Report

Assignment 1

February 19, 2025

Student:
Zoë Azra Blei - 2695271
Eleni Liarou - 2868489

Lecturer:
Oliver Champion

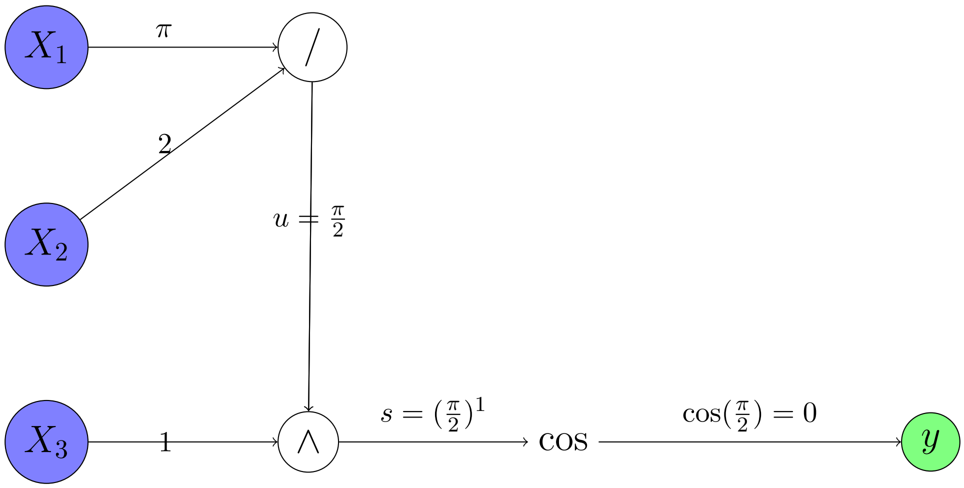
Course:
Deep Learning for Medical Imaging

Course code:
XM_0151

1 Exersice 2 - Feedforward loop calculation by pen

A

Forward pass



Backward pass

We define y, s, and u as follows:

$y = cos(s)$

$u = \frac{x_1}{x_2}$

$s = u^{x_3} = e^{x_3 ln \frac{x_1}{x_2}}$

This yields in the following derivatives:

$\frac{dy}{dx_1} = \frac{dy}{ds} \cdot \frac{ds}{du} \cdot \frac{du}{dx_1} = \frac{d\cos(s)}{ds} \cdot \frac{du^{x_3}}{du} \cdot \frac{d\frac{x_1}{x_2}}{dx_1} = -\sin(s) \cdot x_3 \cdot u^{x_3-1} \cdot \frac{1}{x_2} = -\sin((\frac{\pi}{2})^1) \cdot 1 \cdot (\frac{\pi}{2})^0 \cdot \frac{1}{2} = -1 \cdot 1 \cdot 1 \cdot \frac{1}{2} = -\frac{1}{2}$

$\frac{dy}{dx_2} = \frac{dy}{ds} \cdot \frac{ds}{du} \cdot \frac{du}{dx_2} = -\sin(s) \cdot x_3 \cdot u^{x_3-1} \cdot (-\frac{x_1}{x_2^2}) = -1 \cdot 1 \cdot 1 \cdot (-\frac{pi}{4}) = \frac{\pi}{4}$

$\frac{dy}{dx_3} = \frac{dy}{ds} \cdot \frac{ds}{dx_3} = -\sin(s) \cdot s \cdot (\ln u + \frac{x_3 du}{u dx_3}) = -\sin(s) \cdot s \cdot s(lnu + 0) = -1 \cdot \frac{\pi}{2} \cdot \ln \frac{\pi}{2}$

B

Defining y as follows:

y $= \cos\left(\frac{x_1}{x_2}\right)^{x_3}$

Yields in the following derivatives:

$\frac{dy}{dx_1} = -\sin((\frac{x_1}{x_2})^{x_3}) \cdot x_3 \cdot (\frac{x_1}{x_2})^{x_3-1} \cdot \frac{1}{x_2}$

$\frac{dy}{dx_2} = -\sin((\frac{x_1}{x_2})^{x_3}) \cdot x_3 \cdot (\frac{x_1}{x_2})^{x_3-1} \cdot \frac{x_1}{x_2^2}$

$\frac{dy}{dx_3} = -\sin((\frac{x_1}{x_2})^{x_3}) \cdot (\frac{x_1}{x_2})^{x_3} \cdot \ln \frac{x_1}{x_2} + \frac{x_3 d\frac{x_1}{x_2}}{\frac{x_1}{x_2} dx_3}$

C

$$\begin{aligned} z &= W^T x + b \\ &= \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ 8 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 10 \end{bmatrix} \end{aligned}$$

$$a = \mathrm{ReLU}(z) = \max(0, z) = \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$\text{Output layer: } y = v^T a = \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 10 \end{bmatrix}$$

$$= 20$$

D

We know that:

$y = 20$, $y_{true} = 22$

$L = (y - y_{true})^2 = (20 - 22)^2 = (-2)^2 = 4$

$y = v_1 a_1 + v_2 a_2$

$a_2 = \sigma(x_2 W_{2,2} + x_1 W_{1,2})$

From this it follows that:

$\frac{dL}{dy} = 2(y - y_{true}) = 2(20 - 22) = -4$

$\frac{dy}{da_2} = v_2 = 2$

$\frac{da_2}{dZ_2} = 1$ if $Z_2 \geq 0$, otherwise 0

$\frac{dZ_2}{dW_{2,2}} = x_2 = 3$

Thus:

$\frac{dL}{dW_{2,2}} = \frac{dL}{dy} \cdot \frac{dy}{da_2} \cdot \frac{da_2}{dZ_2} \cdot \frac{dZ_2}{dW_{2,2}} = -4 \cdot 2 \cdot 1 \cdot 3 = -24$