Chain Rule Examples

Compute $\frac{d}{dx}\left[x^{1/4}\right] =$

$\frac{d}{dx}\left[x^{n}\right] = nx^{n-1}$
for $\underline{n \text{ an integer}}$

$(x^{1/4})^4 = x^1$

$\frac{d}{dx}\left[(x^{1/4})^4\right] = \frac{d}{dx}\left[x\right]$

outside$(u) = u^4$ inside $f(x) = x^{1/4}$

outside$'(u) = 4u^3$ inside$'(x) = ?\ \frac{d}{dx}\left[x^{1/4}\right]$

Outside$'(x^{1/4}) = 4x^{3/4}$

$4x^{3/4} \cdot \frac{d}{dx}\left[x^{1/4}\right] = 1$

$\frac{d}{dx}\left[x^{1/4}\right] = \frac{1}{4}x^{-3/4}$ check: $\frac{1}{4} - 1 = -\frac{3}{4}$

$x^r = \left(e^{\ln(x)}\right)^r = e^{r\cdot\ln(x)} \leftarrow$

$\frac{d}{dx}\left[x^r\right] = \frac{d}{dx}\left[e^{r\cdot\ln(x)}\right]$

outside$(u) = e^u$ inside$(x) = r\cdot\ln(x)$

outside$'(u) = e^u$ inside$'(x) = r\cdot\frac{1}{x}$

outside$'($inside$(x)) = e^{r\cdot\ln(x)}$

$= e^{r\cdot\ln(x)}\cdot r\cdot\frac{1}{x}$

$= \frac{x^r\cdot r}{x} = r\cdot x^{r-1}$

Power Rule: for $\underline{\text{any}}$ real number r, $\boxed{\frac{d}{dx}\left[x^r\right] = r\cdot x^{r-1}}$

$\frac{d}{dx}\left[2^x\right] = \frac{d}{dx}\left[\left(e^{\ln(2)}\right)^x\right] = \frac{d}{dx}\left[e^{x\cdot\ln(2)}\right]$

out$(u) = e^u$ inside$(x) = x\cdot\ln(2)$

out$'(u) = e^u$ inside$'(x) = \ln(2)\frac{d}{dx}[x] = \ln(2)$

out$'(x\ln(2)) = e^{x\cdot\ln(2)} = 2^x$

$= 2^x\cdot\ln(2)$

$\boxed{\frac{d}{dx}\left[a^x\right] = \ln(a)\cdot a^x}$