Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x)) \cdot g'(x)$$

$$\lim_{x \to a} \frac{f(g(x)) - f(g(a))}{x-a}$$

$$= \lim_{x \to a} \frac{f(g(x)) - f(g(a))}{g(x) - g(a)} \cdot \frac{g(x)-g(a)}{x-a}$$

$$= \lim_{x \to a} \frac{f(g(x)) - f(g(a))}{g(x)-g(a)} \cdot \underbrace{\lim_{x \to a} \frac{g(x) - g(a)}{x-a}}_{g'(a)}$$

$u = g(x)$, $\lim_{x \to a} u = \lim_{x \to a} g(x) = g(a)$ if g is cts ✓
since $g'(a)$ exists

$$= \left(\lim_{u \to g(a)} \frac{f(u) - f(g(a))}{u - g(a)}\right) \cdot g'(a)$$

$$= \left(\lim_{u \to b} \frac{f(u) - f(b)}{u-b}\right) \cdot g'(a) \qquad \text{Let } b = g(a)$$

$$= f'(b) \cdot g'(a) = \underbrace{f'(g(a)) \cdot g'(a)}$$

"Doutside · Dinside"

Liebniz $y = f(g(x))$ and $u = g(x)$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \qquad \text{"("cancel the du")"}$$

ex) 3.4 #4 $f(x) = (4-3x^2)^{400}$

inside$(x) = 4 - 3x^2$ outside$(u) = u^{400}$

Outside$'(u) = 400u^{399}$ inside$'(x) = 0 - 3 \cdot 2x$

Outside$'$(inside$(x)) = 400 \cdot (4-3x^2)^{399}$ $= -6x$

$$f'(x) = 400(4-3x^2)^{399} \cdot (-6x)$$