Uncertainty, Firm Investment, and Financial Heterogeneity

By Zhenning Zhao*

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This paper compares the performance of firms facing asset-based financial constraints (ABC) versus earning-based financial constraints (EBC) when faced with uncertainty shocks. The findings indicate that firms with EBC perform better than those with ABC when uncertainty levels rise, as they reduce their investment by a smaller margin. This conclusion remains valid even after accounting for the magnitude of the financial constraints. However, the results are reversed when firms face monetary policy shocks. A financial heterogeneity model incorporating both asset-based and earning-based financial accelerators offers an explanation for the divergent behaviors observed in these firms. In comparison to the asset-based financial accelerator, the earning-based financial accelerator incorporates an additional channel that penalizes defaulting firms through the restructuring process. This restructuring process acts as a counteracting force that restrains earning-based firms from decreasing their investment during times of increased uncertainty but amplifies the decline when interest rates rise.

Economic fluctuations are heavily influenced by uncertainty, which can stem from a range of factors including government policies, geopolitical tensions, and natural disasters. When uncertainty reaches excessive levels, it can have a detrimental impact on investment, consumer spending, and economic growth. The COVID-19 pandemic has led to unprecedented levels of uncertainty in the global economy, with both policymakers and markets closely monitoring the situation. The World Bank has identified year 2022 as a year of significant uncertainty[1], and the IMF has observed a surge in global uncertainty in response to the ongoing

* University of Texas at Austin, Economics Department.

[1]The link to the website: https://www.worldbank.org/en/news/podcast/2022/12/01/2022-review-year-uncertainty-forecasts-conflict-inflation-food-pandemic-development-podcast

conflict in Ukraine[2]. As a result, managing uncertainty has become a crucial concern for policymakers, who must balance the need for stability and predictability with the need to respond to rapidly evolving circumstances.

Economic uncertainty often leads to a deterioration of financial conditions, which can amplify the negative effects of uncertainty. In this context, it is important to understand the nuance differences of firm-level responses to uncertainty, as this can provide insight into the mechanisms through which uncertainty affects aggregate economic outcomes. By analyzing the varying responses of different firms to uncertainty, policymakers and researchers can gain a more comprehensive understanding of how uncertainty affects economic activity and develop more effective strategies to manage and mitigate its impact. This paper explores this issue, specifically examining the impact of uncertainty on firms' investment decisions in light of their financial type heterogeneity.

Historically, academic researchers have devoted significant attention to asset-based financial constraints, which permit creditors to claim the liquidation value of specific physical assets (i.e., collateral) in the event of default under Chapter 7 bankruptcy. The work of Kiyotaki and Moore (1997) represents a prominent example of research in this area. However, recent studies, including Lian and Ma (2021), suggest that only a small percentage (20%) of US non-financial corporate debt is collateral-based. Instead, most debts are subject to earnings-based financial constraints, which are governed by Chapter 11 bankruptcy and allow creditors to claim a portion of the future cash flow values of the restructured firm during a default. While the Kiyotaki and Moore borrowing capacity framework is frequently used to compare these two types of financial constraints, it may not be ideal for answering this question. In the case of earnings-based debt, firms generally do not encounter an explicit borrowing limit. The major components of these debts consist of cooperative loans that operate within the credit market, and the primary constraint is the borrowing cost. As a result, the financial accelerator framework, which emphasizes the impact of borrowing costs, developed by Bernanke, Gertler and Gilchrist (1998), may be more relevant for investigating issues linked to earnings-based financial constraints.

This paper provides compelling empirical evidence that firms with a higher earning-based debt ratio are less impacted by uncertainty shocks. Specifically,

[2]The link to the website: https://www.imf.org/en/Blogs/Articles/2022/04/15/global-economic-uncertainty-surging-amid-war-may-slow-growth

when idiosyncratic volatility at the firm-level increases, firms typically scale back their investments. However, those with a higher earning-based debt ratio display a smaller reduction in investment. To identify the uncertainty shocks, I employ the same method as Alfaro, Bloom and Lin (2019) and use the exposure to aggregate economy shocks as an instrument. Next, I utilize the Capital IQ dataset in combination with the Compustat and CRSP dataset to differentiate between earning-based and asset-based loans, following the methodology of Lian and Ma (2021). Finally, I run regressions on the panel data using interaction terms between the instrumented uncertainty shocks and the earning-based debt ratios to explore the different responses of firm-level investment. The results reveal that firms with earning-based financial constraints tend to decrease their investments less when uncertainty increases, as compared to firms with asset-based financial constraints. These results remain robust even after controlling for the size of the financial constraint. This study also reveals evidence suggesting that the cost of funding rises with increasing levels of uncertainty, particularly for firms with higher levels of asset-based debts. These findings offer some insight into the potential underlying mechanisms driving this observed phenomenon.

In addition to analyzing uncertainty shocks, I conducted a similar analysis using monetary policy shocks to further explore the implications of the findings in this paper. Surprisingly, the response to changes in interest rates exhibited a reversed pattern compared to the response to uncertainty shocks. Firms with high asset-based debt ratios demonstrated a larger response to interest rate changes after a period of 2-3 years, which is consistent with previous research (Caglio, Darst and Kalemli-Özcan (2021)). This intriguing and perplexing result, cannot be adequately explained by existing models within the borrowing capacity framework proposed by Kiyotaki and Moore (1997) since these models do not effectively incorporate uncertainties. This calls for the development of a new framework to address this gap in understanding.

To better understand the primary mechanism that underlies this observation, I develop an earning-based financial accelerator within a financial heterogeneity model. This model builds upon the financial accelerator model with risk shocks introduced by Christiano, Motto and Rostagno (2014), incorporating earning-based financial constraints as an additional factor. In contrast to traditional asset-based financial accelerator models, where entrepreneurs lose their entire net worth to the bank in case of default, the earning-based financial accelerator model penalizes defaulting entrepreneurs by modifying the firm's debt structure.

Under the asset based financial accelerator, the banks will punish the firms who default by increasing the credit spread. The increase in the credit spread will directly lower the amount of fundings that the entrepreneurs can use for investment and hence decrease the investment from the demand side. However, under the earning-based financial accelerator model, the banks can punish the defaulting entrepreneurs through an extra channel. The firms that default will experience a loss of control over a portion of the firm, which will be transferred to the banks. The banks will then have a claim on the future earnings of the firm. As direct ownership of equities by banks is less efficient, the banks gradually forgive the entrepreneurs over time, provided they do not default further. This restructuring process offers an additional method of punishment available to banks and can help elucidate the empirical observations.

In the earnings-based financial accelerator, banks have two methods to punish defaulting firms: increasing the credit spread and initiating a restructuring process. This creates a substitution effect between the two punishment, resulting in a lower magnitude of investment decrease caused by the increase of credit spread compared to the asset-based financial accelerator, where banks can only punish defaulting entrepreneurs with credit spread increase. Despite the banks incurring some efficiency losses when punishing entrepreneurs through the restructuring process, if the cost effect is outweighed by the substitution effect, the overall impact will still be smaller for the earnings-based sectors when faced with uncertainty shocks. I further integrate both the asset-based and earning-based financial accelerator models into a financial heterogeneity model to examine the general equilibrium effect. The model categorizes entrepreneurs into two groups based on the types of financial constraints they face. These fractions are predetermined and calibrated to reflect the proportion of financial constraints in the data.

Finally, The financial heterogeneity model offers an explanation for the different behaviors of firms in response to uncertainty shocks and monetary policy shocks. When uncertainty increases, the substitution effect between the two punishment channels weakens the decrease caused by the increase in credit spread. This weakening effect stems from the higher probability of default associated with increased uncertainties. Conversely, when interest rates rise, firms reduce their investment due to the higher external funding costs. This reduction also raises the probability of default, resulting in an additional amplification effect through the restructuring procedure, accompanied by efficiency losses. Through calibrations, it is observed that under monetary policy shocks, the efficiency loss outweighs the substitution effect. Consequently, this dominance can lead to opposing responses from the two types of firms.

In summary, this paper emphasizes the significant difference of firm-level responses to increases in uncertainty, particularly in relation to investment. This finding has critical policy implications, such as highlighting the greater resilience of earning-based financial constraints to uncertainty shocks. Consequently, policymakers should prioritize monitoring and supporting asset-based financially constrained firms during periods of heightened uncertainty to mitigate the potential for widespread financial crises. Furthermore, the paper highlights the differential effects of monetary policy shocks compared to uncertainty shocks. Specifically, when considering interest rate changes, policymakers should be attentive to the needs and vulnerabilities of asset-based firms. This insight emphasizes the importance of tailoring policy interventions based on the specific shock type to effectively address the distinct challenges faced by different types of firms.

The rest of the paper is structured as follows: the next section places this study in the existing literature. Section 2 outlines the primary dataset and methodology used in the empirical analysis. Section 3 presents the empirical evidence. Sections 4 and 5 provide a detailed explanation of the earning-based financial accelerator model and financial heterogeneity model, respectively. Finally, section 6 concludes.

I. Literature Review

Since the publication of Bloom (2009), there has been growing attention among economic researchers to the significance of uncertainty shocks as a driving factor of the business cycle. Various studies have demonstrated that uncertainties have an impact through the channel of inflation expectations in Istrefi and Piloiu (2014), effective demand in Basu and Bundick (2017), monetary policy decisions in Creal and Wu (2017), general equilibrium in Bloom et al. (2018), and the second moment news (Berger, Dew-Becker and Giglio (2020)). To measure aggregate-level uncertainties, researchers have employed different approaches, such as those used in Jurado, Ludvigson and Ng (2015), Ludvigson, Ma and Ng (2015), Baker, Bloom and Davis (2016), Carriero, Clark and Marcellino (2017), Carriero, Clark and Marcellino (2018), and Husted, Rogers and Sun (2019). Prior literature has addressed the endogeneity problem of aggregate uncertainties or focused on the identification of various types of uncertainties, such as those arising from productivity versus those related to policy. More recent studies utilizing micro-level data, such as Coibion et al. (2021) and Kumar, Gorodnichenko and Coibion (2022), have found robust causal evidence that uncertainty shocks have a direct effect on household and firm decisions.

Similarly, financial frictions is another hot topic in economic research. There are two primary approaches to modeling financial frictions in the literature. The first category builds upon the borrowing capacity framework developed in Hart and Moore (1994) and Kiyotaki and Moore (1997). Several subsequent works, such as Shleifer and Vishny (1992), Kocherlakota (2000), Cordoba and Ripoll (2004), Iacoviello (2005), Bianchi and Mendoza (2010), and Jermann and Quadrini (2012), have significantly advanced the model in the traditional DSGE models. Some more recent studies have combined this model with heterogenous firm models to explore the effects of firm size distributions. Examples include Khan and Thomas (2013), Guerrieri et al. (2015), Cao and Nie (2017), and Jensen et al. (2020). This type of model assumes that firm's borrowing capacity is limited by the amount of fixed capital they can provide as collateral, making it particularly suitable for examining asset-based financial constraints. As a result, most recent studies examining the impact of financial constraint types uses this borrowing capacity framework. For instance, all of the literature on financial type heterogeneity that I am aware of, including Greenwald (2018), Lian and Ma (2021), Drechsel (2022), Caglio, Darst and Kalemli-Özcan (2021) amd **?**, utilized this framework.

The financial accelerator framework, which was first developed by Bernanke, Gertler and Gilchrist (1998) and based on costly state verification methods from Townsend (1979) and Gertler and Bernanke (1989), represents the second approach to modeling financial frictions. One of the major advantages of this model is its ability to study the credit spread endogenously within the framework. Researchers such as Carlstrom and Fuerst (1997), Gilchrist and Zakrajšek (2012),Christiano, Motto and Rostagno (2014), and Carlstrom, Fuerst and Paustian (2015) have further developed the financial accelerator model in the DSGE setup to examine the amplification effect of various types of shocks. However, no paper has yet explored the earning-based financial constraint within the financial accelerator model, despite its better alignment with the nature of this type of constraint. This paper aims to fill in this gap in the literature.

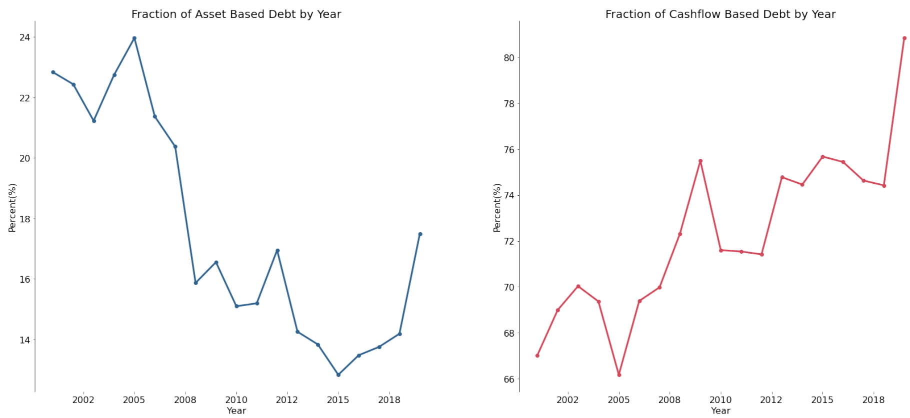
In the literature studying uncertainty shocks and financial constraints, some researchers investigate the interplay between the two. Arellano, Bai and Kehoe (2018), and Fernández-Villaverde and Guerrón-Quintana (2020) are notable examples. Of these, the studies most closely related to this paper are those by Christiano, Motto and Rostagno (2014) and Alfaro, Bloom and Lin (2019). The former presents a financial accelerator framework for examining risk shocks in a straightforward manner, which serves as the foundation for the model proposed in this paper. The latter proposes an empirical analysis framework and suggests the use of exposures to aggregate-level uncertainties as an instrument to address the endogeneity problem arising from the entanglement of first and second moment shocks.

II. Data and Methodology

The data used in this paper consists of two main parts: the classification of debt types and the data on idiosyncratic uncertainty shocks.

The primary dataset used in this paper to identify the debt types is the Capital IQ Capital Structure dataset, which offers a comprehensive view of the debt capital structures of over 60,000 public and private firms worldwide. The dataset includes various attributes of each debt component, such as security type, secured level, interest rate, maturity date, interest type, benchmark, convertible type, issued currency, benchmark spread, and more. The data covers the period from 2001 to 2019, and it provides in-depth information on a firm's debt capital structure based on its 10-K filings for publicly listed companies. The Capital IQ dataset was selected as the main source due to its comprehensive description of a firm's debts. In the previous study, Lian and Ma (2021), the Capital IQ dataset was combined with data from FISD, DealScan, and SDC Platinum for firm-level analysis. The identification procedure in this paper follows a similar approach, with details outlined in the appendix. Given that Lian and Ma (2021) provides a detailed description of asset-based and earnings-based debts, here we will only provide a brief overview of the main components of the two categories.

The asset-based debts primarily consist of commercial mortgages, asset-based loans, capitalized leases, and secured debt. Commercial mortgages are typically debt backed by real estate for corporate purposes. Asset-based loans are non-mortgage business loans that are secured by physical assets, such as inventory, receivables, machinery, equipment, and specialized assets like oil and gas reserves.



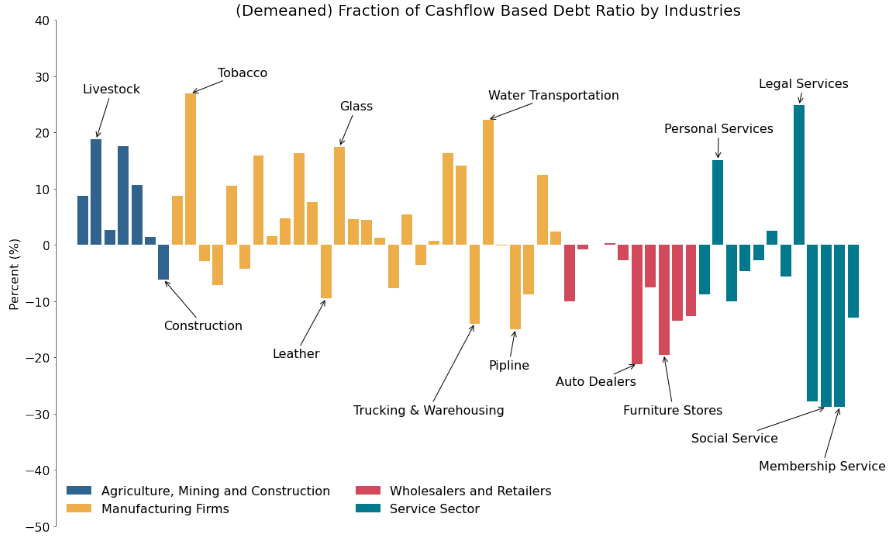
Note: This figure depicts the aggregate fraction of the asset based debt and earning based debt over the years from 2001 to 2019, as identified from the Capital IQ dataset. Notice that the sum of the two by year is not 1, due to the existence of other types of debt.

FIGURE 1. AGGREGATE DECOMPOSITIONS OF DEBTS

A capital lease is a debt where the leased asset appears on the balance sheet. Secured debt is a loan secured by collateral, such as property or assets, that the lender can seize if the borrower defaults on the loan. In summary, all of these debts use physical assets as collateral and can be classified as asset-based debts.

In contrast, earning-based debts primarily consist of corporate bonds and cashflow based loans. Unlike debts secured by physical collateral, corporate bonds are typically unsecured, and if they are secured, it is through a lien on the corporate entity. Similarly, cashflow based loans are backed by the borrower's current cashflow, with creditors paying close attention to the borrower's cashflow and often imposing earning-based covenants to enforce the earning-based constraints.

In addition to asset-based and earning-based debts, firms can also borrow from the government or from private individuals, which are categorized as "other debts." The asset-based, earning based, and other debts will be the three types of debts that will be examined in this study. For each firm in each year, we will calculate the earning-based debt ratio, defined as the amount of earning-based borrowings divided by the total debt a firm borrows, and the asset-based debt ratio in a similar manner. Figure 1 displays the fraction of each type of debt by year on an aggregate level. It demonstrates that earning-based debt is the dominant type of debt among US public firms, and there has been a trend of increasing earning-based debts over the past two decades, rising from around 65%



Note: This figure depicts the fraction of the earning based debt ratio by industries. The number depicted in the figure denotes the demeaned fraction, with highest and lowest earning based debt ratio labeled.

FIGURE 2. AGGREGATE DECOMPOSITIONS OF DEBTS

in the early 2000s to about 80% at the end of 2019. These results align with those found in Lian and Ma (2021).

Additionally, Figure 2 provides a different perspective on the distribution of earning-based debt ratios across industries. The figure reveals considerable variations not only between the first, second, and third industries but also within each major industrial category. Several industries exhibit a high reliance on asset-based debts, such as social services, auto dealers and furniture stores, pipeline and trucking services, and construction. Conversely, industries with significant earning-based debts include tobacco production, water transportation, legal services, and livestock. The figure illustrates a substantial disparity in earning-based debt levels across industries, with a maximum gap of approximately 60%.

Another crucial piece of information that is required is the firm-level uncertainty shocks. I follow the methodology outlined in Alfaro, Bloom and Lin (2019), utilizing both realized and implied uncertainty shocks to reinforce the empirical results. The realized uncertainty shocks are calculated as the changes in the standard error of the firm's realized stock returns for the previous year, and the implied uncertainty shocks are determined through the changes of the implied volatility derived from the firm-level option data. However, due to the high

Table 1—Data Description

	Count	Mean	Standard Error	Min	25 Percentile	Median	75 Percentile	Max
Investment Rate	34334	0.220	0.139	-0.238	0.112	0.186	0.305	0.500
Cashflow Based Debt Ratio	34334	0.678	0.392	0.000	0.339	0.908	0.998	1.000
Δ Cashflow Based Debt Ratio	28754	0.107	0.214	0.000	0.000	0.013	0.097	1.000
Realized Shock	34334	-0.031	0.332	-0.846	-0.259	-0.048	0.169	1.011
Implied Shock	22065	-0.029	0.197	-0.522	-0.156	-0.042	0.072	0.647
Realized Return	34334	0.151	0.617	-0.878	-0.200	0.071	0.349	3.818
Δ Employment	34334	0.028	0.210	-1.000	-0.044	0.019	0.098	1.000
Δ Intangible Assets	34334	0.045	0.375	-1.000	-0.044	0.000	0.078	1.000
Δ Payout	34334	0.033	0.317	-1.000	0.000	0.000	0.040	1.000
Δ Debt	34334	0.052	0.466	-1.000	-0.151	0.000	0.220	1.000
Δ COGS	34334	0.059	0.260	-1.000	-0.032	0.059	0.158	1.000
Δ Sales	34334	0.061	0.253	-1.000	-0.029	0.059	0.157	1.000
Δ Cash	34334	0.042	0.553	-1.000	-0.310	0.046	0.403	1.000
Δ Profit	34334	0.054	0.394	-1.000	-0.048	0.032	0.200	1.000
Tangibility	34334	0.595	0.460	0.002	0.240	0.476	0.860	3.807
Leverage	34334	0.561	0.257	0.039	0.390	0.545	0.693	2.335
Return of Asset	34334	0.040	0.195	-1.997	0.016	0.072	0.126	0.607
Firm Size	34334	0.850	2.001	-5.116	-0.562	0.951	2.230	5.733
Tobin Q	34334	1.584	0.786	0.434	1.061	1.355	1.857	6.100

Note: This table presents the descriptive statistics of the key variables in the Capital IQ-Compustat-CRSP linked dataset. The dataset encompasses 5,865 companies from 2001 to 2019. The definitions of the variables can be found in the appendix.

endogeneity of stock prices and option prices, an OLS regression faces endogeneity problems. To address this, Alfaro, Bloom and Lin (2019) proposed using the firm's exposure to 9 aggregate factors[3] as instruments. The underlying idea is that significant changes in aggregate factors, such as substantial increases or decreases, not only impact firms in terms of their first-moment effects but also have second-moment effects on firm uncertainties. By disentangling the first-order effects from the second-order effects, researchers can isolate the pure second-order effect as an exogenous instrument variable. With this in mind, I adopt the same instrument variables[4], obtaining the annual firm-level shock and instrument variable data until 2019 from the authors, and merge it with the Capital IQ dataset. To ensure the robustness of the findings, I performed additional regressions using quarterly data in this study. Since Alfaro, Bloom and Lin (2019) focused only on annual data, I reconstructed the firm-level instruments for realized uncertainty shocks using quarterly data. The construction process follows a similar methodology to

[3]The aggregate factors employed in this paper includes: the oil price, the growth in the exchange rates of seven major currencies (Euro, Canadian Dollar, Japanese Yen, British Pound, Swiss Franc, Australian Dollar, and Swedish Krona), as designated by the Federal Reserve, and the growth in economic policy uncertainty, as defined by Baker, Bloom and Davis (2016).

[4]The calculation of the instruments and shocks is not covered in this paper, but a brief introduction of the construction of the instruments is provided in the appendix.

the annual-level instrument, and further details can be found in the appendix.

Additionally, I supplemented the dataset with information from the Compustat and CRSP datasets to account for firm-level controls. For most of the variables, I trim the data at the 0.5 and 99.5 percentiles by year to reduce the impact of outliers. However, I made an exception for the variable of interest, the investment rate. The investment rate is defined as the ratio of investment expenditure to the average capital stock between the current year and the previous year, and it was trimmed between -0.5 and 0.5. This upper and lower limit ensures that the observed regression results are not solely driven by the well-known phenomenon of lumpy investment behaviors documented in the literature, for example Bachmann, Caballero and Engel (2013) and Winberry (2021).

The final dataset includes only records with positive values for total assets, total debts, sales, cost of goods, cash, employment, dividends, and fixed capital. Table 1 provides a summary of the data. The analysis incorporates data from 5,383 firms, covering the period from 2001 to 2019. Additional information on the definition of the independent variables can be found in the appendix.

Table 1 reveals that a significant proportion of firms alter their cashflow-based loan ratio over time. More than 25% of the firms adjust their debt structure by increasing or decreasing their cashflow-based loan ratio by more than 10%, while over 50% of them experience changes exceeding 1%. This is noteworthy since the regression analysis utilizes firm fixed effects, and a substantial number of changes in the cashflow-based debt ratio validate the validity of the analysis.

III. Firm Level Empirical Evidence

A. *Effect of Debt Heterogeneity*

What is the impact of a firm's debt type on its investment behavior during times of increased uncertainty? To answer this question, I start with the regression model that includes an interaction term between the uncertainty shocks and the fraction of earnings-based debt. The regression is formulated as follows:

$$\begin{aligned} InvRate_{i,t} &= \beta_0 + \beta_1 VolShock_{i,t-1} + \gamma X_{i,t-1} + \delta_i + \eta_t + \epsilon_{i,t} \\ &\quad + \beta_2 VolShock_{i,t-1} \times EBRatio_{i,t-2} + \beta_3 EBRatio_{i,t-2} \end{aligned} \tag{1}$$

TABLE 2—BASELINE REGRESSION: EFFECT OF THE FIRM DEBT CATEGORY

	(1) OLS	(2) IV	(3) OLS	(4) IV
Realized Shock # Cashflow Based Debt Ratio	0.007	0.028***		
	(0.004)	(0.007)		
Implied Shock # Cashflow Based Debt Ratio			0.010	0.067***
			(0.008)	(0.024)
Realized Shock	-0.016***	-0.069***		
	(0.006)	(0.017)		
Implied Shock			-0.039***	-0.155***
			(0.008)	(0.052)
Cashflow Based Debt Ratio	0.006**	0.006**	0.011***	0.011***
	(0.003)	(0.003)	(0.003)	(0.003)
R-Squared	0.178	0.162	0.221	0.203
Observation	26760	26760	18087	18087

Note: This table displays the results of the first regression. It presents the results of both the OLS and IV regressions with the realized and implied volatility shocks. The coefficients of the control variables have been omitted. Firm-fixed effects and time-fixed effects have been incorporated into each regression. Standard errors are shown in parasynthesis. ***: p<0.01, **: p<0.05, *: p<0.1

where $InvRate_{i,t}$ represents the investment rate for firm i in year t, $VolShock_{i,t-1}$ represents either the realized or implied volatility shocks, instrumented with the firm's exposure to aggregate factors, and $EBRatio_{i,t-2}$ represents the earnings-based debt ratio. $X_{i,t-1}$ includes control variables, separated into two categories: aggregate factor controls and firm-level controls. The aggregate factor controls include changes in the first moment, as uncertainty increases are typically accompanied by a recession of the first moment shocks. The firm-level controls include Tobin's Q, tangibility, leverage, return on assets, log of firm size, realized stock return, investment rate, employment growth, intangible asset growth, debt growth, cost of goods sold growth, sales growth, cash growth, profit growth, and dividend growth.

In this regression analysis, it is important to note that the independent variables and controls are lagged by one year. This lagging is implemented to account for the simultaneous effects of the firm's investment, changes in debt structure, and uncertainty shocks. Additionally, to mitigate the simultaneity issues arising from financial heterogeneity and other key variables, I followed the method outlined in Alfaro et al. (2019) and introduced an additional lag of one year for the cashflow-based loan ratio. Consequently, the subscript of $EBRatio$ in this regression is denoted as $t-2$, instead of $t-1$. In the regression, the firm fixed effects are represented by δ_i, and the yearly fixed effects are represented by η_t. The regression is run on panel data with standard errors clustered at the 2-digit SIC industrial level.

The regression results are displayed in table 2. The table shows that the main effect of uncertainty on a firm's investment is negative, aligning with the main conclusions of Alfaro, Bloom and Lin (2019)[5]. Indeed, when idiosyncratic uncertainty increases, firms tend to become more cautious about the future, resulting in postponed investment decisions. Additionally, heightened uncertainty may also raise the risk of default, leading to further reduction in investment.

A key insight from table 2 is the positive coefficient for the interaction term between the earnings-based debt ratio and the uncertainty shocks. This indicates that if a firm has a higher proportion of earnings-based debt, it will be more resilient to the negative effects of increased uncertainty on investment, as the decline in investment will be less severe. This result is statistically significant for both the regression with the realized and implied uncertainty shocks.

The initial regression results provide a general understanding of the impact of various types of financial constraints on a firm's investment response to uncertainty shocks. However, as demonstrated by Alfaro, Bloom and Lin (2019), financial frictions can interact with uncertainty shocks in a way that intensifies the effect of the shocks on the firm's response. This phenomenon is referred to as the "financial uncertainty accelerator", whereby a tighter financial constraint results in a stronger response of the firm to uncertainty shocks, particularly because larger and publicly traded firms are generally found to have looser financial constraints and also tend to rely more heavily on earning-based debt financing.

To alleviate these worries, I conducted the following regression:

$$\begin{aligned} InvRate_{i,t} = \beta_0 &+ \beta_1 VolShock_{i,t-1} + \gamma X_{i,t-1} + \delta_i + \eta_t + \epsilon_{i,t} \\ &+ \beta_2 VolShock_{i,t-1} \times EBRatio_{i,t-2} + \beta_3 EBRatio_{i,t-2} \\ &+ \beta_4 VolShock_{i,t-1} \times FC_{i,t-2} + \beta_5 FC_{i,t-2} \end{aligned} \tag{2}$$

where $FC_{i,t-2}$ represents proxies for the strength of the financial constraints. This

[5]It is not surprising that the same main effect is found, as the data used in the regression is the same as in Alfaro, Bloom and Lin (2019). The appendix demonstrates that replicating the baseline regression from Alfaro, Bloom and Lin (2019) with a different time period yields results that are very similar to those in the original study.

Table 3—Controlling the Financial Constraint Size: Implied Shocks

Panel A: Realized Uncertainty Shock

	(1) HP Index	(2) WW Index	(3) 1(Invest Grade)	(4) Firm Size	(5) Firm Leverage
Realized Shock # Cashflow Based Debt Ratio	0.014	0.019**	0.026***	0.016*	0.027***
	(0.009)	(0.007)	(0.008)	(0.009)	(0.008)
Realized Shock # Financial Constraint Size Measurment	-0.027**	-0.087*	0.011**	0.005	0.007
	(0.013)	(0.052)	(0.004)	(0.004)	(0.014)
Realized Shock	-0.152***	-0.084***	-0.068***	-0.063***	-0.073***
	(0.044)	(0.026)	(0.017)	(0.012)	(0.016)
Financial Constraint Size Measurment	0.071***	0.179***	-0.001	-0.014***	-0.033***
	(0.010)	(0.038)	(0.002)	(0.003)	(0.005)
Cashflow Based Debt Ratio	0.006**	0.006**	0.006**	0.006**	0.006**
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
R-Squared	0.165	0.166	0.164	0.164	0.162
Observation	26760	26760	26760	26760	26760

Panel B: Implied Uncertainty Shock

	(1) HP Index	(2) WW Index	(3) 1(Invest Grade)	(4) Firm Size	(5) Firm Leverage
Implied Shock # Cashflow Based Debt Ratio	0.050**	0.051**	0.064***	0.050**	0.064**
	(0.023)	(0.022)	(0.024)	(0.019)	(0.025)
Implied Shock # Financial Constraint Size Measurment	-0.070**	-0.275	0.007	0.011	0.031
	(0.027)	(0.188)	(0.008)	(0.009)	(0.020)
Implied Shock	-0.396***	-0.230**	-0.142***	-0.150***	-0.172***
	(0.132)	(0.102)	(0.052)	(0.043)	(0.047)
Financial Constraint Size Measurment	0.068***	0.201***	-0.001	-0.014***	-0.027**
	(0.021)	(0.042)	(0.002)	(0.004)	(0.012)
Cashflow Based Debt Ratio	0.011***	0.010***	0.011***	0.010***	0.011***
	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)
R-Squared	0.202	0.203	0.208	0.207	0.203
Observation	18087	18087	18087	18087	18087

Note: The table presents the regression results controlling for financial constraint size. Panel A shows the regression result with realized uncertain shocks and Panel B represents the regressions using implied volatility shocks. WW and HP index denote the two financial constraint size index developed in Whited and Wu (2006) and Hadlock and Pierce (2010). A higher value of the WW and HP index indicates a better financial situation for the firm. $1(InvestGrade)$ denotes a dummy variable that will take a value of 1 when the firm's long term bond have a credit rating higher than or equal to "A" grade. Firm size is defined as the log of the employee numbers. Firm leverage is defined as the ratio of the firm's total liability over the firm's total asset. Both firm-fixed effects and time-fixed effects have been included in each regression. The standard errors are displayed in parentheses. ***: p<0.01, **: p<0.05, *: p<0.1

regression specifically controlled for the interaction term between the uncertainty shocks and the measurement of the size of the financial constraint, hence remove the financial constraint size effect from the consideration.

In this study, several different proxies were selected to measure the size of the financial constraint. The first two options were drawn from past empirical regressions in the literature. The first is the index proposed in Whited and Wu (2006), denoted as WW index, and the second is the index proposed in Hadlock and Pierce (2010), denoted as HP index[6]. Both the index developed by Whited and Wu (2006) and the one by Hadlock and Pierce (2010) take into account not only a firm's size and leverage, but also other factors such as profitability, age, and industry. A higher value on either index suggests that a firm has less stringent financial constraints and is more likely to secure debt financing. In addition to these indices, we examined several other measures such as the credit ratings of the company's long-term debt, firm size, leverage ratio, and found that the results remain robust when using these alternative measurements.

The regression results are presented in table 3. The results show that uncertainty shocks are amplified by financial frictions, as demonstrated in the study by Alfaro, Bloom and Lin (2019). However, the addition of controls for the financial uncertainty accelerator does not alter the impact of financial constraint type heterogeneity for most of the measurements for the size of the financial constraint. Specifically, the coefficient for the interaction between the earnings-based debt ratio and uncertainty shocks remains significantly positive for both realized and implied uncertainty shocks. This result indicates that financial constraints mitigate the overall effect of uncertainty shocks on investment, particularly in the case of implied uncertainty shocks.

B. Robustness Check

In addition to the baseline regressions, I conducted several variations of the regression to ensure the robustness of the results. This section provides a brief overview of the main regression results performed in the paper[7].

The first regression to enhance the robustness of the analysis is to use dummy variables to categorize the firms based on their earnings-based debt ratio. The

[6]The definitions of WW index and HP index are shown in the appendix.

[7]Due to the limit of space only the first robustness regression result is shown in the text. The rest of the regression results can be found in the appendix.

Table 4—Dummy Variable Regression

	(1) OLS	(2) IV	(3) OLS	(4) IV
Realized Shock # 1(High Cashflow Based Debt Ratio)	0.006*	0.013***		
	(0.003)	(0.005)		
Implied Shock # 1(High Cashflow Based Debt Ratio)			0.002	0.021*
			(0.006)	(0.011)
Realized Shock	-0.014***	-0.058***		
	(0.004)	(0.015)		
Implied Shock			-0.032***	-0.115***
			(0.006)	(0.038)
1(High Cashflow Based Debt Ratio)	0.003**	0.003**	0.005***	0.005***
	(0.001)	(0.001)	(0.002)	(0.002)
R-Squared	0.178	0.163	0.220	0.204
Observation	26760	26760	18087	18087

Note: The table presents the result of the robustness check with the dummy variable indicating whether the earnings-based debt ratio was above the yearly median or not. The standard errors are displayed in parentheses. ***: p<0.01, **: p<0.05, *: p<0.1

result of this regression is shown in Table 4. The dummy variables were set to 1 if the earnings-based debt ratio was above the yearly median, and 0 if not, thereby controlling for any influence from abnormal firms. This also allowed for the separation of the quantitative results for firms with higher earnings-based debt ratios and those with lower earnings-based debt ratios. The regression results showed that a 10 percent increase in uncertainty led to a decrease in investment rate by approximately 0.58% to 1.15% for firms with a low earnings-based debt ratio and 0.13% to 0.21% less for firms with a high earnings-based debt ratio. The decrease in investment was approximately 18.26% to 22.41% less for firms with a high earnings-based debt ratio. This empirical result will be compared with the results from the model simulation.

Other than using dummy variables, I also performed other robustness tests. First, the regressions were tested using the asset-based debt ratio, which is nearly equal to 1 minus the earnings-based debt ratio, ignoring other forms of debt that only account for a small fraction of the total debt borrowed by firms. The results of this test showed that the coefficient of interest had an opposite sign compared to the baseline regression, as predicted. This exercise helped confirm that the results were not significantly impacted by other forms of debt that firms borrow. Secondly, there may be concerns regarding the firm fixed effects. While firms may experience changes in the proportion of their earnings-based debt ratios over time, the magnitude of this variation is minimal. A simpler and more robust approach would be to incorporate controls for the industrial-level fixed effects. This enables a comparison between firms with different debt structures and provides more reliable results. In fact, the regression results indicate that including the industrial-level fixed effects enhances the performance of the empirical analysis.

The last robustness test I conducted was to control for the interplay between the idiosyncratic uncertainty shock and aggregate factors. This is important because the fraction of the earnings-based debt ratio may increase during certain periods, such as when financial constraints become tighter, the dominance of earnings-based loans increases, or aggregate uncertainty levels rise. The goal of this test was to eliminate these concerns. I used the credit spread between Moody BAA and AAA bonds as a proxy for overall financial conditions, the aggregate earnings-based debt ratio to indicate the type of financial constraint, and the VIX Index to represent the level of aggregate uncertainty. The results showed that even after controlling for the interplay between the idiosyncratic uncertainty shock and these factors, the main conclusion remained unchanged.

C. Other Firm Level Variables

After reviewing the investment rate results, we now shift our focus to the impact of uncertainty shocks on the cost of funding. Since the dataset does not provide the credit spread, I approximated the cost of funding as the net paid interest over the total debt. While this is not the most accurate measurement compared to the credit spread, it allows us to conveniently derive the definition from the Compustat dataset.

The cost of funding is widely recognized as being closely linked to a firm's ability to repay its obligations. To eliminate the effect of this relationship, we conducted a regression analysis using regression 2 with the cost of funding as the dependent variable, controlling for industrial-level fixed effects. The results shown in table 5 indicate a positive level effect of uncertainty shocks on the cost of funding, while the interaction term between the firm's cashflow-based loan ratio and uncertainty shocks is negative.

This suggests that as the level of uncertainty increases, the cost of funding tends to rise. However, the increase is less pronounced for firms with a higher proportion of earnings-based loans. Although the level effect is not statistically significant, the interaction effect is. The lack of significance in the level effect may

Table 5—Evidence on the Cost of Funding

	(1)	(2)	(3)	(4)
	Realized Shock		Implied Shock	
	HP Index	WW Index	HP Index	WW Index
Uncertainty Shock # Cashflow Based Debt Ratio	-0.056*	-0.041	-0.166**	-0.135*
	(0.030)	(0.028)	(0.082)	(0.078)
Uncertainty Shock # Financial Constriant Size	-0.012	0.119	-0.024	0.560*
	(0.021)	(0.098)	(0.048)	(0.281)
Uncertainty Shock	-0.011	0.050	0.019	0.271**
	(0.071)	(0.037)	(0.172)	(0.128)
Financial Constriant Size	-0.016	-0.022	0.071	-0.065
	(0.033)	(0.098)	(0.058)	(0.091)
Cashflow Based Debt Ratio	-0.012	-0.011	-0.024**	-0.022*
	(0.009)	(0.009)	(0.011)	(0.011)
R-Squared	0.016	0.016	0.032	0.031
Observation	26760	26760	18087	18087

Note: The table presents the result regressing the finance cost using equation 2. For simplicity only WW index and HP index were used as the measurement for the size of the financial constraint. The standard errors are displayed in parentheses. ***: $p<0.01$, **: $p<0.05$, *: $p<0.1$

be attributed to two factors. First, the measurement of the funding cost only considers the interest paid annually, rather than the actual interest obligations of the firm. Second, the magnitude of variation in the cost of funding is relatively small, making it challenging to fully detect its effect compared to the impact on investment.

Nevertheless, the limited evidence regarding the cost of funding reveals a potential mechanism through which a firm's debt structure can influence its response to interest rate changes. It suggests that firms relying more on cashflow-based debt exhibit a smaller reaction in terms of their funding cost, as well as their investment.

In addition to investment rates and the funding cost, I also examined the impact of financial constraint heterogeneity on firm-level variables. For example, when uncertainty increases, firms tend to hold more cash, and this effect is even stronger for firms with earnings-based financial constraints. This suggests that in general, firms facing earnings-based financial constraints perform better than those facing asset-based constraints when faced with uncertainty shocks. I also investigated the effects of uncertainty shocks on the firm's profits, stock returns, sales, cost of goods, intangible capital growth, employment growth, the payoff to equity holders, and the amount of debt borrowed, but the response of these variables was not significantly impacted by the type of financial constraint.

D. Dynamic Effect and Further Discussions

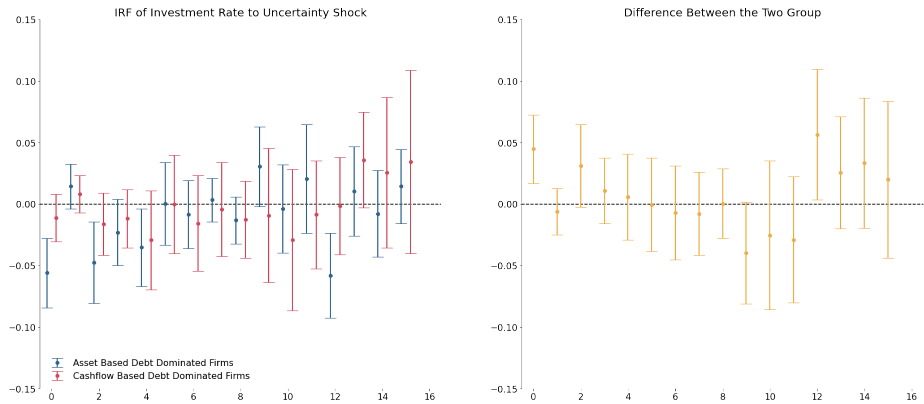
Next, we check the robustness of the key result by examining the dynamic effect. Since the uncertainty have a relatively small effect on the firm's investment, the persistance of the effect is also quite low. Most of the effect of the shock returns to zero within 1 year. This becomes a challange for detection since we are using annual data. To reconcile this problem, I repeat the key regression analysis with the quarterly data. I first created the quarterly realized volatility shocks following the simiar procedure as described in Alfaro, Bloom and Lin (2019). Then, using the quarterly data, I run the following regression:

$$\begin{aligned} (3) \qquad InvRate_{i,t+h} =& \beta_{0,h} + \beta_{1,h} 1(High\ EBDRatio)_{i,t-2} \times VolShock_{i,t-1} \\ &+ \beta_{2,h} 1(Low\ EBDRatio)_{i,t-2} \times VolShock_{i,t-1} \\ &+ \beta_{3,h} 1(High\ EBDRatio)_{i,t-2} + \gamma_h X_{i,t-1} + \epsilon_{i,t} \end{aligned}$$

Estimation 3 resembles the widely used local projection method (Jordà (2005)) in macroeconomics, which is employed to analyze the dynamic effects of exogenous shocks. In this regression, $1(High\ EBDRatio)_{i,t-2}$ and $1(Low\ EBDRatio)_{i,t-2}$ indicate whether a firm's earnings-based debt ratio is higher or lower than the yearly median, respectively. The parameter h represents the number of periods ahead. Essentially, this regression allows us to examine the separate responses of high cashflow-based loan firms and low cashflow-based loan firms.

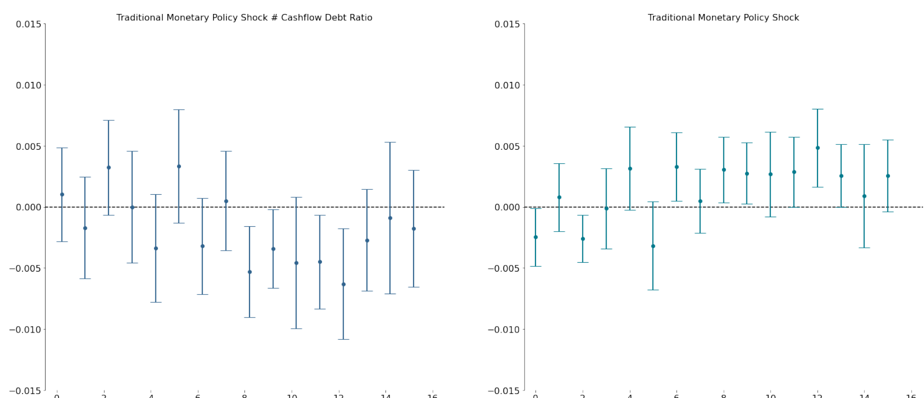
The estimated responses for these two groups of firms are illustrated in the left panel of Figure 3. Consistent with the findings from the baseline regression analysis, firms with lower earnings-based debt ratios exhibit stronger responses to uncertainty shocks. This effect persists for approximately three quarters, after which the difference between the two groups diminishes and becomes statistically insignificant. We can directly assess whether the difference between the two responses is statistically significant or not. The results are presented in the right panel of Figure 3. We can observe that although the difference between the two groups is significant in the first period of the regression, it rapidly diminishes and becomes statistically insignificant.

Lastly, the analysis can be extended beyond the scope of risk shocks. Empirical evidence from Caglio, Darst and Kalemli-Özcan (2021) suggests that firms with



Note: The left panel of the graph shows simultaneously the response of the two groups of the firm to risk shocks. The vertical bar around each estimation point denotes the 90% confidence interval. The right panel of the graph denotes the difference between the responses of the two groups. Again 90% confidence interval is plotted. Firm fixed effect, yearly fixed effect and quarterly fixed effects are included.

FIGURE 3. AGGREGATE DECOMPOSITIONS OF DEBTS



Note: The left panel of the graph shows the difference between the IRFs of the high cashflow based loan ratio firms and the low cashflow based loan ratio firms, while the right panel shows the baseline IRF of the firm's investment in response to a contractionary monetary policy shocks.

FIGURE 4. AGGREGATE DECOMPOSITIONS OF DEBTS

higher earnings-based debt ratios tend to be more responsive to monetary policy shocks. At first glance, this may appear to contradict our analysis of risk shocks. To test the consistency of our data, I re-estimate regression (1) using quarterly data, but with monetary policy shocks instead of risk shocks. The regression focuses on the investment rate in period $t+h$ to derive impulse response functions (IRFs) of firm investment to monetary policy shocks.

The monetary policy shocks used in this study are sourced from Swanson (2021), where the authors employ high-frequency market data to construct traditional and unconventional monetary policy shocks. For simplicity, this paper solely focuses on the traditional monetary policy shocks, as they yield the most effective results.

The IRFs for contractionary monetary policy are depicted in Figure 4. It is evident that when interest rates increase, firms initially tend to reduce their investment, with no discernible difference between high and low cashflow-based loan firms. However, after 2 to 3 years, firms with higher earnings-based debt ratios continue to experience an impact on their investment, while firms relying more on asset-based debts begin their recovery process in terms of investment. This result aligns with the findings of Caglio, Darst and Kalemli-Özcan (2021).

In conclusion, the results of the empirical analysis indicate that firms with a higher proportion of earnings-based debt tend to be less sensitive to changes in investment due to uncertainty shocks. The impact of the type of financial constraint is substantial, even after taking into account the size of the financial constraint. As uncertainty increases, the cost of funding rises less for firms with a higher proportion of earnings-based debt, indicating a potential mechanism behind the differential investment responses. It is important to note that these findings pertain specifically to risk shocks. Conversely, for monetary policy shocks, the relationship is reversed. Hence, a further question arises regarding how to reconcile these two empirical results within a unified theoretical framework.

IV. Asset-Based and Earning-Based Financial Constraint in Model

The results of the empirical study indicate that during periods of heightened uncertainty, firms with earnings-based constraints have better performance compared to those with asset-based financial constraints. However, what is the underlying reason for this outcome? To address this question, in this section, I will integrate the concept of earnings-based financial constraints into the financial accelerator model, based on the works of Bernanke, Gertler and Gilchrist (1998) and Christiano, Motto and Rostagno (2014). I will start by summarizing the traditional financial accelerator model and explaining why it can be considered as the framework for asset-based financial constraints. Next, I will incorporate the earnings-based financial accelerator by modifying the formal contracting problem to fit this framework. Finally, I will compare the impulse response functions of both models and demonstrate that the earnings-based constraint model predicts a lower response to risk shocks. A more comprehensive model that accounts for heterogeneous firms will be presented in the following section.

A. Asset-Based Financial Accelerator

We begin with the traditional financial accelerator framework, as illustrated in the upper panel of figure 5. In this model, the fixed capital is produced by a fixed capital producer and held by entrepreneurs. There is one representative entrepreneur who owns a continuum of projects. The entrepreneur combines the internal funds, represented by the net worth N_t, and external funds B_t, obtained through borrowing from the bank, to purchase the fixed capital on the market to funding the investment for these projects. The interest rate Z_t for the borrowed funds is negotiated with the bank. After acquiring K_{t+1} units of fixed capital at a price of Q_t in period t, the entrepreneur distributes the fixed capital evenly among the projects.

Each project experiences an idiosyncratic effectiveness shock ω, drawn from a log-normal distribution with a mean of 1 and standard deviation σ_t. We can imagine each project is led by a different manager, whose capabilities are different and random over time. As explained by Christiano, Motto and Rostagno (2014), the effectiveness shock represents the different levels of effectiveness that different entrepreneurs may exhibit in utilizing the same unit of fixed capital. For instance, a capable entrepreneur such as Elon Musk or Steve Jobs may produce higher returns from the same unit of fixed capital compared to less skilled entrepreneurs. The shock to the standard error σ_t of the effectiveness shock represents uncertainty shocks. Suppose the aggregate capital return is represented by R^k_{t+1}, then a project with an effectiveness shock of ω will have an idiosyncratic capital return of ωR^k_{t+1}.

We assume that each project manager will make their own decisions of whether to default or repay the borrowed funds, basing on their realized ω. If the project manager decides to repay, he will return B_tZ_t to the bank and keep the re-

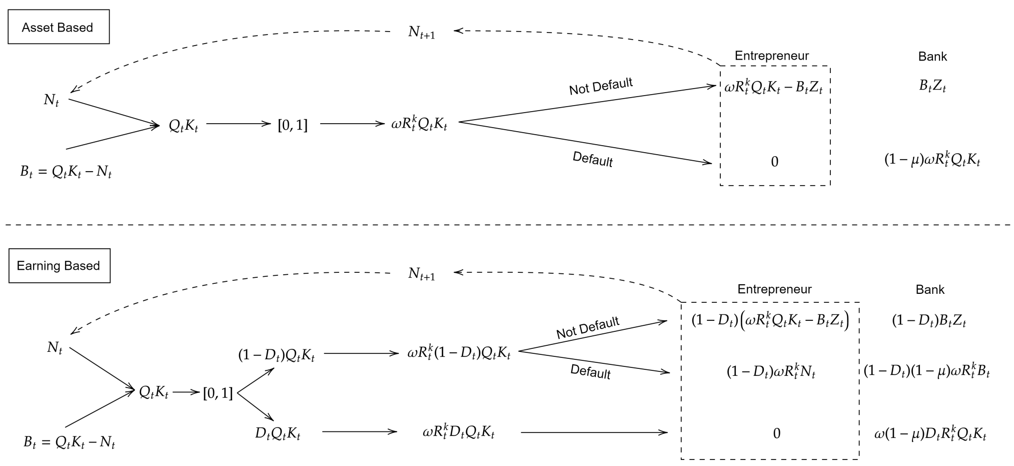


FIGURE 5. ASSET-BASED AND EARNING BASED FINANCIAL ACCELERATOR

maining returns $\omega R_{t+1}^k Q_t K_{t+1} - B_t Z_t$. If the entrepreneur defaults, they declare bankruptcy and receive nothing, while the bank can claim $(1-\mu)\omega Q_t K_{t+1}$, after accounting for monitoring costs μ. Bankrupted managers will die and new born manager will replace them such that there are always a continuum of projects in each period. Since ω is independent for each entrepreneur, their decisions are also independent. At the end of each period, the entrepreneur collects the funds from each project manager and uses them as the net worth for the next period. Since wealth is redistributed among the managers in each period independently, this is a static problem and the formal contracting problem can be solved easily.

We can observe that when the manager's effectiveness is strong enough, they will be able to generate sufficient returns to pay back the debt to the bank. However, if the manager is not so effective, the repayment of the debt will be a negative revenue, then the entrepreneur will opt for default. There is a threshold value $\bar{\omega}_{t+1}$, such that when $\omega_{t+1} > \bar{\omega}_{t+1}$, the entrepreneur will not default, but if $\omega_{t+1} \leq \bar{\omega}_{t+1}$, the entrepreneur will default. The threshold $\bar{\omega}_t$ satisfies the following equation:

$$Z_t(Q_t K_{t+1} - N_t) = \bar{\omega}_{t+1} R_{t+1}^k Q_t K_{t+1}$$

The left-hand side of the equation represents the cost of external funds, while the right-hand side represents the returns on fixed capital. The expected payoff for entrepreneurs can be expressed using the relationship between $\bar{\omega}_t$ and Z_t as follows:

$$\begin{aligned}&\int_{\bar{\omega}_{t+1}}^{+\infty}[\omega_{t+1}R^k_{t+1}Q_tK_{t+1} - Z_t(Q_tK_{t+1} - N_t)]d\Phi(\omega_{t+1})\\ =&\int_{\bar{\omega}_{t+1}}^{+\infty}(\omega_{t+1} - \bar{\omega}_{t+1})d\Phi(\omega_{t+1})R^k_{t+1}Q_tK_{t+1}\end{aligned}$$

Similarly, the expected payoff for the banks can be expressed as:

$$\begin{aligned}&\int_0^{\bar{\omega}_{t+1}}\omega_{t+1}(1-\mu)R^k_{t+1}Q_tK_{t+1}d\Phi(\omega_{t+1}) + \int_{\bar{\omega}_{t+1}}^{+\infty}Z_{t+1}(Q_tK_{t+1} - N_t)d\Phi(\omega_{t+1})\\ =&[\int_0^{\bar{\omega}_{t+1}}\omega_{t+1}(1-\mu)d\Phi(\omega_{t+1}) + \int_{\bar{\omega}_{t+1}}^{+\infty}\bar{\omega}_{t+1}d\Phi(\omega_{t+1})]R^k_{t+1}Q_tK_{t+1}\end{aligned}$$

where the second line is a result of the relationship between $\bar{\omega}_{t+1}$ and Z_{t+1}. We can define $f(\bar{\omega}_{t+1}) = \int_{\bar{\omega}_{t+1}}^{+\infty}(\omega_{t+1} - \bar{\omega}_{t+1})d\Phi(\omega_{t+1})$ and $g(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}}\omega_{t+1}(1-\mu)d\Phi(\omega_{t+1}) + \int_{\bar{\omega}_{t+1}}^{+\infty}\bar{\omega}_{t+1}d\Phi(\omega_{t+1})$, where $\Phi(\omega_{t+1})$ is the cumulative probability function of a log-normal distribution. Under the log-normal distribution, both functions $f(.)$ and $g(.)$ have closed-form solutions[8].

The managers aim to maximize their expected returns, which are defined as the ratio of payoffs to net worth. Let $L_t = Q_tK_{t+1}/N_t$ represent the leverage ratio and R_t denote the risk-free interest rate. The formal contracting problem that the managers solve can then be expressed as follows:

$$\begin{aligned}&max_{\bar{\omega}_{t+1},L_t}E_t[f(\bar{\omega}_{t+1},\sigma_t)R^k_{t+1}L_t]\\ &\quad s.t.\ E_t[\frac{g(\bar{\omega}_{t+1},\sigma_t)R^k_{t+1}L_t}{L_t-1}] = R_t\end{aligned}$$

The constraint ensures that the expected return received by the bank is equal to the risk-free interest rate, as free entry conditions dictate that the expected

[8]Under the assumption of log-normal distribution, we have that

$$\begin{aligned}f(\omega,\sigma) &= 1 - \Psi(\frac{ln\omega}{\sigma} - \frac{1}{2}\sigma) - \omega(1 - \Psi(\frac{ln\omega}{\sigma} + \frac{1}{2}\sigma))\\ g(\omega,\sigma) &= (1-\mu)\Psi(\frac{ln\omega}{\sigma} - \frac{1}{2}\sigma) + \omega(1 - \Psi(\frac{ln\omega}{\sigma} + \frac{1}{2}\sigma))\end{aligned}$$

return must be sufficient to pay the households. The managers take this into account when solving for the optimal contracts that maximize their own returns. The solution to the problem results in the first-order conditions, which can be combined into a single equation through log-linearization:

$$r^k_{t+1} - r_t = \nu l_t + \chi \hat{\sigma}_t \tag{4}$$

The equation presented captures the key features of financial accelerator models. Bernanke, Gertler and Gilchrist (1998) found that ν is positive, indicating that an increase in a firm's leverage leads to an increase in credit spread. Higher borrowing costs reduce entrepreneurs' demand for capital and hence, investment. An increase in uncertainty, as demonstrated by Christiano, Motto and Rostagno (2014), increases the likelihood of default, which in turn leads to lower repayments for banks. This prompts banks to increase the credit spread ($\chi > 0$), causing a rise in the financial cost of the firm and a decrease in investment.

Lastly, the entrepreneur determines the equation governing the dynamics of net worth N_t as follows:

$$N_t = \gamma V_t + W^e_t$$
$$V_t = R^k_t Q_{t-1} K_t - R_{t-1}(Q_{t-1}K_t - N_{t-1}) - \mu \int_0^{\bar{\omega}_t} \omega_t R^k_t Q_{t-1} K_t d\Phi(\omega_t)$$

In the given equations, the parameter γ represents the retention rate of the entrepreneurs' net worth. The entrepreneurs will transfer a fraction of $1-\gamma$ from the current enterprise value V_t back to the household. The value V_t corresponds to the total profits generated by all child entrepreneurs, subtracting the profits transferred to the bank and the losses incurred due to additional monitoring costs in case of firm defaults. The variable W^e_t denotes the wage paid to the entrepreneurs for their labor input, specifically for their management services. Considering that W^t_e, μ, and the probability of default are all small numbers close to zero, the equation governing the dynamics of N_t can be expressed as follows:

$$N_t = \gamma \left[R^k_t Q_{t-1} K_t - R_{t-1}(Q_{t-1}K_t - N_{t-1}) \right] \tag{5}$$

While the traditional financial accelerator model has straightforward logic, it only applies to firms that are financially constrained based on their assets. In this model, if managers choose to default, they are forced to declare bankruptcy and earn no income. The entire value of the firm is transferred to the bank after liquidation. The bank has no memory of this event and does not punish the entrepreneurs in the future. These features align closely with asset-based financial constraints, where managers use the entire value of a project in the current period as collateral. To compare the earning-based financial accelerator with the asset-based financial accelerator, it is crucial to develop a model that better fits earning-based financial constraints in reality.

B. Earnings-Based Financial Accelerator

Earnings-based debts have the following features: Firstly, in case of default, the company typically does not liquidate its assets. Secondly, the company undergoes restructuring, with debt holders having claims on a portion of its future earnings instead of the collaterals. Lastly, this type of debt is mostly obtained from credit markets, where financing costs, such as credit spreads, are used to control borrowing behavior, instead of limiting it through maximum borrowing capacity. The earnings-based financial accelerator framework, depicted in the lower panel of figure 5, effectively captures these characteristics.

In the earnings-based financial accelerator framework, the entrepreneurs has access to the same funding sources as in a traditional financial accelerator. The negotiated interest rate is again represented by Z_t. The entrepreneur still divides the fixed capital evenly among project managers each period, maintaining the static nature of the formal contracting problem.

Project managers, who draw idiosyncratic shocks ω from the same log-normal distribution, however, are unable to obtain all the earnings from holding K_{t+1} amount of fixed capital on average as in the previous case. Suppose the project manager hasn't defaulted in the past, then the entrepreneur has claims on the returns of the project led by this project manager if this manager didn't default today neither. This means this project manager must pay the interest $B_t Z_t$ to the bank. However, if the manager experiences a shock such that the return from fixed capital is insufficient, he will choose to default.

In this scenario, instead of being forced to liquidate all assets as in the asset-based financial constraint and being banned from future operations, the company undergoes a restructuring process. The bank will now claim the ownership of the defaulting project, verifying the realized returns from this project in each period starting from now. The verification cost is still μ. Then, starting from the next period, the bank will obtain the cash flow directly from this project without the involvement of the entrepreneur.

Since the entrepreneur contributed in the current period, the entrepreneur will be able to get a fraction of returns back from this project in the current period, and the fraction of returns that the entrepreneur can get is proportional to the fraction of net worth to the total capital, which is exactly the leverage. However, starting from the next period, the bank will keep all of the returns as a punishment to the entrepreneur and the defaulting manager.

This mechanism captures the essence embedded in the covenant of earning-based debt. Since each entrepreneur owns a continuum of projects, we can treat the discussed scenario as earning-based debenture. Since each project draws the effectiveness shock independently from the same distribution in each period, the specific project owned by the bank doesn't actually matter. Instead, what matters is the fraction of projects owned by the bank, denoted as D_t. By penalizing firms for past defaults with a higher D_t, the bank imposes a burden on entrepreneurs, essentially requiring them to work for free. Consequently, the bank becomes a free-rider, benefiting from a significant portion of the penalty earnings.

However, if the banks were to punish defaulting projects indefinitely, eventually all projects would be punished. This would render the financial market non-existent since every entrepreneur would lose everything to the bank. To avoid such an unrealistic situation, we introduce a forgiving mechanism for the bank. At the end of each period, the bank forgives a fraction of projects, allowing them to return to the ownership of the entrepreneur. Defaulting projects will be owned by the bank for at least one period as a punishment, but then there is a chance for them to be returned to the entrepreneur. We assume that the forgiveness rate is ψ. This parameter controls the size of the earning-based financial constraint.[9] The entire mechanism is captured by the following transitional equation for D_t:

$$D_{t+1} = (1 - \psi)D_t + (1 - D_t)\Phi(\bar{\omega}_{t+1}, \sigma_t)$$

where ψ is the forgiveness rate and $\Phi(\bar{\omega}_{t+1}, \sigma_t)$ represents the current probability

[9]The existence of the ψ parameter ensures that not all funds are punished and serves a similar role as the parameter N in Zhao (2022) N-period enforcement model.

of default. This procedure mimics the restructuring process that firms go through when they default on their earning-based debt. If there is a prolonged period without any defaults, the bank will not punish entrepreneurs, and D_t will be 0. However, if a large proportion of entrepreneurs decide to default, the bank will respond by obtaining ownership of the defaulting projects in the following periods. This ad-hoc law of motion for D_t is analogous to the situation in Chapter 11, where the restructuring process determines the value of a firm and the payouts of cash flow-based debt. By taking the log-linearization, we obtain:

$$(6) \quad d_{t+1} = [1 - \psi - \Phi(\omega)]d_t + \frac{(1-D)\Phi_\omega(\bar{\omega}, \sigma)\bar{\omega}}{D}\hat{\omega}_{t+1} + \frac{(1-D)\Phi_\sigma(\bar{\omega}, \sigma)\sigma}{D}\hat{\sigma}_t$$

Given the fraction of earnings hold by the bank, D_t, we observe that the default decision only depends on the returns from projects still owned by the entrepreneur. The cutoff value of ω, represented by $\bar{\omega}$, now satisfies the following equation:

$$\bar{\omega}_{t+1}R^k_{t+1}(1 - D_t)N_t = \bar{\omega}_{t+1}R^k_{t+1}(1 - D_t)Q_tK_{t+1} - Z_{t+1}(1 - D_t)(Q_tK_{t+1} - N_t)$$

The left-hand side of the equation represents the return in the event of default, while the right-hand side represents the return when the project manager chooses not to default. It is worth noting that in the case of default, the entrepreneur receives returns as if leverage had not been employed. The equation can be re-written in a form that is comparable to a similar equation in the traditional financial accelerator model, as $Z_{t+1} = \bar{\omega}_{t+1}R^k_{t+1}$. Next, we analyze the project manager's expected payoffs, expressed as:

$$(1 - D_t)R^k_{t+1}[\int_0^{\bar{\omega}_{t+1}} N_t\omega_{t+1}d\Phi(\omega_{t+1}) + \int_{\bar{\omega}_{t+1}}^{+\infty}(\omega_{t+1}Q_tK_{t+1} - Z_{t+1}B_t)d\Phi(\omega_{t+1})]$$

The first part of this expression represents the return in the event of default, while the second part represents the return of not defaulting. By using the relationship between $\bar{\omega}_{t+1}$ and Z_t that was established previously, we can eliminate the variable Z_t, similarly to the traditional financial accelerator model. By dividing the expression by the net worth, the expected return for the project managers can be expressed as follows:

$$\int_0^{\bar{\omega}_{t+1}} \omega_{t+1} d\Phi(\omega_{t+1}) + \int_{\bar{\omega}_{t+1}}^{\infty} \bar{\omega}_{t+1} d\Phi(\omega_{t+1}) + \int_{\bar{\omega}_{t+1}}^{+\infty} (\omega_{t+1} - \bar{\omega}_{t+1}) d\Phi(\omega_{t+1}) L_t$$

Notice that the last term features the function $f(.)$. To simplify the expression of the return[10], we can define $h(\bar{\omega}_{t+1}, \sigma_t) = \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} d\Phi(\omega_{t+1})$.

Similarly the expected return for the bank can then be represented as follows:

$$(1-\mu) D_t R^k_{t+1} Q_t K_{t+1} + (1-D_t) B_t \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} d\Phi(\omega_{t+1})(1-\mu) R^k_{t+1}$$
$$+ (1-D_t) B_t \int_{\bar{\omega}_{t+1}}^{+\infty} Z_t d\Phi(\omega_{t+1})$$

The first term in the above equation represents the payoff from directly holding claims to the firm's earnings, the second term indicates the payoff from the projects in the case of default, and the third term represents the payoff from the projects that didn't default. By eliminating the variable Z_t and calculating the returns by dividing B_t, we obtain:

$$(1-D_t)[\int_0^{\bar{\omega}_{t+1}} \omega_{t+1} d\Phi(\omega_{t+1})(1-\mu) + \int_{\bar{\omega}_{t+1}}^{+\infty} \bar{\omega}_{t+1} d\Phi(\omega_{t+1})] R^k_{t+1}$$
$$+ \frac{(1-\mu) R^k_{t+1} D_t L_t}{L_t - 1}$$

Here, the function $g(.)$ appears again. By combining the returns for entrepreneurs and banks, we can formulate the formal contracting problem for entrepreneurs in the earnings-based financial accelerator model:

$$max_{\bar{\omega}_{t+1}, L_t} (1-D_t) R^k_{t+1} h(\bar{\omega}_{t+1}, \sigma) + (1-D_t) R^k_{t+1} f(\bar{\omega}_{t+1}, \sigma) L_t$$
$$s.t.\ (1-\mu) R^k_{t+1} D_t \frac{L_t}{L_t - 1} + (1-D_t) g(\bar{\omega}_{t+1}, \sigma) R^k_{t+1} = R_t$$

[10]Notice that when the monitoring cost is sufficiently small, the function $h(.)$ will be very similar to the function $g(.)$. Also notice that $E[R^k_{t+1}]$ and $1 - D_t$ are dropped off from this equation since it is independent from the entrepreneurs's optimization problem. Under log-normal distribution, we have

$$h(\omega, \sigma) = \Psi(\frac{ln\omega}{\sigma} - \frac{1}{2}\sigma) + \omega(1 - \Psi(\frac{ln\omega}{\sigma} + \frac{1}{2}\sigma))$$

In comparison to the traditional financial accelerator model, both the objective function and the constraint are different, leading to completely different First-Order Conditions (FOCs)[11]. By combining and log-linearizing the FOCs, a distinct relationship is established between the credit spread, leverage, and risk:

$$r^k_{t+1} - r_t = \tilde{\nu} l_t + \tilde{\chi}\hat{\sigma}_t + \tilde{\varphi} d_t \tag{7}$$

From the FOCs, we can also determine the expression for the cutoff as:

$$\hat{\omega}_{t+1} = o(l_t, \sigma_t, d_t, r^k_{t+1} - r_t) \tag{8}$$

The derivation of the above equation is tedious and hence will be shown in the appendix. We should notice that in comparison to the traditional financial accelerator, not only are $\tilde{\nu}$ and $\tilde{\chi}$ different from ν and χ, but also an additional term, d_t, which represents the log-linearized fraction of earnings hold by the bank, appears in the equation.

Lastly, the dynamics of net worth will also be changed. Notice that the value of the firm is now represented by:

$$\begin{aligned} V_t =& (1-\mu D_t) R^k_t Q_{t-1} K_t - R_{t-1}(Q_{t-1}K_t - N_{t-1}) \\ & -\mu \int_0^{\bar{\omega}_t} \omega_t \phi(\omega_t) R^k_t Q_{t-1} K_t d\omega_t \end{aligned} \tag{9}$$

Comparing this to the asset-based financial accelerator, the key difference is that now the entrepreneur will lose a portion of earnings due to efficiency loss resulting from the costly verification procedure that the bank must undergo for the portion of projects directly owned by the bank. While the probability of default is a small number close to zero, the fraction of earnings held by the bank is not. This implies that we can omit the last term from the equation, but not the efficiency loss arising from the earning-based financial constraint. We will see later that this efficiency loss plays an equally important role as the restructuring procedure in explaining the different responses to various shocks.

[11]By taking the FOCs with respect to $\bar{\omega}_{t+1}$ and L_t, two FOCs are obtained, while the third FOC serves as a constraint. To solve the model, I used the three FOCs directly rather than combining them into one, as is done in the traditional financial accelerator model.

C. General Equilibrium

The previous two sections have provided a description of the model setup within the financial sector. In order to evaluate and compare the impact of the two financial accelerators, we incorporate them into a general equilibrium model framework. The model operates similarly to traditional financial accelerator models, as explained below and depicted in figure 6.

The model consists of three main blocks. The first block represents the standard household, whose utility function is given by $U = \sum_t \beta^t E_t[\log(C_t) + \xi \log(1 - H_t)]$, where C_t represents consumption and H_t represents labor. The household has access to a risk-free bond provided by the central bank and a savings account at a bank. By the law of one price, the banks must offer the risk-free interest rate to households. The household earns income through wages received for providing labor to wholesale goods producers and through savings in the bank and government bond. Finally, the household consumes the final goods produced by final goods producers.

The second block represents the standard New Keynesian producer. This block follows the traditional New Keynesian model, where wholesale goods producers rent capital and labor to produce intermediate goods. The producers face the typical Calvo-type price stickiness. The intermediate goods they produce are then transformed into final goods by the final goods producers using a constant elasticity of substitution technology. The final goods are sold to households for consumption or to fixed capital producers to produce fixed capital.

The third and final block represents the fixed capital producers. The economy assumes the presence of fixed capital producers who operate in a perfectly competitive market with free entry. They borrow existing capital, K_t, from entrepreneurs and purchase final goods at a price of 1. They use the technology $\Phi(\frac{I_t}{K_t})K_t$ to produce the investment goods. After production, they return all the borrowed capital goods to the entrepreneurs without cost and sell the newly produced investment goods at the market price, Q_t.

Every period, the entrepreneurs will allocate a fraction of their net worth, specifically $1 - \gamma$, for personal consumption. Additionally, they will work as managers for the wholesale firms, providing labor. Therefore, their net wealth in the next period will be a combination of their remaining net worth from the previous period and the income they earned from providing labor as managers

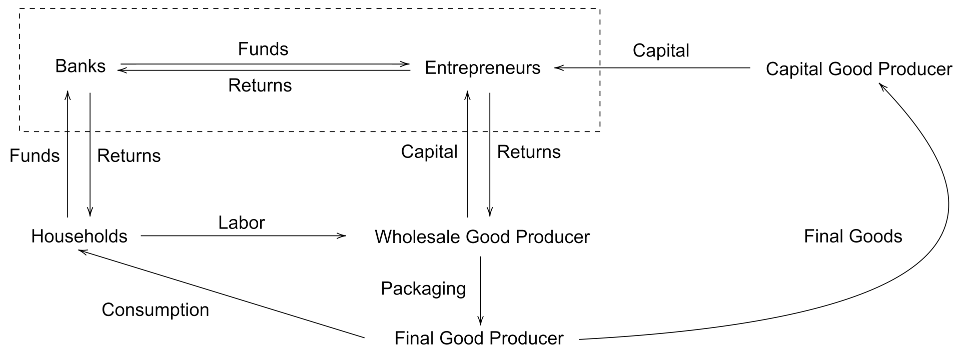


FIGURE 6. GENERAL EQUILIBRIUM MODEL

for the wholesale good producers.

The model reaches an equilibrium when market clearing conditions are satisfied in the final goods market, labor market, and fixed capital market. To complete the model, we assume that the central bank sets monetary policy based on a Taylor rule with interest rate persistence, as described by the following equation:

$$r_t = \rho r_{t-1} + \zeta \pi_{t-1} + e_t^r \tag{10}$$

To solve the two general equilibria separately, we need to combine all the partial equilibrium optimal conditions and the market clearing conditions. The complete system of equations can be found in the appendix.

D. Model Comparison

To demonstrate the impact of different types of financial accelerators, I compare the earning-based financial accelerator model to the asset-based financial accelerator model separately. This means including only one type of financial constraint in the model each time.

In the empirical analysis, there is evidence suggesting that the magnitude of financial constraints has a significant impact on how firms react to shocks of uncertainty. To disentangle the specific effects of different types of financial constraints, it is necessary to mitigate the influence of constraint size. My approach to address this issue involves two parts.

Firstly, I calibrate the asset-based financial accelerator model. Since this is

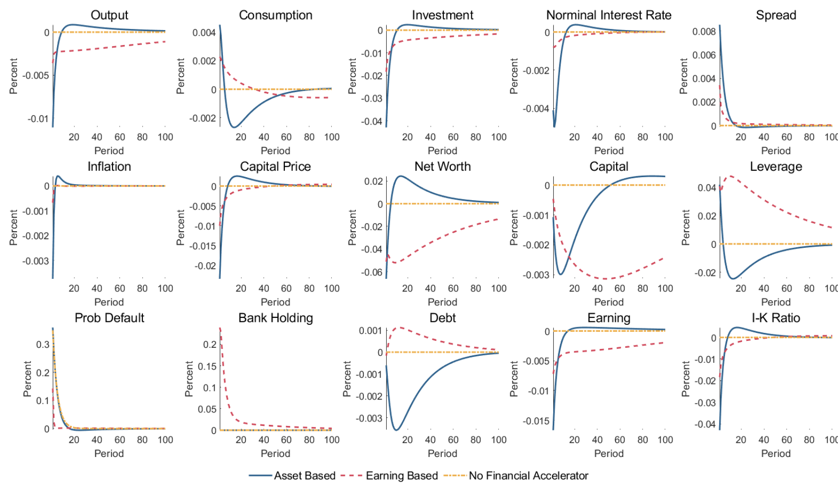


FIGURE 7. IRF TO RISK SHOCK

a well-established financial accelerator model, existing literature provides ample guidance on how to calibrate its parameters. By utilizing parameters that can be directly applied to the log-linearized model, I can derive the underlying parameters necessary for specific calibrations. These same set of deep parameters will then be used for the earnings-based financial accelerator. Among these deep parameters, the most crucial one is the steady-state level of the standard error of capital efficiency shocks. By employing the same steady-state standard error, we ensure that the size of the risk shock remains consistent across both models.

Secondly, once most of the deep parameters have been calibrated, the size of the asset-based financial accelerator model becomes determined. However, the size of the earnings-based financial accelerator model will depend on the forgiveness rate (ψ). I calibrate it in a manner that targets the same probability of default for both models at the steady state. This enables a direct comparison between the two models, at least in terms of their steady-state characteristics, thereby minimizing the impact of size-related effects as much as possible.

Given that the primary objective of this section is to provide a direct comparison between the two financial accelerators rather than presenting quantitative results or examining the interaction between the two financial sectors, I will defer the detailed calibrations to the next section. Here, I directly present the impulse response functions (IRFs) of the two models to risk shocks in figure 7, which have already clearly illustrated the results and mechanisms. We observe that as uncertainty levels increase (i.e., σ increases), investment decreases in both models, but the earnings-based financial accelerator model experiences a smaller drop. To provide a comparison, I also include the results of the no-financial-accelerator model as a baseline, in which the credit spread is irrelevant to the leverage ratio and risk shocks. As the risk shocks only affect the model through the credit spread channel, it is natural to see that the risk shock has no effect in the absence of a financial accelerator. By examining the IRFs of most of the endogenous variables, we can see that the asset-based financial accelerator model experienced a much larger decrease at the beginning, but the effect was quickly reversed. However, the earnings-based financial accelerator model lies between the asset-based financial accelerator and the baseline New Keynesian model, indicating that earnings-based financial constraints have a weaker effect than asset-based financial constraints.

So what accounts for the earning-based financial accelerator's comparatively smaller yet more persistent decline in investment when uncertainty rises? According to the earning-based financial constraints, the transition equation of D_t requires banks to impose stricter penalties on entrepreneurs when there are more defaulting entrepreneurs. Figure 7 shows that when a positive risk shock hits the economy, entrepreneurs have a higher likelihood to default due to the existence of a financial accelerator. Under the asset-based financial accelerator, banks can only raise the interest rate as a form of punishment. However, with the earnings-based financial accelerator, banks can also increase the fraction of earnings that the banks directly hold on to punish entrepreneurs. Since risk shocks have some persistence, banks punish entrepreneurs more severely in the earnings-based financial accelerator than in the asset-based financial accelerator when the level of uncertainty increases in subsequent periods. This, in turn, reduces the entrepreneurs' incentive to default in the following periods, resulting in a smaller impact on investment.

However, adopting this alternative method to penalize entrepreneurs is not costless. By directly holding claims to a firm's future earnings, banks incur verification costs in each period. As a result, this form of punishment diminishes the overall financial efficiency of the market. The ultimate impact arises from the interplay of two counteracting forces, with the first force outweighing the second in the case of uncertainty shocks.

In summary, the earnings-based financial accelerator model exhibits a weaker yet more persistent response to risk shocks compared to the traditional asset-based financial accelerator model. The direction of the model prediction aligns well with the empirical evidence, prompting us to delve deeper into the quantitative results in the subsequent analysis.

V. Heterogeneous Financial Accelerator Model

In the previous session, we examined the asset-based and earnings-based financial accelerator models independently without accounting for general equilibrium effects. This raises the question of how financial heterogeneity can help us better explain the different responses in the firm's investment to risk shocks. To explore this question, I developed a heterogeneous financial accelerator model and calibrated the IRFs with the quarterly empirical analysis. First, I will explain the setup of the heterogeneous firm model, followed by a discussion of the calibration strategy and the simulation results. Additionally, I will delve into the implications of introducing the earnings-based financial accelerator by incorporating monetary policy shocks into the model. This will allow us to examine how the model responds to a broader range of shocks.

By considering financial heterogeneity and integrating both risk and monetary policy shocks, we aim to gain a more comprehensive understanding of the different investment responses observed and the potential implications of the earnings-based financial accelerator.

A. Model Setup

The model setup is dipicted in figure 8. In the heterogeneous firm model, the entrepreneurs, the wholesale good producers and the capital good producers operates in separate sectors. Each sector is characterized by a different financial accelerator: one with an asset-based financial accelerator and the other with an earnings-based financial accelerator. the capital and labor employed in the two markets differ, as do the external funding sources. The parallel sector setup enables independent capital returns for each sector, facilitating the incorporation of the main results from the separate models into this general framework. The final goods producer combines the intermediate goods produced by the two sectors to generate final goods. The fraction of inputs sourced from the asset-based financial accelerator sector is denoted as J. Since the final good producer uses a

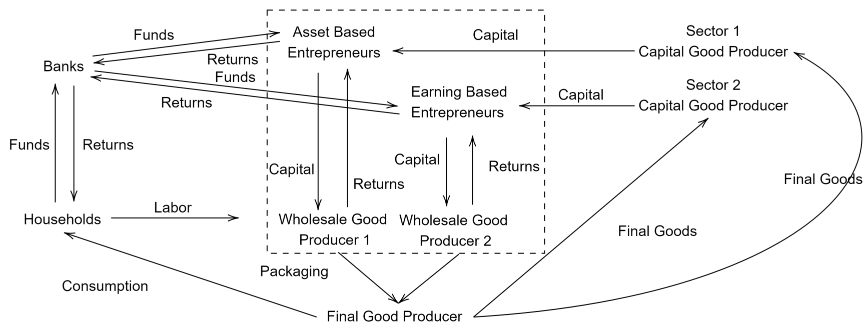


FIGURE 8. FINANCIAL HETEROGENEITY MODEL

Cob-Douglas production function, J also denotes the technology of the final good producer.

In the model, J is treated as a parameter as set to be 30% to match the empirical fraction of the asset based debt ratio from the Capital IQ dataset. Although the assumption of a constant and exogenously given fraction of asset-based financial accelerator firms is unrealistic, abstracting from the endogenous choice of firm type allows us to deal with a simple and computationally tractable model.

The entrepreneurs in the asset-based financial accelerator sector follow equations (4) and (5), while those in the earnings-based sector adhere to equations (7), (8), and (9). On the sector level, the wholesale good producers and capital good producers solve their respective first-order conditions (FOCs). Consequently, a set of parallel equations emerges to characterize the equilibrium. It is worth noting that all sector-level variables in these equations are denoted with subscripts A and E to distinguish between the two sectors.

Furthermore, the parameter J plays a role in determining the calculation of aggregate-level variables. J acts as a weight on the asset-based sector, allowing us to calculate the aggregate variables. The appendix provides a comprehensive overview of the complete model, including further details and information.

B. Calibration and Model Simulation

To match the empirical result, we need to calibrate the model parameters. We have talked a little bit about how to calibrate the model separately, and the heterogeneous financial sector model follows most of the rules. The models are

Table 6—Parameter Calibration

Variable	Name	Value	Target
β	Utility Discounting Factor	0.990	4 Percent Annual Interest Rate
δ	Quarterly Depreciation Rate	0.025	10 Percent Annual Depreciation Rate
α	Labor Share	0.350	35 Percent Labor Share in the US
Ω	Entrepreneur Labor Share	0.985	64 Percent of Entrepreneur Labor Share
η	Elasticity of Substitution Between Goods	11.000	10 Percent Steady State Markup
γ	Entrepreneur Survival Rate	0.973	2.72 Percent Quarterly Natual Net Worth Shrinking Rate
φ	Fixed Capital Producer Technology	0.545	Calibrated From the Data
θ	Price Stickiness	0.750	25 Percent of Price Changing
σ	Steady State ω Standard Error	0.312	2 Percent Credit Spread
ρ	Taylor Rule Persistence	0.900	Common Value
ζ	Taylor Rule Inflation Reaction	1.100	Common Value
ξ	Labor Preference Parameter	5.455	25 Percent of Working Hours of a Day
μ	State Verification Cost	0.015	Common Value
$\bar{G}/\bar{Y}$	Steady State G/Y ratio	0.200	20 Percent Goverment Expenditure to GDP Ratio
J_A	Fraction of the Asset Based Financial Accelerator	0.300	Average of 30% from the data
J_E	Fraction of the Earning Based Financial Accelerator	0.700	Average of 70% from the data

Note: The table displays the parameter values used to solve the impulse response functions (IRFs) of heterogeneous financial accelerator models in response to risk shocks. The calibration of the models is mostly based on Eric Sims' notes.

calibrated quarterly, with the parameter values listed in Table 6. The discounting factor β is set to 0.99 to achieve a target annual interest rate of 4 percent. The depreciation rate δ is set to 0.025 to achieve a target annual depreciation rate of 10 percent for fixed capital. The technology parameters α and Ω are set to 0.35 and 0.9846, respectively, to achieve a target labor share of 35 percent and entrepreneur share of 64 percent in the US. The elasticity of substitution between goods η is set to 11 to achieve a steady-state markup of 10 percent. The entrepreneur survival rate γ is set to 0.9728 to match a quarterly natural net worth shrinking rate of 2.72 percent. The price stickiness parameter θ is set to 0.75 to match a 25 percent price changing rate. The labor preference parameter ξ is set to 3.3122 to match a 25 percent share of working hours in a day. The steady-state government expenditure ratio $\bar{G}/\bar{Y}$ is 20 percent. Most of the other parameters are set according to common calibrations provided in the notes of Eric Sims, with a few exceptions: the steady-state standard error of the effectiveness shock, σ, the forgiveness rate ψ, the investment adjustment cost parameter φ and the persistence of the risk shock ρ_s.

To set the first parameter σ, I target a credit spread of 2 percent in the steady state for the traditional asset-based financial accelerator model. Using the same set of deep parameters and calibrating the forgiveness rate ψ, I then determined the steady state for the earnings-based financial accelerator model to achieve the same probability to default. Table 7 shows that most of the targeted and

Table 7—Steady State

Variable	Aggregate Variables Name	Steady State	Variable	Sector Variables Name	Steady State	
			$\bar{R}^k$	Capital Return	1.015	1.021
$\bar{X}$	Markup	1.100	$\bar{R}^k - \bar{R}$	Credit Spread	0.005	0.011
$\bar{H}$	Working Hours of a Day	0.250	$\bar{I}/\bar{Y}$	I-Y Ratio	0.198	0.172
$\bar{R}$	Riskfree Interest rate	1.010	$\bar{K}/\bar{Y}$	K-Y Ratio	7.935	6.871
$\bar{C}/\bar{Y}$	C-Y Ratio	0.320	$\bar{W}^e/\bar{N}$	Entrepreneur Wage to N	0.003	0.007
$\bar{I}/\bar{Y}$	I-Y Ratio	0.370	$\bar{K}/\bar{N}$	Leverage	2.891	5.300
$\bar{W}/\bar{Y}$	W-Y Ratio	2.327	$\bar{\omega}$	Default Cutoff	0.663	0.663
			$\bar{D}$	Fraction of Buffer Fund	-	0.600

Note: This table list the value of the key endognous variables at the steady state. The left panel shows the aggregate variables at the steady state, while the right panel compares the two sectors at the steady state.

untargeted steady-state variables are identical between the two sectors. Notably, the probability to default in the earnings-based sector coincides with that of the asset-based sector, eliminating the effect of different financial friction sizes.

There are two parameters remaining to be determined in the calibration process: φ and ρ_s. As these parameters relate to the dynamic effects of the model, I utilize the impulse response functions (IRFs) estimated from quarterly data to calibrate them. In the empirical analysis, the investment rate is defined as the net investment divided by the total capital. I simulate the model with a 1 percent increase in the standard error of the capital effectiveness shock to match the scale of the risk shocks observed in the data.

Let $\gamma = \{\varphi, \rho_s\}$ denote the parameters to be calibrated. The calibration process aims to find the values of γ that minimize the following problem:

$$\min_{\gamma}[\hat{\Phi} - \Phi(\gamma)]' V [\hat{\Phi} - \Phi(\gamma)]$$

Here, $\hat{\Phi}$ represents the estimated IRFs, $\Phi(\gamma)$ denotes the model-predicted IRFs, and V is a diagonal matrix with the variances of the empirical IRFs on its diagonals. It is important to note that figure 3 displays the estimated quarterly IRFs for the two different types of financially constrained firms. Therefore, the calibration process aims to choose the parameter values that minimize the distance between the model-predicted IRFs and the empirical data.

The comparison between the model-predicted IRFs and the empirical data is depicted in figure 9. It is evident that the model successfully captures the funda-

Figure 9. IRF to Risk Shock: Compare Data to Model

Note: This figure provides a visual comparison between the model-predicted impulse response functions (IRFs) of the investment rate and the estimated IRFs based on quarterly data. The two panels represent different sectors: the left panel corresponds to the asset-based sector, while the right panel represents the earnings-based sector. In both panels, the horizontal lines indicate the estimated points obtained from Figure 3, accompanied by the colored area denoting the 90% confidence intervals. The dashed lines correspond to the model-predicted IRFs.

mental shape of the empirically estimated IRFs. Specifically, the earnings-based sector aligns well with the data, exhibiting a similar pattern of response. However, in the case of the asset-based sector, the model predicts a smaller contemporaneous effect compared to the observed data.

While the model generally captures the key dynamics observed in the data, the discrepancy in the asset-based sector highlights the need for further analysis and potential refinements. Nonetheless, the overall similarity between the model-predicted IRFs and the data provides confidence in the model's ability to capture the essential dynamics and support further exploration of the effects of financial accelerators on firm investment behavior.

C. Policy Implication with Monetary Policy Shocks

In our empirical analysis, we discovered that financially constrained firms relying on earnings-based constraints exhibit a higher response to monetary policy shocks compared to asset-based financially constrained firms. This surprising observation can be explained by the financial heterogeneity model. To investigate

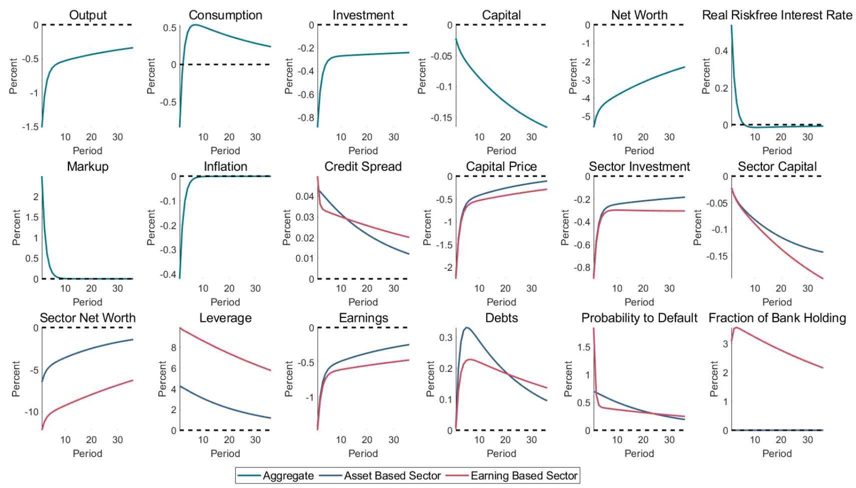


FIGURE 10. IRF TO CONTRACTIONARY MONETARY POLICY SHOCK

further, we conducted model simulations by subjecting the system to a significant increase in the risk-free interest rate. The results, depicted in figures 10, clearly demonstrate that the investment rate of the earnings-based sector responds more prominently to contractionary monetary policy shocks. This finding aligns with the latest empirical evidence derived from our data analysis.

How is it possible for the same model to respond differently to different shocks? This question arises due to the existence of two forces that drive the distinctions between the two types of financial constraints. In the case of earning-based financial constraints, banks impose punishment through two channels, while asset-based financial constraint firms experience only one. Consequently, when uncertainties increase, more defaults occur, leading to a direct follow-on effect of restructuring, which results in harsher punishment for earning-based firms. However, this punishment comes at the cost of reduced overall financial efficiency. This is where the second channel comes into play. By paying verification costs, the bank-owned earnings naturally have fewer funds available for investment. This interplay between different factors helps explain the divergent responses of the model to various shocks.

In the context of increasing uncertainties, the first force aims to mitigate the impact of the shock. When this force dominates the second, we observe a smaller response from the earning-based sector. However, this is not the case with monetary policy shocks. When the central bank raises the interest rate, the cost of external funding, represented by the credit spread, increases. This makes it more challenging for entrepreneurs to borrow money, leading to a higher probability of default. As a consequence, banks impose harsher punishment on the firm, incurring a high financial efficiency cost. This, in turn, further reduces the firm's investment, as both forces align in the same direction, amplifying the effect of the shock for a monetary policy shock.

This mechanism provides a reasonable explanation for the divergent responses of firm investment to different shocks. Figure 10 supports the validity of the mechanism we just described. To the best of my knowledge, this paper is the first to incorporate financial type heterogeneity into the financial accelerator framework and successfully reconcile the disparate responses observed for both risk shocks and monetary policy shocks.

VI. Conclusion

This paper presents compelling evidence that firms with a higher earning-based debt ratio experience smaller impacts in response to uncertainty shocks. Specifically, when firm-level idiosyncratic volatility increases, firms tend to reduce their investment scale, but those with a higher earning-based debt ratio exhibit a smaller reduction in investment. This finding highlights the importance of financial constraints, even when controlling for their size, and underscores the significant role played by the type of financial constraint a firm faces.

In this study, I propose an earning-based financial accelerator, by introducing an additional method of punishment for entrepreneurs who are more likely to default. I show that such constraints exhibit a negative feedback mechanism that plays a self-constraining role when the risk in the economy increases. This finding sheds light on the reason behind the empirical observations. Moreover, the financial heterogeneity model not only explains firms' behavior in response to increased uncertainty but also provides insights into their response to monetary policy shocks.

Scholars are paying more and more attention to the interactions among uncertainty, financial heterogeneity, and firm investment. This paper contributes to this growing body of research, but there are still opportunities for further improvement in this field. One possible extension is to incorporate a dynamic formal contracting problem for the earning-based financial accelerator model. Currently, each firm in the model is assumed to be the same and without a history. However, due to the nature of earning-based debts, there is a naturally occurring dynamic effect that this paper ignores for simplicity. A promising direction for future research is to combine the earning-based financial accelerator model with firms of heterogeneous capital size, and investigate the relationship between firm size distribution and uncertainties.

REFERENCES

Alfaro, Iván, Nicholas Bloom, and Xiaoji Lin. 2019. "The Finance Uncertainty Multiplier." Social Science Research Network SSRN Scholarly Paper ID 3093871, Rochester, NY. (document), I, II, III.A, III.A, 5, III.A, III.D, A.A2, A.A3, A.A4, A1, A.A5

Arellano, Cristina, Yan Bai, and Patrick J. Kehoe. 2018. "Financial Frictions and Fluctuations in Volatility." *Journal of Political Economy*, 127(5): 2049–2103. I

Bachmann, Rüdiger, Ricardo J. Caballero, and Eduardo M. R. A. Engel. 2013. "Aggregate Implications of Lumpy Investment: New Evidence and a DSGE Model." *American Economic Association.* II

Baker, Scott R., Nicholas Bloom, and Steven J. Davis. 2016. "Measuring Economic Policy Uncertainty." *The Quarterly Journal of Economics*, 131(4): 1593–1636. I, 3

Basu, Susanto, and Brent Bundick. 2017. "Uncertainty Shocks in a Model of Effective Demand." *Econometrica*, 85(3): 937–958. I

Berger, David, Ian Dew-Becker, and Stefano Giglio. 2020. "Uncertainty Shocks as Second-Moment News Shocks." *The Review of Economic Studies*, 87(1): 40–76. I

Bernanke, Ben, Mark Gertler, and Simon Gilchrist. 1998. "The Financial Accelerator in a Quantitative Business Cycle Framework." National Bureau of Economic Research Working Paper 6455. (document), I, IV, IV.A

Bianchi, Javier, and Enrique G. Mendoza. 2010. "Overborrowing, Financial Crises and 'Macro-prudential' Taxes." I

Bloom, Nicholas. 2009. "The Impact of Uncertainty Shocks." *Econometrica*, 77(3): 623–685. I

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry. 2018. "Really Uncertain Business Cycles." *Econometrica*, 86(3): 1031–1065. I

Caglio, Cecilia R., R. Matthew Darst, and Șebnem Kalemli-Özcan. 2021. “Collateral Heterogeneity and Monetary Policy Transmission: Evidence from Loans to SMEs and Large Firms.” (document), I, III.D

Cao, Dan, and Guangyu Nie. 2017. “Amplification and Asymmetric Effects without Collateral Constraints.” *American Economic Journal: Macroeconomics*, 9(3): 222–266. I

Carlstrom, Charles T., and Timothy S. Fuerst. 1997. “Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis.” *The American Economic Review*, 87(5): 893–910. I

Carlstrom, Charles T., Timothy S. Fuerst, and Matthias Paustian. 2015. “Inflation and Output in New Keynesian Models with a Transient Interest Rate Peg.” *Journal of Monetary Economics*, 76: 230–243. I

Carriero, Andrea, Todd E. Clark, and Massimiliano Giuseppe Marcellino. 2018. “Endogenous Uncertainty.” Social Science Research Network SSRN Scholarly Paper ID 3152579, Rochester, NY. I

Carriero, Andrea, Todd E. Clark, and Massimiliano Marcellino. 2017. “Measuring Uncertainty and Its Impact on the Economy.” *The Review of Economics and Statistics*, 100(5): 799–815. I

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. “Risk Shocks.” *American Economic Review*, 104(1): 27–65. (document), I, IV, IV.A, IV.A

Coibion, Olivier, Dimitris Georgarakos, Yuriy Gorodnichenko, Geoff Kenny, and Michael Weber. 2021. “The Effect of Macroeconomic Uncertainty on Household Spending.” I

Cordoba, Juan-Carlos, and Marla Ripoll. 2004. “Credit Cycles Redux*.” *International Economic Review*, 45(4): 1011–1046. I

Creal, Drew D., and Jing Cynthia Wu. 2017. “Monetary Policy Uncertainty and Economic Fluctuations.” *International Economic Review*, 58(4): 1317–1354. I

Drechsel, Thomas. 2022. “Earnings-Based Borrowing Constraints and Macroeconomic Fluctuations.” I

Fernández-Villaverde, Jesús, and Pablo A. Guerrón-Quintana. 2020. "Uncertainty Shocks and Business Cycle Research." *Review of Economic Dynamics*, 37: S118–S146. I

Gertler, Mark, and Ben Bernanke. 1989. "Agency Costs, Net Worth and Business Fluctuations." In *Business Cycle Theory.* , ed. Finn Kydland. Edward Elgar Publishing Ltd. I

Gilchrist, Simon, and Egon Zakrajšek. 2012. "Credit Spreads and Business Cycle Fluctuations." *American Economic Review*, 102(4): 1692–1720. I

Greenwald, Daniel. 2018. "The Mortgage Credit Channel of Macroeconomic Transmission." I

Guerrieri, Luca, Matteo Iacoviello, Francisco Covas, John C. Driscoll, Mohammad R. Jahan-Parvar, Michael T. Kiley, Albert Queralto, and Jae Sim. 2015. "Macroeconomic Effects of Banking Sector Losses Across Structural Models." I

Hadlock, Charles J., and Joshua R. Pierce. 2010. "New Evidence on Measuring Financial Constraints: Moving Beyond the KZ Index." *The Review of Financial Studies*, 23(5): 1909–1940. 3, III.A

Hart, Oliver, and John Moore. 1994. "A Theory of Debt Based on the Inalienability of Human Capital*." *The Quarterly Journal of Economics*, 109(4): 841–879. I

Husted, Lucas, John Rogers, and Bo Sun. 2019. "Monetary Policy Uncertainty." *Journal of Monetary Economics.* I

Iacoviello, Matteo. 2005. "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle." *American Economic Review*, 95(3): 739–764. I

Istrefi, Klodiana, and Anamaria Piloiu. 2014. "Economic Policy Uncertainty and Inflation Expectations." Social Science Research Network SSRN Scholarly Paper ID 2510829, Rochester, NY. I

Jensen, Henrik, Ivan Petrella, Søren Hove Ravn, and Emiliano Santoro. 2020. "Leverage and Deepening Business-Cycle Skewness." *American Economic Journal: Macroeconomics*, 12(1): 245–281. I

Jermann, Urban, and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review*, 102(1): 238–271. I

Jordà, Òscar. 2005. "Estimation and Inference of Impulse Responses by Local Projections." *American Economic Review*, 95(1): 161–182. III.D

Jurado, Kyle, Sydney C. Ludvigson, and Serena Ng. 2015. "Measuring Uncertainty." *American Economic Review*, 105(3): 1177–1216. I

Khan, Aubhik, and Julia K. Thomas. 2013. "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity." *Journal of Political Economy*, 121(6): 1055–1107. I

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles." *Journal of Political Economy*, 105(2): 211–248. (document), I

Kocherlakota, Narayana R. 2000. "Creating Business Cycles Through Credit Constraints." *Federal Reserve Bank of Minneapolis Quarterly Review*, 24(3): 2–2. I

Kumar, Saten, Yuriy Gorodnichenko, and Olivier Coibion. 2022. "The Effect of Macroeconomic Uncertainty on Firm Decisions." I

Lian, Chen, and Yueran Ma. 2021. "Anatomy of Corporate Borrowing Constraints*." *The Quarterly Journal of Economics*, 136(1): 229–291. (document), I, II, II, A.A1

Ludvigson, Sydney C, Sai Ma, and Serena Ng. 2015. "Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?" National Bureau of Economic Research Working Paper 21803. I

Shleifer, Andrei, and Robert W. Vishny. 1992. "Liquidation Values and Debt Capacity: A Market Equilibrium Approach." *The Journal of Finance*, 47(4): 1343–1366. I

Swanson, Eric T. 2021. "Measuring the Effects of Federal Reserve Forward Guidance and Asset Purchases on Financial Markets." *Journal of Monetary Economics*, 118: 32–53. III.D

Townsend, Robert M. 1979. "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory*, 21(2): 265–293. I

Whited, Toni M., and Guojun Wu. 2006. "Financial Constraints Risk." *The Review of Financial Studies*, 19(2): 531–559. 3, III.A

Winberry, Thomas. 2021. "Lumpy Investment, Business Cycles, and Stimulus Policy." *American Economic Review*, 111(1): 364–396. II

Zhao, Dongchen. 2022. "Low Interest Rates, Debt Heterogeneity, and Firm Dynamics." 9

Appendix A: Appendix to the Empirical Analysis

A1. Identification of the Debt Types

While Lian and Ma (2021) relies on different data sources, this paper primarily uses the Capital IQ dataset. However, the process for identifying the types of financial debt is similar. For this study, debt structure data was collected from the 10-K filings of firms between the years 2001 to 2019. The identification of the types of debts followed the steps outlined below:

Step 1: Identify asset-based loans using the following criteria:

- Debt descriptions that contain certain words related to asset-based loans such as asset-based, ABL, borrowing base, mortgage, real estate, building, equipment, machine, aircraft, fixed asset, inventory, receivable, working capital, automobile, vehicle, capital lease, SBA, small business, oil, drill, rig, reserve-based, factoring, industrial revenue bond, finance company, capital lease, construction, and project finance.
- Secured resolvers
- Mortgage loans or mortgage notes
- Commercial lease structures

Step 2: Identify other types of loans using the following criteria:

- Loans that do not meet the criteria for asset-based loans in step 1
- Debt descriptions that contain certain words related to other loan types, such as MR, MRS, MS, director, executive, chairman, founder, shareholder, related parties, government, pollution control bond, insurance-related, vendor, seller, supplier, landlord, and affiliated.

Step 3: Identify earning-based loans using the following criteria:

- Debt descriptions that contain certain words related to earning-based loans, such as cash flow-based, cash flow, cash flow loan, debenture, first lien, second lien, third lien, term facility, term loan facility, term loan, syndicated, tranche, acquisition line, bridge loan, bonds, senior subordinated, senior notes, notes due, private placement, and medium-term notes.

- Debt types including debenture, note payable, corporate bond, or term loan.
- Debt that is not first lien or secured debt.

Step 4: All other secured debts are classified as asset-based loans, and all other debts are classified as the other type of loans.

A2. Instrument Variable Construction

The instrument variable used in this paper is based on the approach described in Alfaro, Bloom and Lin (2019). For a more detailed explanation, readers are referred to that paper. Here, I provide a brief overview of how the instrument variable is constructed.

To obtain firm-level exposure to aggregate factors, the following two steps are followed:

Step 1: Using firm-year level data, a regression is run as follows:

$$r^a dj_{i,t} = \alpha_j + \sum_c \beta_j^c r_t^c + \epsilon_{i,t}$$

This regresses the daily risk-adjusted stock return of firm i on changes in the prices of aggregate factors c for each industry j. The regression is clustered at the industrial level to improve the robustness of the estimation. The coefficients β_j^c represent the industrial-level exposure to the aggregate factor c.

Step 2: Using the estimated exposures β, the instrument variables are constructed as follows:

$$z_{i,t-1}^c = |\beta_{j,t-3}^c| \Delta\sigma_{t-1}^c$$

For each factor c, a corresponding instrument is constructed, where $\Delta\sigma_{t-1}^c$ represents the variation of factor c. When the aggregate factor has higher uncertainty, industries with higher exposure to that factor will have greater idiosyncratic volatility.

A3. Quarterly Instrument Variable for Uncertainty Shocks

In the empirical analysis, our primary emphasis lies on annual data. To complement this analysis, we have also incorporated quarterly data. However, we encountered a limitation in the study conducted by Alfaro, Bloom and Lin (2019), as they only provided annual-level exposure to uncertainty shocks. To address this limitation, we followed the procedure outlined in their paper and constructed an instrumental variable that exhibits similar behaviors to the annual instrument variables. This allowed us to extend our analysis and enhance the robustness of our findings.

The process of constructing the instrumental variable follows a similar procedure to that of building the annual data. Initially, we download the daily stock returns for each firm included in the Compustat-CRSP linked dataset, specifically for each business day. Subsequently, we calculate the standard error for each stock on a rolling window of 252 business days.

In the next step, we perform a regression, similar to the first step of constructing the annual instrument variable. It is important to note that, unlike the original paper, in this study, we define the adjusted return as the residual of a regression in which the firm's excess return is regressed on the five stock market factors. These five stock market factors are sourced from Fama and French (2014). After obtaining the adjusted return of the firm, I followed the first step in the previous section and run the daily adjusted stock return on the 9 aggregate factors daily returns on the date when we observe the quarterly financial report.

Finally, we estimate the beta coefficients for each firm with respect to the nine aggregate factors. We proceed with the second step outlined in the previous section, constructing nine instrument variables for each firm at each quarter. To simplify the process, we have disregarded the implied volatility shocks and their corresponding instruments. Additionally, we have omitted the weights used in constructing the instruments, as the paper by Alfaro et al. (2019) suggests that the results remain robust even without weighted instrument variables. By adopting these simplifications, we aim to maintain the integrity of the analysis while reducing complexity.

A4. Definition of the Firm Level Variables

The definitions of the firm level variables are listed as follows:

- Investment Rate:

$$InvRate_{i,t} = \frac{2capx_{i,t}}{ppent_{i,t} + ppent_{i,t-1}}$$

To eliminate the effect of outliers, the investment rate has been winsorized at -0.5 and 0.5. However, even if we do not winsorize the data at these values, the main result remains unchanged. Winsorization has brought the replication of the Alfaro, Bloom and Lin (2019) closer to their original results.

- Employment Growth Rate:

$$Emp_{i,t} = \frac{2(Emp_{i,t} - Emp_{i,t-1})}{Emp_{i,t} + Emp_{i,t-1}}$$

- Intangible Capital Growth Rate:

$$Intan_{i,t} = \frac{2(Intan_{i,t} - Intan_{i,t-1})}{Intan_{i,t} + Intan_{i,t-1}}$$

- Payout Growth Rate:

$$Payout_{i,t} = \frac{2(Payout_{i,t} - Payout_{i,t-1})}{Payout_{i,t} + Payout_{i,t-1}}$$

where $Payout = dvc + dvp$.

- Debt Growth Rate:

$$Debt_{i,t} = \frac{2(Debt_{i,t} - Debt_{i,t-1})}{Debt_{i,t} + Debt_{i,t-1}}$$

where $Debt = dlc + dltt$.

- Cost of Goods Growth Rate:

$$COGS_{i,t} = \frac{2(COGS_{i,t} - COGS_{i,t-1})}{COGS_{i,t} + COGS_{i,t-1}}$$

- Sales Growth Rate:

$$Sales_{i,t} = \frac{2(Sales_{i,t} - Sales_{i,t-1})}{Sales_{i,t} + Sales_{i,t-1}}$$

- Cash Holdings Growth Rate:

$$che_{i,t} = \frac{2(che_{i,t} - che_{i,t-1})}{che_{i,t} + che_{i,t-1}}$$

- Profit Growth Rate:

$$ebitda_{i,t} = \frac{2(ebitda_{i,t} - ebitda_{i,t-1})}{ebitda_{i,t} + ebitda_{i,t-1}}$$

- Tangibility:

$$ebitda_{i,t} = \frac{ppegt_{i,t}}{at_{i,t-1}}$$

- Leverage:

$$leverage_{i,t} = \frac{ppegt_{i,t}}{at_{i,t}}$$

- Return on Asset:

$$ROA_{i,t} = \frac{ebit_{i,t}}{at_{i,t-1}}$$

- Firm Size:

$$Size_{i,t} = log(Emp_{i,t})$$

- WW index:

$$\begin{aligned} WWIndex_{i,t} = & -0.091(oibdp_t)/at_{t-1} - 0.062 \times 1(payout_t > 0) \\ & + 0.021(dltt_t/at_{t-1}) - 0.044log(at_{t-1}) \\ & + 0.102 * \Delta(IndSALE_t) - 0.035\Delta(sales_t) \end{aligned}$$

- HP index:

$$HPIndex_{i,t} = -0.737log(at_t)^2 + 0.043log(at_t)^2 - 0.040age_t$$

Table A1—First Stage Regression and Sanity Check

Panel A: First Stage		
	(1) Realized Shock	(2) Implied Shock
F-Statistics	94.682	58.696
P Value of F-Stats	0.000	0.000
R-Squared	0.099	0.117
Observation	32094	20795

Panel B: Sanity Check				
	(3) Realized OLS	(4) Realized IV	(5) Implied OLS	(6) Implied IV
Uncertainty Shock	-0.010***	-0.048***	-0.030***	-0.102**
	(0.003)	(0.016)	(0.006)	(0.039)
Hansen's J: P-Value		0.612		0.785
R-Squared	0.177	0.163	0.215	0.200
	32094	32094	20795	20795

Note: This table presents the results of the first-stage regression and a sensitivity check regression, which aims to replicate the findings reported in Alfaro, Bloom and Lin (2019). The regression procedure closely follows the methodology used in the baseline regression.

A5. Baseline Regression Result

The regression analysis in this study replicates the regression conducted in Alfaro, Bloom and Lin (2019). Table A1 presents the regression results for the same variables used in their paper. As the dataset used in this study covers different time period than the original paper, this replication analysis is necessary.

The table shows that the estimation effect closely aligns with the results in the original paper. In the baseline regression, the estimated coefficients for realized and implied volatility shocks in the original paper are -0.041 and -0.058, respectively. In this replication analysis, using a subset of time periods from the data, the estimated coefficients are -0.048 and -0.102, respectively. These estimates are also in close proximity to the original regression results.

Moreover, the F-test conducted on the instruments used in the first-stage regressions reveals a strong correlation between the exposure to aggregate uncertainty

Table A2—Baseline Regression with Asset Based Debt Ratio

	(1) OLS	(2) IV	(3) OLS	(4) IV
Realized Shock # Asset Based Debt Ratio	-0.010*	-0.031***		
	(0.005)	(0.009)		
Implied Shock # Asset Based Debt Ratio			-0.002	-0.061**
			(0.008)	(0.026)
Realized Shock	-0.008***	-0.042***		
	(0.003)	(0.013)		
Implied Shock			-0.030***	-0.091**
			(0.005)	(0.035)
Asset Based Debt Ratio	-0.006*	-0.006*	-0.011**	-0.011**
	(0.003)	(0.003)	(0.005)	(0.004)
R-Squared	0.178	0.162	0.221	0.203
Observation	26760	26760	18087	18087

Note: This table displays the results of the first regression but with asset based loan ratio. Standard errors are shown in parasynthesis. ***: $p<0.01$, **: $p<0.05$, *: $p<0.1$

shocks and idiosyncratic volatility shocks. This indicates that the instrument variables are valid for both realized and implied uncertainty shocks.

Finally, I conducted Hansen's J test on the two instrumental variable (IV) regressions to assess the exogeneity hypothesis. Given that there are a total of 9 instruments used in the analysis, Hansen's J test can be employed to validate the instrument variables. The results of the Hansen's J test for both the realized and implied IV regressions indicate high p-values. Therefore, we fail to reject the null hypothesis, confirming the validity of the instrument variables. Although we cannot definitively claim that the IVs are perfectly valid, passing the Hansen's J test alleviates concerns regarding this issue.

A6. Robust Regression and Further Discussions

In order to enhance the robustness of the empirical analysis, I also conducted robust regressions alongside the regular regressions. Table A2 presents the regression results using the asset-based debt ratio instead of the earnings-based debt ratio. Table 9 displays the regression results while controlling for industrial-level effects rather than firm fixed effects. Additionally, I accounted for interaction terms between firm-level uncertainty shocks and aggregate factors, and the corre-

Table A3—Regression Interacted with Aggregate Factors

	(1)	(2)	(3)
		Realized Uncertainty Shocks	
	Average Credit Spread	Aggregate Uncertainty	Aggregate Cashflow Loan Ratio
Uncertainty Shock # Cashflow Based Debt Ratio	0.030***	0.030***	0.027***
	(0.008)	(0.008)	(0.007)
Uncertainty Shock # Aggregate Factor	0.036***	0.001	0.955
	(0.013)	(0.001)	(0.639)
Uncertainty Shock	-0.128***	-0.106***	-0.770
	(0.027)	(0.030)	(0.465)
Cashflow Based Debt Ratio	0.005	0.006	0.006
	(0.003)	(0.003)	(0.003)
R-Squared	0.149	0.145	0.148
Observation	25594	25594	25594
	(4)	(5)	(6)
		Implied Uncertainty Shocks	
	Average Credit Spread	Aggregate Uncertainty	Aggregate Cashflow Loan Ratio
Uncertainty Shock # Cashflow Based Debt Ratio	0.071***	0.073***	0.069***
	(0.023)	(0.026)	(0.022)
Uncertainty Shock # Aggregate Factor	0.020	0.000	-0.381
	(0.019)	(0.001)	(1.464)
Uncertainty Shock	-0.187***	-0.176***	0.141
	(0.048)	(0.050)	(1.067)
Cashflow Based Debt Ratio	0.010**	0.010**	0.011***
	(0.004)	(0.004)	(0.004)
R-Squared	0.205	0.199	0.215
Observation	17119	17119	17119

Note: This table displays the results of the regression controlling for the aggregate factors. Standard errors are shown in parasynthesis. ***: p<0.01, **: p<0.05, *: p<0.1

sponding regression results are presented in table. The aggregate factors considered in our analysis are the Moody BAA-AAA bond credit spread (as a proxy for aggregate credit spread), the aggregate cashflow-based debt ratio (as a proxy for aggregate financial constraint type), and the VIX index (as a proxy for aggregate level uncertainty).

Across all the regression results, we consistently observe the same pattern, indicating that the earnings-based debt ratio significantly influences the magnitude of firms' investment reduction when faced with increased uncertainty.

In addition to conducting robust regressions, I also explored the impact of various types of financial constraints on firm-level variables beyond investment. I examined several firm-level variables, including the growth rate of employment, intangible assets, debt, cost of goods, sales, cash, profit, payout, and stock return. Among these variables, cash and employment are the two variables affected by cashflow-based debts. Specifically, when uncertainty increases, the effects on cash and employment are mitigated for firms with higher levels of cashflow-based debts.

Table A4—Impact on Other Firm-Level Variables

Panel A: Realized Uncertainty Shock

	(1) Employment Growth	(2) Intangible Asset Growth	(3) Debt Growth	(4) Cost of Good Growth	(5) Sales Growth	(6) Cash Growth	(7) Profit Growth	(8) Payout Growth	(9) Stock Return
Realized Shock # Cashflow Based Debt Ratio	0.027	0.001	0.047	0.036	0.039	0.129***	0.070	-0.018	0.052
	(0.018)	(0.028)	(0.052)	(0.029)	(0.034)	(0.039)	(0.051)	(0.039)	(0.095)
Realized Shock	-0.058***	0.034	-0.120**	-0.162***	-0.252**	-0.026	-0.267*	-0.017	0.321***
	(0.017)	(0.040)	(0.054)	(0.036)	(0.099)	(0.065)	(0.142)	(0.039)	(0.108)
Cashflow Based Debt Ratio	0.003	-0.015	-0.079***	0.017**	0.017**	-0.074***	-0.016	0.017**	-0.020
	(0.007)	(0.009)	(0.014)	(0.007)	(0.008)	(0.015)	(0.012)	(0.007)	(0.020)
R-Squared	0.082	0.044	0.072	0.132	0.124	0.095	0.074	0.016	0.096
Observation	25594	25594	25594	25594	25594	25594	25594	25594	25594

Panel B: Implied Uncertainty Shock

	(1) Employment Growth	(2) Intangible Asset Growth	(3) Debt Growth	(4) Cost of Good Growth	(5) Sales Growth	(6) Cash Growth	(7) Profit Growth	(8) Payout Growth	(9) Stock Return
Implied Shock # Cashflow Based Debt Ratio	0.097**	0.097	0.181	0.096	0.103	0.254**	0.189	-0.087	-0.049
	(0.047)	(0.075)	(0.122)	(0.079)	(0.113)	(0.125)	(0.158)	(0.092)	(0.165)
Implied Shock	-0.183**	0.031	-0.384***	-0.516***	-0.651**	-0.047	-0.597	-0.072	1.426***
	(0.079)	(0.099)	(0.144)	(0.137)	(0.282)	(0.179)	(0.427)	(0.095)	(0.294)
Cashflow Based Debt Ratio	0.005	-0.006	-0.086***	0.013	0.015	-0.092***	-0.011	0.008	-0.004
	(0.009)	(0.013)	(0.019)	(0.010)	(0.010)	(0.020)	(0.016)	(0.012)	(0.021)
R-Squared	0.088	0.049	0.070	0.100	0.071	0.098	0.075	0.025	0.005
Observation	17119	17119	17119	17119	17119	17119	17119	17119	17119

Note: This table displays the results of the first regression but with asset based loan ratio. It presents the results of both the OLS and IV regressions with the realized and implied volatility shocks. The coefficients of the control variables have been omitted. Firm-fixed effects and time-fixed effects have been incorporated into each regression. Standard errors are shown in parasynthesis. ***: $p<0.01$, **: $p<0.05$, *: $p<0.1$

Appendix B: Appendix to the Financial Accelerator Model

B1. Optimal Conditions for the Asset Based Financial Accelerator

Solving the contracting problem will give us the key relationship in the traditional financial accelerator model. Denote the Lagrange multiplier as Λ_{t+1}. Solving the first order condition will give us:

$$\begin{aligned}
[\omega] :& E_t[f_\omega(\bar{\omega}_{t+1}, \sigma_t) + \Lambda_{t+1} g_\omega(\bar{\omega}_{t+1}, \sigma_t)] = 0 \\
[L] :& E_t[R^k_{t+1} f(\bar{\omega}_{t+1}, \sigma_t) + \Lambda_{t+1} R^k_{t+1} g(\bar{\omega}_{t+1}, \sigma_t) - \Lambda_{t+1} R_t] = 0 \\
[\Lambda] :& E_t[\frac{g(\bar{\omega}_{t+1}, \sigma_t) R^k_{t+1} L_t}{L_t - 1}] = R_t
\end{aligned}$$

After log-linearization The first FOC becomes

$$\begin{aligned}
\bar{\omega} f_{\omega\omega}(\bar{\omega}, \sigma)\hat{\omega}_{t+1} + \Lambda g_\omega(\bar{\omega}, \sigma)\lambda_{t+1} + \Lambda \bar{\omega} g_{\omega\omega}(\bar{\omega}, \sigma)\hat{\omega}_{t+1} \\
+ \sigma f_{\omega\sigma}(\bar{\omega}, \sigma^2)\hat{\sigma}_t + \Lambda \sigma g_{\omega\sigma}(\bar{\omega}, \sigma)\hat{\sigma}_t = 0
\end{aligned}$$

Use the fact of $\Lambda = -f_\omega(\bar{\omega}, \sigma)/g_\omega(\bar{\omega}, \sigma)$, we have

$$\lambda_{t+1} = [\frac{\bar{\omega} f_{\omega\omega}(\bar{\omega}, \sigma)}{f_\omega(\bar{\omega}, \sigma)} - \frac{\bar{\omega} g_{\omega\omega}(\bar{\omega}, \sigma)}{g_\omega(\bar{\omega}, \sigma)}]\hat{\omega}_{t+1} + [\frac{\sigma f_{\omega\sigma}(\bar{\omega}, \sigma)}{f_\omega(\bar{\omega}, \sigma)} - \frac{\sigma g_{\omega\sigma}(\bar{\omega}, \sigma)}{g_\omega(\bar{\omega}, \sigma)}]\hat{\sigma}_t$$

Define the first term and the second term as Φ_ω and Φ_σ, then

$$\lambda_{t+1} = \Phi_\omega \hat{\omega}_{t+1} + \Phi_\sigma \hat{\sigma}_t \tag{B1}$$

The second and the third FOC becomes

$$\begin{aligned}
r^k_{t+1} - r_t + \frac{\bar{\omega} f_\omega(\bar{\omega}, \sigma)}{f(\bar{\omega}, \sigma)}\hat{\omega}_{t+1} + \frac{\sigma f_\sigma(\bar{\omega}, \sigma)}{f(\bar{\omega}, \sigma)}\hat{\sigma}_t = \lambda_{t+1} - l_t \\
\frac{\bar{\omega} g_\omega(\bar{\omega}, \sigma)}{g(\bar{\omega}, \sigma)}\hat{\omega}_{t+1} + \frac{\sigma g_\sigma(\bar{\omega}, \sigma)}{g(\bar{\omega}, \sigma)}\hat{\sigma}_t + r^k_{t+1} = r_t + \frac{1}{L-1} l_t
\end{aligned}$$

Define $\zeta_f = \frac{\sigma f_\sigma(\bar{\omega},\sigma)}{f(\bar{\omega},\sigma)}$ and $\zeta_g = \frac{\sigma g_\sigma(\bar{\omega},\sigma)}{g(\bar{\omega},\sigma)}$, the second and the third FOC becomes

$$r^k_{t+1} - r_t + \theta_f \hat{\omega}_{t+1} + \zeta_f \hat{\sigma}_t = \lambda_{t+1} - l_t \tag{B2}$$

$$r^k_{t+1} - r_t + \theta_g \hat{\omega}_{t+1} + \zeta_g \hat{\sigma}_t = \frac{1}{L-1} l_t \tag{B3}$$

Combine the three FOC equations, we have

$$(r^k_{t+1} - r_t) = \frac{\Phi_\omega - \theta_f - \theta_g(L-1)}{(L-1)(\Phi_\omega - (\theta_f - \theta_g))} l_t - \frac{\theta_g(\zeta_f - \Phi_\sigma) + (\Phi_\omega - \theta_f)\zeta_g}{\Phi_\omega - (\theta_f - \theta_g)} \hat{\sigma}_t$$

Lastly, use the relationship $\theta_g(L-1) = -\theta_f$, we have

$$r^k_{t+1} - r_t = \frac{\Phi_\omega}{(L-1)\Phi_\omega - \theta_f L} l_t - \frac{\theta_g(\zeta_f - \Phi_\sigma) + (\Phi_\omega - \theta_f)\zeta_g}{\Phi_\omega - (\theta_f - \theta_g)} \hat{\sigma}_t$$

B2. Optimal Conditions for the Earning Based Financial Accelerator

The process of solving the earning based financial accelerator is not significantly distinct from the first-order conditions we have previously resolved for the conventional asset-based financial accelerator. The first order conditions are

$$\begin{aligned}
[\omega] &: h_\omega + f_\omega L_t = \Lambda_t g_\omega \\
[L] &: (1 - D_t) f(\bar{\omega}_{t+1}, \sigma)(L_t - 1)^2 + (1-\mu)\Lambda_t D_t = 0 \\
[\lambda] &: (1 - D_t) g(\bar{\omega}_{t+1}, \sigma) R^k_{t+1} = R_t - (1-\mu) R^k_{t+1} D_t \frac{L_t}{L_t - 1}
\end{aligned}$$

log-linearize the first equation we have:

$$\text{(B4)} \qquad \begin{aligned}
&\left(\frac{\omega h_{\omega\omega}}{f_\omega L} - h_\omega \frac{\omega g_{\omega\omega}}{g_\omega f_\omega L}\right)\hat{\omega}_t + \left(\frac{\sigma h_{\omega\sigma}}{f_\omega L} - h_\omega \frac{\sigma g_{\omega\sigma}}{g_\omega f_\omega L}\right)\sigma_t \\
&\qquad + \Phi_\omega \hat{\omega}_t + \Phi_\sigma \hat{\sigma}_t + l_t = \frac{\Lambda}{\Lambda - h_\omega / g_\omega} \lambda_t
\end{aligned}$$

Notice that with μ close to 0, we can simplify the equation into $\Phi_\omega \hat{\omega}_t + \Phi_\omega \sigma_t + l_t = \frac{\Lambda}{\Lambda - 1}\lambda_t$.

The second FOC will give us

$$\frac{\omega f_\omega}{f}\hat{\omega}_t + \frac{\sigma f_\sigma}{f}\sigma_t + 2\frac{L}{L-1} l_t = \lambda_t + \frac{1}{1-D} d_t$$

which can be written as

$$\text{(B5)} \qquad \theta_f \hat{\omega}_t + \zeta_f \sigma_t + 2\frac{L}{L-1} l_t = \lambda_t + \frac{1}{1-D} d_t$$

The third FOC will give us:

$$r^k_{t+1} - r_t = \frac{DL(1-\mu)}{(L-1)^2}\frac{R^k}{R}l_t - (1 - g\frac{R^k}{R})d_t - (1 - \frac{R^k}{R}\frac{LD(1-\mu)}{L-1})(\theta_g \hat{\omega}_t + \zeta_g \sigma_t) \tag{B6}$$

B3. The Full Financial Accelerator Model

Resource Constraint Block:

The market clearing condition in the model is

$$Y_t = C_t + I_t + G_t + C^e_t$$

Notice that $T_t = G_t$. The equation after log-linearization is

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{G}{Y}g_t + \frac{C^e}{Y}c^e_t \tag{B7}$$

Household:

The household solves the following problem:

$$max \sum_t \beta^t E_t[log(C_t) + \xi log(1 - H_t)]$$
$$s.t.\ C_t + B_{t+1} - T_t = W_t H_t + R_{t-1}B_t$$

The constraint is the budget constraint. The household receives labor income and interest rate income from varies assets. Notice that the households do not have access to the fixed capital. They can only buy the government bond, or save using the bank, and they will only face the risk-free interest rate.

Solving the first order condition first gives us the Euler's equation:

$$\frac{1}{C_t} = \beta R_t E_t[\frac{1}{C_{t+1}}]$$

which after the log-linearization gives us:

$$c_t = -r_t + E_t c_{t+1} \tag{B8}$$

The second equation comes from the household FOC is the intratemporal tradeoff between consumption and labor. After the log-linearization we have:

$$\frac{H}{1-H}h_t = w_t - c_t \tag{B9}$$

Raw Capital Producer and Aggregate Capital Accumulation:

Notice that the fixed capital is produced by the raw capital good producer and hold by the entrepreneurs. The following equation is the first important piece for the financial accelerator model. With the existence of the financial friction, we should use the definition of the return of fixed capital:

$$E[R^k_{t+1}] = E_t[\frac{RR_{t+1} + (1-\delta)Q_{t+1}}{Q_t}]$$

Notice that this implies that $E[R^k_{t+1}] \neq R_t$ because fixed capital is not directly hold by the households. We now solve for the the log-linearized version of this equation.

$$E_t r^k_{t+1} = \frac{RR}{RR+(1-\delta)}E_t rr_{t+1} + \frac{1-\delta}{RR+(1-\delta)}E_t q_{t+1} - q_t$$

We can define $\epsilon = \frac{1-\delta}{RR+(1-\delta)}$ as a parameter. Then we have

$$E_t r^k_{t+1} = (1-\epsilon)E_t rr_{t+1} + \epsilon E_t q_{t+1} - q_t \tag{B10}$$

In the economy, we assume there are raw capital producers who produce in a perfect competition market with free entry. They borrow the existing capital K_t from the household and buy final goods at a price of 1. They produce using the technology of $\Phi(\frac{I_t}{K_t})K_t$. After the production they return all the existing capital goods back to the households and sell the newly produced investment good at the market price Q_t. The raw capital producers solve the following problem:

$$max\ Q_t\Phi(\frac{I_t}{K_t})K_t - I_t$$

They also sell all the existing fixed raw capital and buy new capitals in each period at the price Q_t. We assume that $\Phi(0) = 0, \Phi(\delta) = 1, \Phi'(\delta) = 1$. Their choosing variable is I_t. The FOC of this problem will give us the definition of

Tobin's Q:

$$Q_t = [\Phi'(\frac{I_t}{K_t})]^{-1}$$

We can take logs and solve for $q_t = -\Phi''(\delta)\delta(i_t - k_t)$. Notice that we define $-\Phi''(\delta)\delta = \varphi > 0$ as a parameter. Hence the log-linearized equation is:

$$q_t = \varphi(i_t - k_t) \tag{B11}$$

Lastly, the capital accumulation equation evolves according to the following equation, which can be transferred by taking logs:

$$k_{t+1} = \delta k_t + \frac{I}{K} i_t - \delta k_t + (1-\delta)k_t$$

which means the log-linearized equation is:

$$k_{t+1} = (1-\delta)k_t + \delta i_t \tag{B12}$$

The FOC with respect to K_{t+1} will give us that $\lambda_t = \Phi'(\frac{I_t}{K_t})\mu_t$.

The New Keynesian Block:

The final good producer use the Dixit-Stiglitz packaging technology. The final good producer solves the following problem:

$$\begin{aligned} &max\ P_t Y_t - (\int_0^1 P_{it} Y_{it} di) \\ &s.t.\ Y_t = (\int_0^1 Y_{it}^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}} \end{aligned}$$

The first order condition gives us that

$$Y_{it} = (\frac{P_{it}}{P_t})^{-\eta} Y_t,\ P_t = (\int_0^1 P_{it}^{1-\eta} di)^{\frac{1}{1-\eta}}$$

Notice that the demand function implies that $Y_{it} = \Upsilon_t Y_t$, where Υ_t denote the price dispersion:

$$\Upsilon_t = (1-\theta)(\frac{P_{it}^{\star}}{P_t})^{-\eta} + \theta(\frac{P_{it-1}}{P_t})^{-\eta}$$

with probability $1-\theta$, the intermediate firm will change the price to the optimal level $\Pi_t^\star = \frac{P_{it}^\star}{P_t}$. With probability θ, the price is fixed at the previous level. Hence:

$$\Upsilon_t = (1-\theta)(\Pi_t^\star)^{-\eta} + \theta \Pi_t^\eta D_{t-1}$$

By taking log-linearization we notice that $\upsilon_t = 0$, hence $y_{it} = y_t$ and that is why we can ignore y_{it}. Next, the wholesale firms use effective capital and labor to produce. Notice that the wholesale firms borrow capital from the entrepreneurs, and will return the capital back to the entrepreneurs. Their outputs are the intermediate goods Y_t^M. Suppose they are monopolistic competitors, and the markup is defined as $X_t = P_t/MC_t$:

$$\begin{aligned} min\ RR_t K_t + W_t H_t + W_t^e H_t^e \\ Y_t^M \leq A_t K_t^\alpha (H_t^\Omega H_t^{e1-\Omega})^{1-\alpha} \end{aligned}$$

The first order condition gives us that:

$$\begin{aligned} P_t RR_t &= MC_t \alpha \frac{Y_t}{K_t} \\ P_t W_t &= MC_t (1-\alpha)\Omega \frac{Y_t}{H_t} \\ P_t W_t^e &= MC_t (1-\alpha)(1-\Omega) \frac{Y_t}{H_t^e} \end{aligned}$$

Using the definition of the real markup, we Can change P/MC into X. We assume that the entrepreneurs have 1 unit of labor endowment and will not have any disutility from working. Hence we assume that $H_t^e = 1$, and it will simplify the equation. After log-linearization, we get:

$$rr_t = y_t - k_t - x_t \tag{B13}$$

$$w_t = y_t - h_t - x_t \tag{B14}$$

$$w_t^e = y_t - x_t \tag{B15}$$

The aggregate production function after log-linearization gives us that

$$y_t = a_t + \alpha k_t + (1-\alpha)\Omega h_t \tag{B16}$$

The retailer who are allowed to change price in period t solves the following problem:

$$maxE_t \sum_{k=0}^{\infty} \theta^k \beta^k \frac{C_t}{C_{t+k}} [(\frac{P_{it}}{P_{t+k}})^{1-\eta} Y_{t+k} - X_{t+k}^{-1} (\frac{P_{it}}{P_{t+k}})^{-\eta} Y_{t+k}]$$

where θ denotes the probability of not changing the price. θ denotes the price stickiness. When $\theta = 1$ it corresponds to the most sticky case. When $\theta = 0$, we are in the flexible price case.

The optimal relative reset price $\Pi_t^\star = \frac{P_{it}^\star}{P_t}$ satisfies

$$\begin{aligned}
\Pi_t^\star &= \frac{\eta}{\eta - 1} \frac{X_{1,t}}{X_{2,t}} \\
X_{1,t} &= X_t^{-1} Y_t + \theta E_t \Lambda_{t,t+1} \Pi_{t+1}^{\eta} X_{1,t+1} \\
X_{2,t} &= Y_t + \theta E_t \Lambda_{t,t+1} \Pi_{t+1}^{\eta-1} X_{2,t+1} \\
1 &= (1-\theta)(\Pi_t^\star)^{1-\eta} + \theta(\Pi_t)^{\eta-1}
\end{aligned}$$

Following the standard procedure we can obtain the usual NKPC function:

(B17) $$\pi_t = -\kappa x_t + \beta E_t \pi_{t+1}$$

where $\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta}$.

External Finance Premium and Entrepreneur's Problem:

Each period, $1-\gamma$ fraction of entrepreneurs die and consume their net worth. Hence the aggregate consumption of entrepreneurs is:

$$C_t^e = (1-\gamma) V_t$$

where V_t is entrepreneurial equity. After the fraction of entrepreneurs die, the left will form the new net worth by:

$$N_t = \gamma V_t + W_t^e$$

Since γ is close to 1 and W_t^e is almost 0, we have $V_t \approx N_t$, and:

(B18) $$c_t^e = n_t$$

For the asset-based financial accelerator model, notice that the value of the entrepreneurial equity should be the following:

$$V_t = R_t^k Q_{t-1} K_t - R_{t-1}(Q_{t-1}K_t - N_{t-1}) - \mu \int_0^{\bar{\omega}_t} \omega_t \phi(\omega_t) R_t^k Q_{t-1} K_t d\omega_t$$

The first term is the entire return from sending capitals to the wholesale firm. The second term is the fraction obtained by the bank. The third term is the monitoring cost that vanished. Hence, the entire return is divided by three pieces, the expected bank profit, the expected entrepreneur profit, and the monitoring cost. We can transfer the monitoring cost back to household such that there is no waste in the economy. Hence, the aggregate evolution equation of N_t is

$$N_t = \gamma[R_t^k Q_{t-1} K_t - R_{t-1}(Q_{t-1}K_t - N_{t-1}) - \mu \int_0^{\bar{\omega}_t} \omega_t \phi(\omega_t) R_t^k Q_{t-1} K_t d\omega_t] + W_t^e$$

Notice that we can ignore the integral part of the model which is super small due to the fact that μ is close to 0. Take logs and linearize the equation will result in the following aggregate net worth transition equation:

$$\begin{aligned} n_t =& \frac{\gamma RK}{N}(r_t^k - r_{t-1}) + \gamma R(r_{t-1} + n_{t-1}) \\ &+ (R^k - R)\frac{\gamma K}{N}(q_{t-1} + k_t + r_t^k) + \frac{W^e}{N} w_t^e \end{aligned} \tag{B19}$$

For the earning-based financial accelerator model, notice that the aggregate evolution equation of N_t is:

$$\begin{aligned} N_t =& \gamma[(1-\mu)R_t^k Q_{t-1} K_t - R_{t-1}(Q_{t-1}K_t - N_{t-1}) \\ &- \mu(1 - D_t) \int_0^{\bar{\omega}_t} \omega_t \phi(\omega_t) R_t^k (Q_{t-1}K_t - N_t) d\omega_t] + W_t^e \end{aligned}$$

where for the last term, only the high order terms are affected, hence the aggregate transition equation of N_t after log-linearization is not affected by the last term. However, the first term is also affected by $(1-\mu)$, which denotes the loss of financial efficiency coming from directly holding claims to the future capital earnings by the bank. As a result the final aggregate net worth transition equation after log-linearization for the earning based financial accelerator can be written as:

$$
\begin{aligned}
n_t =& \frac{\gamma RK}{N}(r_t^k - r_{t-1}) + \gamma R(r_{t-1} + n_{t-1}) - \mu D R^k L(d_t + r_t^k + q_{t-1} + k_{t-1}) \\
&+ (R^k - R)\frac{\gamma K}{N}(q_{t-1} + k_t + r_t^k) + \frac{W^e}{N} w_t^e
\end{aligned}
$$

Finally, to close the asset-based and earnings-based financial accelerator models, we will incorporate the corresponding external finance premium equation derived from formal contracting problems, and the monetary policy rule.

B4. Steady State

In order to determine the parameters put into the model, we will need to first solve for the steady state. From solving the steady state we can also gather all the equations that will be used in the DSGE model. At the steady state we have:

Equation (B7) becomes $Y = C + I + G + C^e$.

Equation (B8) becomes $R = \frac{1}{\beta}$.

Equation (B9) becomes $\frac{\xi}{1-H} = \frac{W}{C}$.

Equation (B10) becomes $R^k = RR + (1 - \delta)$.

Equation (B11) becomes $Q = 1$.

Equation (B12) becomes $I = \delta K$.

Equation (B13)-(B15) becomes:

$$
\begin{aligned}
XRR &= \alpha \frac{Y}{K} \\
XH &= (1 - \alpha)\Omega \frac{Y}{W} \\
X &= (1 - \alpha)(1 - \Omega)\frac{Y}{W^e}
\end{aligned}
$$

Equation (B16) becomes $Y = AK^\alpha (H^\Omega)^{1-\alpha}$.

Equation (B17) becomes $X = \frac{\eta}{\eta - 1}$.

Equation (B18) becomes $C^e = (1 - \gamma)N$.

Equation (B19) becomes:$N = \gamma[R^k K - R(K - N)] + W^e$.

For the asset-based financial accelerator models, equation (B1) - (B3) becomes

$$f_\omega(\bar{\omega},\sigma)+\Lambda g_\omega(\bar{\omega},\sigma)=0$$
$$R^k f(\bar{\omega},\sigma)+\Lambda R^k g(\bar{\omega},\sigma)=\Lambda R$$
$$\frac{g(\bar{\omega},\sigma)R^k L}{L-1}=R$$

Also by definition, $L=K/N$. From the equations above, solve for the endogenous variables at the steady state for the asset-based financial accelerator. For the earning-based financial accelerator models, equation (B4) - (B6) becomes

$$h_\omega(\bar{\omega},\sigma)+f_\omega(\bar{\omega},\sigma)L=\Lambda g_\omega(\bar{\omega},\sigma)$$
$$(1-D)f(\bar{\omega},\sigma)(L-1)^2+\Lambda D=0$$
$$(1-D)g(\bar{\omega},\sigma)R^k=R-R^k D\frac{L}{L-1}$$

Lastly, equation equation (6) becomes $\psi D=(1-D)\Phi(\bar{\omega},\sigma)$.

From the 18 equations above, solve for the endogenous variables at the steady state for the earning-based financial accelerator.

B5. Definition of the Shocks

- Monetary Policy Shock: $r_t^n=\rho r_{t-1}^n+\zeta\pi_{t-1}+e_t^{rn}$, where r_t^n denotes the nominal interest rate.
- TFP Shock: $a_t=\rho_a a_{t-1}+e_t^a$.
- Government Spending Shock: $g_t=\rho_g g_{t-1}+e_t^g$.
- Risk Shock: $\sigma_t=\rho^s\sigma_{t-1}+e_t^s$.
- Net Worth Shock: $n_t=\gamma\bar{R}\bar{K}/\bar{N}(r_t^k-r_{t-1})+\gamma*\bar{R}*(r_{t-1}+n_{t-1}+e_t^n)+(\bar{R}^k-\bar{R})\gamma\bar{K}/\bar{N}(q_{t-1}+k_t+r_t^k)+\bar{W}^e\bar{N}*w_t^e$, where e_t^n denotes the shock.
- Labor Supply Shock: $w_t-c_t=(\bar{H}/(1-\bar{H}))h_t+e_t^h$.
- Markup Shock: $\pi=-\kappa x_t+\beta\pi_{t+1}+e_t^x$.
- Observation Shock: $y_t^{obs}=y_t+e_t^{obs}$.

B6. Financial Heterogeneity Model

The financial heterogeneity model is the assembling of the asset based and earning based the financial accelerator. Let's denote the asset based sector with a superscript of A, and denote the earning based sector with a superscript of E. Then the equations to be included in the model are composed of the following:

First, the sector level equations that comes from the raw capital producers, the wholesale good producers:

- Capital return:

$$r_t^{kA} = (1-\epsilon)rr_t^A + \epsilon q_t^A - q_{t-1}^A$$
$$r_t^{kE} = (1-\epsilon)rr_t^E + \epsilon q_t^E - q_{t-1}^E$$

- Capital good producer FOC:

$$q_t^A = \varphi(i_t^A - k_{t-1}^A),\ q_t^E = \varphi(i_t^E - k_{t-1}^E)$$

- Capital Accumulation:

$$k_t^A = \delta i_t^A + (1-\delta)k_{t-1}^A$$
$$k_t^E = \delta i_t^E + (1-\delta)k_{t-1}^E$$

- Wholesale good production technology:

$$y_t^A = a_t + a_t^A + \alpha k_{t-1}^A + (1-\alpha)\Omega h_t^A$$
$$y_t^E = a_t + a_t^E + \alpha k_{t-1}^E + (1-\alpha)\Omega h_t^E$$

- Wholesale labor FOC:

$$w_t = y_t^A - h_t^A - x_t^A,\ w_t = y_t^E - h_t^E - x_t^E$$

- Return to capital:

$$rr_t^A = y_t^A - k_t^A - x_t^A,\ rr_t^E = y_t^E - k_t^E - x_t^E$$

Second, the equations that combines the sector level variables to define the aggregate level variables, along with the equations of the packaging technology:

- Definition of the total final goods produced:

$$y_t = Jy_t^A + (1-J)y_t^E$$

- Definition of the total labor:

$$h_t = Jh_t^A + (1-J)h_t^E$$

- Definition of the aggregate inflation:

$$\pi_t = J\pi_t^A + (1-J)\pi_t^E$$

- Definition of the overall markup:

$$x_t = Jx_t^A + (1-J)x_t^E$$

- FOC of the packaging technology:

$$\pi_t^A - \pi_t^E = -(y_t^A - y_t^E - (y_{t-1}^A - y_{t-1}^E))$$

Third, the equations of the entrepreneurs inside the asset based and the earning based financial accelerator. This includes equation (4) and (5) for the entrepreneurs operating in the asset based sector, and equation (6), (7), (8) and (9) for the earning based sector.

Lastly, the equations from the general equilibrium. This includes the FOCs of the consumers and the market clearing conditions.