Price Stickiness, Capital Adjustment, and Monetary Policy Shocks

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What is the cost of the price stickiness in terms of real variables? In this paper, I show with industrial-level and firm-level price stickiness data that the heterogeneity in the price stickiness leads to the different responses of a firm's capital adjustment and other real variables to an exogenous monetary policy shock. In response to a contractionary MP shock, the investments of the firms with stickier prices drop more. This result is in line with a traditional multi-sector new Keynesian model with investment. The firms with stickier prices cannot react in price as much as the others to the contractionary MP shock, and a higher relative price leads to lower demand. With lower demand, the capital and labor inputs are both affected more, resulting in heterogeneous responses.

Price stickiness is the heart of macroeconomics. It is a phenomenon well-documented by the literature. In the early 2000s, Bils and Klenow (2004) and Nakamura and Steinsson (2008) provided detailed facts about the price setting patterns of the US firms using the micro-level price data underlying the component of the Consumer Price Index (CPI) and the Producer Price Index (PPI). Not only did they find a relatively high price stickiness, with a median frequency of monthly price change of finished producer goods in 1998-2005 of 10.8%, but also a huge variation across major groups. For example, the median price change frequency of the farm product over the same period was about 87.5%, while the lumber and wood products have a price changing frequency as low as 1.3%.

The huge gap in the price stickiness between sectors, or even firms, motivates the question of how costly the price stickiness is. Obviously, to answer this question it is crucial to define the word "cost" properly. In fact, Gorodnichenko and Weber (2016) has already looked into this question and measured the cost of price stickiness with the volatility in the stock price. They found that the stock price for firms with stickier prices will become more volatile after an exogenous monetary policy shock (MP shock). However, a similar question is whether the other real variables, for example, investment, react differently because the firms have different price stickiness.

Why is investment or capital change also an important measure of the cost of price stickiness? Compared to the asset prices, the capital change of a firm represents the decision of the firm manager of the firm and has a more direct impact on the real variables instead of the nominals. Investment is an important component of the GDP, and it is not only super-sensitive to the business cycle fluctuation but also highly heterogeneous across sectors and firms. Recently, there is a huge amount of literature focusing on the heterogeneity in the reaction of the investment to the MP shocks. With the growing access to more and more detailed granular micro-level data, people have found margins that contribute to the heterogeneous behavior of investment across firms. For example, people have tried to give credit to idiosyncratic shocks and nonconvexities in the adjustment cost (Khan and Thomas (2008), Winberry (2021)), the financial constraint (Ottonello and Winberry (2020), Jeenas (2018), Howes (2020)), the cash flow effect (Gürkaynak, Lee and Karasoy Can (2019)), or the difference in the firm-level uncertainty (Kroner (2021)). One of the goals of this paper is to quantify the effect of price stickiness on the heterogeneous response in firms' investment to the exogenous MP shock.

One reason that this question hasn't been answered yet by the literature is that the micro-level price change data is not easy to access. In this paper, I depend on two datasets containing the price-changing data. The first and the most widely used dataset, the PPI Research Database from the production files underlying the U.S. PPI, has been cleaned by Nakamura and Steinsson (2008) and Pasten, Schoenle and Weber (2017), and the price changing frequency data is available at the industrial level. I complement this data with a second novel dataset, the Neilsen consumer panel data. In the Neilsen consumer panel data the consumption quantity and the total price for each good, identified with a unique UPC barcode, can be observed. Using the GS1 GEPIR dataset, the good price data can be linked to its producer. GS1 is the business entity that provides barcodes to products and records the firm name for each UPC available in the Nielsen data. The GEPIR allows me to have access to the names of the company producing a certain good observed from the Neilsen consumer panel data, and the linkage is from Afrouzi, Drenik and Kim (2021). To my best knowledge, this is the first paper using the Neilsen data to answer the question about price stickiness.

Both the industrial level cleaned PPI data and the Neilsen-GS1 linked data can be combined with the Compustat dataset, which contains the quarterly financial report of all the listed firms in the US, and hence I can observe the firm's capital data on a quarterly frequency. The industrial level cleaned PPI data is linked to the Compustat data using the NAICS code, while the Neilsen-GS1 linked data is manually linked to the Compustat data using the company names. With the linked firm-level price changing frequency data and the capital adjustment data, I first leverage the method from Gorodnichenko and Weber (2016) and show that the unpredictable industrial level capital change volatility is negatively related to the industrial level pricing changing frequencies, which means the industries that have higher price changing frequency have less capital changes that cannot be explained by the aggregate factors and the lag of the capital. This result provides a basic taste of the real cost of the price stickiness.

Next, I combine the price changing frequency data from different sources with the exogenous MP shock data identified using the high-frequency method from the literature (Bu, Rogers and Wu (2021) and Swanson (2021)) and perform a local projection exercise as in Jordà (2005) to estimate the impose response function (IRF) of the firm-level capital in response to the contractionary MP shocks. The advantage of using exogenously identified monetary policy and the local projection method is that it can provide a certain amount of exogeneity, and hence make the causality more clear.

The local projection analysis shows that although the firms' capital tends to drop in response to a contractionary MP shock, the firms operating in the stickier-price sector respond more. When the unified MP shocks increase the 2-year treasury yield by 100 bps, the capital response of the 20 percent of firms that operate in the most sticky-price industries drops by about 5-10 percent, while the capital of the 20 percent firms having the least price stickiness almost has no response. The difference between the gap is significant, and the interaction between the firm's price changing frequency and the MP shock is significantly positive, indicating that a higher price changing frequency leads to a less response in the capital adjustment.

To ease the concern about the endogeneity due to the simultaneity of price and capital choosing, I perform a certain amount of robustness checks of the baseline result. The idea is to include as many firm-level controls as possible, such that the firm-level idiosyncratic behaviors have the least confounding effect. I show that the heterogeneous response is robust to many concerns. First, I show that the main result is not affected by the sample period changes or the data source changes. The result is also robust to further controlling the interaction between the MP shocks and the firm-level controls, or the interactions between the shocks and the proxies of the financial constraint, which is one important margin that has been detected by the literature. The result is robust to adding the controls of the firm fixed effect and the time fixed effect. The first digit NAICS code or whether the firm is a durable good producer also has no effect on the relationship between price stickiness and the response of capital to the MP shocks. Third, not only the conventional monetary policy shocks but also the forward guidance shock and the large-scale asset purchase shocks that are more and more common in the post-08 period have bigger effects on the stickier firms. What's more, the heterogeneous response of firms with different price stickiness is not limited to the firm's investment. We can see that the firms operating in stickier price industries respond more also in terms of sales and employment.

Lastly, I move from the industrial-level PCF data to the firm-level PCF data calculated from the Neilsen dataset. The advantage of using the Neilsen data is to allow me to further control the interaction of the more detailed industry effect with the MP shock. It further alleviates the concerns that the baseline result is driven by the firm's endogenous response to the factors that are common among industries. We can see that the main result preserves even for the firm-level price changing frequency data. In a nutshell, the real effect of price stickiness is strong, at least in terms of investment.

To show the intuition of the empirical results, I build a medium-scale multi-sector new Keynesian model with investment. Based on Carvalho, Lee and Park (2021), I allow the firm owners to accumulate and frictionally adjust the capital. The link between different price stickiness and different responses in the capital input is clear, and the basic logic follows exactly from Carvalho, Lee and Park (2021). Given a contractionary MP shock, the firms tend to set lower prices. Due to the separate market, the price settings are strategically complementary to each other. Since the firms operate in sectors of different price stickiness, the prices of the firms in a stickier sector drop less, which makes their goods to be more expensive. With a higher relative price, the demand for their goods is relatively lower, and the inputs of labor and capital will be lower. The model thus predicts that the firms with higher price stickiness have relatively higher costs in the sense that the capital will drop more in response to a contractionary MP shock. After showing the basic intuition using a 2-sector model, I generalize the model to 180 sectors to quantitatively match the empirical result. However, the model shows that the amount of the different responses of the capital in MP shocks is limited. A model more general than a basic new Keynesian model is needed to fully interpret the gaps between the firms with different price stickiness. Even though the DSGE model didn't match entirely the heterogeneity in the capital response to the contractionary MP shock, the price stickiness heterogeneity in the new Keynesian model still manages to simulate about 50% of the variation on average observed in data for the first 8 quarters after the shock. One implication of the multi-sector model is that it generates larger responses than the one-sector model. Compared to the standard one-sector model, the multi-sector model in this paper has larger responses in the aggregate capital, output, and consumption.

In general, the answer to the question of whether price stickiness has any real cost is yes. The cost, in terms of investment responses to the MP shocks, is significant and cannot be ignored by the macroeconomics researcher.

The rest of this paper is organized as below: the next section discusses the related literature, section 2 shows the main empirical result using the industrial level price changing frequency data from the PPI, and section 3 describes the Neilsen data and the complementary empirical evidence using the firm-level price changing data, section 4 provides the basic intuition behind the evidence and discusses the multi-sector new Keynesian model with investment, and section 5 concludes.

I. Literature Review

This paper answers a similar question raised in Gorodnichenko and Weber (2016), which is closely related to the literature trying to empirically determine the degree of price stickiness and its impact. Other than the two papers mentioned in the introduction, the literature also tried different ways to measure the menu cost. For example, Zbaracki et al. (2004) and Anderson, Jaimovich and Simester (2015) directly measure the price stickiness with the observed menu cost. This paper follows the most widely used definition from Nakamura and Steinsson (2008) and defines the price stickiness with the price changing frequency, and leverages a novel dataset to answer this question.

For the dependent variable, this paper focuses on the investment changes over the business cycle. Gourio and Kashyap (2007) pointed out that the extensive margin contributes more to the investment changes over the business cycle, but they didn't dig deep into the granular correlation between a firm's investment and the firm-level characteristics. This paper also contributes to the recent growing literature quantifying the heterogeneous effect on a firm's capital dynamics of the monetary policy shocks. On one hand, the identification of the MP shock using high-frequency data is widely used in the literature now. Gürkaynak, Sack and Swanson (2005) used this method to depart the forward guidance shocks from the traditional MP shocks, and this method is generalized in Swanson (2021) to include the Large Scale Asset Purchase (LSAP) shocks. A similar method has been used to study the different reactions of different asset prices to different MP shocks, for example, Campbell et al. (2012), or the different reactions of people's expectations and the information effect, for example, Nakamura and Steinsson (2018). The baseline MP shock used in this paper comes from Bu, Rogers and Wu (2021), which generated a unified measurement of the conventional and unconventional MP shocks. On the other hand, the different responses in the investment draw more and more attention. Besides the channel mentioned in the introduction, this paper provides a price-stickiness channel that is consistent with the theoretical framework of the traditional new Keynesian model.

The last strand of literature that this paper is related to develops the multi-sector new Keynesian model framework. Christiano, Eichenbaum and Evans (2005) used the medium-scale DSGE models to quantitatively estimate the drives of the business cycle. Justiniano, Primiceri and Tambalotti (2011) included the investment-specific shocks and studied its impact. People also have tried to include capital into the multi-sector new Keynesian model, such as Carvalho and Nechio (2017) and Carvalho and Nechio (2016). Some recent studies like Pasten, Schoenle and Weber (2017) showed that the heterogeneity in price stickiness has an amplification effect on the idiosyncratic shocks. Rubbo (2020) studied how the optimal monetary policy trade-off the inflation between the sectors. This paper follows directly from Carvalho and Nechio (2017) and shows that the difference in the price stickiness could lead to different responses in the capital, but the scale of the response difference is hard to be quantitatively matched to the data, implying the room for including some extra channel into the DSGE model.

II. Empirical Evidence with Industrial Level Price Stickiness

A. Data

Price stickiness is defined as the price-changing frequency (PCF) of a given industry over a certain period. The main price-changing data used in the paper is based on the micro-level data underlying the PPI from the BLS to calculate the frequency of price adjustment at the industry level. The BLS measures PPI from the view of the producers directly by sending surveys to the participating establishments on a monthly frequency.[1] Even though I don't directly have access to the confidential microdata from BLS, the data aggregated at the industrial level is available from Pasten, Schoenle and Weber (2017). According to the authors, the price change frequency is defined as the number of price changes to the number of sample months at a good level. Then they aggregate the good-based price changing data to the BEA industry classification level. Hence, for each industry, there is only one PCF data for the entire sample period. The data covers from the year 2005 to 2011. I link the BEA industry classification to the NAICS codes using the official crosswalk provided by the BEA.[2] For each BEA industry classification code, there is a NAICS 2-6 digit codes to which can be matched. After deleting the data for the utilities and the finance department (7 industries), I end up with 334 industries.

The first panel of figure 1 listed the PCF for the industries in the data. As can be observed from the graph, most of the data covers the manufacturing industries, but the data also covers a certain amount of agriculture, mining, construction, and services industries. The PCF data have a large variation, with a minimum of 2.74% and a maximum of 89.59%.

Another PCF data that I choose is the industrial level data aggregated by Nakamura and Steinsson (2008). Based on the same PPI confidential good level PCF data, they aggregate the PCF data into the 4-digit BLS product codes, covering the year 1999 to 2005. Similar to Pasten, Schoenle and Weber (2017), they also have one observation for the entire sample period for each industry. Since the BLS product codes don't have a one-to-one relationship with the existing industrial code such as the NAICS or the SIC codes, I link the BLS product code

[1]At this point the PPI data has been widely used by the literature. A detailed discussion of the procedure of the PPI data collection can be found in Nakamura and Steinsson (2018).

[2]The link between the BEA industry classification and the NAICS code can be found in the BEA website: https://apps.bea.gov/scb/pdf/2007/10%20October/007_benchmark_io.pdf

with the NAICS codes by manually checking the product names from the BLS product codes and the NAICS codes. It allows me to aggregate the PCF data into about 90 groups, covering mostly manufacturers and the first industry, but no service providers. The PCF varies from 2.10% to almost 100%, which is similar to the data from Pasten, Schoenle and Weber (2017). The full detail of the link is discussed in the appendix. Because the data is more ancient in history, and the linkage between the codes is not one-to-one, I only use this data as a complement to the baseline analysis.

The firm-level capital input data and other firm-level controls come from the Compustat dataset, which contains the quarterly financial report of the listed firms in the US. Following the standard procedure, I keep the firms that are reported in US dollars and delete the firms operating in the finance and utilities department. Then, I choose the firms that cover the years that can be matched to the different sample sources of the PCF data. As a result, the sample size may vary when the analysis period varies. As in Kroner (2021) and Jeenas (2018), I choose the changes in the firm's capital stock to measure the investment impulse responses, instead of using the observable investment rate directly from calculating using the capital expenditures, due to the volatility in the firm's investment and the difficulty that it raised in the estimation procedure. The capital stock is calculated using the perpetual inventory method, and the nominal variable is translated to the real term using the investment deflator. For the baseline regression, I end up with 3,255 firms for the period from 2005 to 2011. I linked them to the industrial PCF data according to the NAICS codes that those firms belong to. Panel B of figure 1 plots the industrial level PCF from the lowest to the highest and the number of firms in each industry in the data. All the firm-level data are winsorized at the 1-th and the 99-th percentile for each quarter. The data description of the key variables is listed in table 1.

Lastly, I also use the aggregate monetary policy shock identified using the high-frequency method from the literature. For the baseline estimation, I choose the unified MP shock identified by Bu, Rogers and Wu (2021). In their paper, they constructed a unified monetary policy shock that combined both the conventional and the unconventional monetary policy shock, based on the cross-sectional regression method from Fama and MacBeth (1973). The exogenous MP shock that they identified was scaled such that 1 unit increase in the shock corresponds to 100 bp increase in the 2-year treasury yield. Since the monetary policy changes only happen during the FOMC announcement, I manually add the shocks to quarterly frequency such that it has the same frequency as the financial report in Compustat. In addition to the relatively new measurement of the monetary policy, I also use the monetary policy shock from Swanson (2021), in which the author generalized the first principal component decomposition procedure in Gürkaynak, Sack and Swanson (2005) and provided a detailed conventional and unconventional MP shock covering from 1994 to 2019.

B. Industrial Level Evidence

First, with the industrial level PCF data I can check the relationship between the investment changes and the industrial price stickiness. Similar to the logic in Gorodnichenko and Weber (2016), the sectors with lower price stickiness tend to be more volatile in their responses to the shocks, and the shocks are not necessarily monetary policy shocks. I first calculate the industrial level investment changes, which are defined using the error term from the following regression at the firm level:

$$\Delta log(k_{i,t}) = \beta_0 + \beta_1 log(k_{i,t-1}) + \sum_c \beta_c X^c_{i,t} + \epsilon_{i,t}$$

where $\Delta log(k_{i,t}) = log(k_{i,t}) - log(k_{i,t-1})$, and X^c_t are the control variables. The controls of the regression include 3 parts. The first group of controls are the aggregate controlling variables including the 4 quarter lags of the inflation, GDP growth, unemployment rate, the excess bond premium from Gilchrist and Zakrajšek (2012), The second groups of controls are the firm-level financial indicators, including the lag of the firm's log size, the cash flow ratio, the leverage ratio, liquid asset ratio, capital share, Tobin's q, the log of firm's capital stock, and the sale growth rate. The last group in the controls are the firm fixed effect, a linear yearly effect, and a quarterly effect. After calculating the firm-level estimation errors from the above regression, I calculate the standard error of the firm-level investment change that cannot be explained by the above regression over the sample period 2005 to 2011, and aggregate the variations to the industrial level, using the median of all the firms in a given industry, such that it can be linked to the PCF data from Pasten, Schoenle and Weber (2017). Using the median level of the capital fluctuation across firms prevent an overestimation of the volatility caused by some extreme big or small firms with industries.

In figure 2, I plot the industrial level investment volatility and the industrial level PCF. we can see that there is a negative relationship between the investment volatility and the PCF when we directly regress the industrial level median of the standard error of the unexpected investment change on the industrial level PCF. The negative relationship is stronger when we weight the regression with the industrial level firm number or the capital share of each industry over the sample period. A higher PCF implies a lower price stickiness, and hence a stickier industry tends to have a more volatile investment.

C. *Firm Level Investment and Monetary Policy Shock*

The exercise above provides a basic taste of the relationship between the investment and the price stickiness, however, the regression without explicitly specifying the shock to the economy is exposed to the endogeneity problems. At a certain time, the firms might be exposed to different shock, and hence might have different reactions. To ease this concern, I choose one specific aggregate shock, the monetary policy shocks, and perform a local projection using the firm-level data following Jordà (2005). More specifically, I first run the following regression to check the IRF of the firm's capital stocks in response to an MP shock:

$$\Delta log(k_{i,t+h-1}) = c_{i,h} + \beta_{0,h} m_t + \alpha_h X_{i,t} + e_{i,t+h}$$

where $\Delta log(k_{i,t+h-1})$ is defined as $log(k_{i,t+h}) - log(k_{i,t-1})$, m_t is the identified MP shocks, for the baseline I choose the unified MP shocks identified from Bu, Rogers and Wu (2021), and $X_{i,t}$ are similar to the regression in the previous regression without the firm fixed effect while adding the 4-digit NAICS code industrial fixed effect. The IRF of the firm's capital changes are $\beta_{0,h}$, and it should be negative, since the traditional business cycle theory predicts that the contractionary MP shock should lead to a shrink in the capital stock. Then, I run the regression adding the interaction between the firm's price changing frequency and the MP shocks:

$$\Delta log(k_{i,t+h-1}) = c_{i,h} + \beta_{0,h} m_t + \beta_{1h} m_t PCF_i + \alpha_h X_{i,t} + e_{i,t+h}$$

The main focus of this regression is $\beta_{1,h}$, which proxies the heterogeneous effect on the investment of firms with different price stickiness.[3] The main effect $\beta_{0,h}$ should still be negative. I cluster the regression error at the industrial level for all the above regressions and run the local projections for 0 to 7 quarters ahead, corresponding to the capital changes in 2 years following an MP shock. For the baseline regression, I keep the sample period between 2005 and 2011 to match the PCF data from Pasten, Schoenle and Weber (2017).

Figure 3 shows the main result of this paper. First, when we run the local projection on the monetary policy shocks only, the main effect is negative, indicating a negative change in the firm's investment following the contractionary MP shock. However, the second panel shows that when we include the interaction term of the firm's PCF and the MP shock, the coefficient β_{1h} is significantly positive. This means a firm that changes price more frequently will have a much less response in their investment when hit by the contractionary MP shock. Quantitatively a 10 percent increase in the price changing frequency decrease the response of the firm's capital adjustment by about 2.5 percentage points in response to the 1 unit increase in the unified MP shocks. The scale of the effect seems to be large, considering that the main effect shows that the average drop in the investment is about 6-8 percentage points. Hence, the heterogeneous effect of price stickiness seems to be relatively large. One might be concerned that the effect of the price stickiness is not linear and might wonder if the effect of the price stickiness could change the direction of the investment response. Hence, in panel C I directly compare the 20% of the firms that have the highest PCF with the 20% having the lowest PCF. We can see that for the low PCF group, the investment responds negatively to the interest rate increase, while the high PCF group barely has significant changes. In Panel D we can see that the IRF difference between the high PCF group and the low PCF is significantly positive. I draw from this exercise the conclusion that the effect of price stickiness is large, but not crazily large such that it changes the direction of the main effect on investment.

[3] I omit the level controls of the PCF since the PCF will be the same for the firms operating in the same industry and the industry fixed effect has been controlled in the regression. The coefficient in front of the PCF will be absorbed by the industry fixed effect.

D. Further Discussions

The effect of price stickiness is not only large but also robust. In this section I will show that the key result of this paper is robust to the sample period selection, the change of data source, further controlling for additional confounding variables, the change in the format of monetary policy shocks, or the response of other real variables.

Sample Period Selection: In the baseline local projection, the regression is limited to the sample period between 2005 and 2011 to match the firm-level financial report data to the industrial level PCF data from Pasten, Schoenle and Weber (2017). However, the identified monetary policy shock begins in the 1990s, and the traditional local projection method requires a relatively long period to obtain more credible IRFs. Suppose we generalize the data under the assumption that the price changing the frequency of the industry does not change much over time, we will be able to run the local projections of the firm's capital adjustment for the full sample. Panel A of figure 4 plots the regression result using the full sample, covering from the year 1994 to 2019. The figure again shows that the coefficient in front of the interaction between the PCF and the monetary policy shocks is significantly positive.

Data Source Change: The baseline regression uses the PCF data aggregated to the BEA industry classification level from Pasten, Schoenle and Weber (2017). This is not the only PCF data that I have access to. In Nakamura and Steinsson (2008) the authors also used the confidential micro-level PPI data and aggregated the PCF data into the PPI product code level. For simplicity, I denote the first as Pasten's data and the second as NS data. Compared to Pasten's data, NS data covers from 1998 to 2005, providing a nice complement to the baseline estimation. Panel B of figure 4 shows the result of the same regression using NS data. The regression remains to give us a significantly positive coefficient in front of the interaction term in the first and second quarters ahead, and the scale of the effect doesn't change much compared to the baseline regression.

Confounding Controls: Other than the price stickiness, there are many margins that people have already discovered that could lead to heterogeneous responses to the firm's investment. How do these margins interact with the heterogeneity in the firm's price stickiness? Can we still observe the significant relationship between the firm's PCF and the response in the firm's capital adjustment once we control for the effect of these confounding margins? After a massive amount of robustness checks, the answer is yes. The heterogeneous effect that the price stickiness brings in is robust.

The first and most important channel leading to heterogeneity in the responses of investment to MP shocks is the financial constraint channel. Howes (2020) have found that the different financial constraint that the manufacturing firms and the service firms have could lead to different investment responses. Ottonello and Winberry (2020) showed that the firms with high "distance to default" are the most responsive to monetary shocks. In order to control for the effect of the different financial constraint, I further control different proxies of the firms' financial constraint or financial frictions. I run the following regression:

$$\Delta log(k_{i,t+h-1}) = c_{i,h} + \beta_{0,h}m_t + \beta_{1h}m_tPCF_i + \beta_{2h}m_tP_{i,t} + \alpha_hX_{i,t} + e_{i,t+h}$$

in which $P_{i,t}$ is the proxy of the firm's financial constraint and it is contained in $X_{i,t}$. The main results are shown in figure 5.

The firm's size is one of the most widely used proxies for the firm's financial constraints. In panel A I showed that the baseline correlation between the PCF and the response in the firm's investment is robust to controlling the interactions between the firm's size and the monetary policy shocks. It also shows that the result is robust to further controlling all the interactions between the firm-level controls and the monetary policy shocks. In panel B I use the probability to default that have been used in Ottonello and Winberry (2020) and Rezghi (2022)[4]. In panel C I choose the firm's dependency on external finance and the investment intensity used in Rajan and Zingales (1998) as the proxies of the firm's financial constraint. In both panel B and panel C I tried to further control the interactions including all the other firm-level controls and all the regressions give me similar results. In conclusion, although the financial constraint plays an important role in the firm-level investment dispersion, the price stickiness is an equally important factor that drives the different reactions.

Another proxy for the firm's financial constraint is the industry in which the firm operates. As discussed in Howes (2020), the manufacturing firms have more

[4]The probability to default is defined following the definition of distance to default calculated in Bharath and Shumway (2008). The distance to default model is based on Merton (1974) and has been widely used in the financial literature. Since the calculation of the probability to default requires the volatility of the daily stock price, I obtain the data from the CRSP and the sample size is smaller than the baseline regression.

fixed capital than the service sector, and hence these firms face different financial constraints.[5] Controlling for this interaction not only alleviates the concern about the confounding effect of the firm's financial constraint but also controls other effects that are common among industries and haven't been picked up by the baseline regression. Panel A of figure 6 shows the regression result controlling the heterogeneous effect of the firms operating in different 1-digit NAICS code industries.

The second concern that people might have is about the type of producer. For example, Barsky, House and Kimball (2003) pointed out that when durable goods have flexible prices the multi-sector sticky-price models might behave differently. In response to these concerns, I further control the interaction between whether a firm is a durable good producer and the MP shocks on top of the controlling of the industrial effect.[6] As shown in panel A of figure 6, neither the industrial effect nor the type of the producers changes the relationship between the firm's price stickiness and the response in their investments.

Other than the firm's financial constraint, the industrial effect, and the type of goods that the firm is producing, people might still be concerned that some other unobservable firm-level characteristics can also be mixed with the firm's price stickiness. Hence, as the last robustness check, in panel B of figure 6 we can see that even if we further control the firm's fixed effect or the time fixed effect[7], the estimation of β_{1h} is still significantly positive.

Conventional and Unconventional MP Shocks: In the baseline regression we use the unified monetary policy identified from Bu, Rogers and Wu (2021), which combined the conventional and unconventional MP shocks together. As pointed out in Barakchian and Crowe (2013), the forward guidance shocks has to be included such that the responses to the aggregate level capital adjustment can have the correct direction. A question raised naturally is whether there are any difference in response to different monetary policy shocks. We change from the unified monetary policy shock to the recently identified MP shock in Swanson

[5]Both the Pasten's data and the NS data correspond to PCF at the industrial level, so we can only control the interactions between the industrial effect and the monetary shock for the 1-digit NAICS code. A more precise interaction will absorb the key coefficient $\beta_{1,h}$.

[6]A firm is defined as a durable good producer according to the NAICS code that the firm belongs to. If a 4-digit NAICS code of the manufacturing industry can correspond to a department in the durable good wholesaler, we treat it as a durable good manufacturing industry.

[7]Notice that the main focus of our regression is the interaction term between the aggregate MP shock and the firm's PCF, so we can still identify β_{1h}, but not β_{0h} when we control for the time fixed effect.

(2021). The author generalized the method in Gürkaynak, Sack and Swanson (2005) such that the MP shocks are separated into conventional MP shock, forward guidance shock, and Large Scale Asset Purchasing(LSAP) shocks. Doing regression similar to the baseline, we can separate the response to different type of MP shocks, and the result is plotted in figure 7. The result shows that the effect of price stickiness is the strongest for the traditional MP shock and the effect is weaker for the forward guidance shocks and the LSAP shocks, even though for all three different types of shocks the coefficient is significantly positive.

Other Real Variables: Until now we define the cost of price stickiness as the response of the firm's capital adjustment to the MP shocks, yet other real variables could also have different reactions when the firm is operating in sectors that can change their prices at different frequencies. Figure 8 shows the response of the firm's real sales and the firm's employment. Both the firm's sale and the employment have less impact when the firm's price stickiness is lower, corresponding to a higher PCF. The effect of the real sales is large and more sensitive, showing an overshooting effect after several quarters, partly due to the high volatility of the firm's sales. The estimated effect on the employment is more persistent, mainly because the employment data can only be observed at an annual frequency and linear interpolation are used.

In general, the impact of price stickiness is strong, robust, and not limited to the firm's investment, showing concrete empirical evidences of the real cost of the price stickiness.

III. Firm Level Price Changing Frequency Evidence

In section 2 I used the industrial level price changing frequency obtained from the literature. Both Pasten's data and NS data aggregate the good level PCF data from the PPI microdata. As a complement to them, I use a novel dataset, the Neilsen household panel data, to provide more detailed price-changing data at the firm level. In this section, I will first discuss the Neilsen dataset and then use it to rerun the baseline empirical local projection exercise, further controlling for a more detailed industrial effect.

A. Data

The Neilsen consumer panel data is available at the Kilts Marketing Data Center at the University of Chicago Booth School of Business. The data surveyed consumers at a weekly frequency for an average of 60,000 households each year in the US, and it contains the total spending on each good and the quantity of each purchase. Hence, we can use the purchase data to derive the price of the goods in the dataset.[8] A good is identified using a barcode, which is a unique universal product code (UPC). According to Neilsen, the consumer panel data covers more than 4.5 million different UPCs and covers about 30 percent of the consumption in the CPI basket. The sample data starts in 2004 and ends in 2019, right before the pandemic period.

Besides the Neilsen consumer panel data, I also used the GS1 GEPIR dataset to determine the relationship between a barcode and the firm that produced it. The GS1 is the official company that provides barcodes to products and records the firm name for each UPC available in the Nielsen data. The GEPIR dataset allows me to manually check and link the company name with the product, identified with the UPC, that the firm produces. Following the linking procedure in Afrouzi, Drenik and Kim (2021),[9] I manually link the UPC to the name of the firms and then using the company names and the address record in GEPIR and Compustat, I construct the link between the products and the firms that produced them. The linkage gives me a sample of 233 firms that coexists in Compustat, GEPIR, and the Neilsen consumer panel data. Although the number of firms seems to be small, the sales of the products cover about 1/4 of the total sales, according to Afrouzi, Drenik and Kim (2021).

Since the main focus of the paper is the price of the good, the survey weight assigned to the consumer becomes less relevant to the analysis. There are several issues worth mentioning. First, most firms provide more than only one product. For each product, the firm could set the price differently, and hence the price changing frequency should be an aggregated summary of all the products that the firm produces. Second, notice that we can only observe the price of a product if there is a purchase of the good during the observing week. Hence, we need to make an assumption when we do not observe the purchase of the good. I assume

[8]Nakamura and Steinsson (2008) pointed out that the price changing frequency might have different behavior depending on whether we consider sales. In the Neilsen dataset, the total spending is defined as the spending before the discount. Hence, the issue of sales has been considered.‘’

[9]To manually link the UPC with the firms from GEPIR, I obtained the list of firms that can be linked from Afrouzi, Drenik and Kim (2021), according to the author, the firms that have the potential to be linked is small (about 300), which makes the manual linkage possible. Special thanks to Professor Ryan Kim at JHU. Also, the free GEPIR database only shows 20 random matches between the GS1 Company Prefix (the first several digits of the UPC) and the firm name so I assume that the rest data is randomly missing.

that the price of the product does not change when there are no observations, which adds downwards bias to the estimation of the price changing frequency. Also notice that the price of a good with the same UPC can have a different price at different places and stores, due to the pricing strategy that a firm chooses. The method that I choose to overcome this difficulty is to treat the goods sold in different regions differently. Lastly, Nakamura and Steinsson (2008) pointed out that taking into account the fact that some goods are sold on sale might change the price stickiness. Following the tradition of this literature, I take the price before the sales to generate enough price-changing frequencies consistent with the previous literature.

Hence, to take into account all four concerns, the final data processing procedure is as follows. Define a good as a unique UPC in a specific county and a specific state. Calculate the average price for each good for any given week from the purchase data. Then calculate the number of price changes divided by the length of weeks that can be observed for each good, generating the price changing the frequency at the good level. The price change counts if it changes by more than 1 percent of the old price, or if the change is larger than 1 cent. I omit the goods that last less than a month (4 weeks) in the dataset, which are likely to be the goods that fail quickly in the market. Then, I take the median of the PCF for a given UPC over the different regions. Lastly, I calculate the median over the UPCs for a given firm and assign it as a proxy of the firm-level PCF. The average PCF obtained using the Neilsen data is about 0.15 (scaled to monthly frequency), which is similar to the Pasten's data, but the variation is much smaller (0 to about 0.5 at a monthly frequency).

B. Local Projection with Firm-Level PCF

After cleaning the data and linking the Neilsen dataset to Compustat data, I run the baseline local projection again for the firm's investment. The monetary policy is still the unified MP shock identified in Bu, Rogers and Wu (2021). Keeping the assumption that the price changing frequency doesn't change much over time, I checked the local projection using the data from 2004 to 2019. As a robustness check, I also run the regression using the full sample, covering the year 1994 to 2019. As can be seen in panel A of figure 9, both regressions give me a significant positive coefficient in front of the interaction term, showing a robust regression result.

The firm-level PCF data from linking Neilsen and the Compustat dataset is a good complement to the industrial-level PCF data that I obtained from the literature. The first advantage is that it provides a different data source, making the baseline regression result more credible. The second and more important advantage the new data can provide is that with the variation at a more detailed firm level, we can further control the interaction between the 4-digit NAICS code and the monetary policy shock, while before we can only do so for the 1-digit NAICS code. A more detailed industry control allows me to give more credit to the effect of price stickiness. As shown in panel B of figure 9, after controlling for the interactions between the 4-digit NAICS code and the MP shock, the regression becomes more significant and the coefficient remains to be positive. The regression using the firm-level PCF data thus provides consistent evidence showing that the firms with sticker prices responses more in their investment to a contractionary monetary policy shock.

IV. DSGE Model

In the previous sections, I have shown the empirical evidence of the real cost of the sticky price. In this section, I will use a multi-sector new Keynesian model to show the linkage from the heterogeneity in the price stickiness to the different responses of the investment to a contractionary monetary policy shock. The multi-sector medium scale new Keynesian model with frictional investment has been studied by the literature, for instance, see Carvalho and Nechio (2016) and Carvalho, Lee and Park (2021) for details. The main purpose of leveraging the model in this paper is to show the transmit channel and attempt to quantitatively match the empirical result with the theory.

In the following I will first discuss the structure of the model, then put the model in a two-sector setup to show the basic intuition of the transmit channel. Lastly, I will generalize the model back to an M-sector model and try to match the result I obtained in the previous sections.

A. Model Structure

Households: There is one representative worker who owns the capital and lends the capital to type i firms separately. We have the optimization problem for the consumer:

$$
\begin{aligned}
max \ & \sum_{t=0}^{+\infty} \beta^t [\frac{C_t^{1-\sigma}}{1-\sigma} - \sum_{i=1}^{M} \phi_i \frac{n_{it}^{1+\eta}}{1+\eta}] \\
s.t. \ & P_t C_t + B_t + P_t \sum_{i=1}^{M} I_{it} \\
& = R_{t-1} B_{t-1} + P_t \sum_{i=1}^{M} w_{it} n_{it} + P_t \sum_{i=1}^{M} R_{it} K_{it-1} + \sum_{i=1}^{M} \int_0^1 \Pi_{ijt} dj \\
& K_{it} = (1-\delta) K_{it-1} + (1 - \frac{\Omega}{2} (\frac{I_{it}}{I_{it-1}} - 1)^2) I_{it}
\end{aligned}
$$

households demand the final good consumption C_t. There is one risk free asset B_t with one period of maturity, which have the nominal risk-free interest rate R_t. The consumer frictionally invest $I_{i,t}$ in the capital $K_{i,t}$ for each sector i, and the capital stock works separately for different sectors. The friction of the investment is defined with the cost function $\Omega/2(I_{it}/I_{it-1} - 1)^2$. The real capital return for sector i capital is $R_{i,t}$.

The household first-order conditions solve the following equations:

$$
\begin{aligned}
(1) \quad & Q_{t,t+1} = \beta E_t [\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}(1+\pi_{t+1})}] \\
(2) \quad & w_{it} = \phi n_{it}^{\eta} C_t^{\sigma} \\
(3) \quad & q_t = \beta E_t [\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} [R_{t+1} + (1-\delta) q_{t+1}]] \\
(4) \quad & 1 = q_t (1 - \frac{\Omega}{2} - \frac{3\Omega}{2} \frac{I_t}{I_{t-1}}^2 + 2\Omega \frac{I_t}{I_{t-1}}) + \beta \Omega E_t [q_{t+1} \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{I_{t+1}^2}{I_t^2} (\frac{I_{t+1}}{I_t} - 1)] \\
(5) \quad & K_{it} = (1-\delta) K_{it-1} + (1 - \frac{\Omega}{2} (\frac{I_{it}}{I_{it-1}} - 1)^2) I_{it}
\end{aligned}
$$

where the first equation defines the stochastic discount factor and the second determines the labor supply. In the third equation we define the real Tobin's Q, q_t, as the ratio of the market value of the investment divided by the asset replacement cost.[10]

Final good Producer: The final good producers operate in a perfect compe-

[10]Suppose the Lagrange multiplier of the budget constraint is defined as Λ_t and the Lagrange multiplier of the capital transition equation is μ_t, then Tobin's q is defined as $q_t = \mu_t / P_t \Lambda_t$.

tition market. They take the M different types of intermediate goods produced by the firms in different sectors and generate a final good using the CES technology. The optimization problem solves that

$$max\ P_tY_t - \sum_{i=1}^{M} P_{it}Y_{it}$$
$$s.t.\ Y_t = (\sum_{i=1}^{M} \mu_i^{\frac{1}{\gamma}} Y_{it}^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}}$$

where $\sum_{i=1}^{M} \mu_i = 1$ denote the share of GDP of industry i at the steady state. The parameter γ denotes the elasticity of substitution between sectors. Solving the first order conditions gives us the demand function of the intermediate goods and the aggregate price index:

$$Y_{it} = \mu_i(\frac{P_{it}}{P_t})^{-\gamma}Y_t \tag{6}$$

$$P_t^{1-\gamma} = \sum_{i=1}^{M} \mu_i P_{it}^{1-\gamma} \tag{7}$$

Intermediate Good Producer: The intermediate good firms also produce in a perfect competition market. For each sector i, the intermediate good producer has a CES technology and uses all the raw materials produced by the raw material firms operating in sector i. Suppose there is a continuum of raw good producer in each sector. The optimization problem for the intermediate good producer is:

$$max\ P_{it}Y_{it} - \int_0^1 P_{ijt}Y_{ijt}$$
$$s.t.\ Y_{it} = (\int_0^1 Y_{ijt}^{\frac{\theta-1}{\theta}} dj)^{\frac{\theta}{\theta-1}}$$

in which θ is the elasticity of substitution within sectors. The first order condition gives us the demand function of the raw material good firms:

$$Y_{ijt} = (\frac{P_{ijt}}{P_{it}})^{-\theta}Y_{it} \tag{8}$$

$$P_{it}^{1-\theta} = \int_0^1 (P_{ijt})^{1-\theta}dj \tag{9}$$

Raw Good Producer: The raw good producers operating in sector i compete with other firms within sectors under monopolistic competition. For firm $j \in [0,1]$, the firm combines the labor and the capital input using a Cobb-Douglas technology and first solve the cost minimization problem:

$$tc(Y_{ijt}) = min\ w_{ijt}n_{ijt} + R_{ijt}K_{ijt}$$
$$s.t.\ A_{ijt}n_{ijt}^{1-\alpha}K_{ijt}^{\alpha} = Y_{ijt}$$

The first order condition solves the ratio of the labor capital input, and defines the marginal cost:

$$\frac{w_{it}}{R_{it}} = \frac{(1-\alpha)K_{ijt}}{\alpha n_{ijt}} \tag{10}$$

$$mc_{it} = (1-\alpha)^{-(1-\alpha)}\alpha^{-\alpha}\frac{(w_t^i)^{1-\alpha}(R_t^i)^{\alpha}}{A_t^i} \tag{11}$$

After solving the cost minimization problem, we will solve for the profit maximization problem for firm j in type i industry. Calvo (1983) type price stickiness exists in the model. The firms in different industries have different price stickiness, i.e. for the firm operating in sector i, the probability that the firm can change the price of the product is $1-\lambda_i$. Suppose a firm can change the price in period t, the firm will solve the following problem, choosing the optimal price $P^\star$:

$$max\sum_{k=0}^{+\infty}(\lambda_i)^k Q_{t,t+k}[P^\star - P_{t+k}mc_{it+k}]Y_{ijt+k}$$

The first order condition solves that:

$$\sum_{k=0}^{+\infty}(\lambda_i)^k Q_{t,t+k}[(1-\theta)\frac{P_t^\star}{P_{t+k}} + \theta mc_{it+k}](\frac{P_{it+k}}{P_{t+k}})^{-1}(\frac{P_t^\star}{P_{t+k}})^{-\theta-1}Y_{it+k} = 0 \tag{12}$$

Government and Monetary Policy Shocks: The government in the model is a central bank that chooses the risk-free interest rate. The supply of the risk free bond is zero, and the monetary policy follows a standard Taylor principle with interest rate smoothing:

$$\frac{R_t}{\bar{R}} = (\frac{R_{t-1}}{\bar{R}})^{\rho_r}((1+\pi_t)^{\phi_\pi}\frac{Y_t}{\bar{Y}}^{\phi_y})^{1-\rho_r}e^{\epsilon_t} \tag{13}$$

where e_t is the monetary policy shock.

Market Clearing Conditions: In equilibrium all markets are clear. The final good market, the labor market, and the capital market-clearing conditions give us that:

$$C_t + \sum_{i=1}^{M} K_{it} = Y_t + \sum_{i=1}^{M}(1-\delta)K_{1t-1} \tag{14}$$

$$\int_0^1 n_{ijt} = n_{it} \tag{15}$$

$$\int_0^1 K_{ijt} = K_{it-1} \tag{16}$$

An equilibrium is defined as a set of endogenous variables such that given the price, the consumers, the final good producers, the intermediate good producers and the raw good producers solve their optimization problem, the monetary policy holds and the markets clear. The full set of equations to solve the equilibrium, the steady state, and the log-linearized equation system is listed in the appendix.

B. Two-Sector Model

Before we use the DSGE model to match quantitatively the empirical result, I will first set the number of sectors that have different price stickiness to be 2. The purpose of this exercise is to show the intuition about the linkage between the heterogeneous price stickiness and the different responses of the firm's investment.

Suppose the two sectors have different price stickiness. From the first order condition of the raw good producer and the definition of the price index we can derive the multi-sector Phillips curve after log-linearization:

$$\pi_{it} =(1-\lambda_i)(1-\lambda_i\beta)mc_t + \lambda_i\beta\pi_{it+1} \tag{17}$$

$$+(1-\lambda_i)\lambda_i\beta(p_{it}-p_t) - (1-\lambda_i)(p_{it-1}-p_{t-1}) + (1-\lambda_i)\pi_t \tag{18}$$

By definition the inflation in sector i is defined as $\pi_{it} = (p_{it}-p_t)+\pi_t-(p_{it-1}-p_{t-1})$. According to the price index of the final good, we have that the log-linearized price index gives that

$$0 = \sum_{i=1}^{M} \mu_i \frac{P_i}{P}^{1-\gamma} (p_{it} - p_t) \tag{19}$$

where $p_{it} - p_t$ is the log-linearized price index of industry i. This implies that the relative price of the sectors are different. Keeping the number of sectors to be 2, we have that when one sector's relative price is higher the other is naturally lower. From the Phillips curve we can see that when λ is larger, i.e. the price is stickier, the slope in front of the marginal cost is steeper. Hence the inflation change will be lower and more will be transmit to the real marginal cost.

To show the intuition more clearly, I solve the 2-sector model numerically using a very standard choice of parameters.[11] They are relatively unimportant since the main purpose of the 2 sector model is to show the basic intuition. In figure 10, I plot the comparison of the IRFs of the key variables for the two sectors. The shock is an unexpected increase in the current nominal interest rate. From the plot the channel is clear. In response to a contractionary monetary policy shock, the non-neutrality of the new Keynesian model predicts that the optimal price setting should decrease. However, when the two sectors have different price stickiness, the stickier sector will not be able to move its price as much as the other sector. By definition, the relative price of the two sectors will be different, and the stickier sector will now have a higher relative price. A higher relative price will lead to lower demand, and lower demand will drive both the employment and the capital input to be lower compared to the other sector. Hence, a higher price stickiness sector will respond more to the monetary policy shock.

C. Multi-Sector Model

We now move on to the multi-sector model to check if the traditional new Keynesian theory can explain quantitatively the variation in the responses of the capital. The essence and the basic logic of the model interpretation don't change.

We start with the calibration of the model. The main calibration is listed in table 2. In the spirit of simplification, I choose most of the parameters from the literature. The model is tuned to quarterly frequency. The discount factor β is

[11]The choice of the parameters is relatively unimportant since the main purpose of the 2-sector model is not to quantitatively match with the empirical result. Later I will take the calibration more rigorously and list in detail the parameter values.

chosen to match a 3% annual discount rate. We take log utility for consumption. The inverse Frisch elasticity η is chosen to be 2 as in Pasten, Schoenle and Weber (2017). The total labor supply is normalized to 1. For simplicity, I assume that the parameter in the disutility of the labor ϕ is the same across sectors. The elasticity of substitution across industries, θ, is set to be 2 according to Carvalho, Lee and Park (2021). The elasticity of substitution within industries for the raw goods is set to be 6, implying a markup of 20 percent. We assume that the capital share α in the technology is about 0.36, which follows the choice in Justiniano, Primiceri and Tambalotti (2011) and can be matched to the capital share observed in the US. Similarly, we choose the depreciation rate as 0.025, also following the same paper. The coefficient in the Taylor rule $\phi_\pi = 1.688$ and $\phi_y = 0.046$ are set according to Justiniano, Primiceri and Tambalotti (2011).

I choose the number of sectors M to be 180 to match the industries that can be observed from Compustat and Pasten's data. I target the sector price stickiness to Pasten's Data. Since the PCF observed in Pasten's data is at a monthly frequency, I rescale it to quarterly frequency and delete the industries that changes their prices more frequent than once a quarter. At the steady-state, I target the final good production parameter μ_i to the output share from the BEA IO make table (2012). The make table shows the production of commodities by industry. I aggregate the output and compute the GDP shares for any given industry.

Lastly, I choose the persistence of the monetary policy, ρ_m, and the investment adjustment cost Ω to target the empirical evidence shown in the previous sections. More specifically, these two parameters are tuned to match the main effect in the IRF of the capital stock and the gap between the 20 percent of the firms that have the most flexible price and 20 percent of the firms that have the most sticky price. The shock is scaled to match a 100-bp increase in the risk-free nominal interest rate, using the conventional MP shock identified in Swanson (2021).[12] The IRF targeting leverages the technique following Christiano, Eichenbaum and Evans (2005). The first 8 quarters of the IRF and the gap is matched to data, with the inverse of the local projection standard error as weights. After calibrating the parameters I simulate the model by generating an exact number of firms for each industry and running an OLS regression of the responses on the PCF. The

[12] I choose the FFR shock from Swanson (2021) mainly because the conventional MP shock identification is easier to be matched by the model. I rescaled the shock such that if we put the shock in a VAR model, a 1 unit increase in the shock will lead to a 100 bp increase in the FFR rate.

simulation result corresponds to the main focus of the empirical part of the paper, the interaction term between the PCF and the MP shocks. For generality, the regression using the actual data covers from the year 1994 to 2019.

In figure 12, I first plot the aggregate responses of the real variables in the economy. In response to the contractionary monetary policy shock, the new Keynesian model naturally predicts a drop in aggregate output, consumption and inflation.

In figure 11, I plot the IRF of the aggregate capital along with the simulation to match the empirical result. The left panel of figure 11 shows that the aggregate IRF of the capital that the model predicts matches well with the empirical result. However, the right panel shows that the model predicts a relatively smaller effect than the different PCFs could lead to. The coefficient obtained from the model simulation in front of the interaction between the PCF and the MP shock is still positive, implying the correct direction of the relationship as the empirical evidence shows, but the difference in the price stickiness is not enough to generate a sufficiently large heterogeneity that can be observed in the data. Quantitatively, the model predicts about 48 percent of the variation that can be observed from the data on average for the first 8 quarters.

The mismatch of the model and the data indicates that either the Compustat data have too much variation in the response in the capital adjustment, or the model lacks certain features that could interact with the heterogeneity in the price stickiness. For example, the model assumes that the sector-level capital adjustment frictions are the same, which is not true in the real world. The interactions between the heterogeneity in the capital adjustment frictions and the price stickiness can be interesting and potentially solve the problem that the DSGE models leave us with.

One implication of the multi-sector DSGE model is that it will generate larger movement at the aggregate level for the economy compared to the traditional one-sector model. As the last part of the exercise and an implication of the model, I compare the model with multiple sectors to the one-sector model. In the one-sector model, I choose most of the parameters to be the same as in the multi-sector model with only small changes. I choose price stickiness as the median of the sector level price stickiness and plug it into the one-sector model. I then rescale the parameter in the dis-utility function such that the aggregate labor supply is still one. Figure 12 shows the comparison between the two models. As the figure shows, the multi-sector model generates a similar level of responses on the impact on the aggregate output, consumption, and capital, but it also amplifies the effect after several periods. The impact is more persistent and larger in the multi-sector model compared to the one-sector model. The responses in inflation are similar in the two models, considering that the aggregate price stickiness is similar.

As a result, the heterogeneity in the price stickiness could not only help explain the different responses in the firm-level investment but can also amplify the responses at the aggregate level.

V. Conclusion

In this paper, I showed deterministic evidence of the different responses of the capital to a contractionary monetary policy shock. Using both the data widely used in the literature based on the price changing data from the micro level PPI data of the BLS, and a novel dataset combining the Neilsen consumer panel with the Compustat dataset, I showed the firm-level local projection result of the IRFs of the capital adjustment. The firms with stickier prices respond more in their investment, along with some other real variables, to the aggregate shocks such as the monetary policy shocks. The multi-sector new Keynesian model predicts the result and shows an amplification effect in the response of the aggregate capital.

In summary, the answer to the question of whether the price stickiness is costly is yes. Unlike the way answered in Gorodnichenko and Weber (2016), even if we measure the cost of price stickiness with the real variables instead of the asset prices, the cost still exists. In a nutshell, the heterogeneity in the price stickiness plays a non-trivial role in explaining the heterogeneous response of capital adjustment to the monetary policy shock.

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Table 1—Data Description

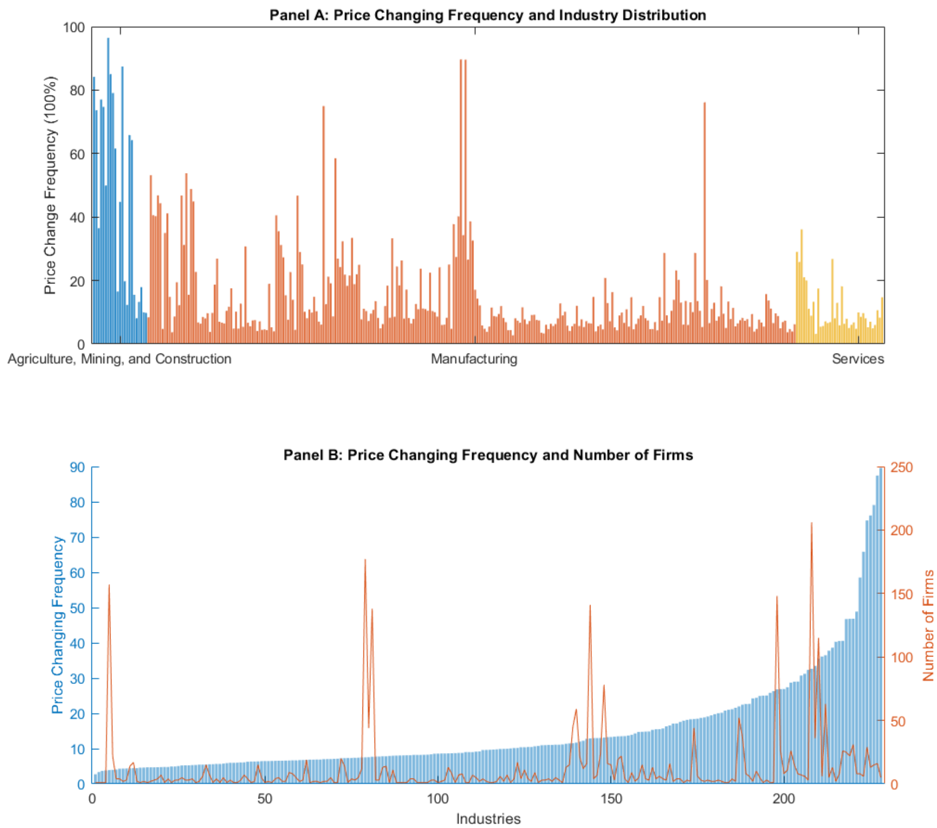
	Count	Mean	Standard Error	Min	Max
$\Delta log(k_{i,t})$	60660	0.0167	0.0884	-0.4734	0.6273
Lag Size	60660	5.4449	2.3018	-0.8981	10.8900
Lag Cash Flow Ratio	60660	-0.0386	0.9698	-130.5333	0.2544
Lag Debt Asset Ratio	60660	0.5510	0.5192	0.0439	5.1274
Lag Liquid Asset ratio	60660	0.2173	0.2334	0.0000	0.9367
Lag Capital Share	60660	0.2465	0.2323	0.0040	0.9170
Lag Tobin's Q	60660	2.5207	3.0198	0.4131	31.0393
Lag Growth of Real Sales	60660	0.0747	0.4090	-1.1453	2.5878
Lag log(Captial)	60660	3.8874	2.7114	-5.5814	11.6692
Lag Probability to Default	48356	0.1569	0.2300	0.0000	1.0000
Lag External Finance Dependence	38911	10.7557	63.6799	-202.0308	1151.5789
Lag Investment Intensity	38911	0.0132	0.0334	-0.2160	0.4519

Note: This table shows the main data description of the baseline regression. For the baseline only the data between the year 2005 and 2011 is used.

Table 2—Calibration of the Multi-Sector DSGE Model

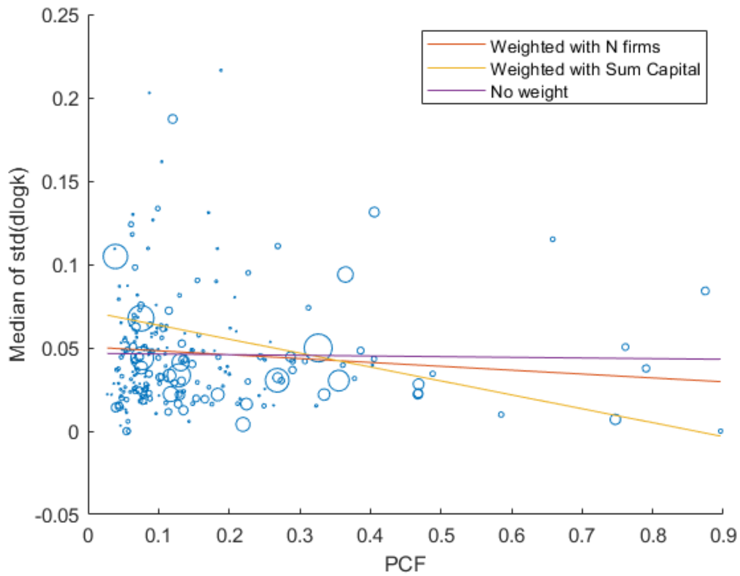
Meaning	Parameter	Value	Target
		Predetermined	
discount rate	β	0.993	3% annual discount rate, quarterly data
inverse Frisch elasticity	η	2.000	Pasten, Schoenle and Weber (2017)
utility parameter for leisure	ϕ	13655.413	normalize total labor to 1
elasticity between 2 sectors	γ	2.000	Pasten, Schoenle and Weber (2017)
capital depreciation rate	δ	0.025	Justiniano, Primiceri and Tambalotti (2011)
elasticity within sector	θ	6.000	Pasten, Schoenle and Weber (2017)
capital share	α	0.360	Justiniano, Primiceri and Tambalotti (2011)
Taylor rule parameter	ϕ_π	1.688	Justiniano, Primiceri and Tambalotti (2011)
Taylor rule parameter	ϕ_y	0.046	Justiniano, Primiceri and Tambalotti (2011)
		Calibration	
average price stickiness	λ	0.656	Pasten's PCF Data
average fraction of industry	μ	0.006	Sector GDP share at steady state
persistency of MP shock	ρ_m	0.911	Calibrated to match the empirical evidence
capital adjustment cost	Ω	0.142	Calibrated to match the empirical evidence

Note: This table shows the main parameter values calibrated for the multi sector Ney Keynesian model. Since there are 181 sectors in the model, the price stickiness and the fraction of the GDP for each industries are omitted from the table. Instead, I list the average price stickiness and the average GDP shares for the industries as a summary.



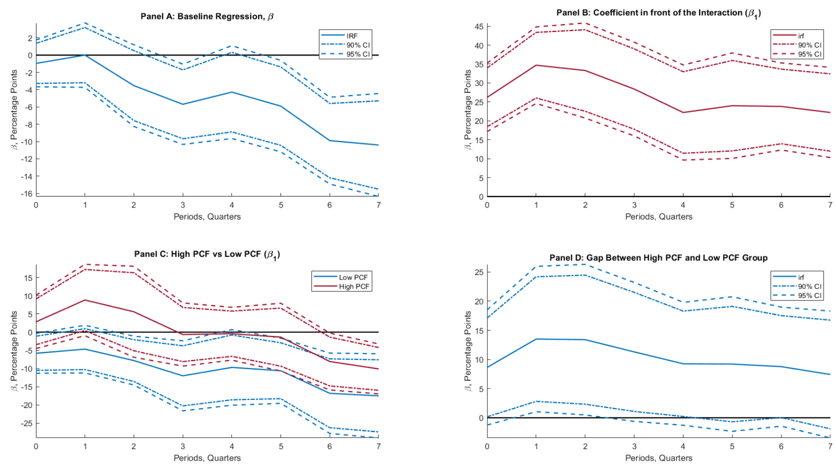
Note: This figure plots the Distribution of the industrial level price changing frequency(PCF). Panel A shows the 1-digit NAICS code that the industry belongs to. Panel B rearrange the industries in an ascending order and plot the number of the firms belonging to each industry that are in the Compustat dataset and can be linked to the data. The PCF data comes from Pasten, Schoenle and Weber (2017).

Figure 1. Industrial Price Changing Frequency Distribution



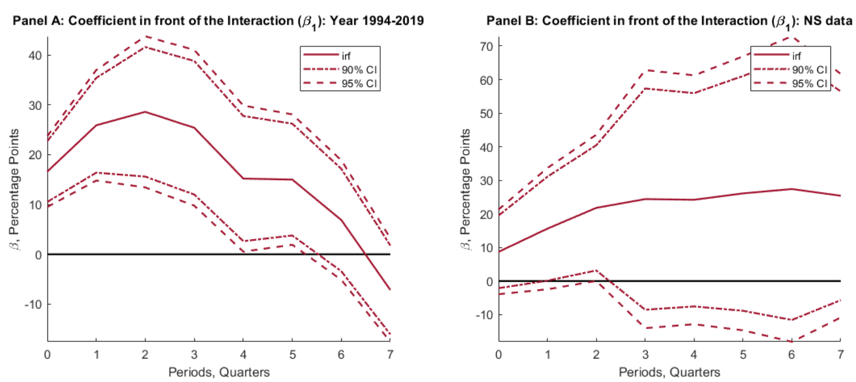
Note: This figure plots the industrial capital investment volatility and the PCF. Industrial investment volatility is defined using the median of the standard error of the unpredictable changes in each firm's investment in each industry. The size of the bubble indicates the number of firms in each industry. The three lines in the graph are the OLS estimation using different weights. The slope of the regression with no weight is -0.019, the regression with the weight of the number of firms is -0.028, and the regression with the weight of the total capital is -0.076.

FIGURE 2. INDUSTRIAL INVESTMENT VOLATILITY AND PCF



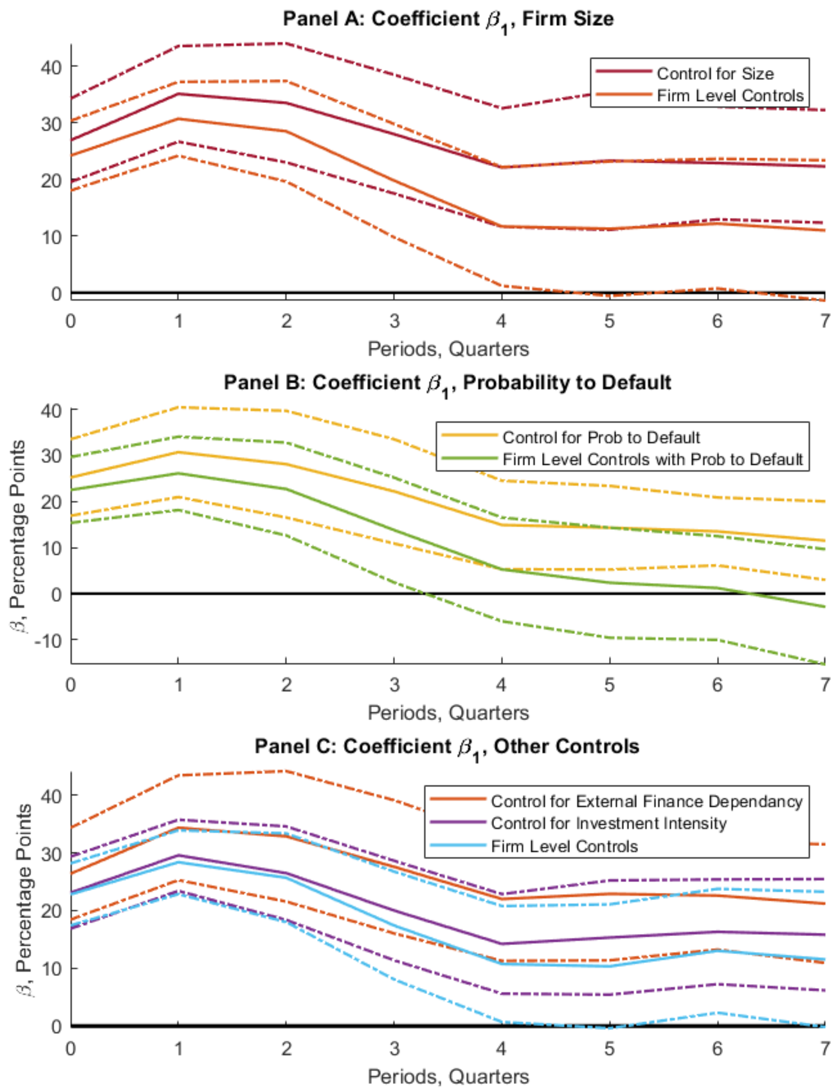
Note: Panel A shows the baseline result of the local projection of the firm's capital in response to a contractionary monetary policy shock, without the interaction term. Panel B shows the coefficient in front of the interaction between the MP shock and the firm's PCF. Panel C compares the IRF of the capital adjustment between the high PCF group and the low PCF group. Panel D plots the gap between the two groups. The sample period is from 2005 to 2011.

FIGURE 3. BASELINE LOCAL PROJECTION, IRFS OF THE CAPITAL



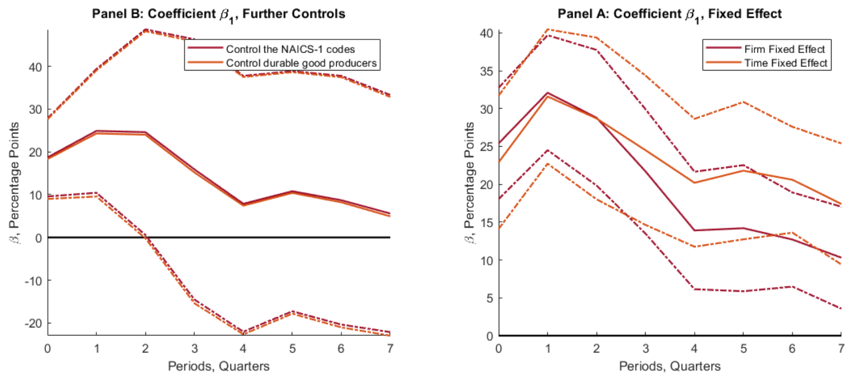
Note: Panel A runs the baseline regression using Pasten's data, but covers the time period from 1994 to 2019, instead of from 2005 to 2011. Panel B changes the PCF data from Pasten's data to NS data. The NS data covers from the year 1998 to 2005. The plot shows only the coefficient in front of the interaction term. The main effect of a contractionary MP shock is still negative in both regressions.

FIGURE 4. CHANGE THE SAMPLE PERIOD AND THE SAMPLE SOURCE



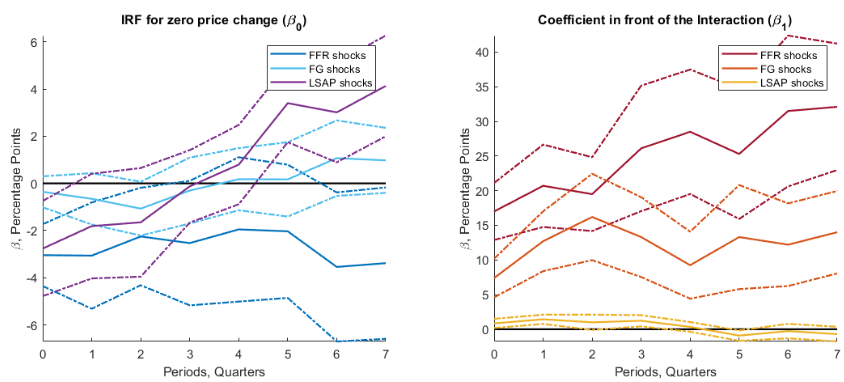
Note: Panel A runs the local projection with further control of the interactions between the firm size and the MP shocks. Panel B uses the probability to default as a proxy and interacts it with the MP shock. Panel C chooses the dependency of external finance and the investment intensity as proxies for the firm's financial constraint. All three panels include another type of regression that further controls the interaction between all the other firm-level controls and the MP shocks. The dashed line shows the 90% confidence interval.

Figure 5. Regression with Further Controls



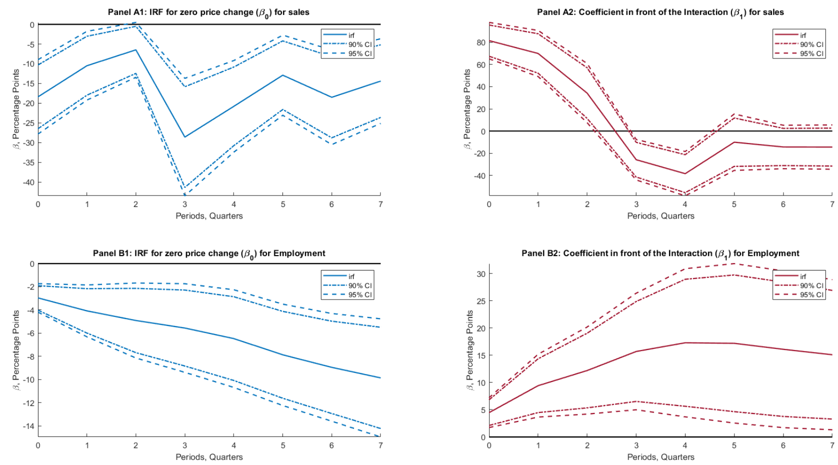
Note: Panel A runs the local projection with further control of the interactions between the dummies of the firm's 1-digit NAICS code and the MP shocks. It also includes the interactions of a dummy variable indicating whether a firm is a durable good producer. Panel B controls the firm fixed effect and the time fixed effect. The dashed line shows the 90% confidence interval.

Figure 6. Firm Fixed Effect and Time Fixed Effect and Further Controls



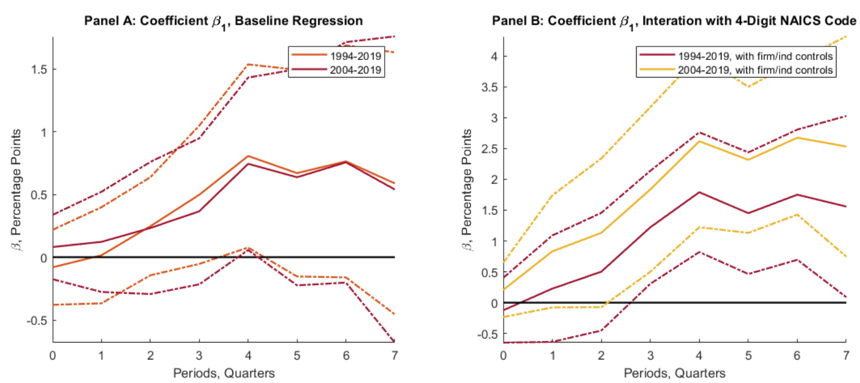
Note: The left subplot shows the main effect β_0 in the baseline regression using the MP shock identified from Swanson (2021), while the right shows the key coefficient β_1 for the three separated MP shocks. The dashed line shows the 90% confidence interval.

Figure 7. Responses to Different MP Shocks



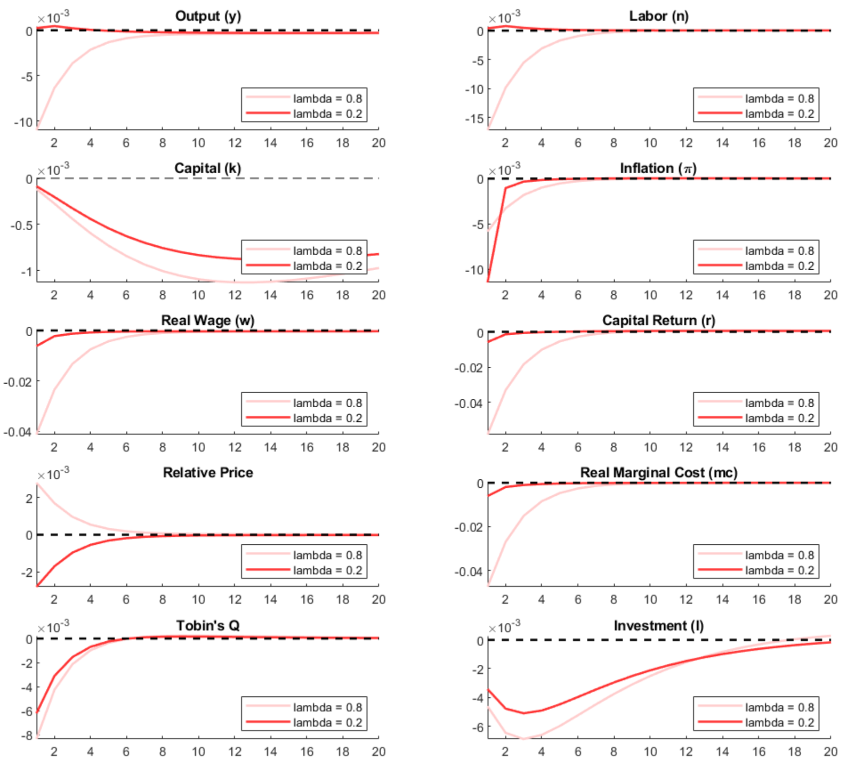
Note: The first line corresponds to the baseline regression with the real sales as the dependent variable, and the second line the employment. The left two graphs shows the main effect β_0, while the right graph shows the interaction term β_1. For the sales of the firm, I adjust the data with the inflation of the quarter using the Urban CPI data obtained from the Fed st. Louis. For the employment data, I used interpolation to update the annual data to quarterly frequency.

Figure 8. Responses of Other Real Variables



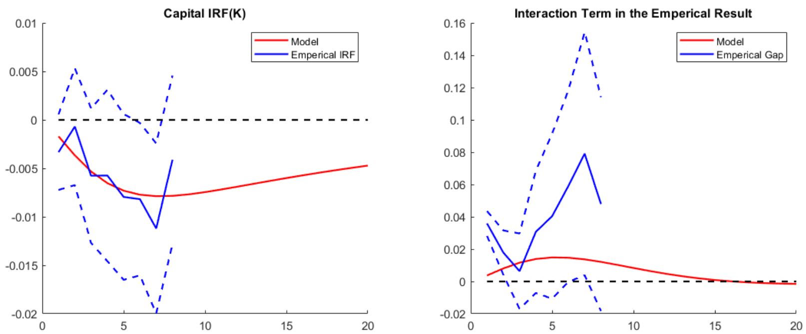
Note: This figure runs the local projection of the firm's capital using the firm-level PCF data obtained from the Neilsen dataset. Panel A shows the key coefficient β_1 in the baseline regression, while panel B runs the regression with the control of the interaction term between the dummy of the 4-digit NAICS code and the MP shock. In both panel, I run regressions with the data between 2004 and 2019 and with the full sample. There are 233 firms left after linking the Neilsen with GS1 data and the Compustat data. The dashed line shows the 90% confidence interval.

Figure 9. Regression with Firm Level PCF Data



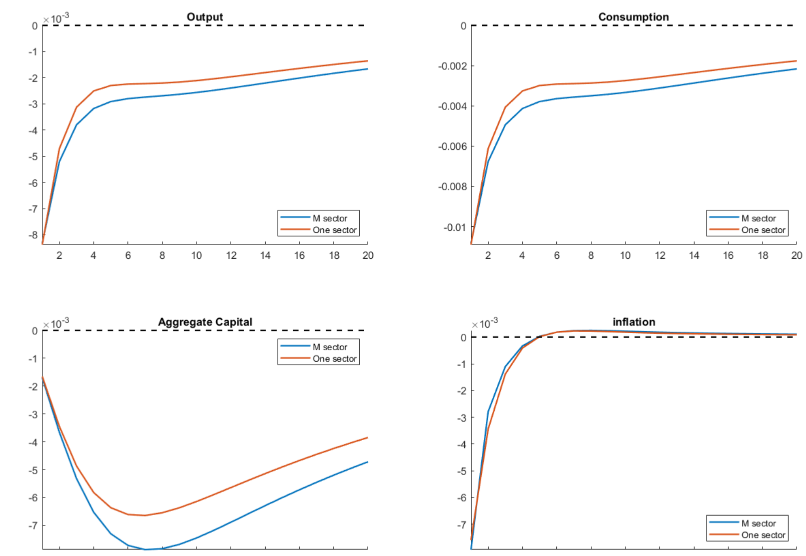
Note: This figure shows the IRF plots for the real variables for the two sectors in the 2-sector model. The parameter choice is listed in the appendix. The red line corresponds to the sector with relatively flexible prices and the pink line is the sticky sector. The main purpose of this exercise is to show the price stickiness channel of heterogeneous responses in the capital stock. The shock to the economy is an increase in the current nominal interest rate.

FIGURE 10. 2-SECTOR DSGE MODEL IRF



Note: This figure shows the IRF plots for the aggregate capital and the regression in front of the interaction term between the PCF and the MP shock in both the data and the model prediction. The blue line corresponds to the empirical regression result of the baseline regression using Swanson's conventional MP shock from 1994 to 2019. The shock is rescaled to match with a 100 bp increase in the annual federal fund rate in a quarterly VAR with 4 quarter lags containing the following variables in the given order: the MP shock, the Federal Fund Rate, the Industrial Production, the unemployment rate, the excess bond premium. The dashed line corresponds to the 90 percent confidence interval. The red line comes from the simulation of the multi-sector DSGE model. The IRF of the aggregate capital corresponds to the IRF of the weighted average of the sector-level capital responses. The interaction term from the model comes from running an OLS of the IRFs of the firm-level responses to the PCF.

Figure 11. M-Sector DSGE Model IRF



Note: This figure compares the IRF of the aggregate variables in the multi-sector model to the one-sector model.

Figure 12. M-Sector DSGE Model IRF, Aggregate Variables

Appendix A: Empirical Evidence

A1. Linkage Between the PCF data and the Compustat Data

In this appendix, I will explain the details of the combinations of the PCF data and the Compustat data. As mentioned there are three different sources of price changing frequency data, Pasten's data, NS data, and the Neilsen data. In the text, I talked about the procedure to generate the price-changing frequency data from the Neilsen consumer panel in detail. In the appendix, I will focus on the linkage between the industrial level PCF and the Compustat.

Pasten's Data: The core data this paper mainly focuses on comes from Pasten, Schoenle and Weber (2017). In their paper, the price changing frequency data is calculated using the good level price data from the survey to form the PPI from the BLS. The frequency of price changes at the good level is defined by the authors as the ratio of the number of price changes to the number of sample months. the author then aggregates the good-based frequencies to the BEA industry classification. Since we don't have access to the microdata from the BLS, the BEA industry classification is the most detailed data that we can get.

The BEA provides a detailed link from the BEA industry classification to the NAICS code, which is observable in the Compustat dataset. However, the linkage between the two classifications is complicated. A 6-digit BEA industry can be linked to either a 2, 3, 4, 5, or 6-digit NAICS code. Hence the strategy that I choose to link between the two classifications is that I first check the linkage between the BEA classification code and the 6-digit NIACS code, and assign the price changing frequency data to the firm that can be linked using the 6-digit code. Then I move on to use the 5-digit NIACS code to link, and so on. The general idea is to use the most detailed industrial code to link the data, and if the most detailed code is not able to link, try to generalize the industry constraint and use the closest industrial definition to assign the price changing frequency.

NS's Data: To complement the data from Pasten, Schoenle and Weber (2017), I also use the industrial level PCF data from Nakamura and Steinsson (2008). In their paper, the price changing frequency data is also calculated using the good level price data from the survey to form the PPI from the BLS. Different from the Pasten's data, the NS data aggregate the good level PCF to a different classification, the 4-digit commodity code used only in BLS. The BLS doesn't provide a crosswalk from the commodity code to the SIC or the NAICS code, so I manually link the BLS commodity code to the NAICS code by the name of the code. Notice that it is possible that one commodity code can be linked to multiple NAICS codes, and a NAICS code may able to be linked to multiple commodity codes.

The whole linking procedure is as follows. First, for each commodity code, I manually check the category name and choose the closest NAICS 6-digit code that contains the keywords of the category name. If a 6-digit code cannot be linked, I choose the closest 4-digit NAICS code. Then I gather all the NAICS 4-digit codes to form a general group. Notice that this group is relatively big in the sense that it can contain multiple commodity groups and multiple NAICS 4-digit codes. Then, for each group, I use the average relative importance of the commodity index provided by the BLS from 2003 to 2005 to calculate a weighted median of the PCF of all the commodity codes that belongs to a certain group. I use the weight from 2003 to 2005 because the NS data covers from 1998 to 2005, but the earliest weight than can be found on the BLS website starts in 2003. Using this method, the PCF data will be the same for all the firms belonging to a certain big group. I end up with 90 groups.

Table A1—Example Group: Vegetables and Fruits

PPI group	PPI code	4/6-digit code	6-digit NAICS group	4-digit code
Canned specialties	0284	311422	Specialty Canning	3114
Jams, jellies, and preserves	0281	311421	Fruit and Vegetable Canning	3114
Frozen specialties	0285	311412	Frozen Specialty Food Manufacturing	3114
Pickles and pickle products	0282	311421	Fruit and Vegetable Canning	3114
Canned vegetables and juices	0244	311421	Fruit and Vegetable Canning	3114
Canned fruits and juices	0241	311421	Fruit and Vegetable Canning	3114
Frozen fruits, juices and ades	0242	311411	Frozen Fruit, Juice, and Vegetable Manufacturing	3114
Frozen vegetables	0245	311411	Frozen Fruit, Juice, and Vegetable Manufacturing	3114
Vegetable cake and meal feeds	0292	3114	Fruit and Vegetable Preserving and Specialty Food Manufacturing	3114
Fresh and dried vegetables	0113	3114	Fruit and Vegetable Preserving and Specialty Food Manufacturing	3114
Fresh and dried vegetables	0113	1112	Vegetable and Melon Farming	1112
Fresh and dried vegetables	0113	1113	Fruit and Tree Nut Farming	1113
Fresh and dried vegetables	0113	1114	Greenhouse, Nursery, and Floriculture Production	1114

Table A1 provides an example group. The group name is "food and vegetables". As can be seen from the table, this group contains multiple commodity codes and multiple NAICS codes. Since the 4-digit NAICS codes gather all the canned fruit and vegetables, it contains multiple commodity codes in this sub-category. On the other hand, the fresh and dried vegetable category in the commodity code also contains several different NAICS 4-digit codes. The result of this N-to-N linkage is we end up a "big" group containing almost all the fresh and canned vegetables and fruits.

A2. Variable Definition

The baseline regression requires the calculation of the firm-level variables. For each of the variables, I winsorize the data at the 1 percentile and the 99 percentile for each quarter.

Firm Size: The size of a firm is defined as the log of a firm's total assets.

Cash Flow Ratio: The cash flow ratio is defined as (income before extraordinary items + depreciation and amortization) / total asset.

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Liquid Asset Ratio: The liquid asset ratio is defined as cash and short-term investments / total asset.

Capital Share: The capital share of a firm is defined as the net property, plant and equipment / total asset.

External Finance Dependence: The external finance dependence of a firm is defined as (capital expenditures - net cash flow of the operating activities)/ capital expenditures.

Investment Intensity: The investment intensity of a firm is defined as capital expenditures / common equity.

Tobin's Q: The Tobin's Q is defined as (total asset + stock price × common shares outstanding - common equity + deferred taxes and investment tax credit) / total asset

Growth of Real Sales: The real sales is defined as the sales of the firm adjusted using the urban CPI. The growth rate of a variable is defined as $(x_t - x_{t-1})/x_{t-1}$.

Growth of Employment: The employment at the quarterly frequency is defined as the linear interpolation using the closest 2 observations in the annual report. The growth rate is defined as before.

Real Capital Stock: The real capital stock is defined using the perpetual inventory method. First, initialize the capital stock using the first available entry of PPEGT. Then iterate forward defining the net investment as $PPENT_t - PPENT_{t-1}$. If PPENT is missing, use the linear interpolation between the two closest observables. Finally, deflate the capital using the investment deflator from the FED st. Louis.

Appendix B: DSGE Model

B1. Equilibrium System

The full system to characterize the model includes the following equations:

- The FOC of the household:

$$Q_{t,t+1} = \beta E_t[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}(1+\pi_{t+1})}] \tag{B1}$$

$$w_{it} = \phi n_{it}^{\eta} C_t^{\sigma} \tag{B2}$$

$$q_t = \beta E_t[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}[R_{t+1} + (1-\delta)q_{t+1}]] \tag{B3}$$

$$1 = q_t(1 - \frac{\Omega}{2} - \frac{3\Omega}{2}\frac{I_t}{I_{t-1}}^{2} + 2\Omega\frac{I_t}{I_{t-1}}) \tag{B4}$$

$$+ \beta\Omega E_t[q_{t+1}\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{I_{t+1}^2}{I_t^2}(\frac{I_{t+1}}{I_t} - 1)] \tag{B5}$$

$$K_{it} = (1-\delta)K_{it-1} + (1 - \frac{\Omega}{2}(\frac{I_{it}}{I_{it-1}} - 1)^2)I_{it} \tag{B6}$$

- The FOC of the final good producer:

$$Y_{it} = \mu_i(\frac{P_{it}}{P_t})^{-\gamma} Y_t \tag{B7}$$

$$P_t^{1-\gamma} = \sum_{i=1}^{M} \mu_i P_{it}^{1-\gamma} \tag{B8}$$

$$Y_t = (\sum_{i=1}^{M} \mu_i^{\frac{1}{\gamma}} Y_{it}^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}} \tag{B9}$$

- The FOC of the intermediate good producer:

$$Y_{it} = (\int_0^1 Y_{ijt}^{\frac{\theta-1}{\theta}} dj)^{\frac{\theta}{\theta-1}} \tag{B10}$$

$$Y_{ijt} = (\frac{P_{ijt}}{P_{it}})^{-\theta} Y_{it} \tag{B11}$$

$$P_{it}^{1-\theta} = \int_0^1 (P_{ijt})^{1-\theta} dj \tag{B12}$$

- The FOC of the cost minimization problem of the raw good producer:

$$\frac{w_{it}}{R_{it}} = \frac{(1-\alpha)K_{ijt}}{\alpha n_{ijt}} \tag{B13}$$

$$Y_{ijt} = A_{it} n_{ijt}^{1-\alpha} K_{ijt}^{\alpha} \tag{B14}$$

$$mc_{it} = (1-\alpha)^{-(1-\alpha)} \alpha^{-\alpha} \frac{(w_t^i)^{1-\alpha} (r_t^i)^{\alpha}}{A_t^i} \tag{B15}$$

- The FOC of the profit maximization problem of the raw good producer:

$$\sum_{k=0}^{+\infty} (\lambda_i)^k Q_{t,t+k} [(1-\theta) \frac{P_t^{\star}}{P_{t+k}} + \theta mc_{it+k}] \\ (\frac{P_{it+k}}{P_{t+k}})^{-1} (\frac{P_t^{\star}}{P_{t+k}})^{-\theta-1} Y_{it+k} = 0 \tag{B16}$$

- Market clearing conditions:

$$C_t + \sum_{i=1}^{M} K_{it} = Y_t + \sum_{i=1}^{M} (1-\delta) K_{1t-1} \tag{B17}$$

$$\int_0^1 n_{ijt} = n_{it} \tag{B18}$$

$$\int_0^1 K_{ijt} = K_{it-1} \tag{B19}$$

B2. Steady State

At the steady state, we apply the symmetry conditions and solve for the endogenous variables $\{C, Y, K_i, n_i, Y_i, mc_i, w_i, r_i, \frac{P_i}{P}\},$. Notice that using the symmetry condition will give us the result that $P_{ij} = P_i$ and $Y_{ij} = Y_i$ at the steady state.

- The FOC of the household:

$$w_i = \phi n_i^{\eta} C_i^{\sigma} \tag{B20}$$

$$1 = \beta(r_i + 1 - \delta) \tag{B21}$$

- The FOC of the final good producer:

$$Y = (\sum_{i=1}^{M} \mu_i^{\frac{1}{\gamma}} Y_i^{\frac{\gamma-1}{\gamma}})^{\frac{\gamma}{\gamma-1}} \tag{B22}$$

$$Y_i = \mu_i(\frac{P_i}{P})^{-\gamma} Y \tag{B23}$$

$$P^{1-\gamma} = \sum_{i=1}^{N} \mu_i P_i^{1-\gamma} \tag{B24}$$

Notice that since these $M+2$ equations are linearly dependent, we only need $M+1$ of them to solve for the steady state. We choose the first $M+1$ and ignore the last equation.

- Intermediate firm's FOC are omitted by symmetry. The raw good firms' cost minimization problem will give us:

$$\frac{w_i}{R_i} = \frac{(1-\alpha)K_i}{\alpha n_i} \tag{B25}$$

$$Y_i = A_i n_i^{1-\alpha} K_i^{\alpha} \tag{B26}$$

$$mc_i = (1-\alpha)^{-(1-\alpha)} \alpha^{-\alpha} \frac{(w_i)^{1-\alpha}(r_i)^{\alpha}}{A_i} \tag{B27}$$

- The raw good firms' profit maximization problem will give us:

$$\frac{P_i}{P} = \frac{\theta}{\theta-1} mc_i \tag{B28}$$

- Market clearing conditions:

$$Y = C + \sum_{i=1}^{M} \delta K_i \tag{B29}$$

The subscription i is omitted due to symmetry.

B3. Log-Linearization of the System

First, under the flexible price setup, we take the log-linearization for the equation system. By the complete market setup, the fraction of consumption that the consumers consume is a stable variable and it is the same as the steady state variable z. Taking the log-linearization of the equations, we will solve for the endogenous variables $\{c_t, y_t, k_{it}, n_{it}, y_{it}, mc_{it}, w_{it}, r_{it}, p_{it} - p_t, q_{it}, i_{it}\}$ using the following equations:

- Consumer's FOC:

$$w_{it} = \eta n_{it} + \sigma c_t \tag{B30}$$

$$0 = \sigma c_t - \sigma c_{t+1} + (1 - \beta(1 - \delta))r_{it+1} \tag{B31}$$

$$q_{it} = -\sigma(c_{t+1} - c_t) + (1 - \beta(1 - \delta))r_{it+1} + \beta(1 - \delta)q_{it+1} \tag{B32}$$

$$0 = q_{it} + \Omega(i_{it-1} - i_{it}) + \beta\Omega(i_{it+1} - i_{it}) \tag{B33}$$

$$k_{it} = (1 - \delta)k_{it-1} + \delta i_{it} \tag{B34}$$

- Final firm's FOC:

$$y_t = \sum_{i=1}^{M} \mu_i^{\frac{1}{\gamma}} \frac{Y_i}{Y}^{\frac{\gamma-1}{\gamma}} y_{it} \tag{B35}$$

$$y_{it} = -\gamma(p_{it} - p_t) + y_t \tag{B36}$$

  Notice that we omit the forth equation in the equation system since it is linearly dependent to the first three.

- The raw good firms' cost minimization:

$$y_{it} = a_{it} + (1 - \alpha)n_{it} + \alpha k_{it-1} \tag{B37}$$

$$w_{it} - r_{it} = k_{it-1} - n_{it} \tag{B38}$$

$$mc_{it} = (1 - \alpha)w_{it} + \alpha r_{it} - a_{it} \tag{B39}$$

- The raw good firms' profit maximization problem will give us:

$$p_{it} - p_t = mc_{it} \tag{B40}$$

- The market clearing condition:

$$\sum_{i=1}^{M}(1 - \delta)K_i k_{it-1} + Y y_t = C c_t + \sum_{i=1}^{M} K_i k_{it} \tag{B41}$$

Second, The only difference will be at equation (B40). When the economy has sticky price, the raw good producer's profit maximization problem give us the following log-linearized FOC:

$$\begin{aligned} 0 =& \sum_{k=0}^{+\infty} (\lambda_i \beta)^k [(p_{ijt} - p_{t+k}) - mc_{it+k}] \\ p_t^\star =& (1-\lambda_i\beta)(p_t + mc_t) + \lambda_i \beta p_{t+1}^\star \\ p_t^\star - p_{it-1} =& (1-\lambda_i\beta) mc_t + \lambda_i\beta(p_{t+1}^\star - p_{it}) \\ &+ \lambda_i\beta(p_{it} - p_t) + (p_{t-1} - p_{it-1}) + \pi_t \end{aligned}$$

Since only $(1-\lambda)$ of the firm can change the price, we have the following relationship:

$$P_{it}^{1-\theta} = (1-\lambda) P_{ijt}^{1-\theta} + \lambda P_{it-1}^{1-\theta} \tag{B42}$$

The log-linearization will give us

$$\begin{aligned} p_{it} &= (1-\lambda_i) p_t^\star + \lambda_i p_{it-1} \\ p_{it} - p_{it-1} &= (1-\lambda_i)(p_t^\star - p_{it-1}) \\ (p_t^\star - p_{it-1}) &= \frac{1}{1-\lambda_i}(p_{it} - p_{it-1}) \end{aligned}$$

Combine with the raw good firm's FOC will give us the New Keynesian Phillips Curve:

$$\begin{aligned} \pi_{it} &= (1-\lambda_i)(1-\lambda_i\beta) mc_t + \lambda_i\beta\pi_{it+1} \\ &+(1-\lambda_i)\lambda_i\beta(p_{it} - p_t) - (1-\lambda_i)(p_{it-1} - p_{t-1}) + (1-\lambda_i)\pi_t \end{aligned}$$

By definition of the industrial inflation, we have:

$$\pi_{it} = (p_{it} - p_t) + \pi_t - (p_{it-1} - p_{t-1}) \tag{B43}$$

Notice that for the final good firms, we have to replace the final firm's technology with the following price index relationship:

$$1 = \sum_{i=1}^{M} \mu_i (\frac{P_{it}}{P_t})^{1-\gamma}$$

After the log-linearization we have:

$$0 = \sum_{i=1}^{M} \mu_i \frac{P_i}{P}^{1-\gamma} (p_{it} - p_t) \tag{B44}$$

Finally, the monetary policy in this model is

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\phi_\pi \pi_t + \phi_y y_t) + e_t$$

Notice that the consumer's problem solves:

$$-r_t = -\sigma(c_{t+1} - c_t) - \pi_{t+1} \tag{B45}$$

which is the dynamic IS curve.