San José State University
Department of Computer Science

Ahmad Yazdankhah
ahmad.yazdankhah@sjsu.edu
www.cs.sjsu.edu/~yazdankhah

Grammars

(Part 2)

Lecture 21

Day 25/31

CS 154

Formal Languages and Computability

Spring 2018

Agenda of Day 25

- Summary of Lecture 20
- Quiz 8
- Lecture 21: Teaching ...
  - Grammars (Part 2)

Summary of Lecture 20: We learned ...

Grammars

- We are looking for a more powerful and practical tool to represent all formal languages.
- Roughly speaking, a set of "production rules" is called grammar.
- A sentence is well-formed if ...
  - ... we can derive it from the production rules.
- Associated language to the grammar G is ...
  - ... the set of all strings generated by it.
  - ... denoted by L(G).

Any Question

NAME	Alan M. Turing		
SUBJECT	CS 154	TEST NO.	8
DATE	04/19/2018	PERIOD	1 / 2 / 3

TEST RECORD	
PART 1	123
PART 2	
TOTAL	

Your list # goes here!

Quiz 8

Use Scantron

Constructing Grammars

Example 11

- Find a grammar that generates the following language over $\Sigma = \{a, b\}$:

  $L = \{w : w \in \Sigma^*\}$

Solution

Constructing Grammars

Example 12

- Find a grammar that generates the following language over $\Sigma = \{a, b\}$:

  $L = \{w : w \text{ contains exactly one } a\}$

Solution

Homework

- Find a grammar that generates the following languages over $\Sigma = \{a, b\}$:

1. $L = \{w : w \text{ contains at least one } a\}$
2. $L = \{w : w \text{ contains at least 2 } a\text{'s}\}$
3. $L = \{w : w \text{ contains no more than 3 } a\text{'s}\}$
4. $L = \{a^{2n} b^n : n \geq 0\}$
5. $L = \{a^{2n} b^m : n, m \geq 0\}$

Definitions

Formal Definition of Grammar

- A grammar G is defined by the quadruple:

$$G = (V, T, S, P)$$

- Where:
  - V is a nonempty finite set of variables.
  - T is a nonempty finite set of symbols (aka terminals) called terminal alphabet.
  - $S \in V$ is a special symbol called start variable.
  - P is a finite set of production rules (or simply rules) of the form $A \to B$ where:
    - $A \in (T \cup V)^* \, V \, (T \cup V)^*$ //Contains at least one variable
    - $B \in (T \cup V)^*$

Formal Definition of Grammar

Set of Variables

Start Variable

$$G = (\ V\ ,\ T\ ,\ S\ ,\ P\ )$$

Set of Terminals

Set of Production Rules

Formal Definition of Grammar

Example 13

- The following grammar generates the language $L = \{a^n b^n : n \geq 0\}$.

  $S \rightarrow aSb \mid \lambda$

- Write V, T, Starting variable, and P

Solution

$V = \{S\}$

$T = \{a, b\}$

Start variable: $S \in V$

$P = \{S \rightarrow aSb\ ,\ S \rightarrow \lambda\}$

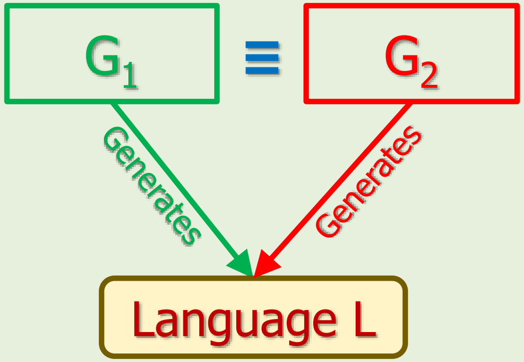
Equivalency of Grammars

- A given language can be generated by many grammars.

Definition

- Two grammars G_1 and G_2 are equivalent iff they generate the same language.

$$G_1 \equiv G_2 \leftrightarrow L(G_1) = L(G_2)$$



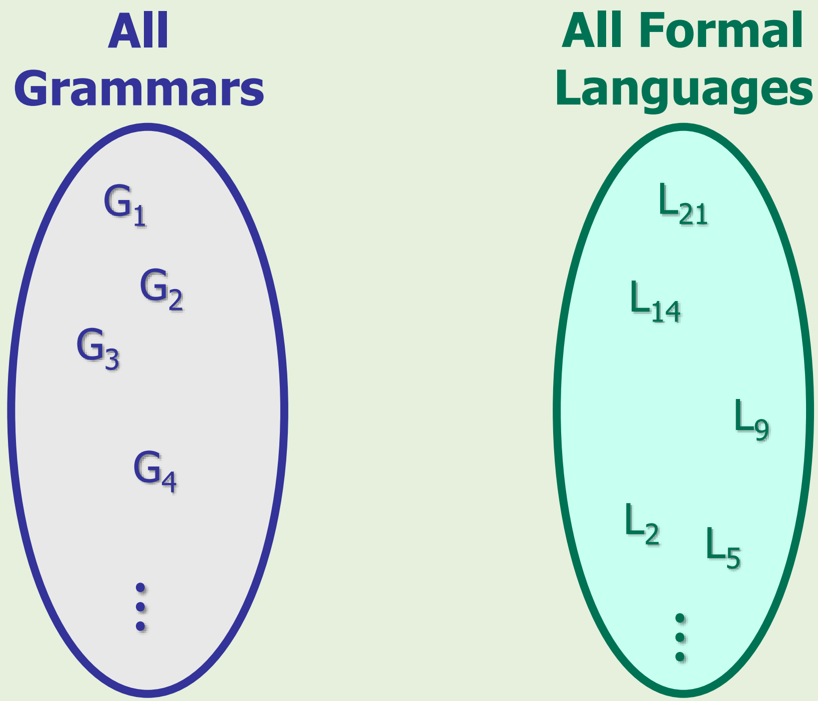
Grammars and Languages Association

Grammars and Languages Association

- What is the relationship between:

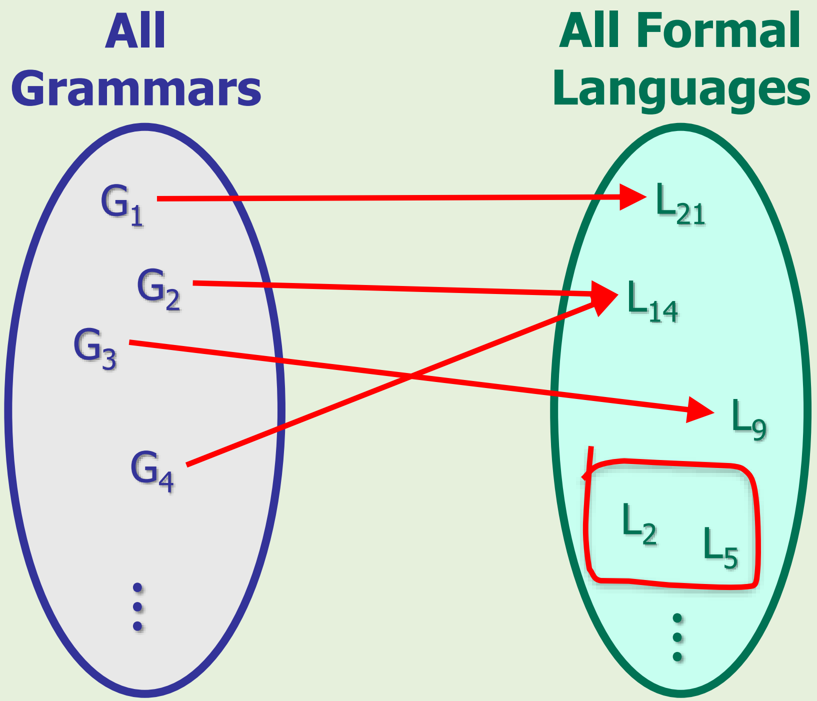
  the set of Grammars, and

  the set of all formal languages?



Grammars and Languages Association

- We know that "every grammar represents a language".
- BUT we don't know yet whether we can represent every language, by a grammar or not!
  - Our knowledge is not enough yet.



Types of Grammars

Linear Grammars

Definition

- A grammar G is linear if the right side of every production rule has at most one variable.

$A \rightarrow x \mid Bx \mid xC$

Where $A, B, C \in V$ and $x \in T^*$

Example 14

- Is the following grammar linear?

  $S \rightarrow A$

  $A \rightarrow aB \mid \lambda$

  $B \rightarrow Ab$

- Yes, because all production rules have at most one variable in the right side.

Right-Linear Grammars

Definition

- A linear grammar is said to be right-linear if all production rules are of the form:

$A \rightarrow x \mid xB$

Where $A, B \in V$ and $x \in T^*$

Example 15

- Is the following grammar right-linear?

$S \rightarrow abS \mid a$

- Yes, it is right-linear.

Left-Linear Grammars

Definition

- A linear grammar is said to be left-linear if all production rules are of the form:

$A \rightarrow x \mid Bx$

Where $A, B \in V$ and $x \in T^*$

Example 16

- Is the following grammar left-linear?

$S \rightarrow Aab$

$A \rightarrow Bab \mid B$

$B \rightarrow a$

- Yes, it is left-linear.

Regular Grammars

Definition

- A grammar is said to be regular if it is either right-linear or left-linear.

Example 17

- Is the following grammar regular?

  $S \rightarrow A$

  $A \rightarrow aB \mid \lambda$

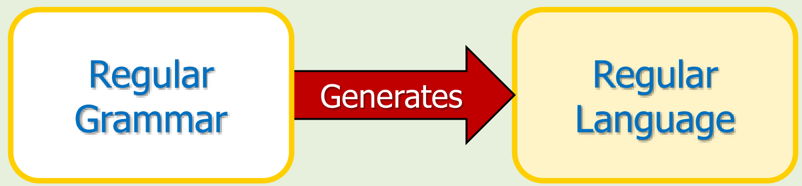
  $B \rightarrow Ab$

- It is NOT regular because it is neither right-linear nor left-linear.

Regular Grammars and Regular Languages

Theorem

- If G is a regular grammar, then L(G) is a regular language.

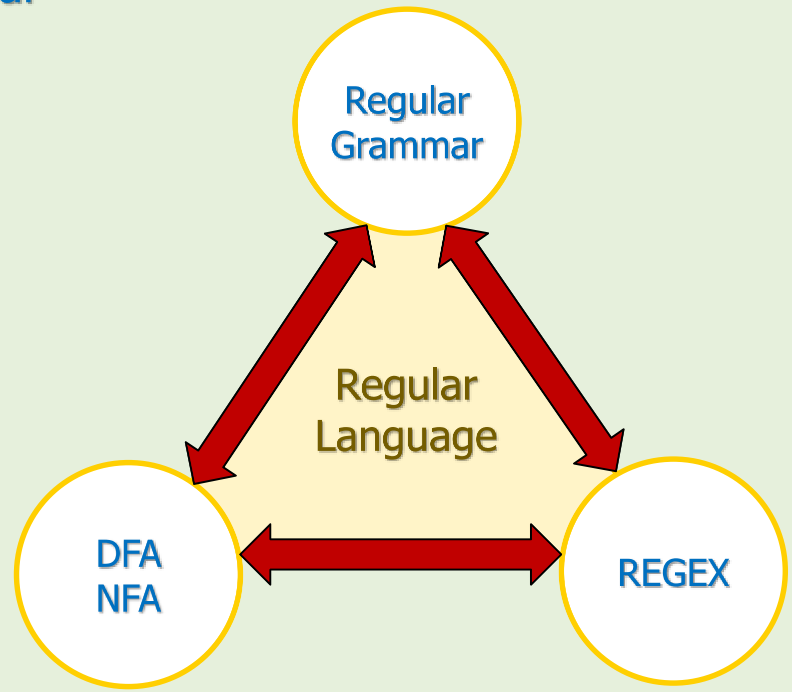


Theorem

- Let L_1 be a regular language on Σ.
  Then there exists a regular grammar G such that $L_1 = L(G)$.

Regular Languages Representations

- Now, We have three ways for representing Regular Languages:
  - DFA / NFA
  - REGEX
  - Regular Grammar



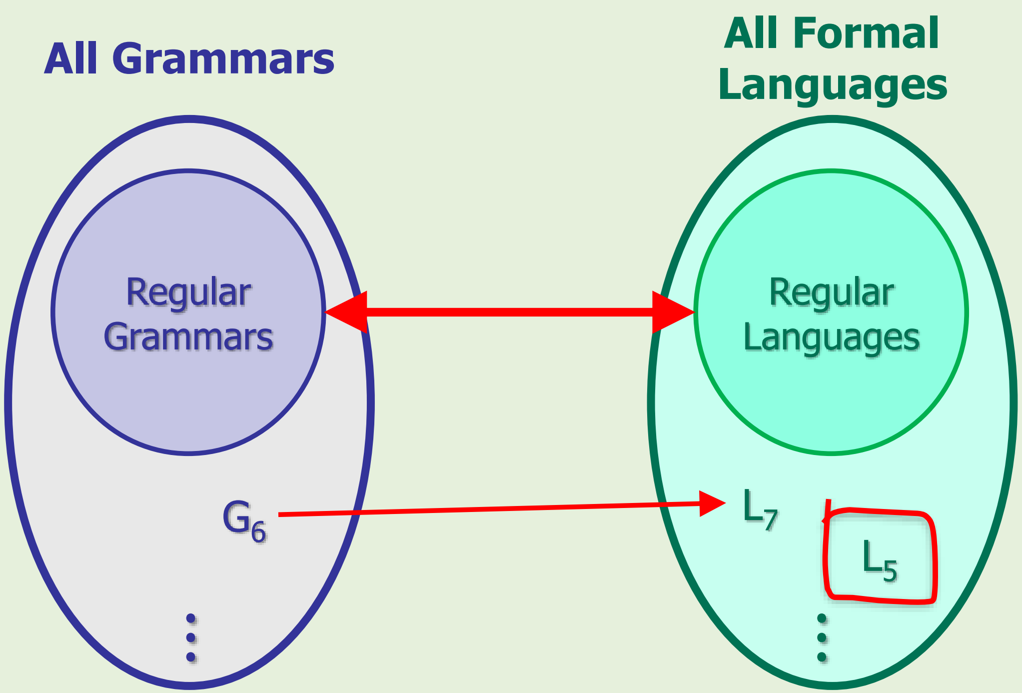
Grammars and Languages Association

Revisited

- We've already known that "every grammar represents a language".
- At this moment we know that:

Regular grammars represent regular languages.

Every regular language can be represented by a regular grammar.



Context-Free Grammars (CFG)

Context-Free Grammars (CFG)

Definition

- A grammar G is said to be context-free (CFG) if all production rules are of the form:

$$A \rightarrow v$$

Where $A \in V$ and $v \in (V \cup T)^*$

Example 18

- Is the following grammar context-free?

$$S \rightarrow a\,S\,b \mid \lambda$$

- Yes, it is a context free grammar.

Context-Free Grammars (CFG)

Example 19

- Consider $L_1 = \{a^n b^n : n \geq 0\}$ over $\Sigma = \{a, b\}$.
- Let's take a different look at this language.

- For example, consider this language:
- $L_2 = \{(^n\ )^n : n \geq 0\}$ over $\Sigma = \{(\ ,\ )\}$

- What strings would this language contain?
- What strings do not belongs to this language?

- What is L_2 representing?

Context-Free Grammars (CFG)

Example 20

- Is the following grammar context-free?

$S \rightarrow a\, S\, a \mid b\, S\, b \mid \lambda$

- What language does it produce?

References

1. Linz, Peter, "An Introduction to Formal Languages and Automata, 5th ed.," Jones & Bartlett Learning, LLC, Canada, 2012
2. Michael Sipser, "Introduction to the Theory of Computation, 3rd ed.," CENGAGE Learning, United States, 2013
ISBN-13: 978-1133187790
3. The ELLCC Embedded Compiler Collection, available at: http://ellcc.org/