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Nondeterministic Finite Automata

(Part 2)

Lecture 10

Day 10/31

CS 154

Formal Languages and Computability

String 2018

Agenda of Day 10

- About Midterm 1
- Solution and Feedback of Quiz 3
- Solution and Feedback of HW 2
- Summary of Lecture 09
- Lecture 10: Teaching ...
  - Nondeterministic Finite Automata (Part 2)

About Midterm 1

2nd Announcement

- Midterm #1 (aka Quiz+)
  - Date: Thursday 03/01
  - Value: 10%
  - Topics: Everything covered from the beginning of the semester
  - Type: Closed y ∈ Material

    Material = {Book, Notes, Electronic Devices, Chat, ... }

- The cutoff for midterm #1 is the end of lecture 09.

Solution and Feedback of Quiz 3 (Out of 27)

Metrics	Section 1	Section 2	Section 3
Average	23	24	22
High Score	27	27	27
Low Score	17	19	15

Summary of Lecture 09: We learned ...

NFAs

- Two violations in DFAs were introduced...
- Violation #1
  During a timeframe, the machine has no choice (zero choice).
  - The transition function is partial function.
- Violation #2
  During a timeframe, the machine has more than one choice.
  - The transition function is a multifunction.
- We introduced a new class of automata.

NFAs Behavior

1. When an NFA has no choice ...
   - ... it halts.
2. When there are more than one choice, ...
   - ... it starts parallel processing.

When NFAs halt

- All input symbols are consumed. ≡ **c**

OR

- It has zero transition. ≡ **z**

$$(\mathbf{c} \vee \mathbf{z}) \leftrightarrow \mathbf{h}$$

Any question?

Summary of Lecture 09: We learned ...

NFAs

- A string is accepted if ...
  - ... at least one process accepts it.
  - For NFAs, **(h ∧ c ∧ f)** ↔ **a** is valid for accepting strings by one process.
  - Because **h** and **c** might have different values.
- A string is rejected if ...
  - ... all processes reject it.

Any question?

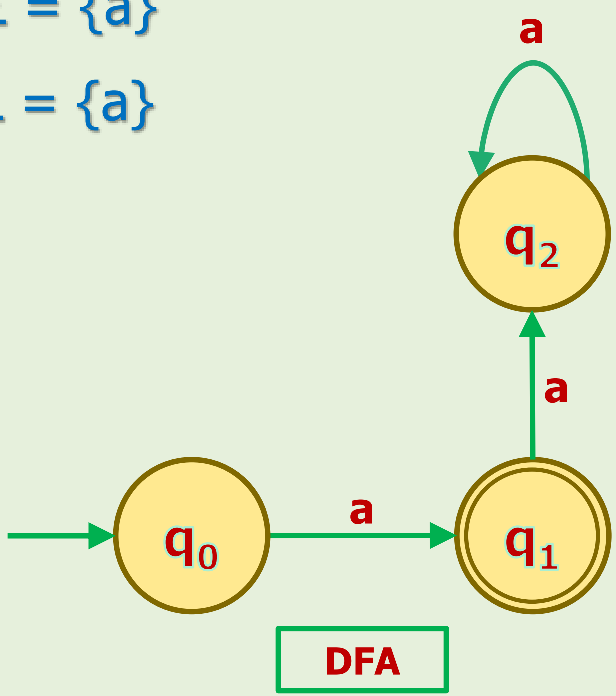
Why We Need a New Class

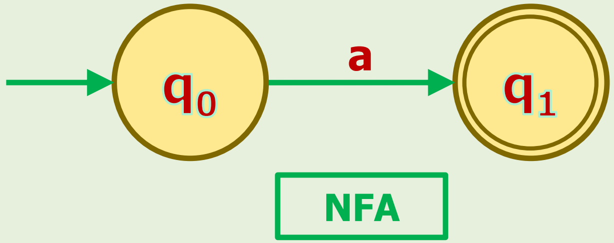
Revisited Question

- NFAs are interesting because their transition graphs are simpler.

Example 7

- $\Sigma = \{a\}$
- $L = \{a\}$





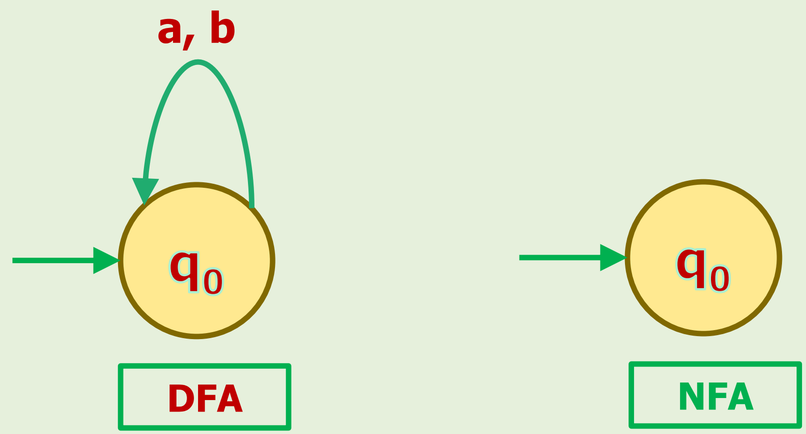
No trap needed!

Why We Need a New Class

Revisited Question

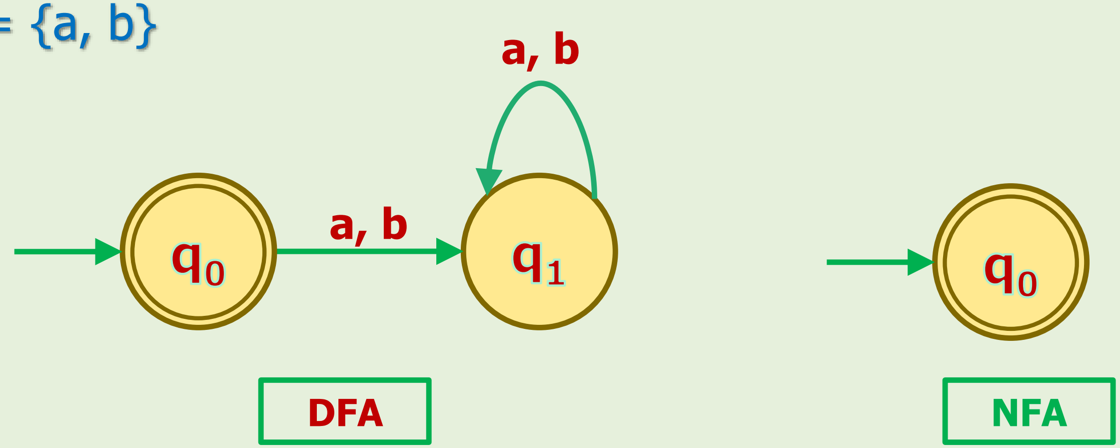
Example 8: Empty Language

$L = \{\ \}$ over $\Sigma = \{a, b\}$



Example 9: Empty String Language

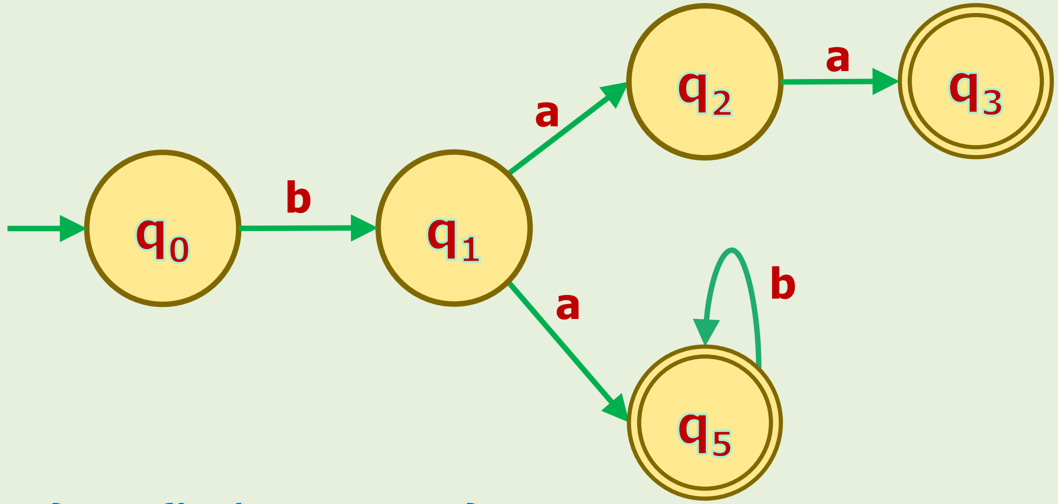
$L = \{\lambda\}$ over $\Sigma = \{a, b\}$



Associated Language to NFAs

Example 10

- What is the associated language to the following automaton over $\Sigma = \{a , b\}$?



- $L = \{baa\} \cup \{bab^n : n \geq 0\}$

- Design this machine by DFA.

Lambda Transition

Introduction

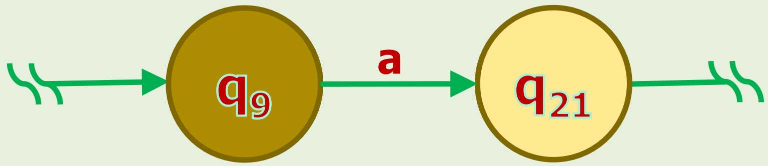
- So far, we learned two violations on DFAs.
- They encouraged us to introduce a new class of machines.
- We named it NFA.

- In this section, we'll talk about a special kind of transition.
- Another possible transitions that are strictly prohibited in DFAs ...
- But is allowed in NFAs.

Let's Shine our Knowledge

Question

- In the following NFA, if the machine is in q_9, what is the "condition" for transition to q_{21}?



Answer

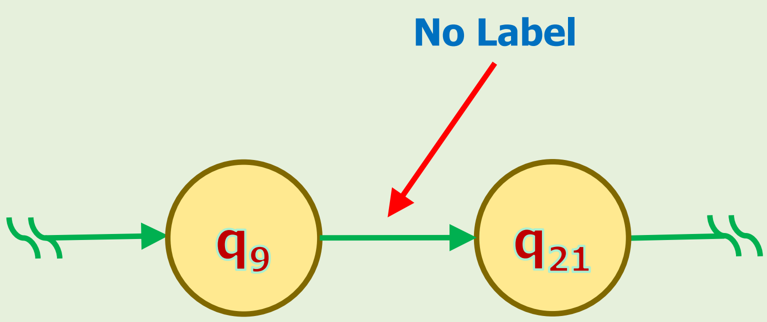
- If the machine is in q_9 AND the next input symbol is 'a', then the machine transits to q_{21}.

Conclusion

- The transition from q_9 to q_{21} is "conditional".

Let's Remove the Condition

- What would happen if we remove the condition?

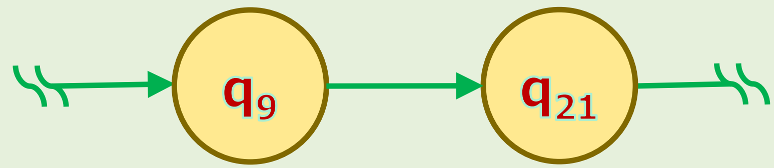


- This possible situation is called "short-circuit".
- A "short-circuit" is an edge with no label (symbol).

- What is the meaning of short-circuit?
- What is the behavior of the machine when it encounters it?

What is the Meaning of Short Circuit?

- If there is no label, then there is NO condition for transition!

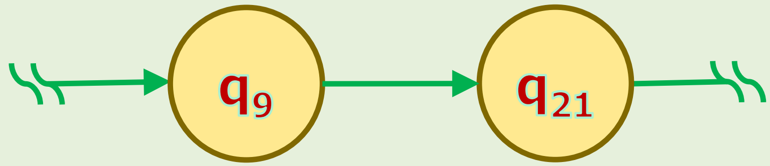


The machine can transit unconditionally!

- What would happen to the input tape?
- Does it need to consume any symbol?
- No, it doesn't!
- In fact, the control unit does not need to wait to receive the input symbol for deciding where to go.

How To Represent the Transition Function

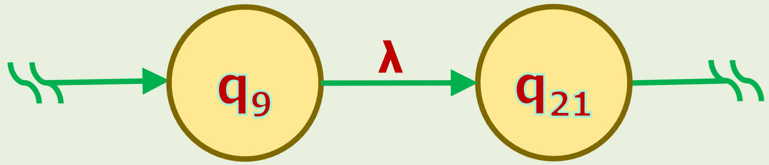
- What is the sub-rule of the following transition?



- The format of the sub-rules is: $\delta\ (q_9\ ,\ ?) = \{??\}$
- We should find out what to substitute for ? and ??.
- One suggestion for '?' is blank: $\delta\ (q_9\ ,\ ) = \{??\}$
- But this is not desirable format in math!
- We need to put something there.

How To Represent the Transition Function

- The symbol "λ" was chosen to represent "short-circuit".



- So, the sub-rule would be:

$$\delta\ (q_9\ ,\ \lambda) = \{??\}$$

- Because of this symbol, this type of transitions are called "lambda transition" or "λ-transition".

- We still need to find out what is the value of ??.

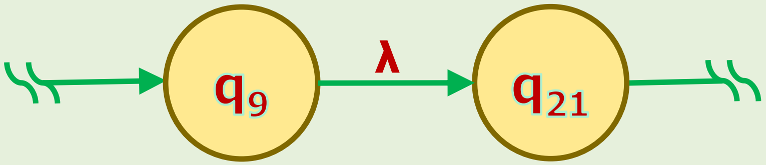
Another Look to Meaning of λ

- Before going further, note that:
- We've already used λ to represent "empty string".
- In fact λ means "NO symbol". (Empty String = NO Symbol)
- And short-circuit has "no symbol".
- That's why the short-circuit is represented by λ.

- Be careful:
  - Using λ as "empty string" and the symbol of "short-circuit" can be confusing but you'll get used to it!

How To Represent the Transition Function

$$\delta(q_9, \lambda) = \{??\}$$



- What is the value of ??.
- Since the machine may transit unconditionally, it means that ...

  it may stay as well.

- So, when a machine encounters a λ-transition,

  it may stay or it may transit.

- Therefore, the sub-rule for the example is:

$$\delta(q_9, \lambda) = \{q_9, q_{21}\}$$

λ-Transition Definition

- It's time to define λ-transition officially.

Definition

- λ-transition in automata theory means:

  The machine may "unconditionally" transit.

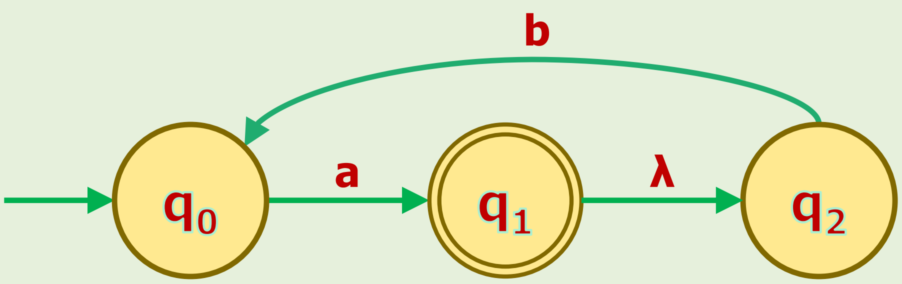
- Note that this is a general definition for all types of automata.
- Also note that λ-transition concept changes our view about sub-rules.

- Let's take an example.

How To Represent the Transition Function

Example 11

- Write the rule of the following transition graph over $\Sigma = \{a, b\}$ by using algebraic notation.



- δ: $\begin{cases} \delta(q_0, a) = \{q_1, q_2\} \\ \delta(q_0, b) = \{\ \} \\ \delta(q_1, a) = \{\ \} \\ \delta(q_1, b) = \{q_0\} \\ \delta(q_1, \lambda) = \{q_1, q_2\} \\ \delta(q_2, a) = \{\ \} \\ \delta(q_2, b) = \{q_0\} \end{cases}$

NFAs Transition Function

- We should change the transition function definition to accommodate λ-transitions.
- Recall that we concluded NFAs' transition function to be:

$$\delta: Q \times \Sigma \rightarrow 2^Q$$

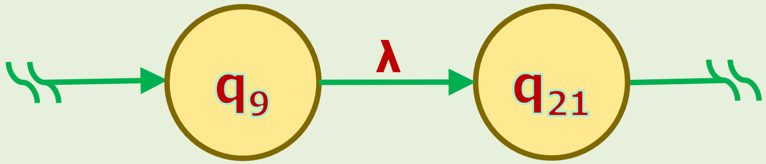
- But now, we have a new symbol for λ-transition.
- So, we just need to add it to Σ.

$$\delta: Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q$$

- We'll mention it again when we are giving the formal definition of NFAs.

How NFAs Behave If They Encounter λ-transitions

- Let's review the example one more time:



$$\delta (q_9 , \lambda) = \{q_9 , q_{21}\}$$

- The NFA has multiple choices.

Stay in q_9, OR transit to q_{21}.

- How should it behave when it has multiple choices?
- It would check all possibilities by "parallel processing".
  - In other words, for every possible choice, it initiates a new independent process and every process independently continues processing the string.

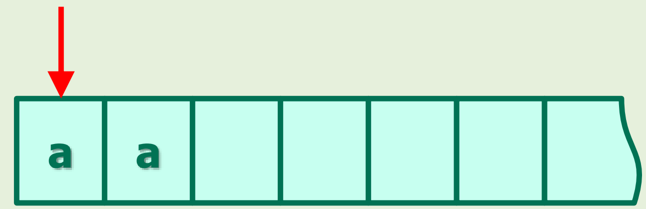
- Let's see some practical examples of λ-Transitions!

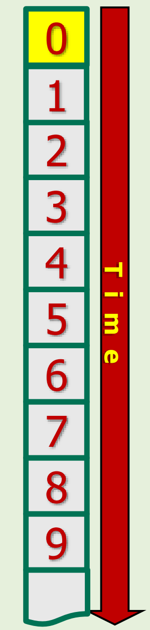
λ-Transitions in Action

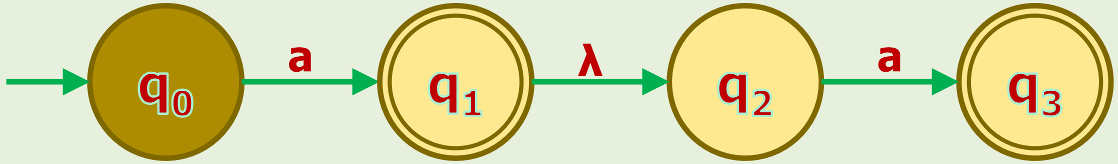
λ-Transitions in Action

Example 12: Process #1

- $\Sigma = \{a\}$
- $w = aa$
- $\delta(q_0, a) = \{q_1\}$



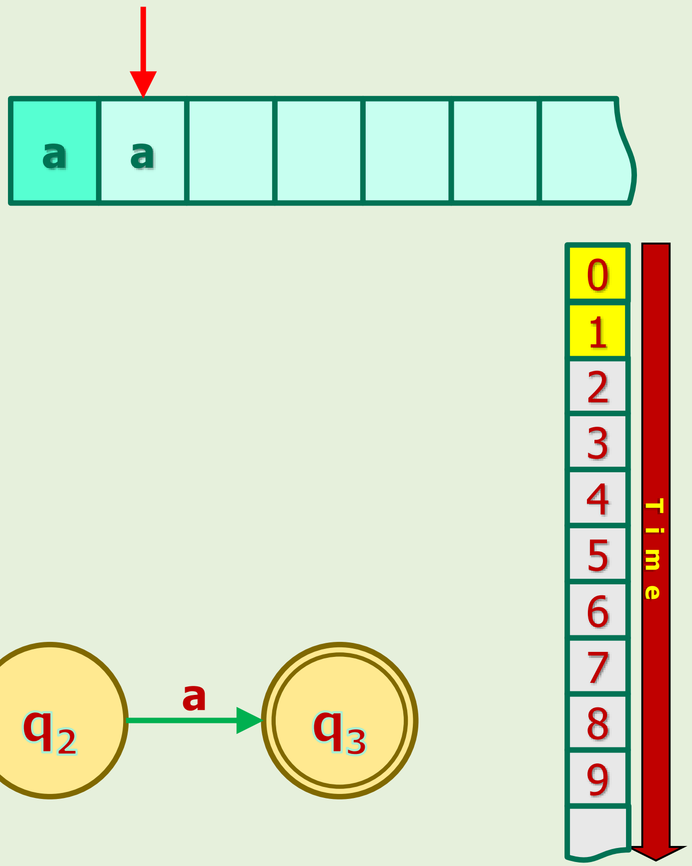


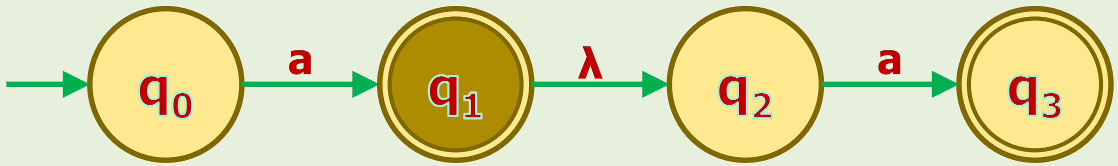


λ-Transitions in Action

Example 12: Process #1

- $\Sigma = \{a\}$
- $w = aa$
- $\delta(q_1, \lambda) = \{q_1, q_2\}$



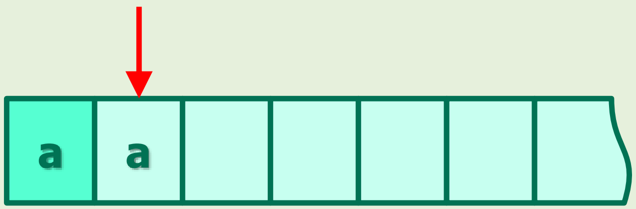


- The NFA encounters a λ-transition.
- Process #2 is initiated.
- Initial configuration: state=q_2, rest of input=a, clock=1

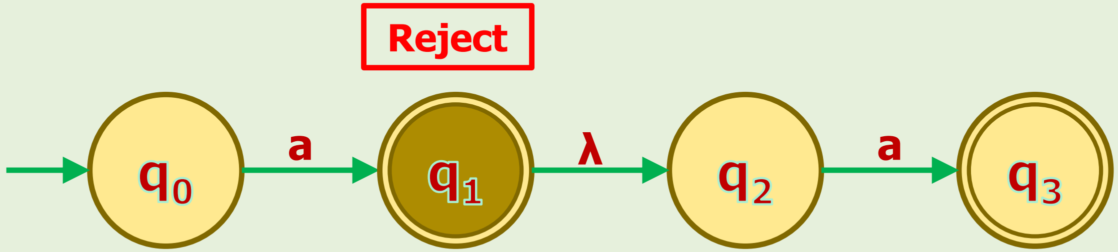
λ-Transitions in Action

Example 12: Process #1

- $\Sigma = \{a\}$
- $w = aa$



- Process #1 cannot continue because it has no choice for symbol 'a' in q_1.
- So, it halts in q_1.

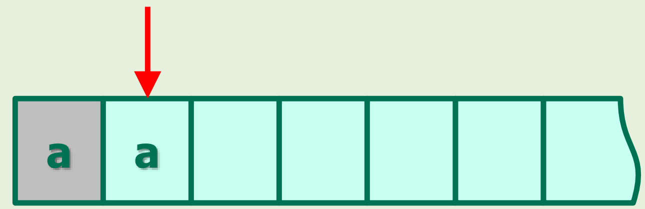


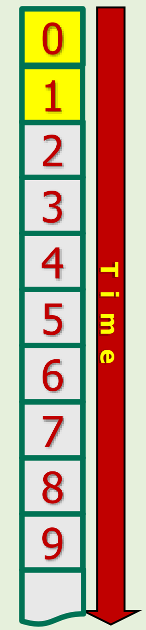
- Some symbols are NOT consumed, so, process #1 rejects the string.

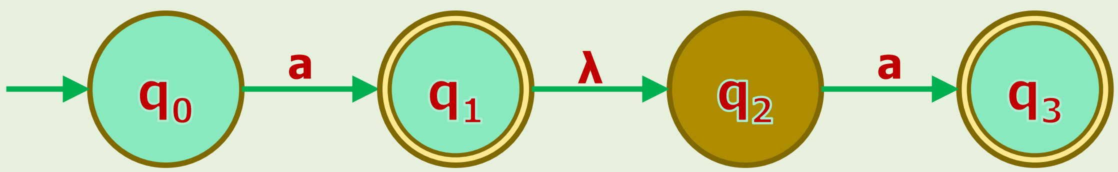
λ-Transitions in Action

Example 12: Process #2

- $\Sigma = \{a\}$
- $w = aa$
- Initial config: state=q_2, rest of input=a, clock=1



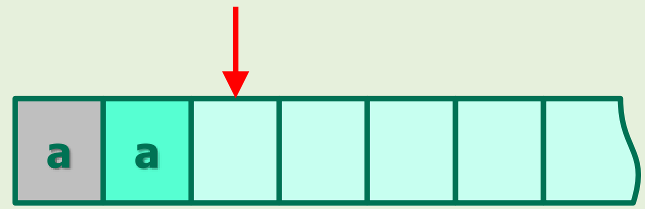


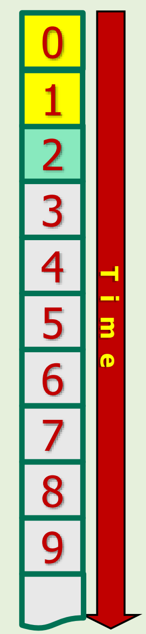


λ-Transitions in Action

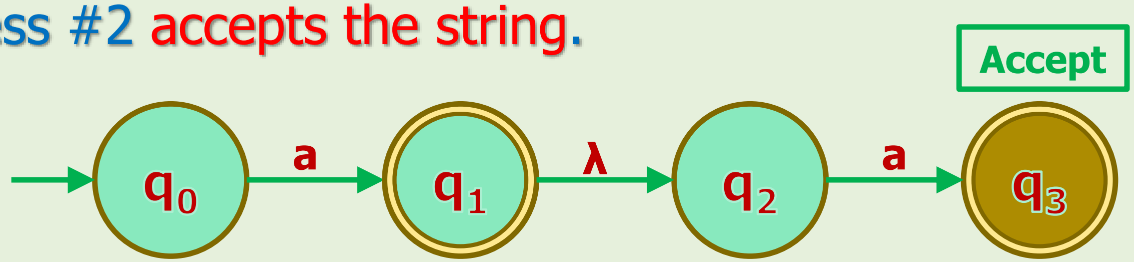
Example 12: Process #2

- $\Sigma = \{a\}$
- w = aa





- All symbols are consumed.
- The machine halts in an accepting state.
- So, process #2 accepts the string.

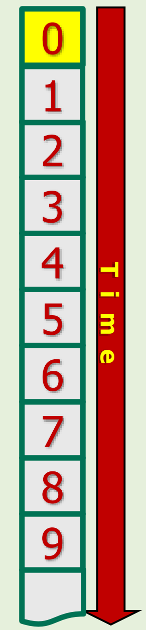


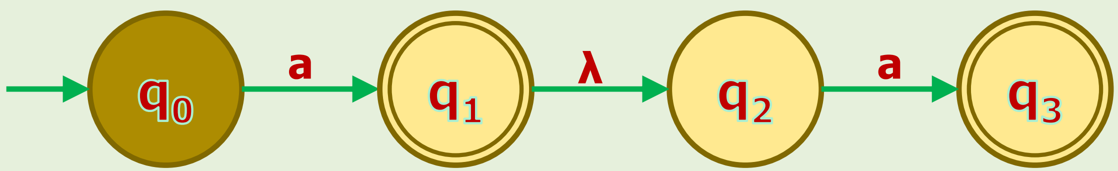
- Recap: if one process accepts a string, then the string is accepted.
- So, overall, the string aa is accepted.

λ-Transitions in Action

Example 13: Process #1

- $\Sigma = \{a\}$
- $w = aaa$

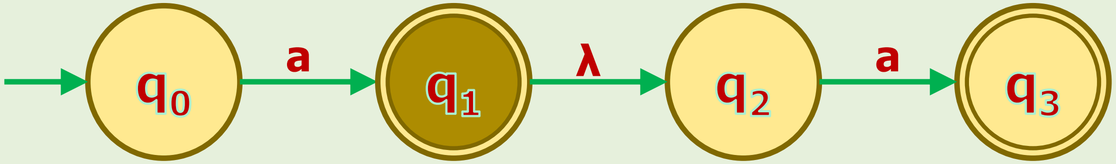




λ-Transitions in Action

Example 13: Process #1

- $\Sigma = \{a\}$
- w = aaa
- Again, the NFA initiates another process because of the λ-transition.
- Initial config: state=q_2, rest of input=aa, clock=1
- Process #1 continues.

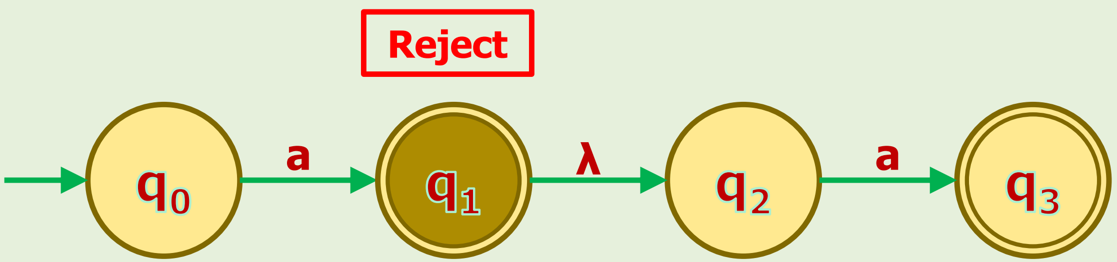


λ-Transitions in Action

Example 13: Process #1

- $\Sigma = \{a\}$
- $w = aaa$

- Process #1 cannot continue any longer because it has no choice for symbol 'a' in q_1.

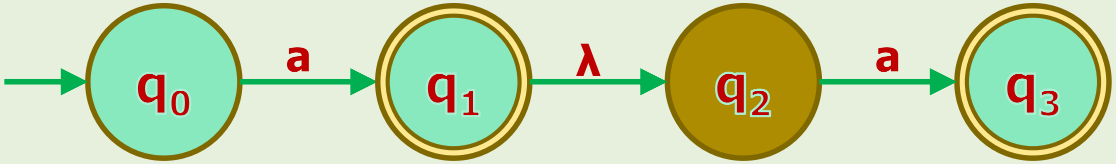


- So, process #1 rejects the string because some symbols are not consumed.

λ-Transitions in Action

Example 13: Process #2

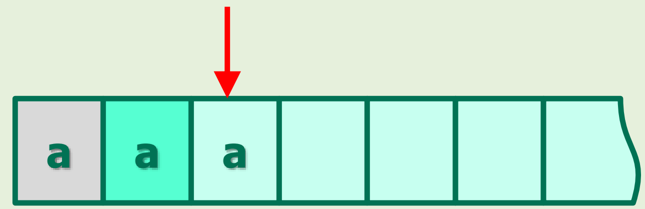
- $\Sigma = \{a\}$
- $w = aaa$
- Initial config: state=q_2, rest of input=aa, clock=1

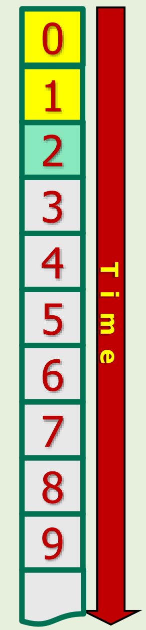


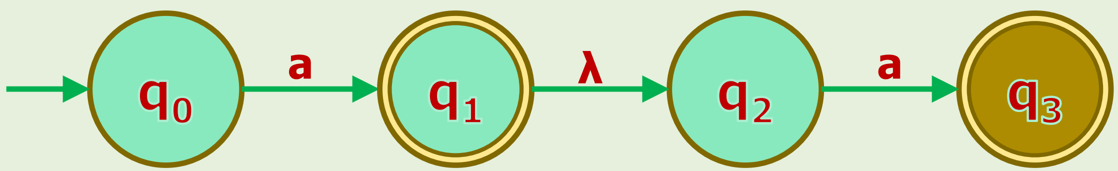
λ-Transitions in Action

Example 13: Process #2

- $\Sigma = \{a\}$
- $w = aaa$



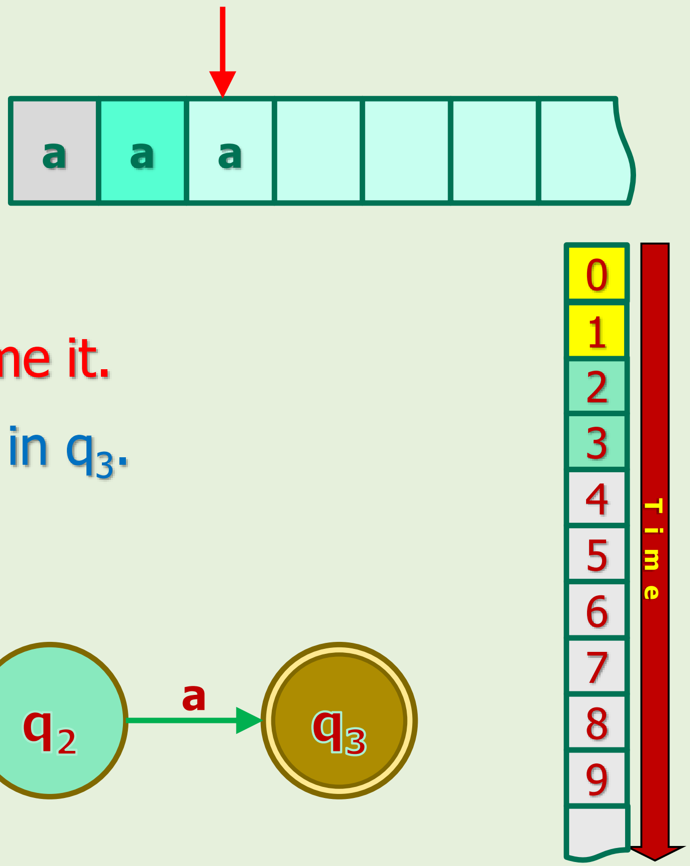




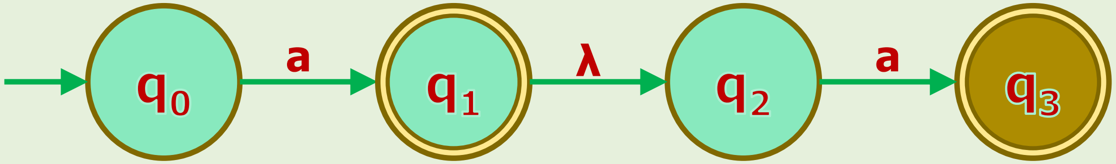
λ-Transitions in Action

Example 13: Process #2

- $\Sigma = \{a\}$
- w = aaa



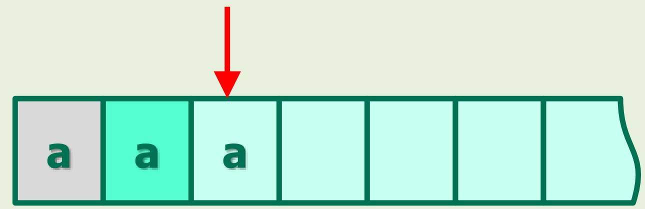
- Process #2 reads 'a' but cannot consume it.
- Because it has no choice for symbol 'a' in q_3.

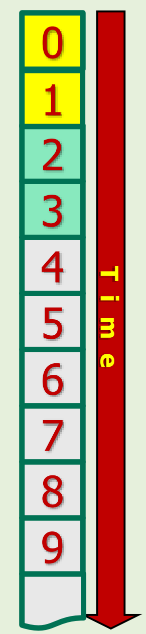


λ-Transitions in Action

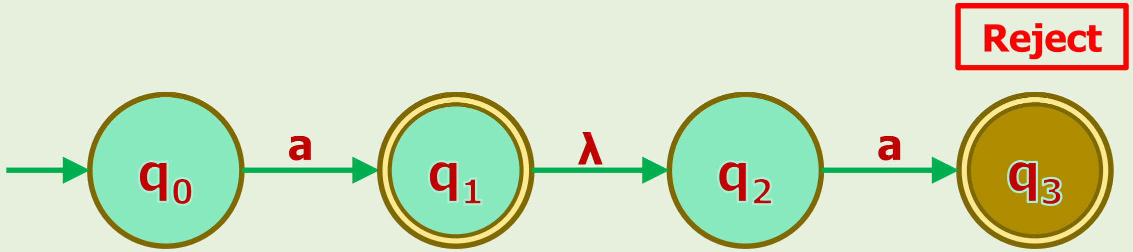
Example 13: Process #2

- $\Sigma = \{a\}$
- $w = aaa$





- Some input symbols are NOT consumed.
- No matter where the machine halted, the string is rejected.

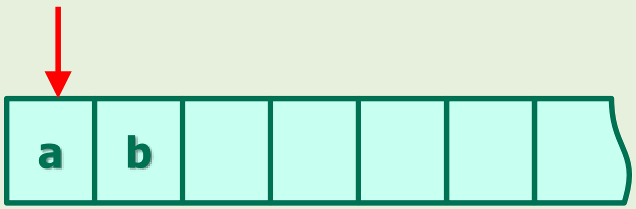


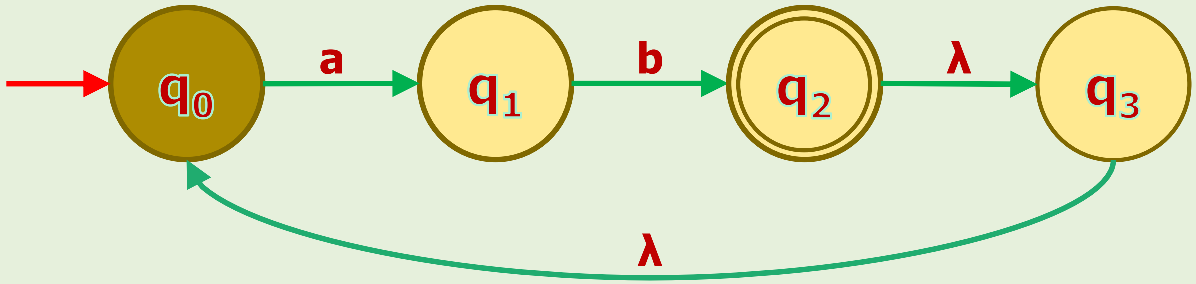
- Both processes rejected the string. So, aaa is rejected.

λ-Transitions in Action

Example 14: Process #1

- $\Sigma = \{a, b\}$
- $w = ab$
- $\delta(q_0, a) = \{q_1\}$

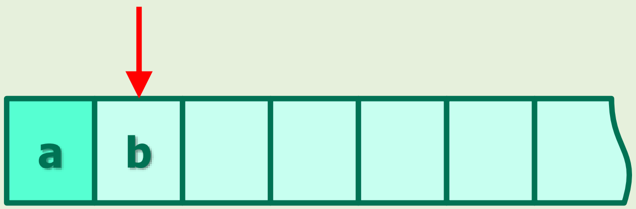


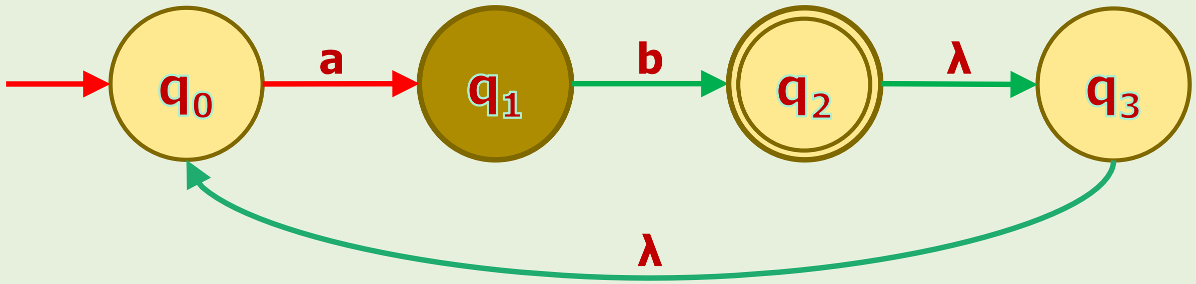


λ-Transitions in Action

Example 14: Process #1

- $\Sigma = \{a, b\}$
- $w = ab$
- $\delta(q_1, b) = \{q_2\}$

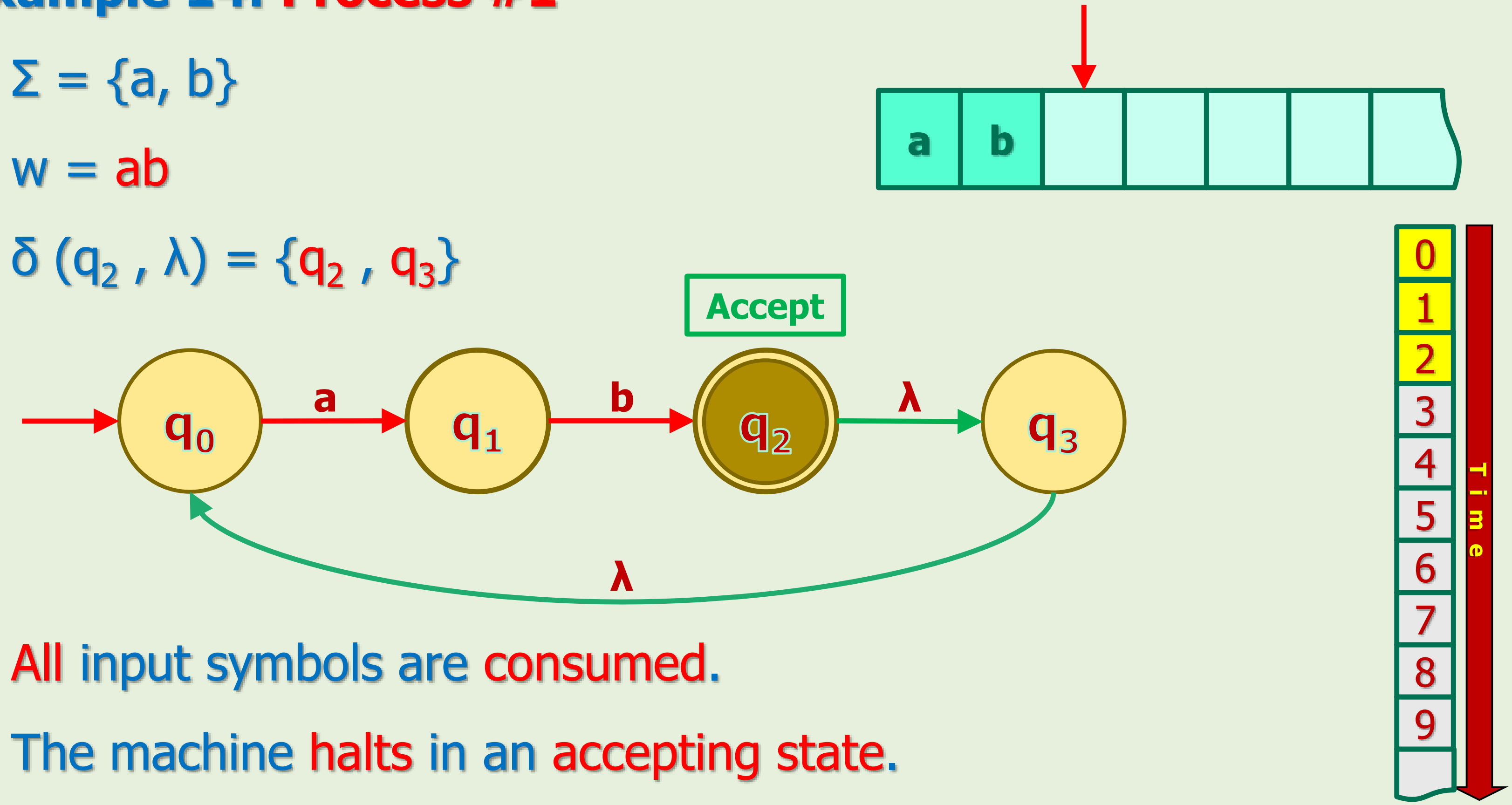




λ-Transitions in Action

Example 14: Process #1

- $\Sigma = \{a, b\}$
- $w = ab$
- $\delta(q_2, \lambda) = \{q_2, q_3\}$

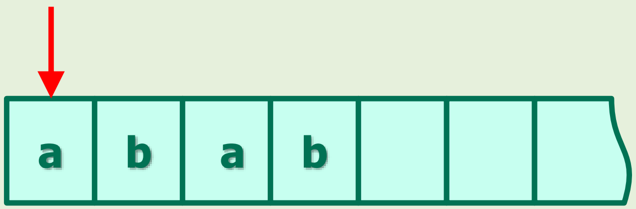


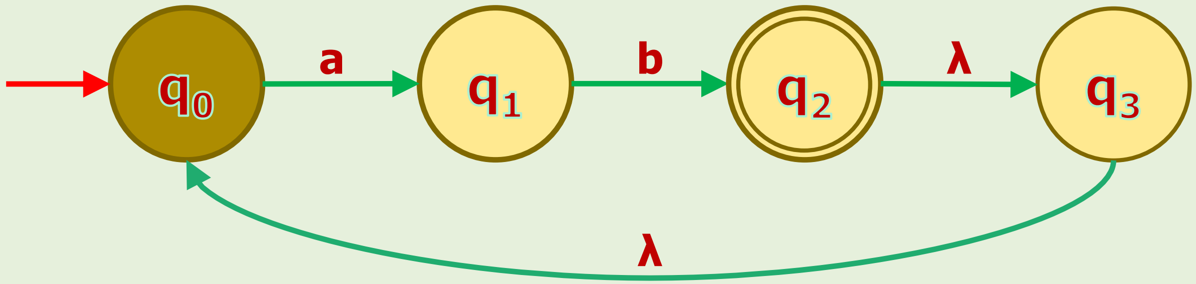
- All input symbols are consumed.
- The machine halts in an accepting state.
- So, the string is accepted.
- It does not need to initiate another process!

λ-Transitions in Action

Example 15: Process #1

- $\Sigma = \{a, b\}$
- $w = abab$
- $\delta(q_0, a) = \{q_1\}$

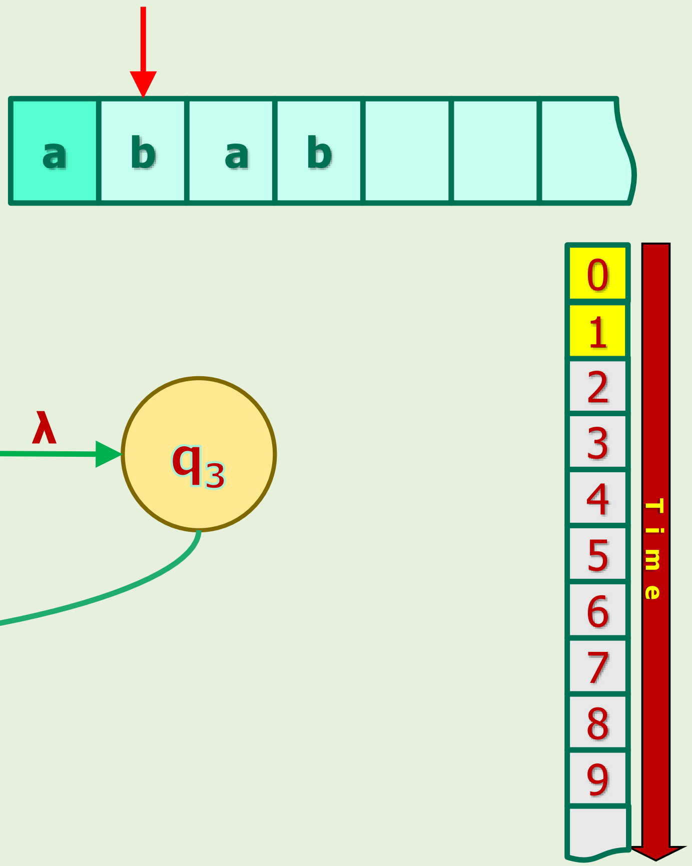


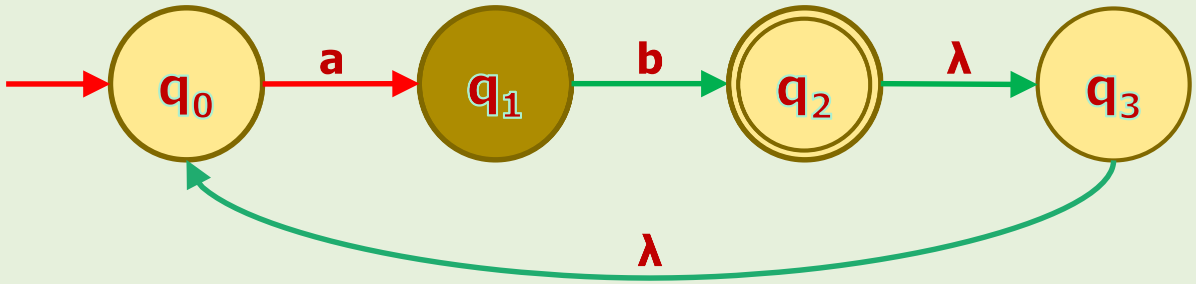


λ-Transitions in Action

Example 15: Process #1

- $\Sigma = \{a, b\}$
- $w = abab$
- $\delta(q_1, b) = \{q_2\}$

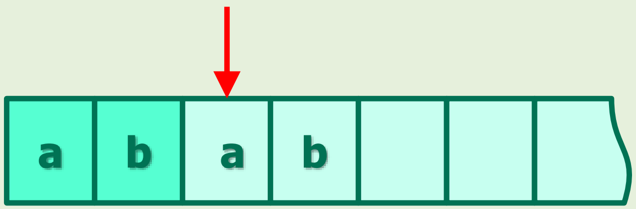


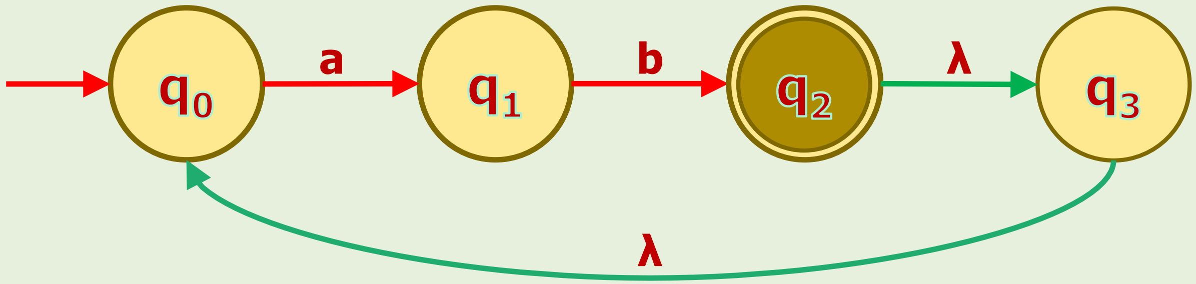


λ-Transitions in Action

Example 15: Process #1

- $\Sigma = \{a, b\}$
- $w = abab$
- $\delta(q_2, \lambda) = \{q_2, q_3\}$



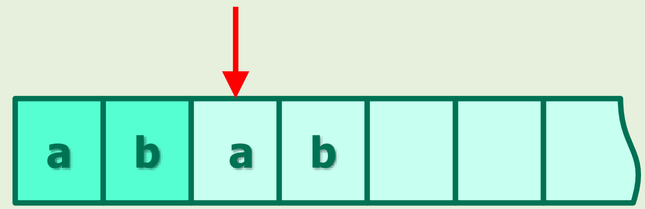


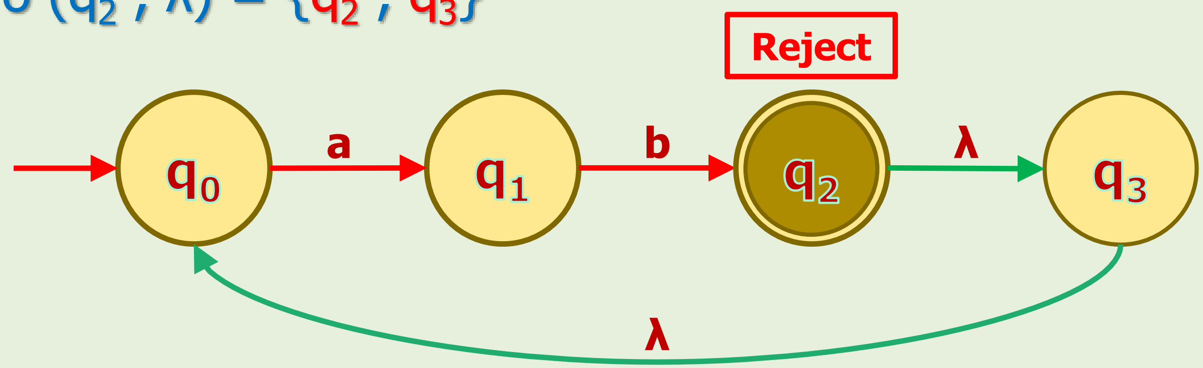
- It initiates process #2 because of λ-transition.
- Initial config: state=q_3, rest of input=ab, clock=2

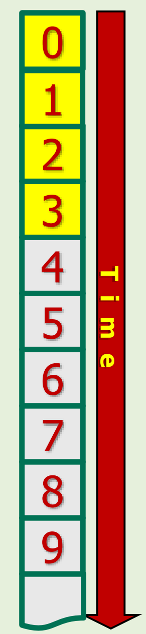
λ-Transitions in Action

Example 15: Process #1

- $\Sigma = \{a, b\}$
- $w = abab$
- $\delta(q_2, \lambda) = \{q_2, q_3\}$



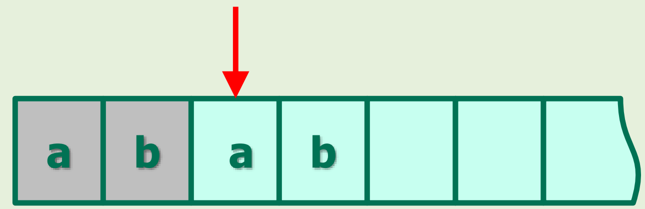




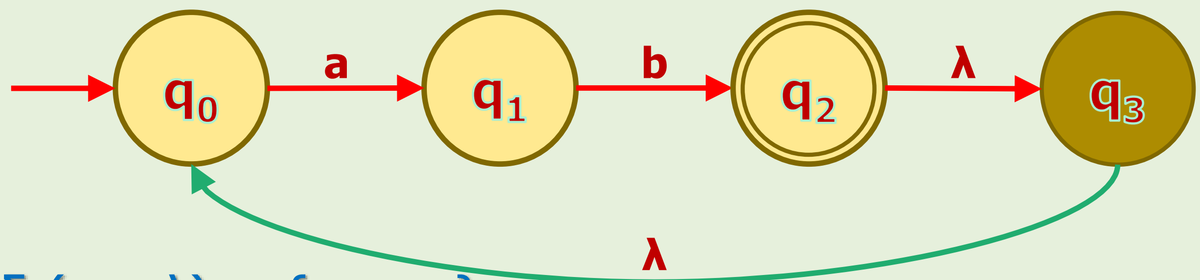
- Process 1 reads 'a' but cannot consume it.
- Some input symbols are NOT consumed.
- So, process #1 rejects the string.

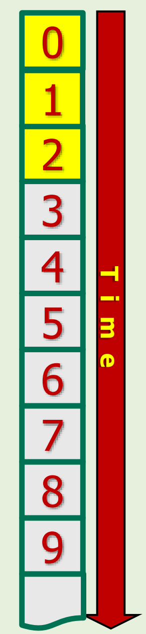
λ-Transitions in Action

Example 15: Process #2



- $\Sigma = \{a, b\}$
- w = abab
- Initial config: state=q_3, rest of input=ab, clock=2



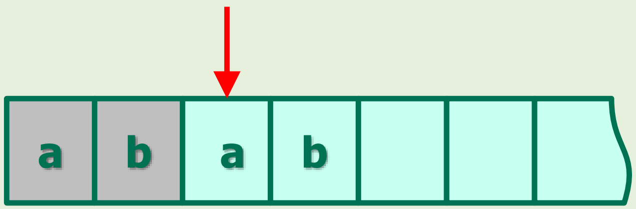


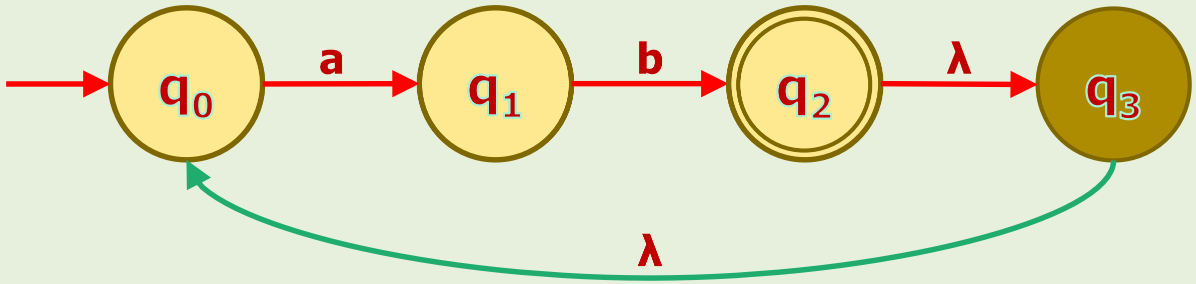
- $\delta(q_3, \lambda) = \{q_3, q_0\}$
- It encounters another λ-transition.
- It initiates process #3 because of λ-transition.
- Initial config: state=q_0, rest of input=ab, clock=2

λ-Transitions in Action

Example 15: Process #2

- $\Sigma = \{a, b\}$
- $w = abab$



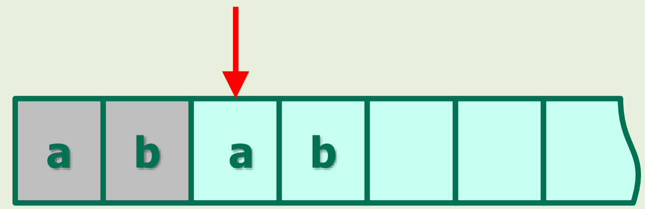


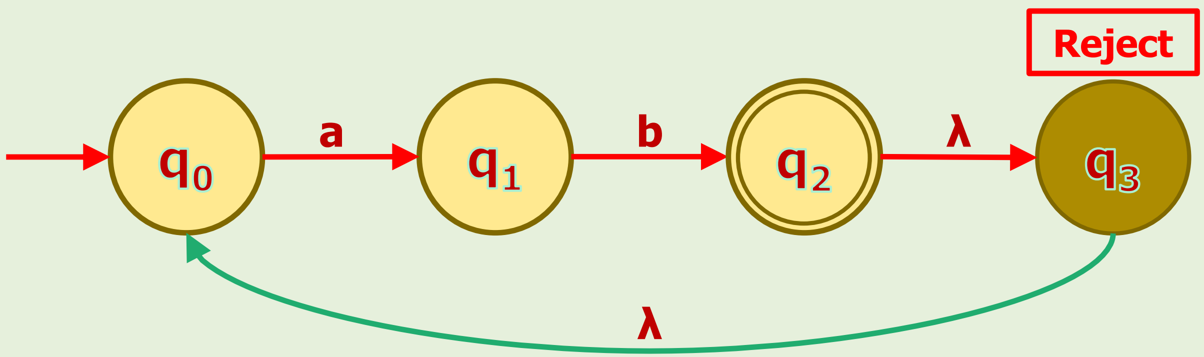
- Process #2 reads 'a' but cannot consume it.
- So, it has to halt.

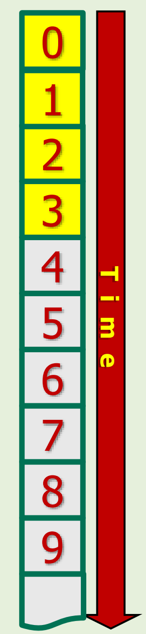
λ-Transitions in Action

Example 15: Process #2

- $\Sigma = \{a, b\}$
- w = abab





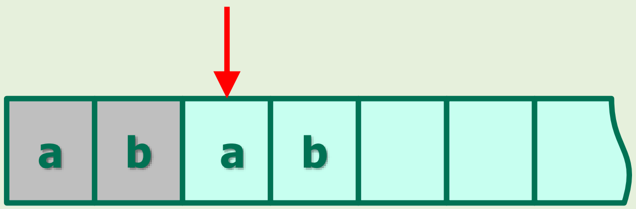


- Process #2 rejects the string because some input symbols are NOT consumed.

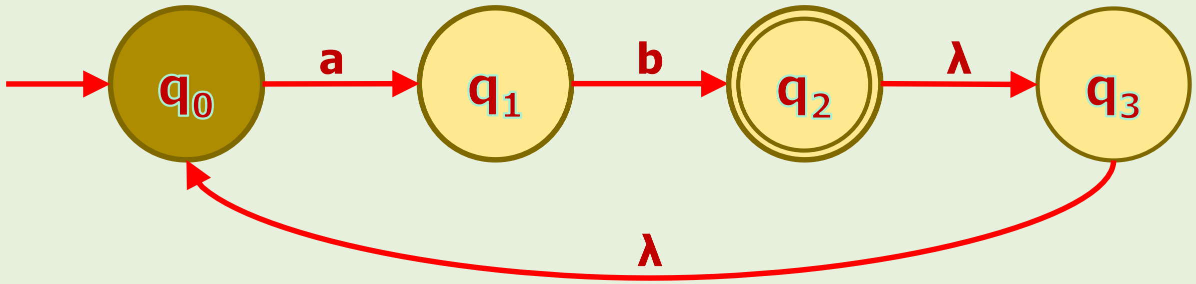
λ-Transitions in Action

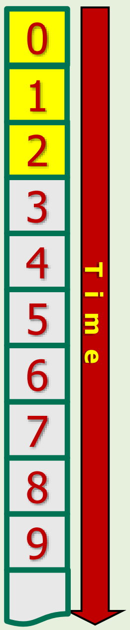
Example 15: Process #3

- $\Sigma = \{a, b\}$
- w = abab



- Initial config: state=q_0, rest of input=ab, clock=2



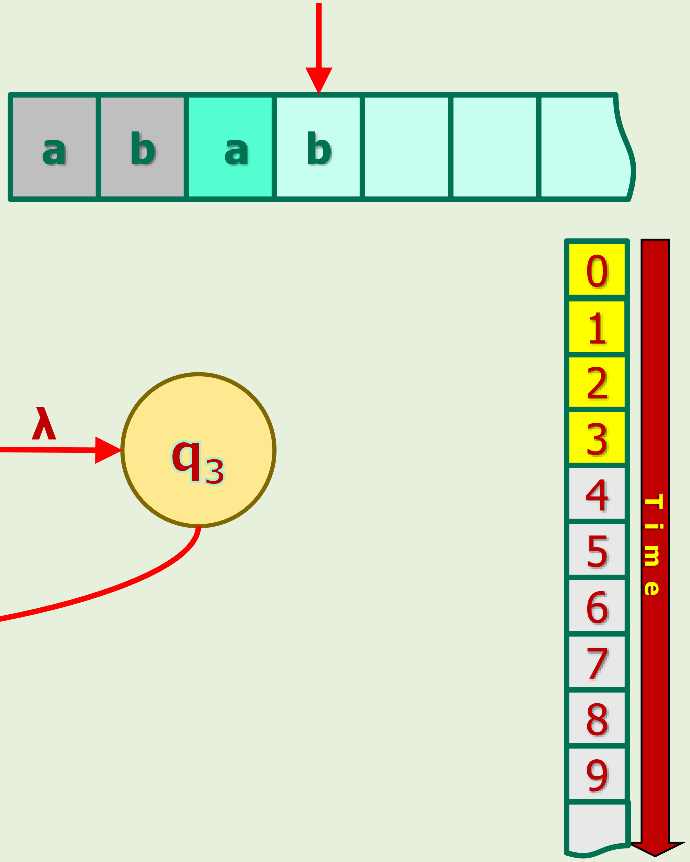


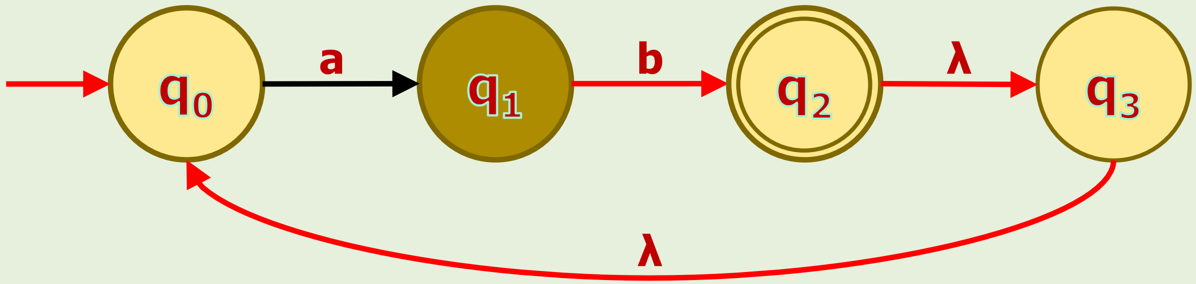
- $\delta(q_0, a) = \{q_1\}$

λ-Transitions in Action

Example 15: Process #3

- $\Sigma = \{a, b\}$
- $w = abab$
- $\delta(q_1, b) = \{q_2\}$

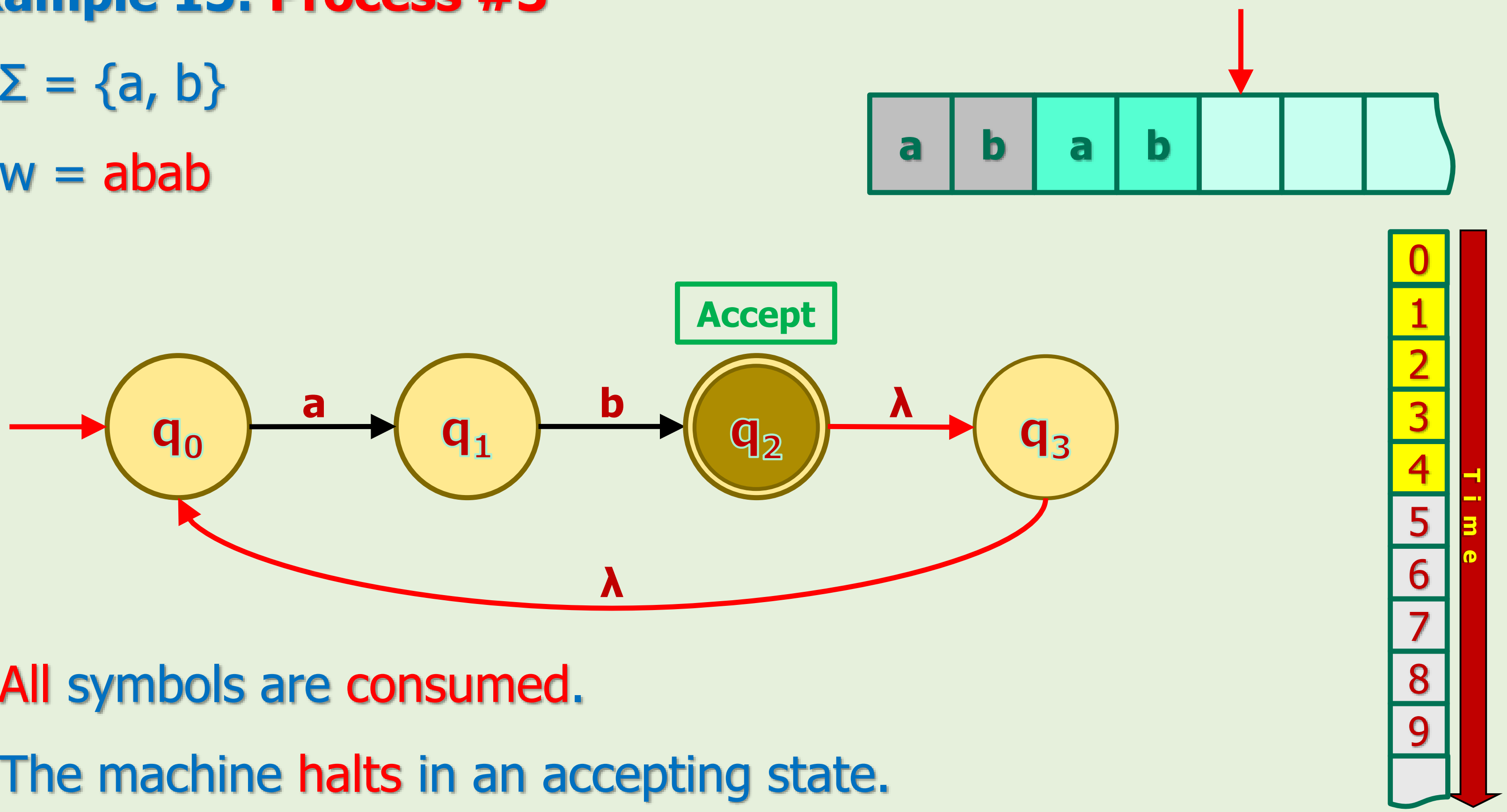




λ-Transitions in Action

Example 15: Process #3

- $\Sigma = \{a, b\}$
- $w = abab$



- All symbols are consumed.
- The machine halts in an accepting state.
- So, the string is accepted.
- It does not need to initiate another process!

References

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