+c) = 5/8$

(ii) $P(+c \mid +a, -d) = 2/3$

b)

Sample	Weight
-a +b +c -d	0.277
+a +b +c -d	0.170
+a +b -c -d	0.025
-a +b -c -d	0.042

c) $P(-a \mid +b, -d)$

$$\frac{1/24 + 10/36}{10/36 + 5/30 + 1/40 + 1/24} = 0.625$$

d) The better suited for likelihood weighting is P(D | A) because it is an upstream evidence and likelihood evidence ~~fely~~ conditions only on it.