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Regular Languages

Lecture 12

Day 13/31

CS 154

Formal Languages and Computability

Spring 2018

Agenda of Day 13

- Summary of Lecture 11
- Quiz 4
- Lecture 12: Teaching ...
  - Regular Languages

Summary of Lecture 11: We learned ...

NFAs Formal Definition

- Formally, we define NFAs by a quintuple $M = (Q, \Sigma, \delta, q_0, F)$
- Except δ, the rest items are similar to DFAs'.

$$\delta : Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q$$

δ is total function.

Machines and Languages Association

- Every machine has an associated language.
- BUT we do NOT know yet whether or not for every language, we can construct a machine!

DFAs vs NFAs

- What is power?
- Automata class A is more powerful than class B iff ...
  - ... the set of languages recongnized by class B is a proper subset of the set of the languages recognized by class A.

Theorem

- The set of languages accepted by NFAs are equal to the set of languages accepted by DFAs.
- NFAs and DFAs have the same power.

Any question?

Quiz 4

No Scantron Needed!

Introduction

Example 1

- Design a DFA/NFA to recognize our famous language:
- $L = \{a^n b^n : n \geq 0\}$ over $\Sigma = \{a, b\}$
- You're struggling!
- Let's forget about this, and take a simpler example!

Example 2

- Design a DFA/NFA to recognize the following language:
- $L = \{ww^R : w \in \Sigma^*\}$ over $\Sigma = \{a, b\}$

You!

Me!

Why We Could NOT Construct Machines

- After some struggling, we realized that we could not construct such machines.

What is the reason?

- Because we need to store some info from one part of the construction and use that info to construct the other parts.

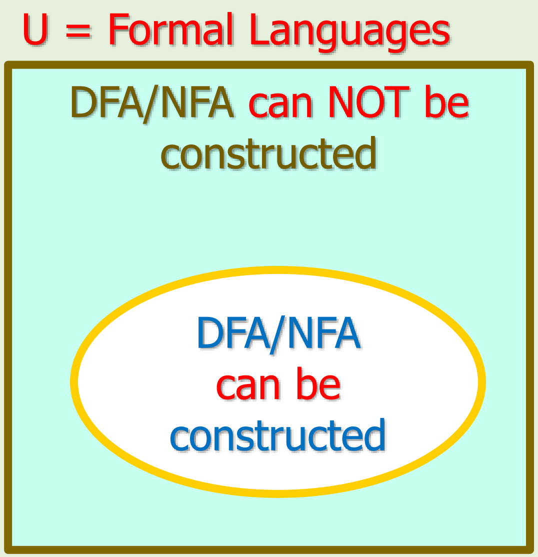
- In example 1, to make sure that the number of a's and b's are equal, we need to count and store the number of a's somehow.
- But we cannot implement counters by DFAs/NFAs!
  - They don't have memory!

- How about example 2?
- Explain why we could not construct the machine?

Categorizing Formal Languages

- We just realized that there are different kinds of formal languages.
  - Some languages are more complex than the others.
- To study formal languages, we need to categorize them.

- Up to this point, we've realized that ...
  - We can construct a DFA/NFA for some languages while we cannot for the others.

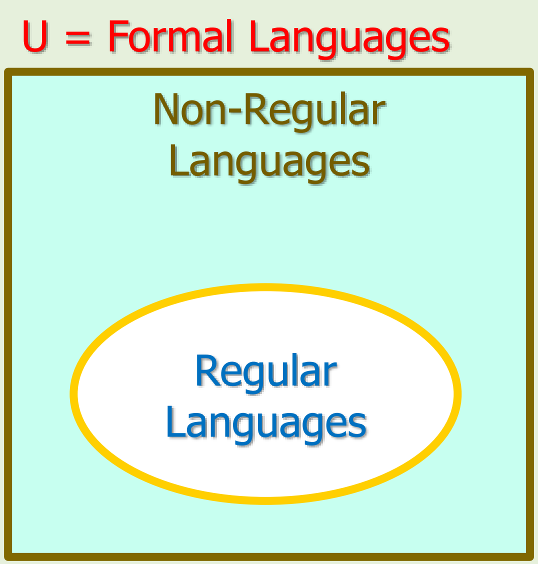


- Let's give a name to these two categories!

Regular Languages

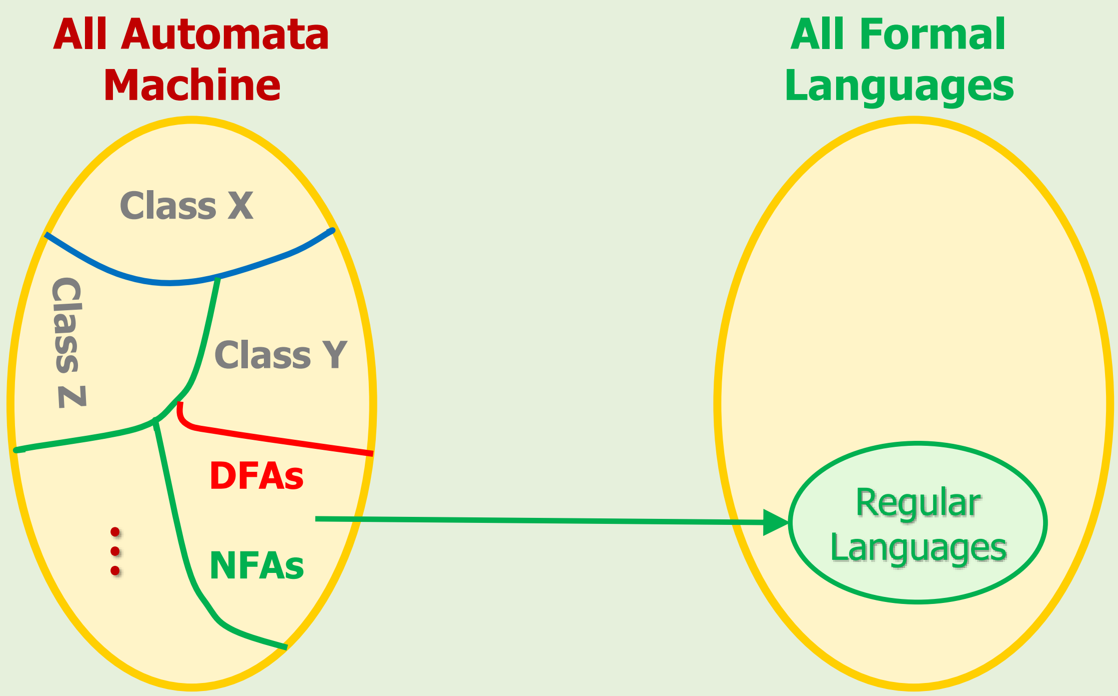
Definition

- A language L is called regular iff there exists a DFA/NFA to recognize it.
- Note that this definition has two sides:
  - If there exists a DFA/NFA, then its associated language is regular.
  - If L is regular, then there exists a DFA/NFA to recognize it.



Machines and Languages Association

- We already saw the association between machines and languages.
- Now we have a name for the languages that DFAs/NFAs recognize.



Categorizing Formal Languages

- Recall that before, we categorized formal languages as "finite" and "infinite".
- And this is our second categorization:
  1. "Regular Languages", and
  2. "Non-Regular Languages"
- Note that the correct English word is "irregular" but in computer science we use "non-regular".
- How can we prove that a language is regular?
- We need to construct a DFA/NFA for it.

Regular Languages

Example 3

- Which of the following languages is regular over $\Sigma = \{a, b\}$?
- $L = \{abbaa\}$
- $L = \{\lambda\}$
- $L = \{ \}$
- $L = \{\lambda, a, abb\}$
- $L = \{a, b\}^*$
- $L = \{a^n b : n \geq 0\}$

- We've already constructed DFAs for almost all of the above languages.
- So, all of them are regular.

Homework

a. Prove that the language $L = \{awa : w \in \{a, b\}^*\}$ is regular.

b. Write a set-builder for L^2.

c. Prove that L^2 is regular.

Recognizing Regular Languages Heuristically

- Sometimes we can heuristically find out whether a language is regular or not. How?
- Let's explain it through some examples.

Example 4

- Which of the following languages are regular?

1. $L = \{ab\ w : w \in \Sigma^*\}$ over $\Sigma = \{a, b\}$
2. $L = \{w\ w : w \in \Sigma^*\}$ over $\Sigma = \{a, b\}$
3. $L = \{w\ abb\ w : w \in \Sigma^*\}$ over $\Sigma = \{a, b\}$
4. $L = \{1^{2k} : k \geq 0\}$ over $\Sigma = \{1\}$
5. $L = \{1^n + 1^m = 1^{n+m} : n, m \geq 1\}$ over $\Sigma = \{1, +, =\}$ (Unary addition)

Finite Languages

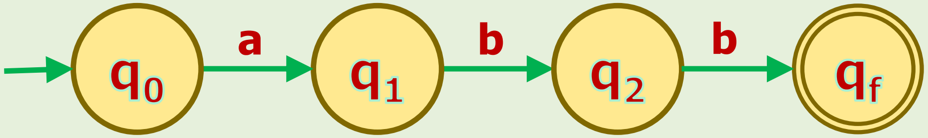
Finite Languages

Theorem

- All "finite" languages are regular.

Proof

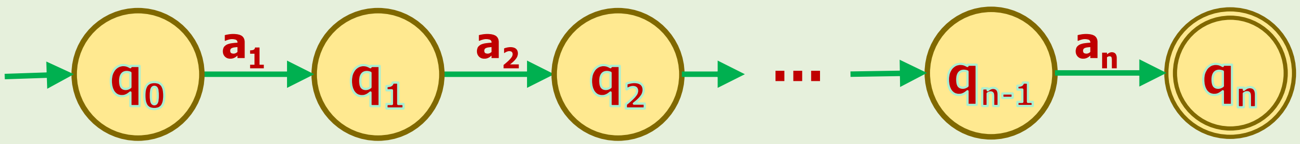
- To prove this theorem, we need to construct an NFA for a general finite language $L = \{w_1, w_2, ..., w_n\}$
  - Where $w_j \in \Sigma^*$ for $j = 1, 2, ..., n$.
- We know that strings are finite sequence of symbols.
- So, we can construct a separate NFA for every string.
- For example if w = abb, then the following DFA can recognize it.



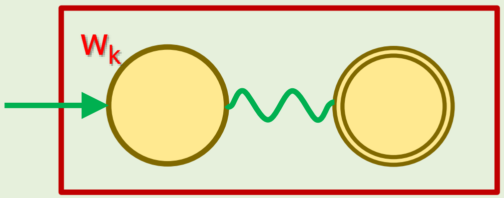
Finite Languages Are Regular

Proof (cont'd)

- Let $w_k = a_1\ a_2 \ldots a_m$ be a general string where $a_i \in \Sigma$ for $i = 1, 2, \ldots, m$.
- We can construct the following NFA to recognize w_k.



- For simplicity, let's put the NFA in a box ...

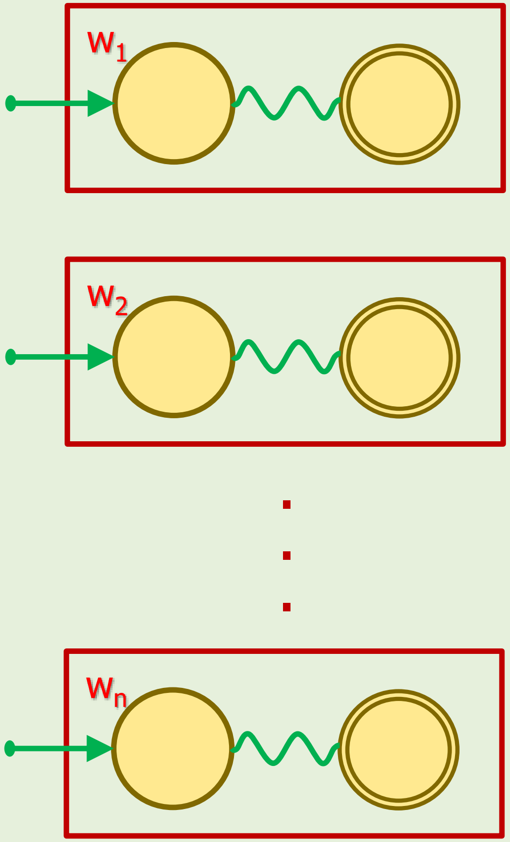


- So, this NFA can accept the string w_k.

Finite Languages Are Regular

Proof (cont'd)

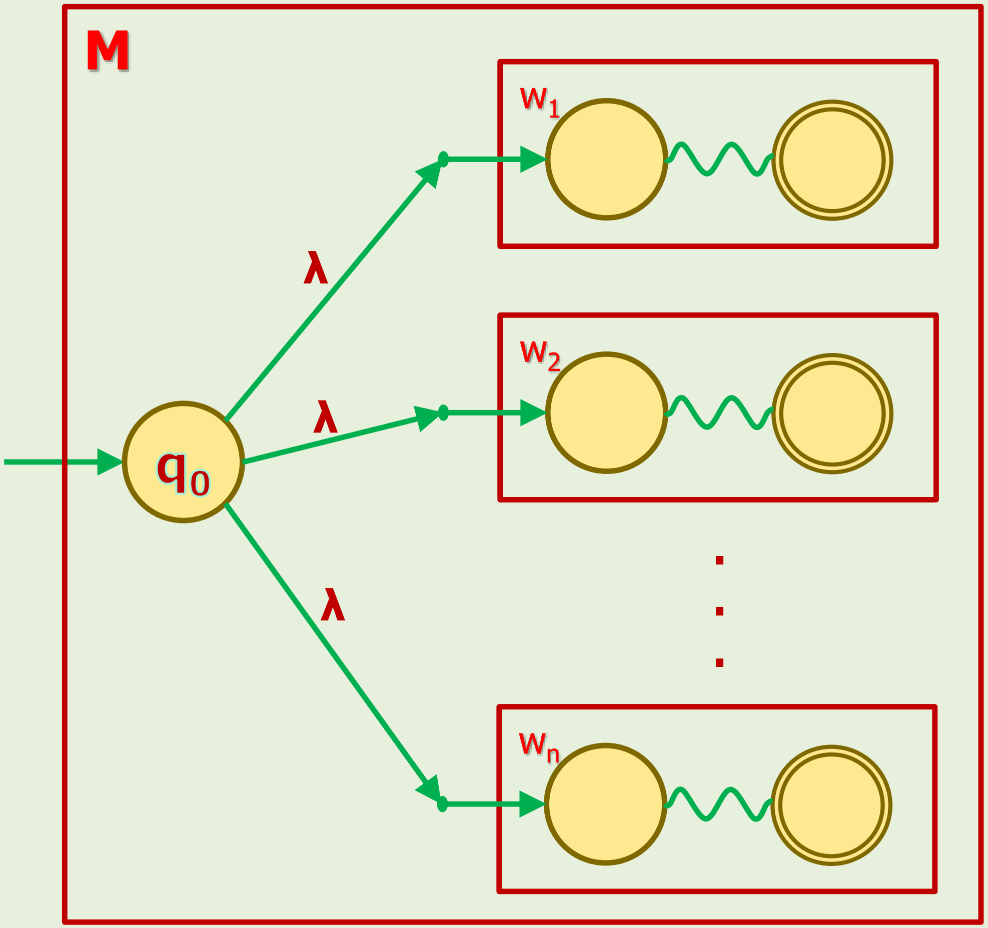
- So, in the similar way, we can construct an NFA for every w_j in the language.
- We should combine these simple NFAs and construct an NFA to recognize L.
- But how?



Finite Languages Are Regular

Proof (cont'd)

- We can combine them by using λ-transitions.
- This new machine recognizes L.
- Explain why?
- Since L is a general finite language, so, we proved all finite languages are regular.



Non-Regular Languages Are Infinite

- The contrapositive of every theorem is also true.

Recap: Contrapositive

$$p \rightarrow q \equiv \sim q \rightarrow \sim p$$

- The theorem we just proved:

If L is finite, then L is regular.

- L is finite. $\equiv$ f
- L is regular. $\equiv$ r

$$\mathbf{f \rightarrow r \equiv \sim r \rightarrow \sim f}$$

- Translation:

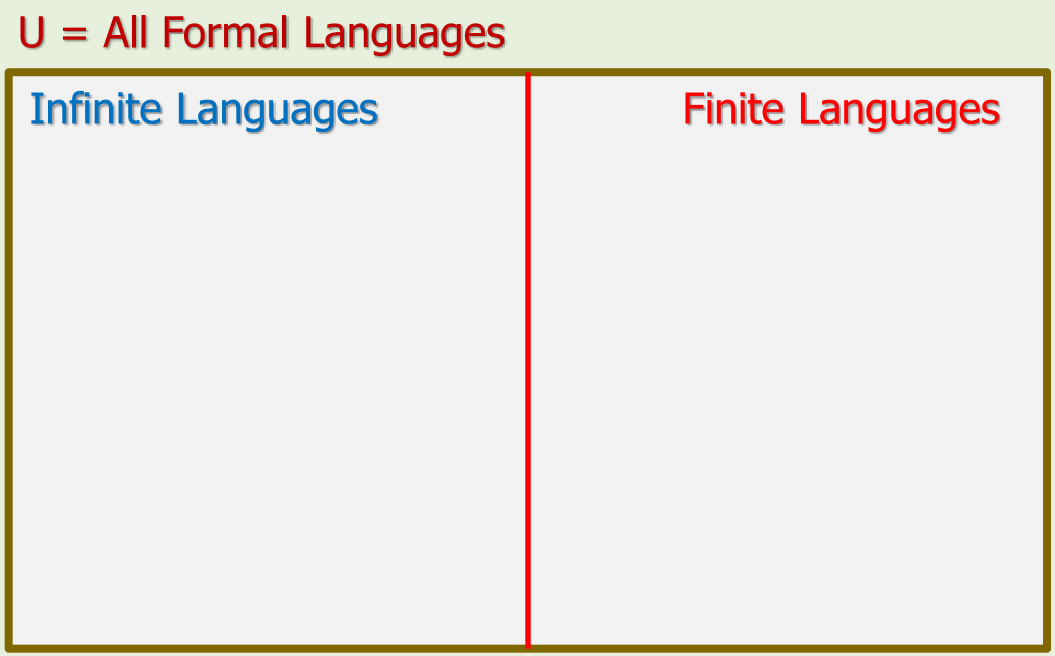
If L is non-regular (= not regular), then L is infinite (= not finite).

- The compact version:

All non-regular languages are infinite.

Categories of Formal Languages

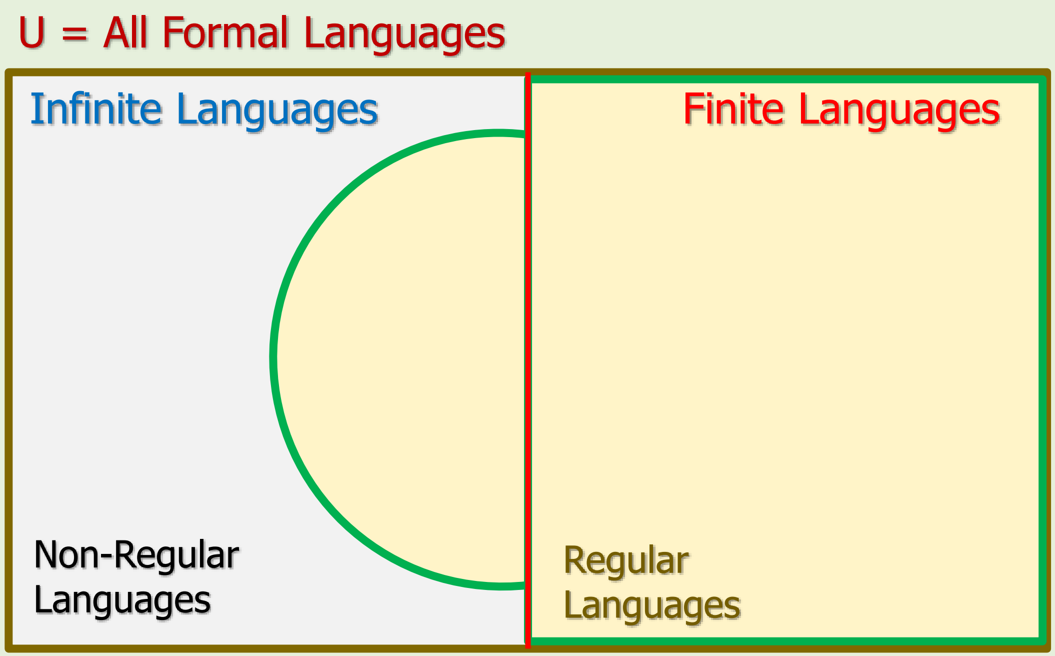
1st Categorization: Finite and Infinite



- Where would you locate "regular" and "non-regular" languages?

Categories of Formal Languages

2nd Categorization: Regular and Non-Regular



Closure Properties of Regular Languages

Theorem

- If L, L_1 and L_2 are all regular languages, then:

Union	$L_1 \cup L_2$
Concatenation	$L_1 L_2$
Star-Closure	L^*
Reversal	L^R
Complement	$\overline{L}$
Intersection	$L_1 \cap L_2$
Minus	$L_1 - L_2$

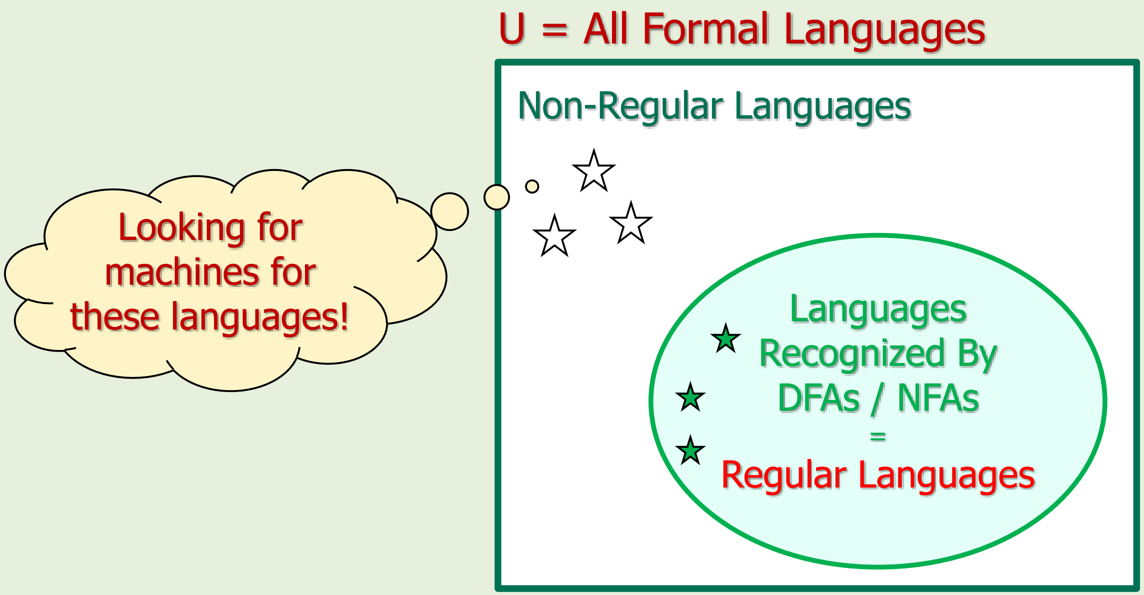
are regular languages too.

- It means: The family of "regular languages" is closed under the above operations.

What is the Next Step?

Conclusion

- NFAs and DFAs recognize "regular languages".
- The next step is to define a new class of machines that recognizes "non-regular languages".



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