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Question B1

(a) $\because N(t) = k\cdot e^{-a\cdot e^{-bt}}$

$\therefore N_0 = k\cdot e^{-a} \Rightarrow \ln N_0 = -a + \ln k$

$\therefore a = \ln(\frac{k}{N_0})$

(b) $\because N_0 < k$ from (a) $a = \ln(\frac{k}{N_0})$

$\therefore a > 0$

$\because b > 0$ $t > 0$ $\therefore -bt < 0$

$\therefore e^{-bt} \in (0,1)$

$\therefore -a\cdot e^{-bt} \in (-a, 0)$

$\therefore e^{-a\cdot e^{-bt}} \in (0,1)$

$\therefore N_t = k\cdot e^{-a\cdot e^{-bt}} \in (0,k)$

$\therefore$ when $t \geq 0 \Rightarrow N_t < k$

(c) $\because N(t) = k\cdot e^{-a\cdot e^{-bt}}$

$\therefore \frac{dN}{dt} = a\cdot b\cdot k\cdot e^{(-a\cdot e^{-bt} - bt)}$

$= a\cdot b N\cdot e^{-bt}$

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$\because \ln N = \ln k - a\cdot e^{-bt}$

$\therefore a\cdot e^{-bt} = \ln\frac{k}{N}$

$\therefore \frac{dN}{dt} = b\cdot N\cdot \ln\frac{k}{N}$

$\frac{d^2N}{dt^2} = -a\cdot b^2 k\cdot e^{(-a\cdot e^{-bt} - 2bt)}$

$= -b\cdot\frac{dN}{dt}\cdot e^{-bt}$

$$\because e^{-bt} = \frac{1}{a}\ln\left(\frac{K}{N}\right)$$

from (4) $a = \ln\left(\frac{K}{N_0}\right)$

$$\therefore e^{-bt} = \frac{\ln K - \ln N}{\ln K - \ln N_0}$$

$$\therefore \frac{d^2N}{dt} = b \cdot \frac{dN}{dt}(\ln K - \ln N - 1)$$

(d) $\because N_0 < k$ from (a) $a = \ln(\frac{k}{N_0})$

$\therefore a > 0$

$\because b > 0 \quad N > 0$

$\therefore \frac{dN}{dt} > 0$

$\boxed{\text{same as } \cancel{\lim_{t\to\infty} N(t) = k}}$

so as $N_0 < k$, $N(t)$ is monotonically increasing

(e) $\because N_0 < k$ from (a) $a > 0$ $\because b > 0$

$\therefore \lim_{t\to\infty} N(t) = \lim_{t\to\infty} k \cdot e^{-a \cdot e^{-bt}}$

$= \lim_{t\to\infty} k \cdot e^{-a \cdot 0} = k$

(f) $\because N_0 < \frac{k}{e}$ $\therefore \ln\frac{k}{e} > \ln N_0$ $(\ln k - 1 > \ln N_0)$

let $\frac{d^2N}{dt^2} = 0 \Rightarrow$

$b^2 N(\ln k - \ln N)(\ln k - \ln N - 1) = 0$

$\because b > 0 \quad N > 0$

$\therefore \ln N = \ln k$ or $\ln N = \ln k - 1 = \ln\frac{k}{e} > \ln N_0$

$\therefore$

| $N$ | $(0, N_0)$ | $N_0$ | $(N_0, \frac{k}{e})$ | $\frac{k}{e}$ | $(\frac{k}{e}, k)$ |
|---|---|---|---|---|---|
| $\frac{d^2N}{dt^2}$ | + | 0 | + | 0 | − |

$\therefore N = \frac{k}{e}$ corresponds to an inflection point

$\frac{k}{e} = k \cdot e^{-a \cdot e^{-bt}}$

$$e^{1-a\cdot e^{-bt}} = 1$$

$$1-a\cdot e^{-bt} = 0$$

$$a\cdot e^{-bt} = 1$$

$$\therefore \ln a - bt = 0$$

$$t = \frac{1}{b}\ln a$$

So $N(t)$ has an inflection point

at $t_1 = \frac{1}{b}\ln a$ when $N_0 < \frac{k}{e}$