

Cobb-Douglas 生產函數  $Q = f(L, K) = L^\alpha K^\beta, \alpha, \beta > 0$

① 產出彈性

勞動平均產量與勞動邊際產量為  $AP_L = \frac{Q}{L} = \frac{L^\alpha K^\beta}{L} = L^{\alpha-1} K^\beta$

$$MP_L = \frac{\partial Q}{\partial L} = \alpha L^{\alpha-1} K^\beta$$

資本平均產量與資本邊際產量為  $AP_K = \frac{Q}{K} = \frac{L^\alpha K^\beta}{K} = L^\alpha K^{\beta-1}$

$$MP_K = \frac{\partial Q}{\partial K} = \beta L^\alpha K^{\beta-1}$$

$$\text{勞動產出彈性} = \varepsilon^L = \frac{MP_L}{AP_L} = \frac{\alpha L^{\alpha-1} K^\beta}{L^{\alpha-1} K^\beta} = \alpha$$

$$\text{資本產出彈性} = \varepsilon^K = \frac{MP_K}{AP_K} = \frac{\beta L^\alpha K^{\beta-1}}{L^\alpha K^{\beta-1}} = \beta$$

② 生產力彈性

勞動與資本要素同時增加  $\phi$  倍對生產函數的影響：

$$Q = f(\phi L, \phi K) = \phi^{\alpha+\beta} L^\alpha K^\beta$$

$$\text{生產力彈性} = \varepsilon^\phi = \frac{\frac{dQ}{d\phi}}{\frac{Q}{\phi}} = \frac{\frac{dQ}{d\phi}}{\frac{Q}{\phi}} = \frac{(\alpha+\beta)\phi^{\alpha+\beta-1} L^\alpha K^\beta}{\frac{\phi^{\alpha+\beta} L^\alpha K^\beta}{\phi}} = \alpha + \beta$$

③ 替代彈性

$$\text{邊際技術替代率} = MRTS = \frac{MP_L}{MP_K} = \frac{\alpha L^{\alpha-1} K^\beta}{\beta L^\alpha K^{\beta-1}} = \frac{\alpha}{\beta} \cdot \frac{K}{L}$$

$$\text{替代彈性} = \varepsilon^{LK} = \frac{d \ln \left[ \frac{K}{L} \right]}{d \ln (MRTS)} = \frac{d \ln \left[ \frac{K}{L} \right]}{d \ln \left[ \frac{\alpha}{\beta} \right] + d \ln \left[ \frac{K}{L} \right]} = 1$$

練習題

假設生產函數  $Q = 3K + 2L$ 。K 為資本，L 為勞動，Q 為產出

(1) 函數呈現固定規模報酬：

若 K 和 L 同時增加  $n$  倍。

$$f(nK, nL) = 3(nK) + 2(nL) = nQ, \text{ 故為正確。}$$

3) MRTS 為一定值，故為正確

(2) 函數呈現資本與勞動的邊際產量遞減：

$$MP_L = \frac{\Delta Q}{\Delta L} = 2, \quad MP_K = \frac{\Delta Q}{\Delta K} = 3$$

兩者皆固定，故為錯誤。