dongguk UNIVERSITY

CSE 2017 Data Structures and Lab

Lecture #10: Heap

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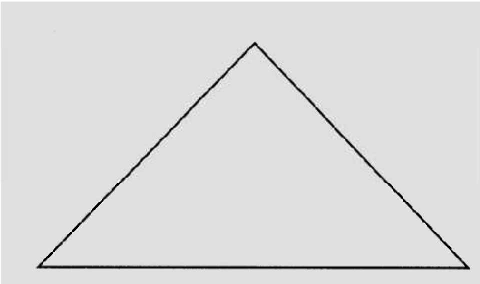
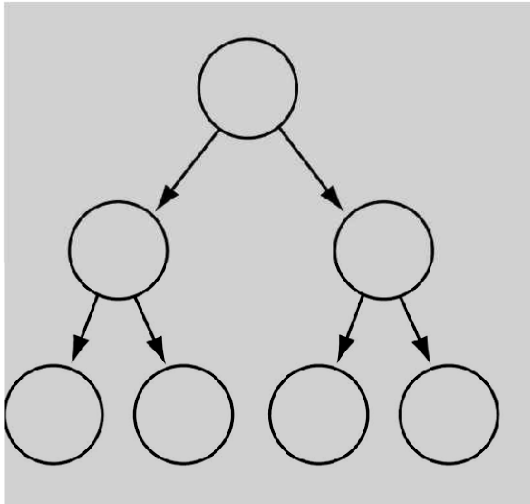
Full Binary Tree

- Every non-leaf node has two children
- Leaves are on the same level



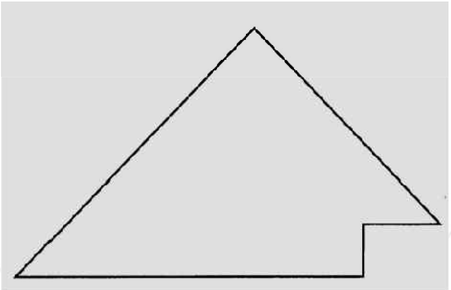
Full Binary Tree

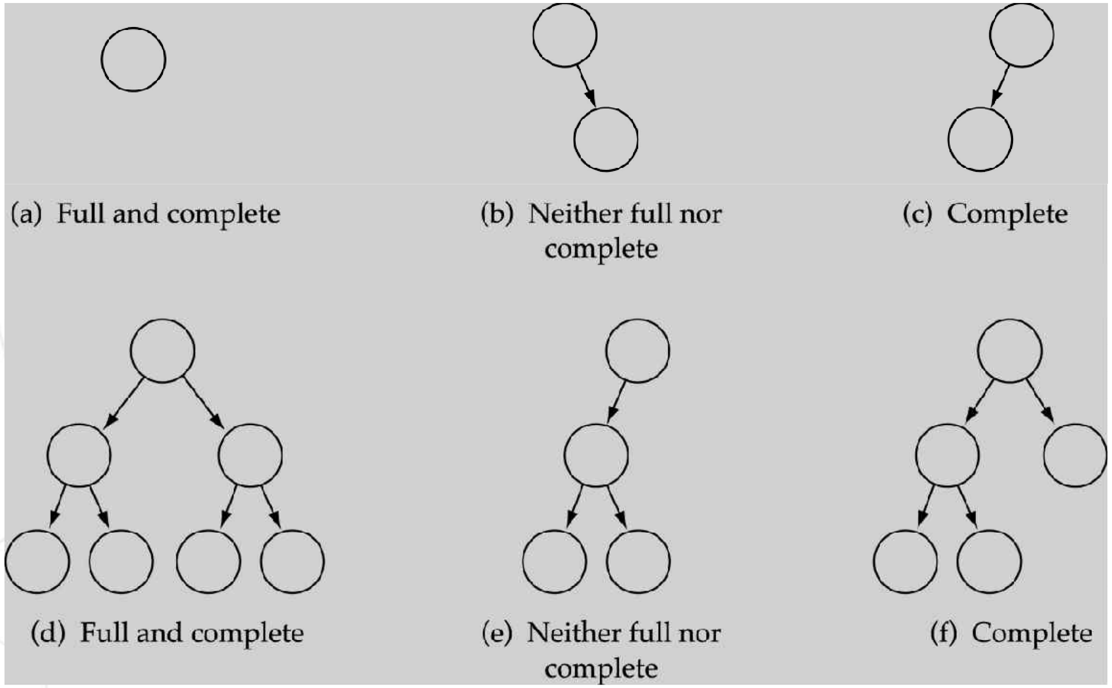


Complete Binary Tree

(1) A binary tree that is either full <u>or</u> full through the next-to-last level

(2) The last level is full from left to right (i.e., leaves are as far to the left as possible)





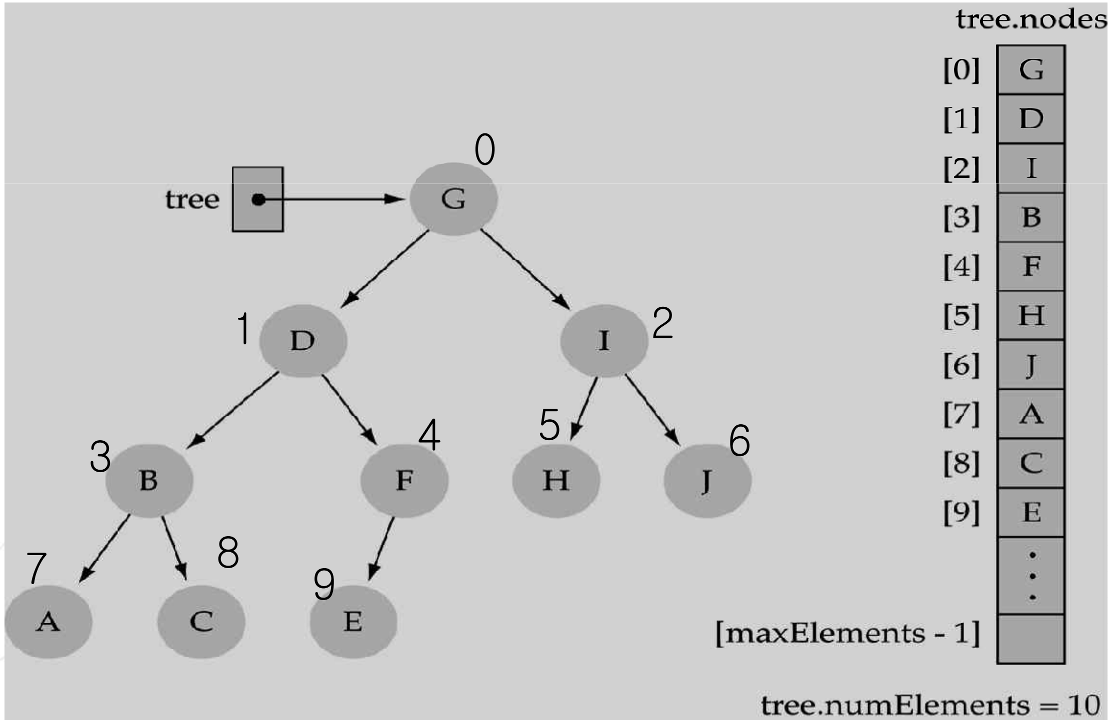
(a) Full and complete (b) Neither full nor complete (c) Complete

(d) Full and complete (e) Neither full nor complete (f) Complete

Array-based representation of binary trees

- **Memory savings (i.e., no pointers)**
- **Preserve parent-child relationships**

<u>Store:</u> *(i)* level by level, and *(ii)* left to right



Array-based representation of binary trees (cont.)

- **Parent-child relationships:**
  - **left child** of ***tree.nodes[index]*** = ***tree.nodes[2*index+1]***
  - **right child** of ***tree.nodes[index]*** = ***tree.nodes[2*index+2]***
  - **parent node** of ***tree.nodes[index]*** = ***tree.nodes[(index-1)/2]***

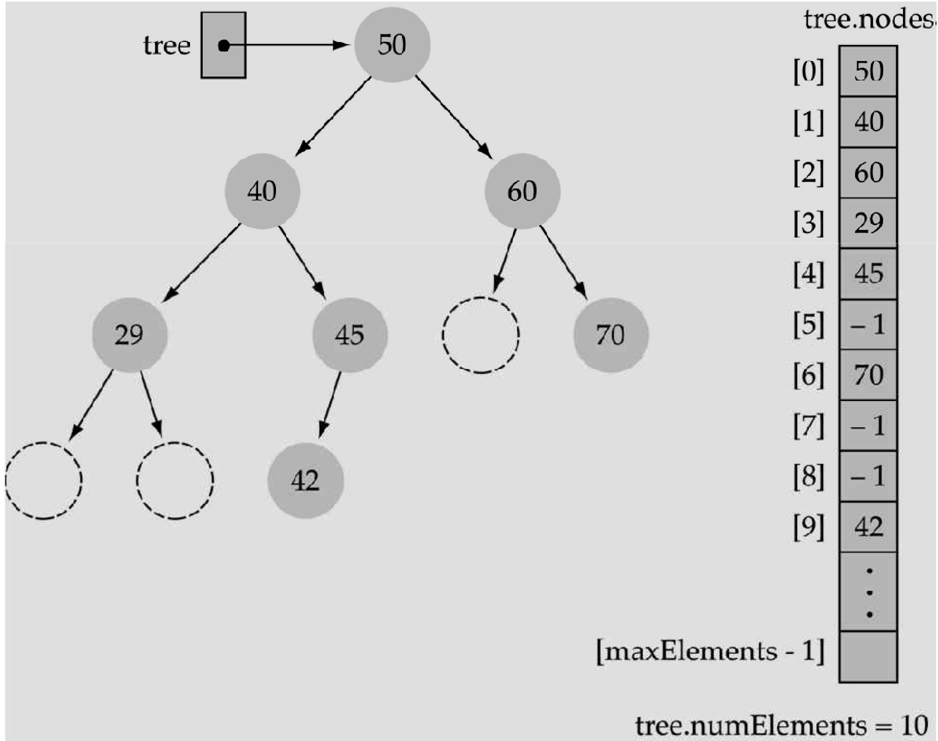
(int division)

- **Leaf nodes:**
  - ***tree.nodes[numElements/2]*** **to** ***tree.nodes[numElements - 1]***

Array-based representation of binary trees (cont.)

- **Full or complete trees can be implemented efficiently using an array-based representation (i.e., elements occupy contiguous array slots).**

"Dummy nodes" are required for trees which are <u>not</u> full or complete.



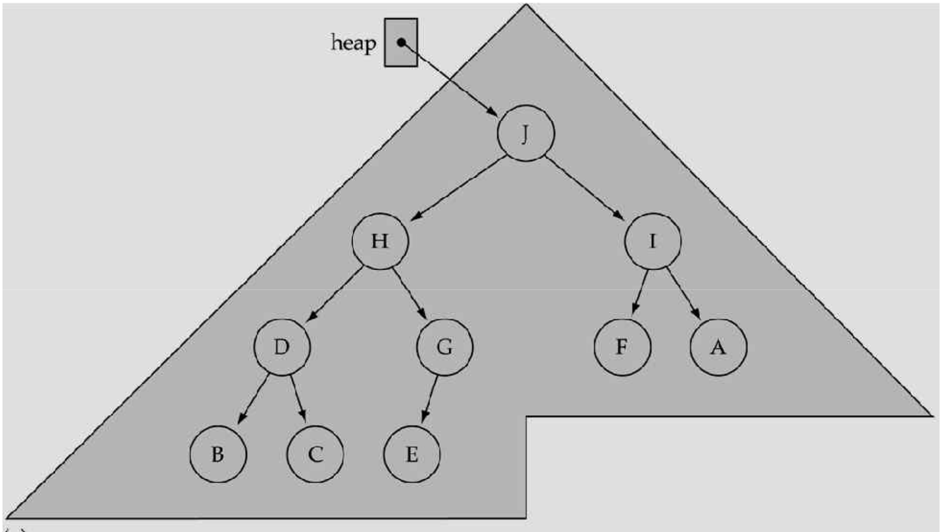
What is a heap?

- **It is a binary tree with the following properties:**

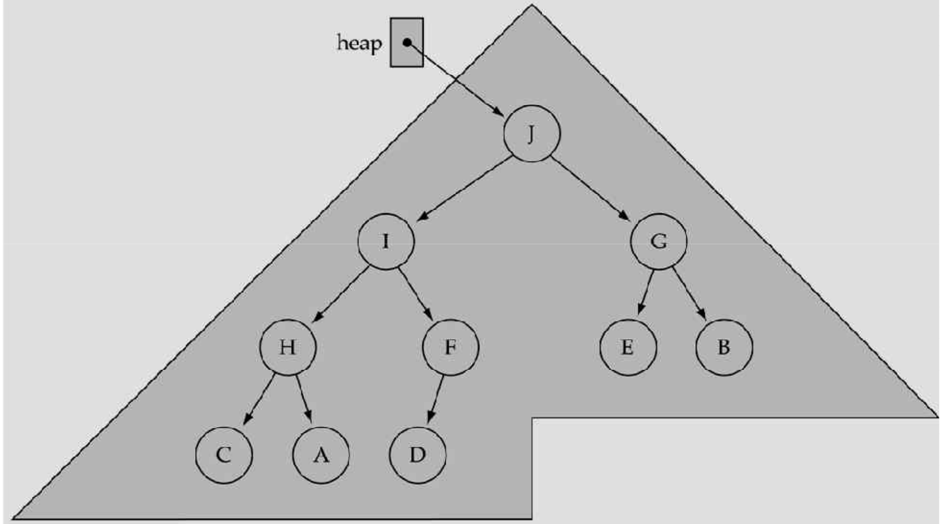
  <u>Property 1</u>: **it is a complete binary tree**

  <u>Property 2</u>: **(heap property): the value stored at a node is greater or equal to the values stored at the children**

Not unique!



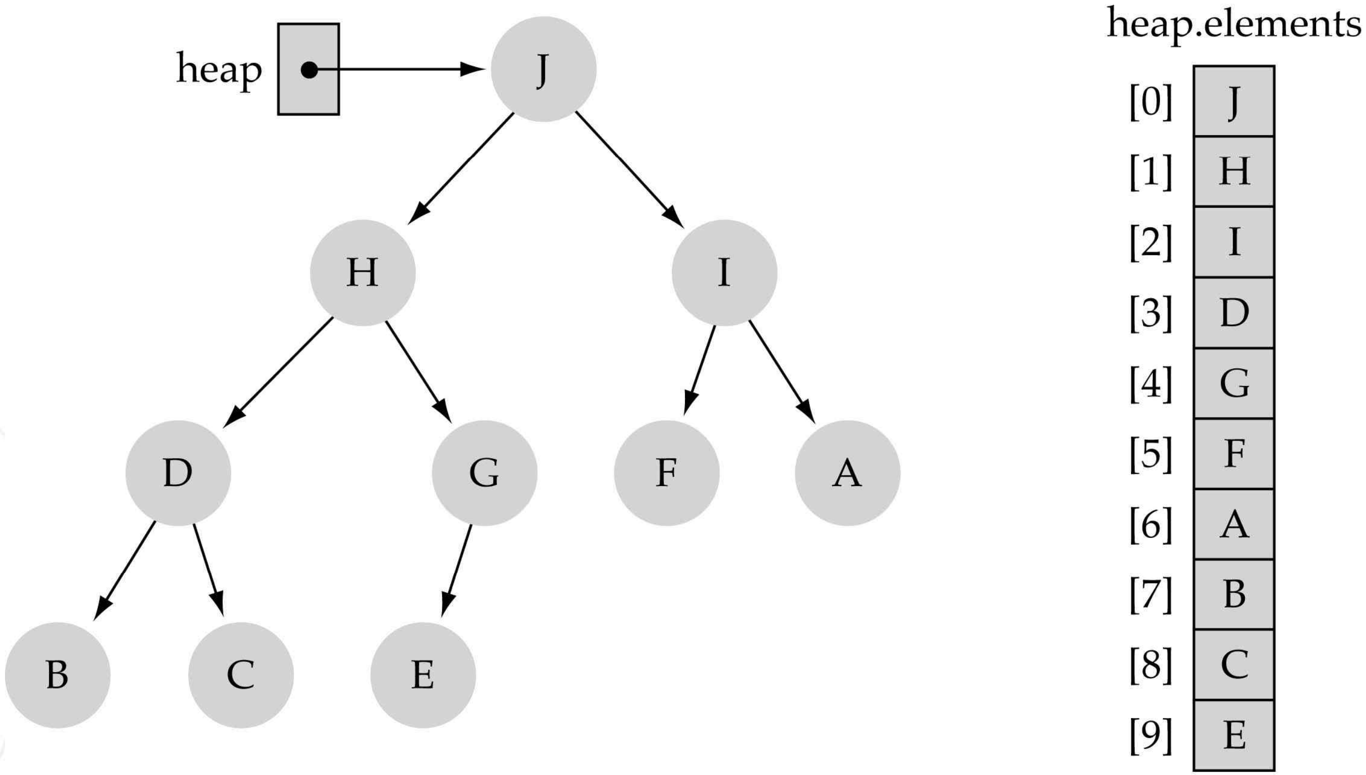
(a)



(b)

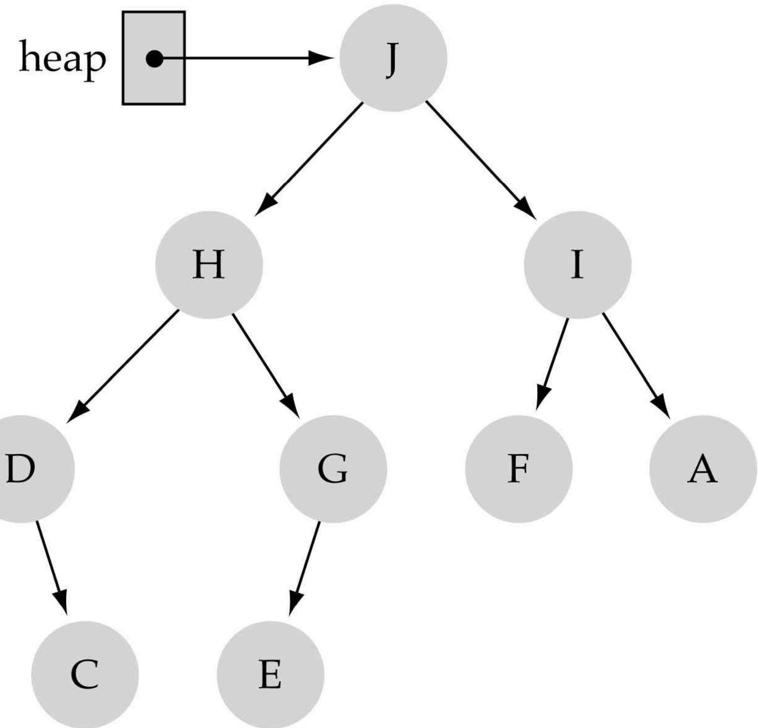
Largest heap element

- From *Property 2*, the largest value of the heap is always stored at the root



Heap implementation

- **Heaps are always implemented as arrays!**



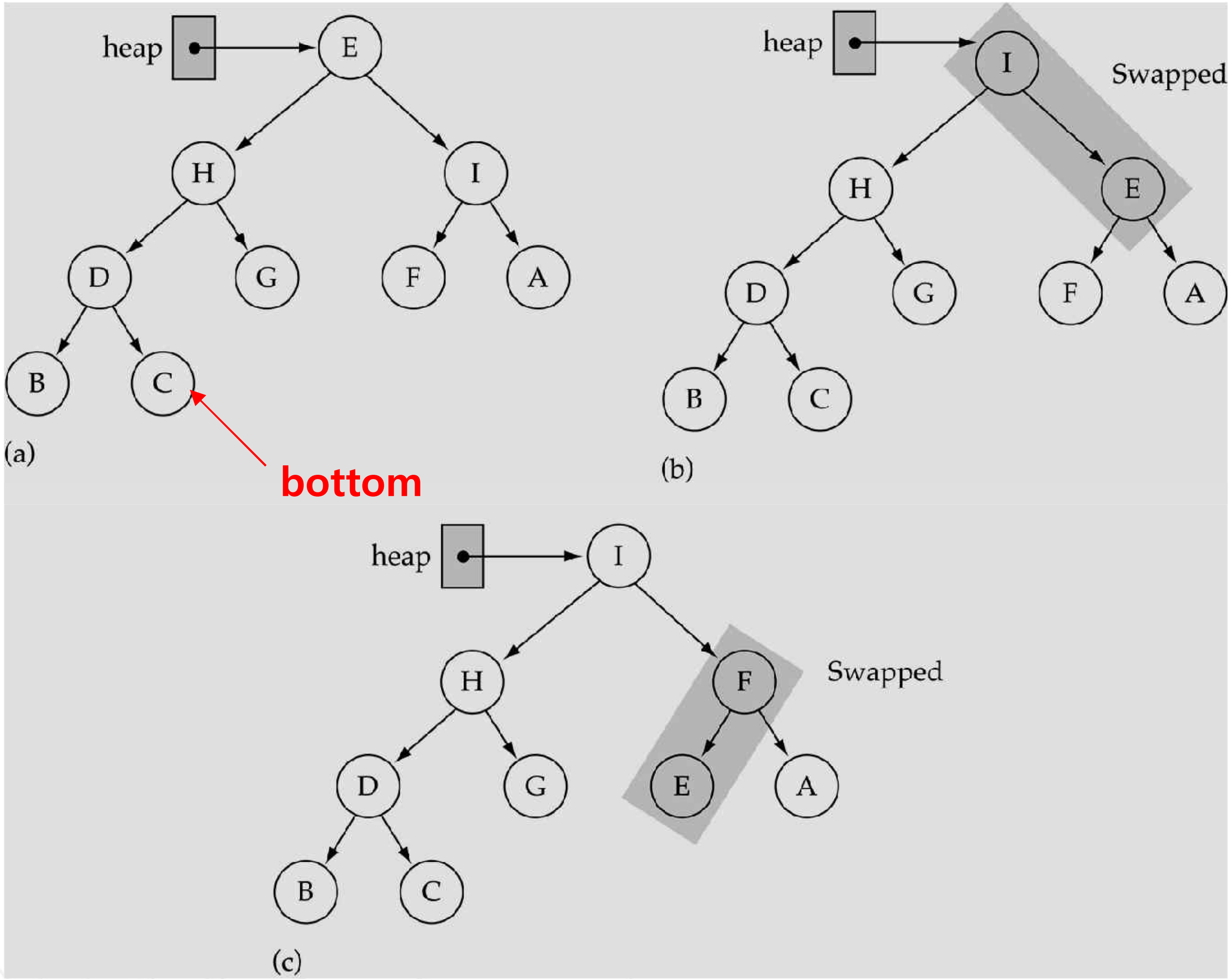
heap.elements

[0]	J
[1]	H
[2]	I
[3]	D
[4]	G
[5]	F
[6]	A
[7]	B
[8]	C
[9]	E

Heap Specification

```
template<class ItemType>
struct HeapType {
    void ReheapDown(int, int);
    void ReheapUp(int, int);
    ItemType *elements; // dynamic array
    int numElements;
};
```

The ReheapDown function



Assumption:

heap property is violated at the root of the tree

ReheapDown function

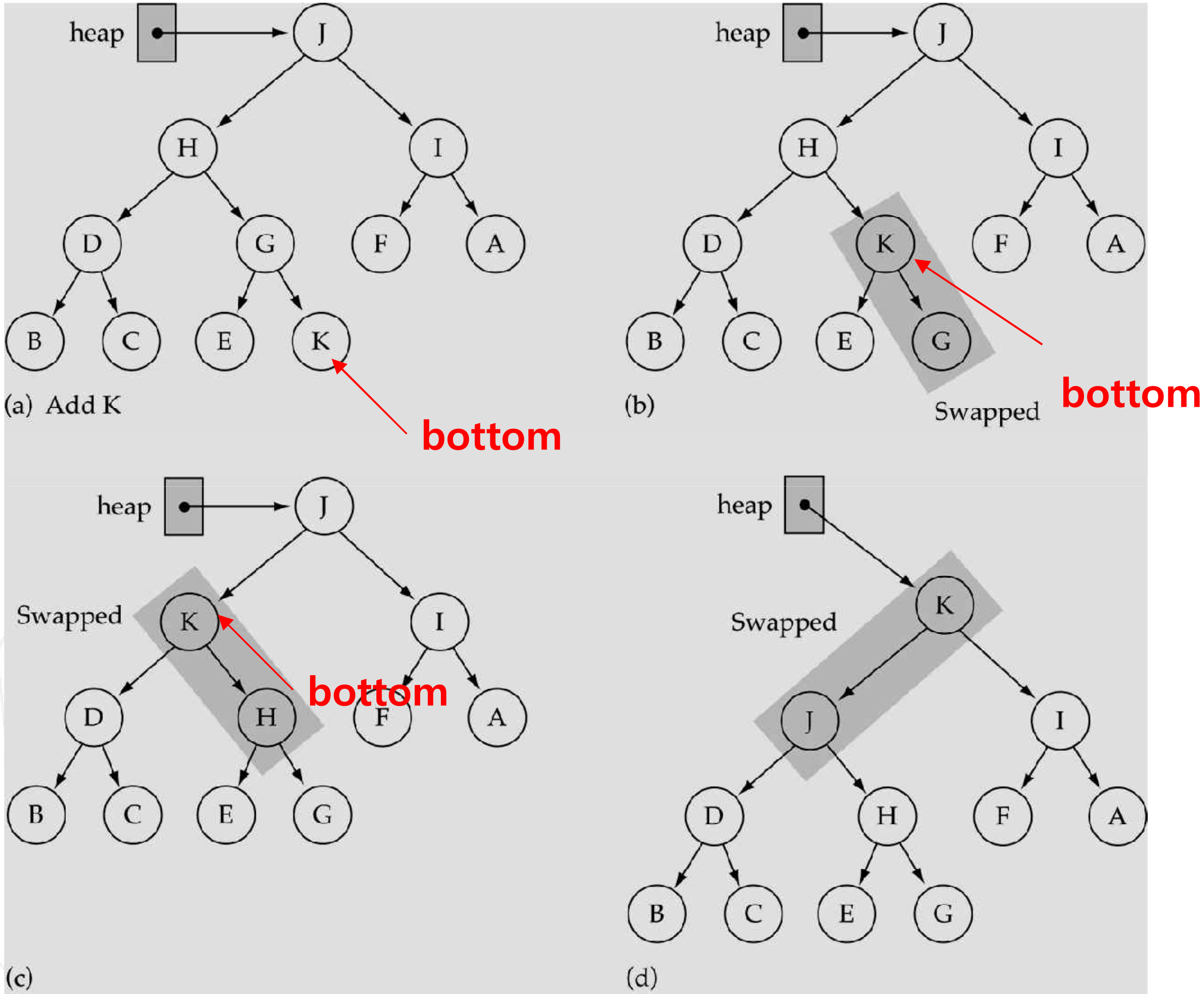
```
template<class ItemType>
void HeapType<ItemType>::ReheapDown(int root, int bottom)
{
 int maxChild, rightChild, leftChild;

 leftChild = 2*root+1;
 rightChild = 2*root+2;

 if(leftChild <= bottom) {  // left child is part of the heap
   if(leftChild == bottom) // only one child
     maxChild = leftChild;
   else { // two children
     if(elements[leftChild] <= elements[rightChild])
       maxChild = rightChild;
     else
       maxChild = leftChild;
   }
   if(elements[root] < elements[maxChild]) {// compare max child with
  parent
     Swap(elements, root, maxChild);
     ReheapDown(maxChild, bottom);
   }
 }
}
```

rightmost node
at the last level

The ReheapUp function



Assumption:

heap property is violated at the rightmost node of the last level of the tree

ReheapUp function

```
template<class ItemType>
void HeapType<ItemType>::ReheapUp(int root, int bottom)
{
 int parent;

 if(bottom > root) { // tree is not empty
   parent = (bottom-1)/2;
   if(elements[parent] < elements[bottom]) {
     Swap(elements, parent, bottom);
     ReheapUp(root, parent);
   }
 }
}
```

rightmost node at the last level

O(logN)

Priority Queues

- What is a priority queue?
  - It is a queue with each element being associated with a "priority"
  - From the elements in the queue, the one with the highest priority is dequeued first

Priority queue specification

```
template<class ItemType>
class PQType {
  public:
    PQType(int);
    ~PQType();
    void MakeEmpty();
    bool IsEmpty() const;
    bool IsFull() const;
    void Enqueue(ItemType);
    void Dequeue(ItemType&);
  private:
    int numItems; // num of elements in the queue
    HeapType<ItemType> heap;
    int maxItems; // array size
};
```

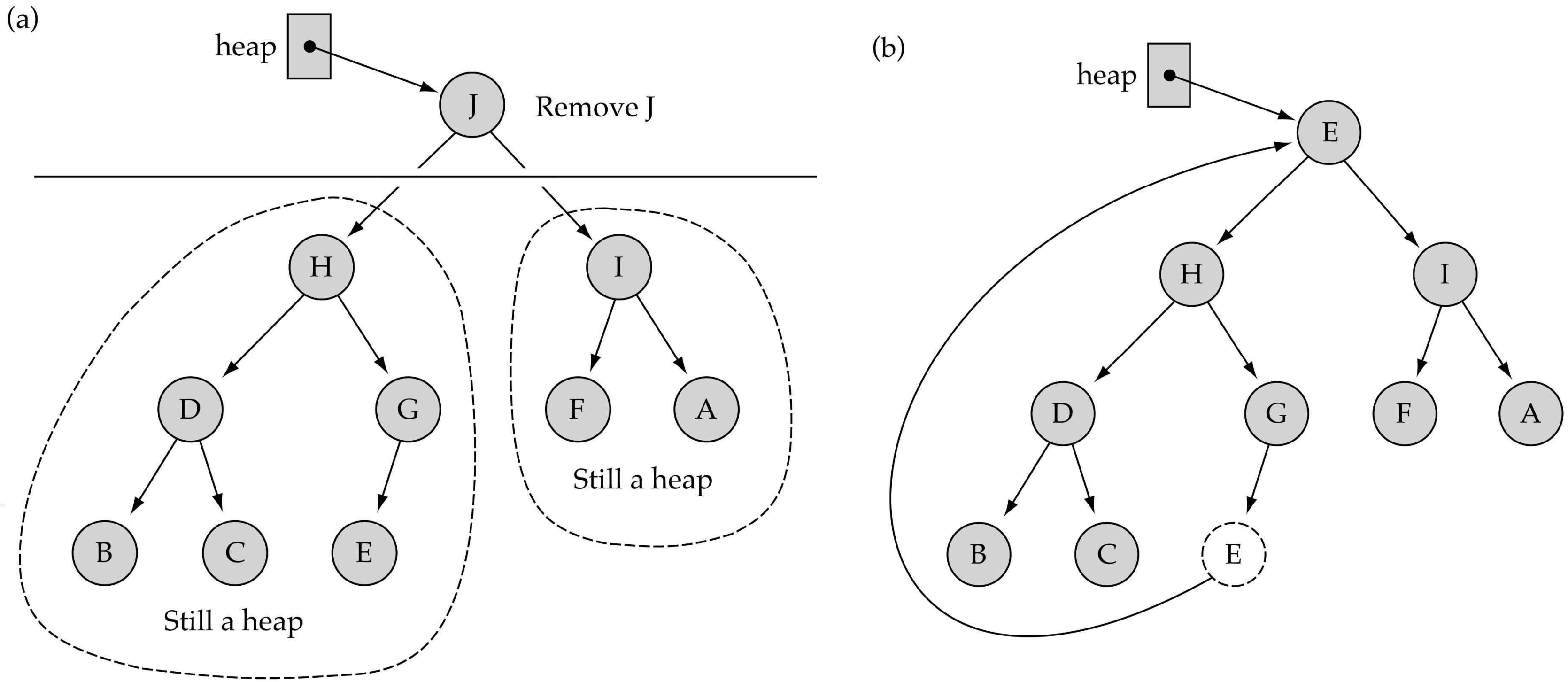
Dequeue: remove the largest element from the heap

(1) Copy the bottom rightmost element to the root

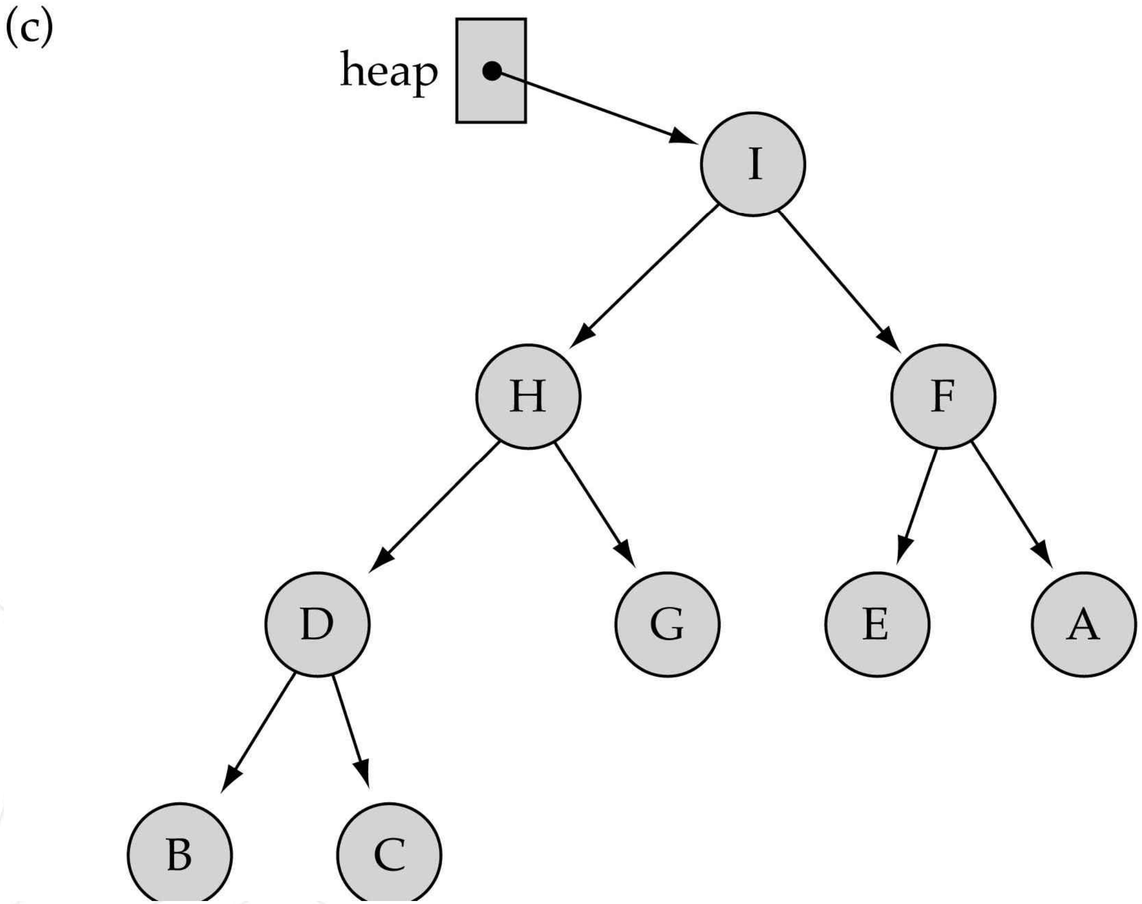
(2) Delete the bottom rightmost node

(3) Fix the heap property by calling *ReheapDown*

Removing the largest element from the heap (cont.)



Removing the largest element from the heap (cont.)



Dequeue

```
template<class ItemType>
void PQType<ItemType>::Dequeue(ItemType& item)
{
 item = heap.elements[0];
 heap.elements[0] = heap.elements[numItems-1];
 numItems--;
 heap.ReheapDown(0, numItems-1);
}
```

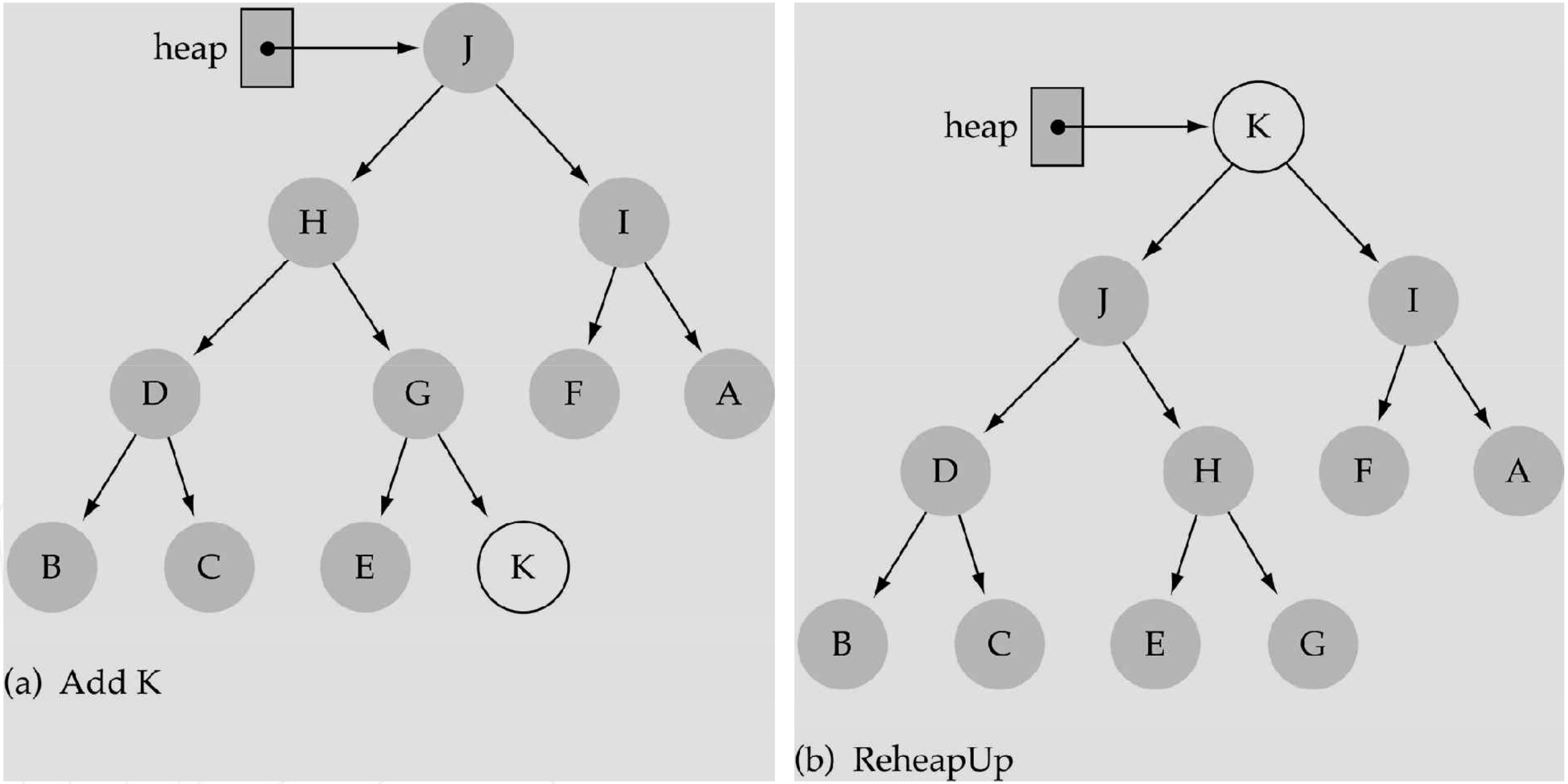
bottom

Enqueue: insert a new element into the heap

(1) Insert new element in the <u>leftmost</u> place at the bottom level (start new level if last level is full).

(2) Fix the heap property by calling *ReheapUp* .

Inserting a new element into the heap (cont.)



Enqueue

```
template<class ItemType>
void PQType<ItemType>::Enqueue(ItemType newItem)
{
 numItems++;
 heap.elements[numItems-1] = newItem;
 heap.ReheapUp(0, numItems-1]);
}
```

bottom

Other Functions

```
template<class ItemType>
PQType<ItemType>::PQType(int max)
{
 maxItems = max;
 heap.elements = new ItemType[max];
 numItems = 0;
}
```

```
template<class ItemType>
PQType<ItemType>::MakeEmpty()
{
 numItems = 0;
}
```

```
template<class ItemType>
PQType<ItemType>::~PQType()
{
 delete [ ] heap.elements;
}
```

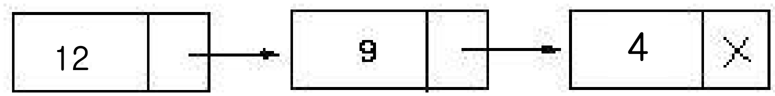
Other Functions (cont.)

```
template<class ItemType>
bool PQType<ItemType>::IsFull() const
{
 return numItems == maxItems;
}
```

```
template<class ItemType>
bool PQType<ItemType>::IsEmpty() const
{
 return numItems == 0;
}
```

Comparing heaps with other implementations

- Priority queue using a sorted list



O(N) on the average!

  - Remove a key in O(1) time
  - Insert a key in O(N) time

- Priority queue using heaps
  - Remove a key in O(logN) time
  - Insert a key in O(logN) time

O(lgN) on the average!