Median-Based Filters for Removing Impulse Noise

“Review and Implementation”

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Abstract:

As the medical images’ quality is of great importance, the technics of denoising these images are of interest. The nature of impulse noise that may mostly contaminate CT and MRI images, is defined in the first section. In the next sections, different denoising algorithms are explored, one of the algorithms is implemented, the evaluation methods and parameters for noise removal are investigated, and some of them are employed to estimate the performance of the implemented algorithm.

1- Introduction:

Noise types

Different factors may be responsible for introduction of noise in the image. Noise types depends on the cause of its occurrence. It can be categorized to Impulse noise, Gaussian noise, Speckle noise, Periodic noise, etc.

The impulse noise that corrupts mainly medical images, is caused by random disturbance of electron component or synchronization errors during digitization. Unstable voltage, malfunctioning of camera sensors, faulty memory locations, and transmition in a noisy channel could also result in development of impulse noise. It is uncorrelated to pixel intensity values. The probability of this type of noise is defined as follows.

$$p(z) = \begin{cases} Pa, & z = a \\ Pb, & z = b \\ otherwise\ 0 \end{cases}$$

It is classified to Salt and Pepper noise that takes only 0 and 255 instead of a random value in range [0,255]. Gaussian noise has a normal distribution and is an additive noise, which means the values of pixels are not replaced, but added to a random value generated in noise matrix.

2- Median-Based Filters

Noise removal is an important preprocessing step. It can be done by order statistics filters that are a branch of Spatial Filtering. A filter is defined as a window of (2*windowsize+1)^2 which is to assure the window has a center (The number of cells are odd). Then the convolution is performed and the result of the convolution is used to replace the center cell value. Min and Max filters can be used to remove salt noise and pepper noise respectively. But if the image is contaminated of both, Mean filter can be employed. This filter does not result in a good quality image since it is in fact ignoring the values at two sides of the array. Single noisy point in the window skews significantly the resulting filtered mask. Median-Based Filters are known as the most efficient ones. Since they are less sensitive to outliers compared to Mean Filters. There are many novel types of these algorithms. Here, we explore some.

Standard Median Filter

Sliding the window all over the image, Median takes all the magnitudes within a window, demands a sorting process that can done in O (n log n). The center of the window, then is computed:

$$\hat{f}(x,y) = Median\{g(s,t)\}\ where\ (s,t)\epsilon S(x,y)$$

The drawback to this algorithm is that even if the pixel is not a noisy one, it is always replaced by the median. Which means the algorithms fails to preserve edges. This, results in deteriorating the overall visual quality. With introducing algorithms that change their behavior depending on the window pixels, the chance of preserving edges will increase.

Adaptive Median Filter

AMF adapts to the statistical characteristics of inside filter region pixels. This is done by checking if the median is a noise itself or not. In some highly contaminated noisy images, median value of a window may be influenced by the high amount of noise. In these windows that pixels are very similar (ex: a high amount of pepper), the median is not trustworthy and we have to increase the window size to find a meaningful behavior. In that appropriate window, then the pixel is checked to be a noise, if so it is replaced by the median that has already passed the noise test in level A. Considering below variables,

Zmin = minimum intensity value in Sxy
Zmax = maximum intensity value in Sxy
Zmed = median of the intensity values in Sxy
Zxy = intensity value at coordinates (x, y)

The algorithm is defined in two levels:

Level A: If Z min < Zmed < Zmax, go to level B
Else increase the window size
If window size < Smax, repeat level A
Else output Zmed

Level B: If Z min < Zxy < Zmax, Output Zxy
Else output Z med

This algorithm results in a better image by not changing some points. Therefore it avoids error propagation that occurs in SMF. The choice of Smax affects the quality of the image, and is determined based on the spatial density of noise.

Weighted Median Filters

Specific pixels are repeated a given number of times to participate in taking the median of the window. These pixels participate in voting so that they have more influence on median.
In Center Weighted median filter, only the center has a weight. Inspiring from the fact that it may be an edge pixel, so to preserve it we have:

$$\hat{f}(x,y) = Median\{g(s,t)\}\ where\ (s,t)\epsilon S(x,y)$$

$$\begin{cases} w\big(g(s,t)\big) = 2k+1\ where\ s,t=0 \\ w\big(g(s,t)\big) = 1\ \ otherwise \end{cases}$$

K is the window size. The weight is difficult to be determined in all types of center weighted masks. Yet all the pixels are replaced with a new value, causing blurring in the final result.
In Adaptive CWMFs the fact that the center pixel changes according to the labeling stage, this problem reduces. One of the proposed algorithms is able to detect noisy pixels iteratively and the output of this level is a consensus that is passed to the denoising level which is a CWMF. The process of labeling is a complex one though, but still fast. Let us assume the noisy image pixels as Xij and the original one Yij. Rij is the random noise with probability r that infected the Yij with probability 1-r. Let $\widehat{Y}\ ij$ be the enhanced image pixel.
During the first phase, ACWMF is applied to detect noisy pixels.
Imposing the w=2k to each window center gives us Y2kij .In which 2k is the weight and ◊ is the repetition operand so that median gets closer to it.

$$Y_{ij}^{2k} = median\{X_{i-u,j-v}, (2k)\ \lozenge\ X_{ij}\ \big|-h \le u,v\ \le h\}$$

The difference between this image and the destroyed one is dk defined below.

$$d_k = \left|Y_{ij}^{2k} - X_{ij}\right|\ \ for\ k = 0,1,\dots,L-1$$

To label the noisy pixels, there should be a threshold (Tk). The threshold varies from one point to another within a window. And in a pixel with difference greater than that threshold, the pixel is considered as a noisy one, then replaced with the median of weight 0 (Y_{ij}^{0}). If the threshold is not passed, the pixel is a signal candidate and does not change. The median of absolute deviations from the median is defined as below and is used to be employed in threshold calculations.

$$MAD = median\{\ |\ X_{i-u,j-v} - Y_{ij}^{0}\ |\ \ |-h \le u,v\ \le h\}$$

MAD is a vector of elements each of which corresponds to a pixel in the image. MAD is an appropriate estimation of threshold, but still needs to be scaled.

$$T_k = s\ .MAD +\ \delta_k\ \ ,0 \le k \le 3$$

Where $[\delta_0\ ,\delta_1\ ,\delta_2\ ,\delta_3] = [40,25,10,5]$, and $0 \le s \le 0.6$

1. Set r = 0. Initialize $X^{(r)}$ to be the observed image.
2. run ACWMF with the thresholds to the image $X^{(r)}$to get the noise candidate set $M^{(r)}$.
3. Let $N^{(r)}$ = $\bigcup_{i=0}^{r} Mi$
4. For all (i, j) ∉ N(r), take $\hat{Y}ij = X_{ij}^{(r)}$. For all pixels $N^{(r)}$, take $\hat{Y}ij$ be the minimizing point of the following function:

$$\sum_{(i,j)\in N^{(r)}} \left|\mathrm{Y_{ij}} - \mathrm{X_{ij}}\right| + \sum_{(\mathrm{m,n})\in \mathrm{V_{ij}}\cap \mathrm{N}} \left|\mathrm{Y_{ij}} - \mathrm{Y_{mn}}\right| + \sum_{(\mathrm{m,n})\in \mathrm{V_{ij}}\backslash \mathrm{N}} 2\left|\mathrm{Y_{ij}} - X_{mn}\right|$$

$(\mathrm{m,n}) \in \mathrm{V_{ij}}\backslash \mathrm{N}^{(\mathrm{r})}$ are the four neighbors of ij that are not noise candidates in rth iteration. ϕ is the edge preserving function that can have various definitions.

5. In the next iterations X is replaced with the enhanced version of itself.
6. If r <rmax, set r =r +1 and go back to step 2.

The minimization of the above mentioned function yields:

$$\hat{Y}_{ij} = median\{X_{ij}, X_{mn}, (m,n) \in \mathcal{V}_{ij}\backslash \mathcal{N}^{(r)}, X_{mn}, (m,n) \in \mathcal{V}_{ij}\backslash \mathcal{N}^{(r)}, Y_{mn}, (m,n) \in \mathcal{V}_{ij} \cap \mathcal{N}^{(r)}\}$$

It means the median of: the affected point, four enhanced version neighbors that are noisy with weight 2, and four affected version neighbors that are not noisy. Median is computed as an average of two middle points for the even numbers of elements.

The method first takes pixels as Red and Black ones alternately. Running the algorithm on odd pixels (R), then $\hat{Y}_{ij}$ is used to comute new $\hat{Y}_{ij}$ for the (B) pixels. So the Red pixels use the same equation above, but Black pixels obey this one:

$$\hat{Y}_{ij} = median\{X_{ij}, X_{mn}, (m,n) \in \mathcal{V}_{ij}\backslash \mathcal{N}^{(r)}, X_{mn}, (m,n) \in \mathcal{V}_{ij}\backslash \mathcal{N}^{(r)}, Y_{mn}, (m,n) \in \hat{Y}_{ij} \cap \mathcal{N}^{(r)}\}$$

ACWMF proposed in is claimed to perform much better than any other algorithm. It has other superiorities too.

Progressive Switching Median Filter

This method has three stages. Impulse detection, progressive filtering that is done iteratively, and boundary updating which is promised to be able to preserve edges. This method also avoids changing all pixels values and detects before replacement. The detection is based on the difference of the pixel's value and the median. This it used to compare and be replaced in next iteration. Edge detection procedure involves computing the difference between the pixel and the mean of two vertical neighbors. Also the difference between the pixel and the mean of two horizontal neighbors, and then comparing them to determine if it is a vertical edge or horizontal one.

Evaluation Methods

Mathematical analysis are proved to be a good tool to measure the quality of the enhanced image and the difference between the original one and the restored one. There are four widely used factors shown below. All are simply computed.

MSE/ MAE

Mean Square Error and Mean Absolute error are the method to evaluate the performance of a denoising algorithm by computing deviation of original and processed image. Effectiveness of an algorithm can be proven if this factor is minimized. It tells us how close to the original image we are. MSE is defined as follows. Where and F(x,y) is the Original image pixel and I(x,y) corresponds to the restored Image pixel.

$$\frac{1}{MN}\sum_{x=1}^{M}\sum_{y=1}^{N}\{F(x,y)-I(x,y)\}^2$$

PSNR

Peak Signal to Noise Ratio is the maximum pixel value of an image (Max power of a signal) over the power of the corrupting noise. It can be computed with both MSE and MAE.

$$\frac{2^{bit}-1=255}{RMSE=\sqrt{1/MN\sum_{x=1}^{M}\sum_{y=1}^{N}\{F(x,y)-I(x,y)\}^2}}$$

IEF

Image Enhancement Factor is the restoration performance, hence calculated with the use of the values in corrupted image (N). The higher this factor, the better visual quality.

$$\frac{\sum_{x=1}^{M}\sum_{y=1}^{N}(N(x,y)-F(x,y))^2}{\sum_{x=1}^{M}\sum_{y=1}^{N}(I(x,y)-F(x,y))^2}$$

CORR

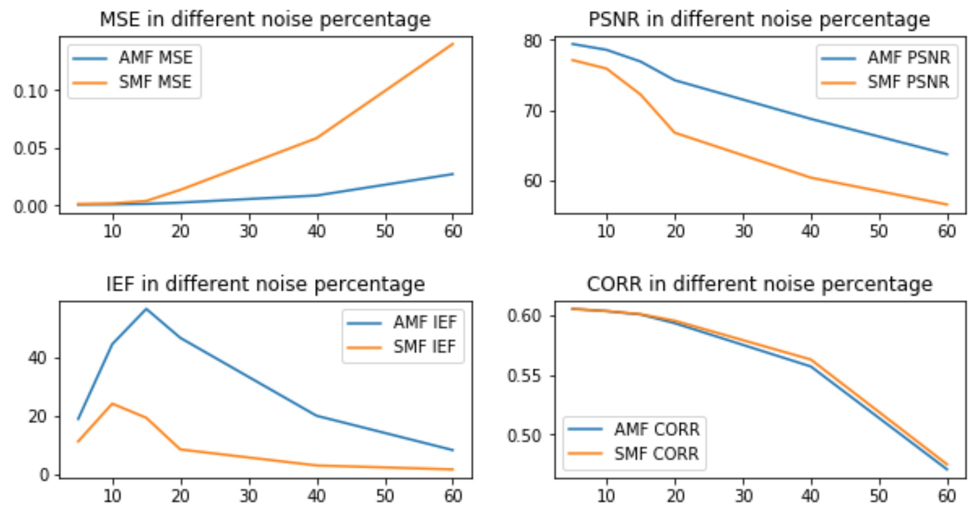
Correlation Ratio measures the degree in which two images vary together or taking similar values from 0.0 to 1 .0. The value of CORR which gets close to 1.0 reflects the better image visualization. μ_F and $\mu_I$describe the mean intensity value of the original image and the enhanced image respectively.

$$\frac{\sum_{x=1}^{M}\sum_{y=1}^{N}(F(x,y)-\mu_F)(I(x,y)-\mu_I)}{\sqrt{\sum_{x=1}^{M}\sum_{y=1}^{N}(F(x,y)-\mu_F)^2\ (I(x,y)-\mu_I)^2}}$$

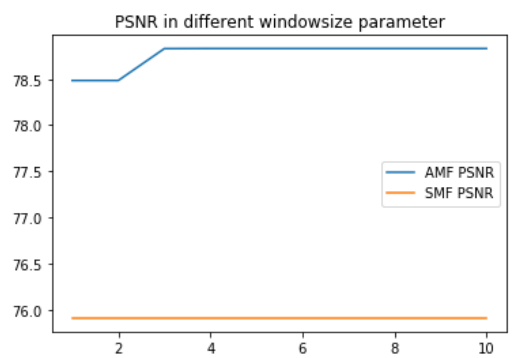
Evaluating Median-Based Filters and Results

In the main paper, Experimenting on a CT image of a liver, it is shown that the AMF is performing better than others significantly. All algorithms encounter problems removing noise from the high density noise images. And they do not preserve the edges, exhibiting blurring and error propagation. In high noise density, SMF's problem with small window size is its insufficient suppression. But in low noise density, SMF, CWMF, and PSMF filters provide a low performance when they are contrasted with AMF filter at various noise densities.

In my implementation results, it can be seen that as the higher noise percentage gets, the inefficiency of two compared algorithms converge. Hence the logical conclusion would be better algorithms should be employed to remove high density noise and still preserve edges.



AMF also requires setting an optimized parameter (Maximum window size) for the iterations. Determining this parameter is a challenge. And my results show that increasing window size, cannot affect the performance after a while. This is more noticeable in higher noise densities.



Another claim is that the AMF does not respond well to Random value noise.

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