

Lecture 5

Friday, 29 January 2021

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How eigenvectors/values can be used to explain your data

If data is normally distributed, mean \pm SD represents about 68% of your data

- However, variance only explains variance seen along a certain axis
- Covariance matrix can be used if variance is diagonal

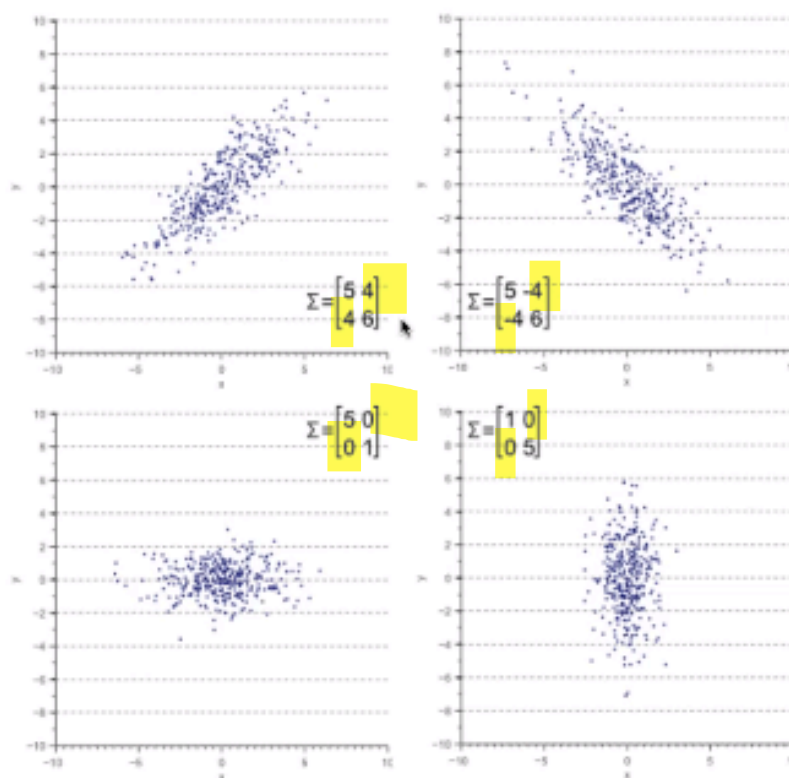


Figure 3. The covariance matrix defines the shape of the data. Diagonal spread is captured by the covariance, while axis-aligned spread is captured by the variance.

$$\Sigma = \begin{bmatrix} \text{Var} & \text{Cov} \\ \text{Cov} & \text{Var} \end{bmatrix}$$

+Ve correlation = +ve cov

-ve corr = -ve cov

No covariance = no corr (no variance on diagonal)

Top 2: eigenvalues are the same as the variances as no covariance

X and y axes are fine to explain variation

Bottom 2: eigenvectors (blue and pink lines) not the same as variances

- X and y axes aren't sufficient, eigenvectors as axes more effectively capture variation

$$\begin{bmatrix} \text{Var}(x) & \text{Cov}(x, y) \\ \text{Cov}(y, x) & \text{Var}(y) \end{bmatrix}$$

convey variation

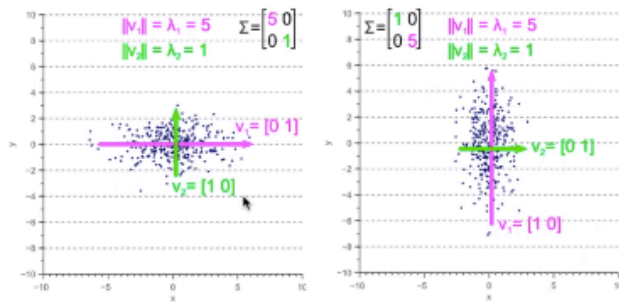


Figure 4. Eigenvectors of a covariance matrix

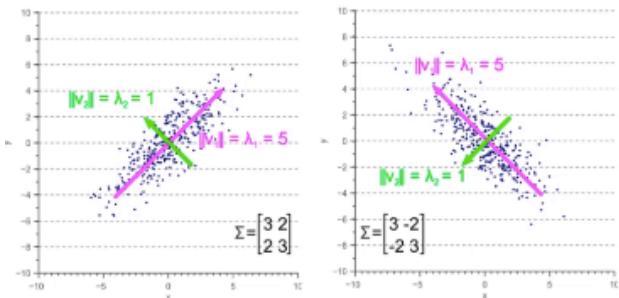
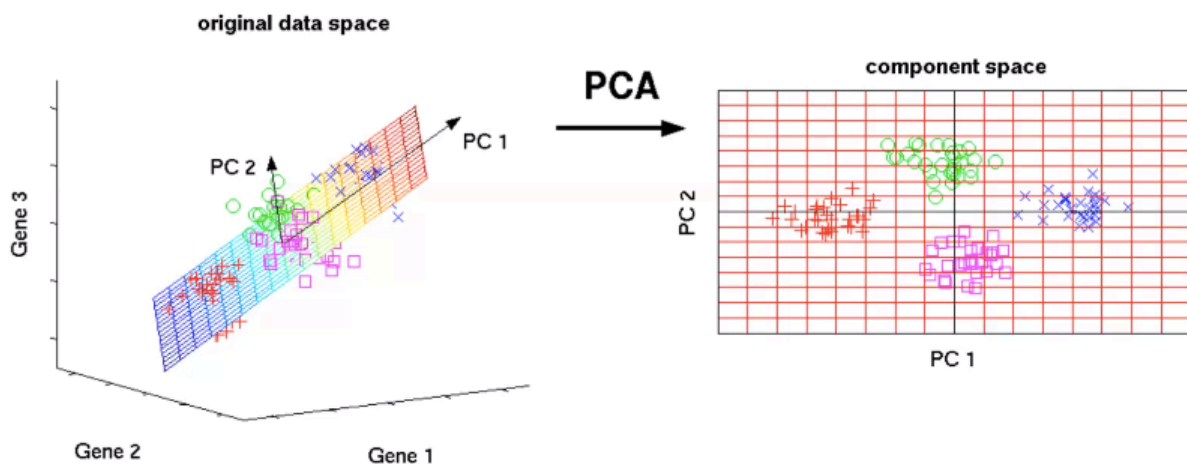


Figure 5. Eigenvalues versus variances

$$\Sigma = \begin{bmatrix} \text{Var}(x) & \text{Cov}(x, y) \\ \text{Cov}(y, x) & \text{Var}(y) \end{bmatrix}$$

$$\Sigma \vec{v} = \lambda \vec{v}$$

Principle Component Analysis



If have very large datasets, will often have many parameters which is hard to interpret

- Num params = num dimensions

Principle component analysis turns 3 dimensional graph on left into 2d on the right, aims to minimise dimension while preserving variability

PC1 crosses through multidimensional mean, and is the line that has the lowest RSS that captures the most variance

PC2 is the line that captures the second most variance

PC1 is eigenvector that gives most variance = highest eigenvector. PC2 =

... eigenvector with highest = highest eigenvector, ...
eigenvector with second highest = second highest number

