Cosmology Tutorial

Journal Club 12/11/20

Overview

- Recap of Standard Model of Cosmology (Following Weinberg’s *Cosmology*)
- Distance measures with Hogg’s *Distance Measures in Cosmology*
- Cosmology in gravitational waves
- How to compute things with Astropy

Geometry of spacetime

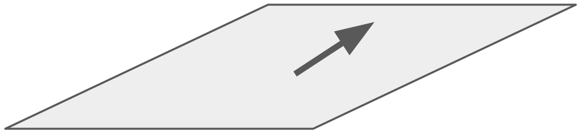
Starting point: assume a spacetime that is homogeneous (same under translations) and isotropic (same under rotations)

Find only three geometries allowed that meet this for three spatial dimensions (up to coord transform): flat, spherical, and hyperbolic

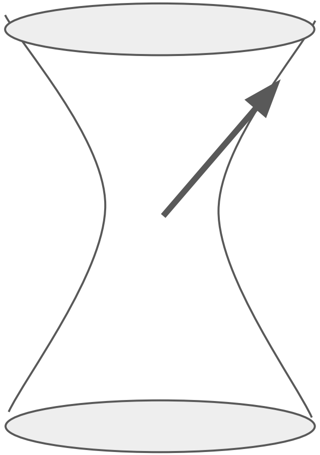
$$ds^2 = a^2 \left[d\mathbf{x}^2 + K \frac{(\mathbf{x} \cdot d\mathbf{x})^2}{1 - K\mathbf{x}^2} \right] ,$$

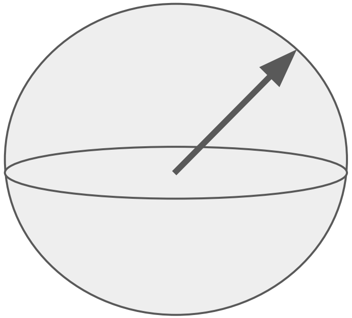
$$K = \begin{cases} +1 & \text{spherical} \\ -1 & \text{hyperspherical} \\ 0 & \text{Euclidean} \end{cases} .$$

$ds^2 = d\mathbf{x}^2$



$ds^2 = d\mathbf{x}^2 - dz^2$





$ds^2 = d\mathbf{x}^2 + dz^2$

Friedmann-Lemaitre-Robertson-Walker (FLRW) metric

$$d\tau^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega\right]$$

Scale factor *a* can change with time coordinate *t*.

These are *co-moving* coordinates, i.e. they are the coordinates in which free-falling particles at rest stay at rest.

Friedmann Equations

$$\boxed{\dot{a}^2 + K = \frac{8\pi G \rho a^2}{3} .}$$

$$\boxed{\dot{\rho} = -\frac{3\dot{a}}{a}(\rho + p) .}$$

Redshift

$$\boxed{1 + z = a(t_0)/a(t_1) \ .}$$

Redshift *z* is related to ratio of scale factors now and then

$$\boxed{H_0 \equiv \dot{a}(t_0)/a(t_0) \ .}$$

Taylor exapnd a(t) and define this

Plug back in to find Hubble-Lemaitre Law

$$\boxed{z = H_0 d + \dots \ .}$$

Important Constants

where h is a dimensionless number parameterizing our ignorance. (Word on the street is that $0.6 < h < 0.9$.) The inverse of the Hubble constant is the *Hubble time* t_H

$$t_H \equiv \frac{1}{H_0} = 9.78 \times 10^9 \, h^{-1} \text{ yr} = 3.09 \times 10^{17} \, h^{-1} \text{ s} \tag{3}$$

and the speed of light c times the Hubble time is the *Hubble distance* D_H

$$D_H \equiv \frac{c}{H_0} = 3000 \, h^{-1} \text{ Mpc} = 9.26 \times 10^{25} \, h^{-1} \text{ m} \tag{4}$$

Spacetime dynamics

Non-relativistic matter: Here $\rho = \rho_0(a/a_0)^{-3}$, and the solution of Eq. (1.5.19) with $K = 0$ is

$$a(t) \propto t^{2/3} \,. \tag{1.5.32}$$

Relativistic matter: Here $\rho = \rho_0(a/a_0)^{-4}$, and the solution of Eq. (1.5.19) with $K = 0$ is

$$a(t) \propto \sqrt{t} \,. \tag{1.5.34}$$

Vacuum energy: Lorentz invariance requires that in locally inertial

$$a(t) \propto \exp(Ht) \tag{1.5.36}$$

where H is the Hubble constant, now really a constant, given by

“de-Sitter spacetime”

$$H = \sqrt{\frac{8\pi G\rho_V}{3}} \,. \tag{1.5.37}$$

Matter and Dark Energy Densities

$$\boxed{\begin{aligned}\Omega_{\mathrm{M}} &\equiv \frac{8\pi\, G\, \rho_0}{3\, H_0^2} \\ \Omega_{\Lambda} &\equiv \frac{\Lambda\, c^2}{3\, H_0^2}\end{aligned}}$$

$$\boxed{\Omega_{\mathrm{M}} + \Omega_{\Lambda} + \Omega_k = 1}$$

There is also a $\mathbf{\Omega}_{\mathrm{R}}$ for radiation energy density, but it is much less than $\mathbf{\Omega}_{\mathrm{M}}$ so is usually ignored.

Comoving Distance

A small *comoving distance* δD_C between two nearby objects in the Universe is the distance between them which remains constant with epoch if the two objects are moving with the Hubble flow. In other words, it is the distance between them which would be measured with rulers at the time they are being observed (the *proper distance*) divided by the ratio of the scale factor of the Universe then to now; it is the proper distance multiplied by $(1 + z)$.

"Distances that move with Hubble flow"

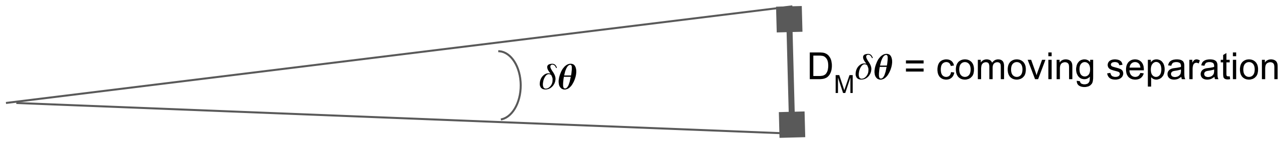
$$E(z) \equiv \sqrt{\Omega_{\rm M}\,(1+z)^3 + \Omega_k\,(1+z)^2 + \Omega_\Lambda}$$

$$D_{\rm C} = D_{\rm H} \int_0^z \frac{dz'}{E(z')}$$

Comoving Distance (Transverse)

The comoving distance between two events at the same redshift or distance but separated on the sky by some angle $\delta\theta$ is $D_{\mathrm{M}}\,\delta\theta$ and the transverse comoving distance D_{M} (so-denoted for a reason explained below) is simply related to the line-of-sight comoving distance D_{C}:

$$D_{\mathrm{M}} = \begin{cases} D_{\mathrm{H}} \frac{1}{\sqrt{\Omega_k}} \sinh\left[\sqrt{\Omega_k}\, D_{\mathrm{C}}/D_{\mathrm{H}}\right] & \text{for } \Omega_k > 0 \\ D_{\mathrm{C}} & \text{for } \Omega_k = 0 \\ D_{\mathrm{H}} \frac{1}{\sqrt{|\Omega_k|}} \sin\left[\sqrt{|\Omega_k|}\, D_{\mathrm{C}}/D_{\mathrm{H}}\right] & \text{for } \Omega_k < 0 \end{cases} \quad (16)$$



Angular Diameter Distance

$$\boxed{D_{\mathrm{A}} = \frac{D_{\mathrm{M}}}{1+z}}$$

Luminosity Distance

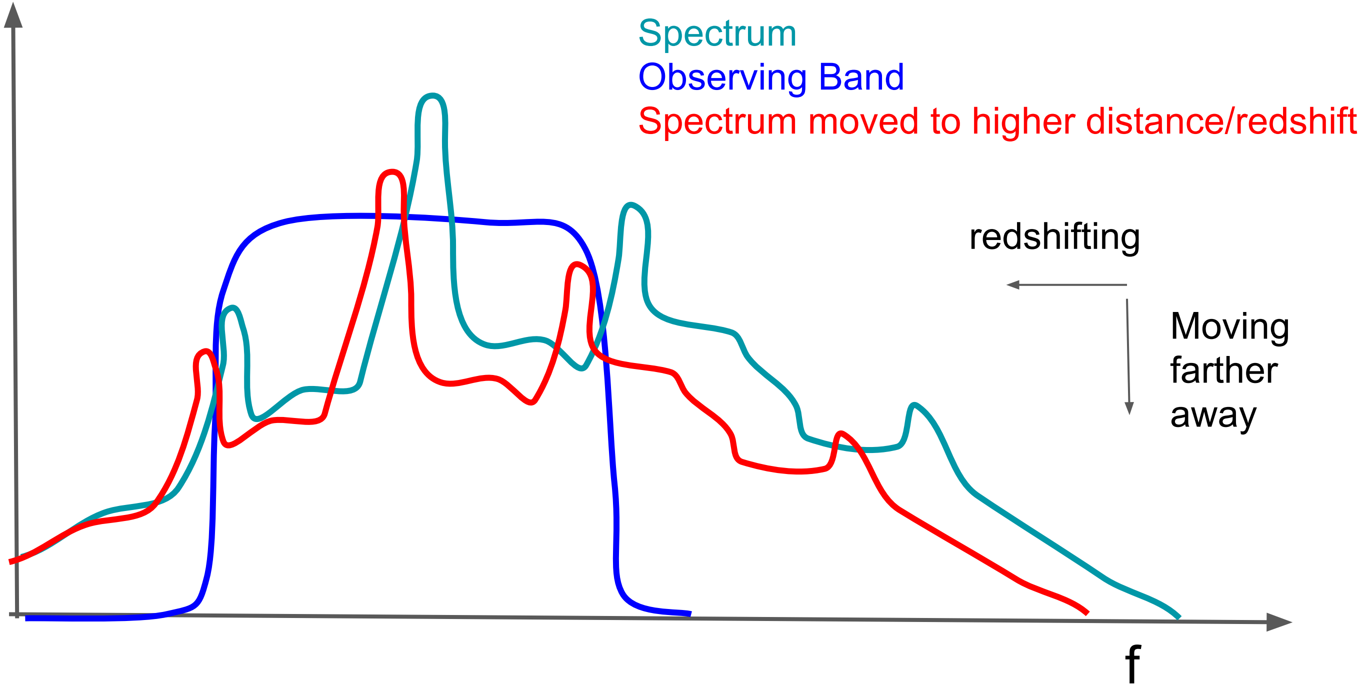
The *luminosity distance* D_{L} is defined by the relationship between bolometric (ie, integrated over all frequencies) flux S and bolometric luminosity L:

$$D_{\mathrm{L}} \equiv \sqrt{\frac{L}{4\pi\, S}} \tag{20}$$

It turns out that this is related to the transverse comoving distance and angular diameter distance by

$$D_{\mathrm{L}} = (1+z)\, D_{\mathrm{M}} = (1+z)^2\, D_{\mathrm{A}} \tag{21}$$

K-correction



Have to adjust observed *specific* fluxes when considering arbitrary spectra

Comoving Volume

The *comoving volume* V_{C} is the volume measure in which number densities of non-evolving objects locked into Hubble flow are constant with redshift. It is the proper volume times three factors of the relative scale factor now to then, or $(1+z)^3$. Since the derivative of comoving distance with redshift is $1/E(z)$ defined in (14), the angular diameter distance converts a solid angle $d\Omega$ into a proper area, and two factors of $(1+z)$ convert a proper area into a comoving area, the comoving volume element in solid angle $d\Omega$ and redshift interval dz is

$$dV_{\mathrm{C}} = D_{\mathrm{H}} \frac{(1+z)^2 \, D_{\mathrm{A}}^2}{E(z)} \, d\Omega \, dz \qquad (28)$$

Lookback Time

The *lookback time* $t_{\rm L}$ to an object is the difference between the age $t_{\rm o}$ of the Universe now (at observation) and the age $t_{\rm e}$ of the Universe at the time the photons were emitted (according to the object). It is used to predict properties of high-redshift objects with evolutionary models, such as passive stellar evolution for galaxies. Recall that $E(z)$ is the time derivative of the logarithm of the scale factor $a(t)$; the scale factor is proportional to $(1+z)$, so the product $(1+z)\,E(z)$ is proportional to the derivative of z with respect to the lookback time, or

$$t_{\rm L} = t_{\rm H} \int_0^z \frac{dz'}{(1+z')\,E(z')} \tag{30}$$

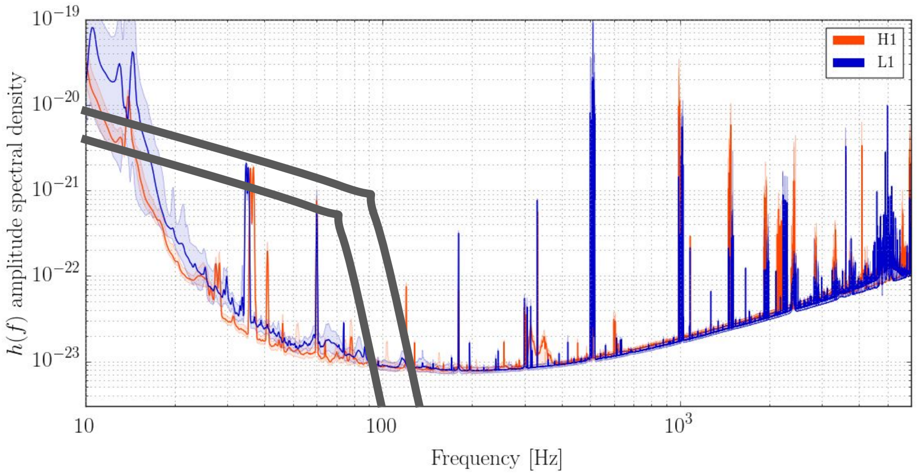
Cosmological Effects on Waveforms

$$h \propto D_L^{-1}$$

$$h(m_1, m_2, D_L) = h(m_1 \times (1+z), m_2 \times (1+z), D_L(z))$$

If we put a source at a cosmological distance D_L, the observed waveform will redshift. Since GR is “scale-free”, a shift in frequencies is equivalent to a change in mass, and we can simply generate a waveform with the *redshifted masses* at luminosity distance D_L

Cosmological Effects on Waveform (GW K-correction)



Brain teaser: Consider an arbitrary GW source at luminosity distance D_L measured to have SNR 8 under an arbitrary noise curve in a single detector. If we move the source to a *farther* distance along the same line of sight, must the SNR go down?

Cosmological Effects on Rates

Redshifting will not only lower the frequencies of GW waveforms: it will also lower the number of sources per time on our clock. I.e. if there are are 100 mergers happening per day in a galaxy at $z=0$, there will be $100/(1+z_1)$ at redshift z_1.

Measuring the Hubble-Lemaitre Constant with GWs

1. Estimate D_L from GW,
2. get redshift from galaxy,
3. use $H_0 D_L = cz$ to solve for H_0.