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Turing Machines

(Part 2)

Lecture 16

Day 17/31

CS 154

Formal Languages and Computability

Spring 2018

Agenda of Day 17

- About Teams
- Summary of Lecture 15
- Lecture 16: Teaching ...
  - Turing Machines (Part 2)
- Quiz 6 (Take-Home Exam)

Summary of Lecture 15: We learned ...

Standard Turing Machines (TMs)

- NPDAs are unable to accept some languages like $a^n b^n c^n$ and ww.
- The limitation of NPDAs is ...
  - ... we lose some data when we access the older data.
  - ... so, stack is not so flexible in storing and retrieving data.
- We introduced standard Turing machines to overcome this limitation.
- Standard TMs are deterministic.

- The main difference between TMs and NPDAs is ...
  - ... we have the ability to move the read/write head to the left or right.
- The transition condition of TMs is ...
  - ... input symbol.
- TMs halt iff ...
  - ... they have zero transition.

Any Question

Summary of Lecture 15: We learned ...

TMs

- The criteria of accepting strings for previous machines are ...

$$(h \wedge c \wedge f) \leftrightarrow a$$

- Consuming all input symbols is meaningless for TMs.
- So, theoretically, the logical representation of accepting strings is ...

$$(h \wedge f) \leftrightarrow a$$

- And for rejecting strings is ...

$$(\sim h \vee \sim f) \leftrightarrow \sim a$$

- But in practice, it is important to note that ...
- ... that is the TMs designers responsibility to make sure that the machine halts in an accepting state when all symbols are consumed.
- Otherwise, it should halts in a non-accepting state.

Any Question

Summary of Lecture 15: We learned ...

TMs

- For the first time, we observed a new phenomenon that happens in Turing machines ...
  - ... Infinite loops!
- This phenomenon never happened in the previous deterministic machines.
- This is the consequence of ...

  ... having freedom of moving the read-write head to the left or right.

- Recall that if a TM is in infinite-loop, the string it is processing, is considered as "rejected".
- When a machine is working for a long time, from an observer's point of view, ...
  - ... is it in an infinite loop? OR
  - ... it is in the middle of a very long computation?
- There is no known algorithm to answer this!

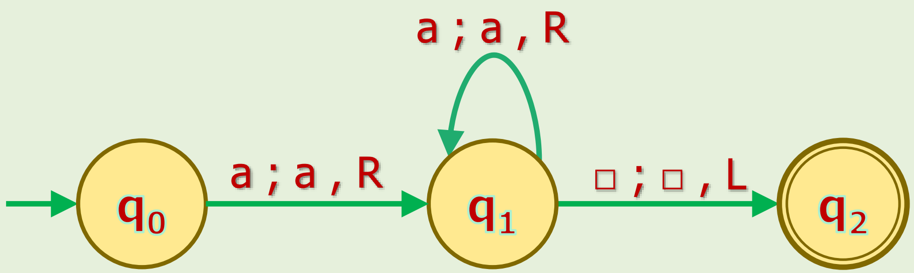
Any Question

TMs Design Examples

TMs Design Examples

Example 7

- Design a TM to accept $L = \{a^n : n \geq 1\}$ over $\Sigma = \{a, b\}$.
- Note that TMs don't like λ!

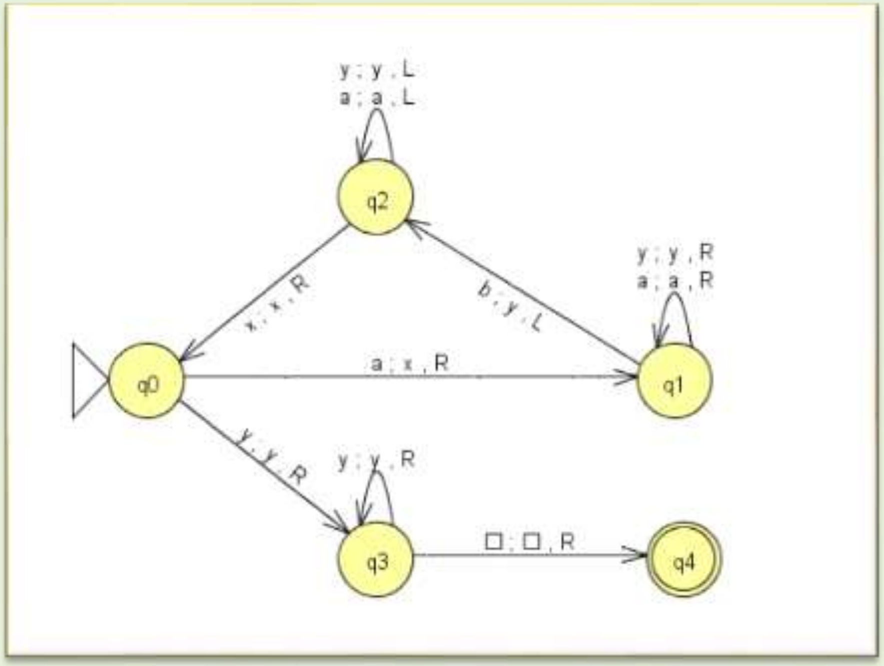


TMs Design Examples

Example 8

- Design a TM to accept our famous language $L = \{a^n b^n : n \geq 1\}$ over $\Sigma = \{a, b\}$.

Solution



- **Strategy**: For every a's, you should find one "b". So, we read the first "a" and mark it as read by replacing it to "x". Then we go right to find a corresponding "b" and mark it as "y".
We continue this process until we don't have any a's.
The string is accepted if then is no "b" as well.

Homework

- Design a TM for the following languages:
  1. $L = \{w \in \{a, b\}^+\}$
  2. $L = \{w \in \{a, b\}^+ : |w| = 2k, K \geq 0\}$
  3. $L = \{w \in \{a, b\}^+ : |w| = 2k+1, K \geq 0\}$
  4. $L = \{1^{2k} : k \geq 1\}$ over $\Sigma = \{1\}$
  5. $L = \{w \in \{a, b\}^+ : n_a(w) = n_b(w)\}$ //number of a's = number of b's
  6. $L = \{a^n b^n c^n : n \geq 1\}$
  7. $L = \{a^n b^m c^{nm} : n \geq 1, m \geq 1\}$
  8. $L = \{w\#w : w \in \{a, b\}^+\}$
  9. $L = \{w \in \{a, b\}^+ : |w| = 2k+1, K \geq 0$, w contains at least one a$\}$
  10. $L = \{ww : w \in \{a, b\}^+\}$

6. Definitions

6. Formal Definition of TMs

- A standard TM M is defined by the septuple (7-tuple):

$$M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$$

- Where:
  - Q is a finite and nonempty set of states of the transition graph.
  - Σ is a finite and nonempty set of symbols called input alphabet.
  - Γ is a finite and nonempty set of symbols called tape alphabet.
  - δ is called transition function and is defined as:

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$

δ may be partial xor total function.

  - $q_0 \in Q$ is the initial state of the transition graph.
  - $\square \in \Gamma$ is a special symbol called blank.
  - $F \subseteq Q$ is the set of accepting states of the transition graph.

6. Formal Definition of TMs

$$M = \left(\overset{\text{Set of States}}{Q}\ ,\ \underset{\text{Input Alphabet}}{\Sigma}\ ,\ \overset{\text{Tape Alphabet}}{\Gamma}\ ,\ \underset{\text{Transition Function}}{\delta}\ ,\ \overset{\text{Initial State}}{q_0}\ ,\ \underset{\text{Blank Symbol}}{\square}\ ,\ \overset{\text{Set of Final States}}{F} \right)$$

6. Formal Definition of TMs: Notes

1. L and R are called "move symbols".
   - They indicate whether the read-write head moves one cell left or right, after the new symbol has been written.

2. $\Sigma \subseteq \Gamma - \{\square\}$.
   - So, the input string cannot contain blank symbol.

3. There is no relationship between "determinism" and "total function".
   - A machine whose transition function is partial can be deterministic as long as it does not violate the definition of determinism.

TMs Transition Function Examples

Example 9

- Write the sub-rule of the following transition.

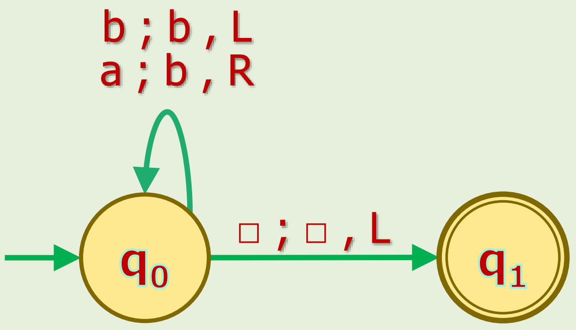
- $\delta(q_1, a) = (q_2, b, R)$

TMs Transition Function Examples

Example 10

- Write the δ of the following transition graph.

$$\delta:\begin{cases}\delta(q_0, a) = (q_0, b, R)\\ \delta(q_0, b) = (q_0, b, L)\\ \delta(q_0, \square) = (q_1, \square, L)\end{cases}$$



- Is the function total or partial?

7. TMs vs NPDAs

Can TMs Do Whatever NPDAs Can Do?

- Let's assume that we've constructed an NPDA for an arbitrary language L.
- Can we always construct a TM for L?

- To compare previous machines (i.e. DFAs, NFAS, NPDAs), we used the "formal definition conversion" technique .
- For this case, it is not so easy to do that.
- But there is another technique called "simulation".

- So, we convert the above question to:

Can we simulate NPDAs operations by TMs?

- Yes! How?

Can TMs Do Whatever NPDAs Can Do?

- Let M be an NPDA for the language L.
- We want to simulate M by an equivalent TM called M' such that:

  L(M) = L(M')

- The NPDA has several transitions and we should be able to simulate all of them by TM.
- We just show the simulation of one simple transition and leave the rest of them for the reader as exercise.

- I put the following document in Canvas for your reference:

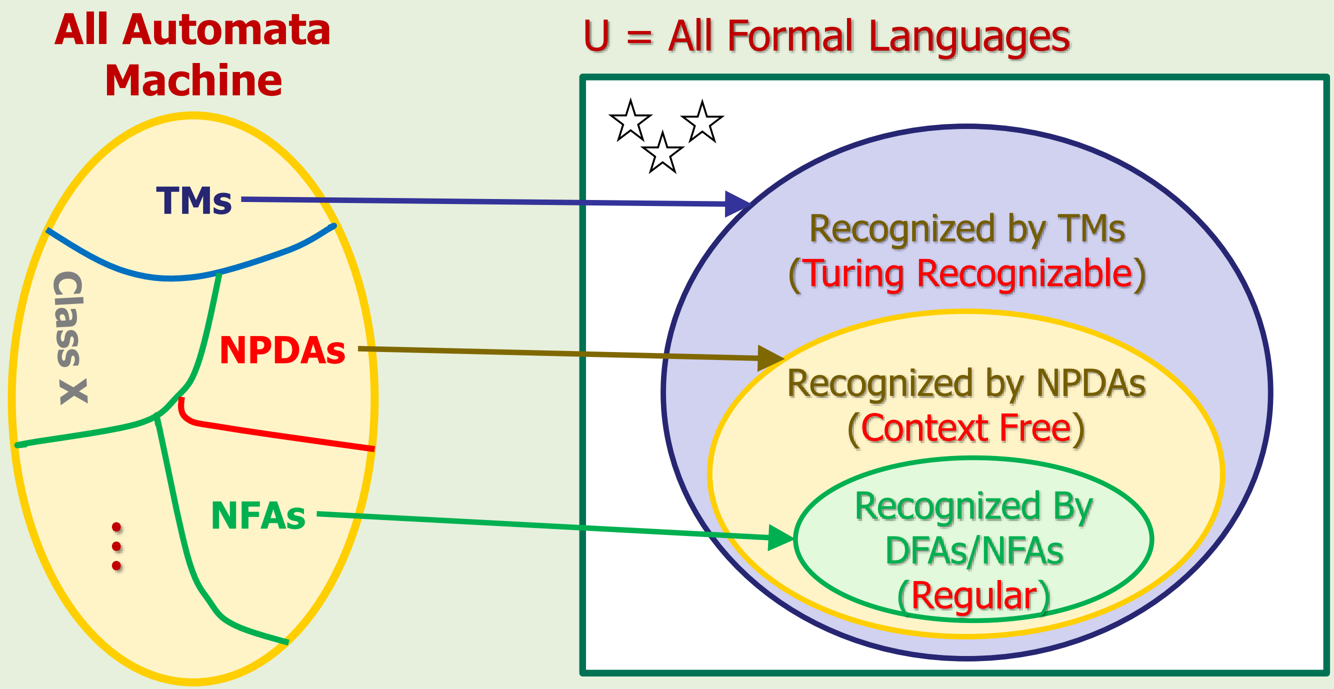
  Canvas → Files → Misc

  S18-Ahmad Y-CS154-NPDAs-Transition-Simulation.pdf

Can NPDAs Do Whatever TMs Can Do?

- Let's assume that we've constructed a TM for an arbitrary language L.
- Can we always construct an NPDA for L?
- No! Why?
- At least we know the following languages for which we constructed TMs but it was impossible to construct NPDAs.
  - $L = \{a^n b^n c^n : n \geq 1\}$
  - $L = \{ww : w \in \Sigma^*\}$
- Let's summarize our knowledge and figure out what would be the next step.

Machines and Languages Association

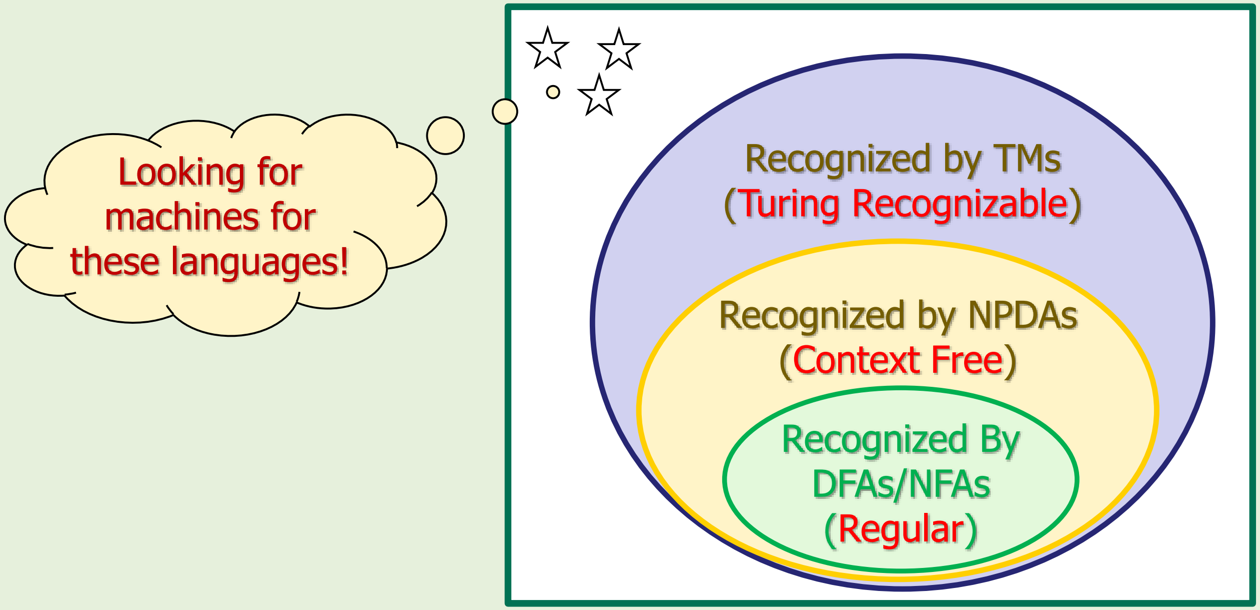


- The set of languages that NPDAs recognize is a proper subset of the set of languages that TMs recognize.
- So, TMs are more powerful than NPDAs.

8. What is the Next Step?

- TMs recognize some other non-regular languages called "Turing recognizable".
- But there are still languages that are not Turing recognizable!
- First, we need to find at least one of them, then we'll think about the next step!

U = All Formal Languages



NAME	Alan M. Turing		
SUBJECT	CS 154	TEST NO.	6
DATE	03/22/2018	PERIOD	1 , 2 , 3

TEST RECORD	
PART 1	123
PART 2	
TOTAL	

Your **list #** goes here!

Quiz 6

No Scantron

Take-Home Exam

Nice Videos

1. Turing machines explained visually
   https://www.youtube.com/watch?v=-ZS_zFg4w5k
2. A Turing machine – Overview
   https://www.youtube.com/watch?v=E3keLeMwfHY

References

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