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Turing Machines

(Part 3)

Lecture 17 - 1

Day 20/31

CS 154

Formal Languages and Computability

Spring 2018

Agenda of Day 20

- Collecting Quiz 6 (Take-Home Exam)
- About Teams
- Summary of Lecture 16
- Lecture 17: Teaching ...
  - Turing Machines (Part 3)
  - Other Models of TMs (Part 1)

Summary of Lecture 16: We learned ...

TMs

- A standard Turing machine M is defined by the septuple:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$$

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$$

- $\square \in \Gamma$ is a special symbol called blank.

- $\delta(q_1, a) = (q_2, b, R)$

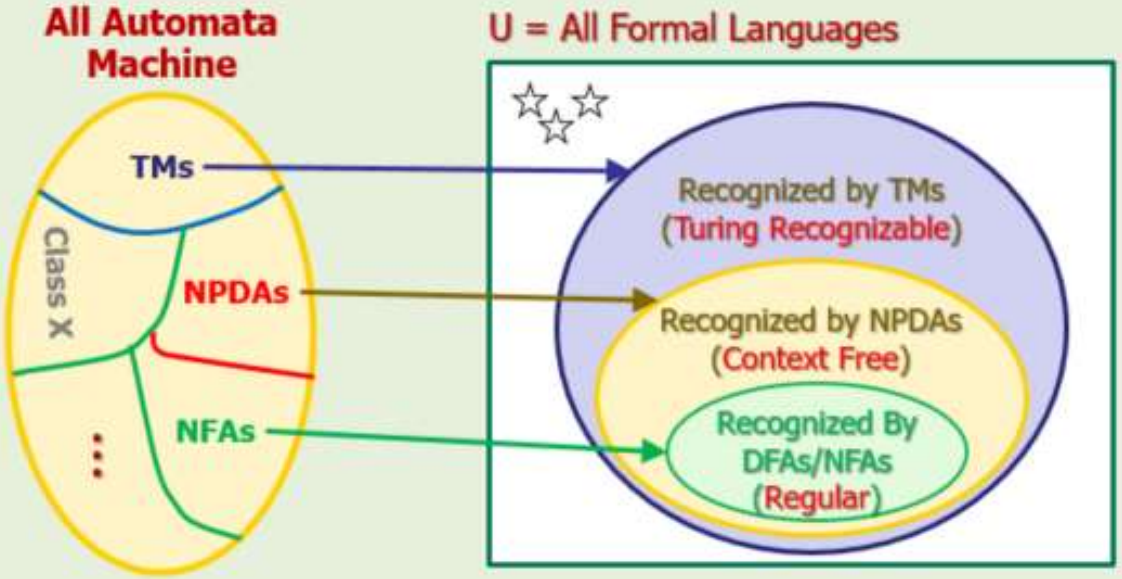
TMs vs NPDAs

- Another way to compare classes of automata is "simulation".
- We can simulate whatever NPDAs do with TMs.
- But not vice versa!
  - At least we know some languages for which we could not construct NPDAs but could construct TMS
  - such as: $a^n b^n c^n$ and ww
- So, TMs are more powerful than NPDAs.

Any Question

Summary of Lecture 16: We learned ...

Machines and Languages Association



- TMs recognize some other non-regular languages called "Turing recognizable".

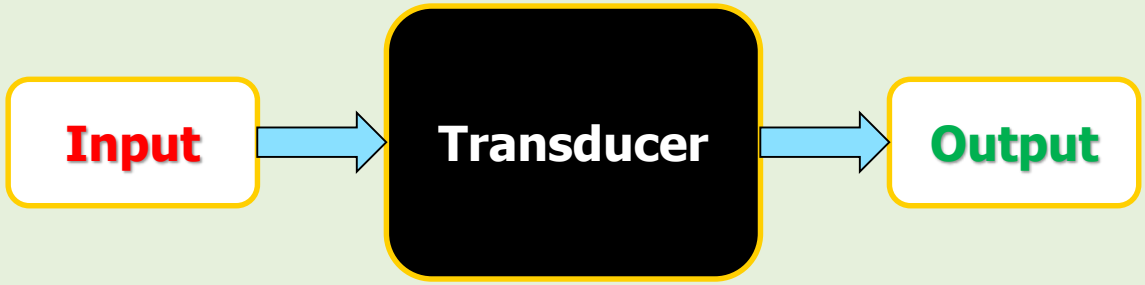
Any Question

TMs as Transducers

What are transducers?

- In physics, transducer is a device that converts the variation in a physical quantity into an electrical signal, or vice versa.
  - The variation could be movement, pressure, brightness, or so forth.

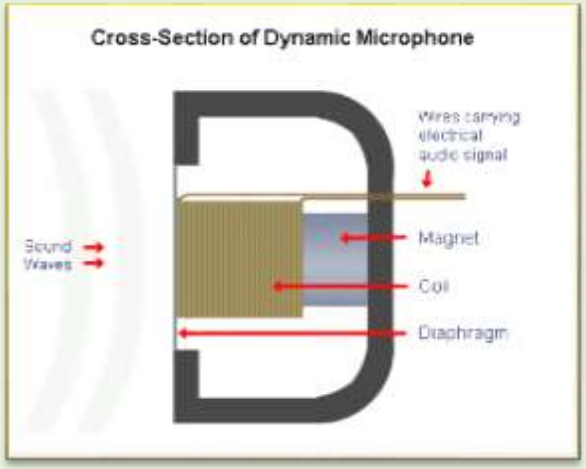
- In a nutshell, transducer is a device that converts an input to an output.



Examples of Transducers

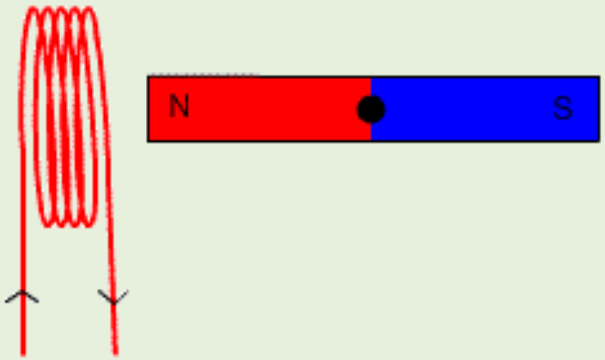
Example 11: Microphone

- A microphone converts your voice to electrical signals.



Example 12: Electric Generator

- An electric generator converts movement to electrical power.



Reference

- Bo Krantz Simonsen at da.wikipedia

Mathematical Model of Transducers

- What is the mathematical model for transducers?
- Functions!

- Recall that a function ...
- ... converts (aka maps) an input (a member of its domain) to an output (a member of its range) based on a rule.

TMs as Transducers

- A TM can act like a transducer. Why?
  - Because it can transform an input to an output.
- What are the input and output of a TM when we run TMs in transducer mode?

Input

- All or part of the nonblank symbols on the tape at the initial time.

Output

- All or part of the tape's content when the machine halts.

- In fact, it's designer's responsibility to define the input and output.

How JFLAP Shows TMs' Outputs

- JFLAP can run TMs in two modes:
  - Regular mode
  - Transducer mode

- In regular mode, JFLAP shows "Accept" or "Reject" for accepting/rejecting strings.
- In transducer mode, it shows the output of the computation too.

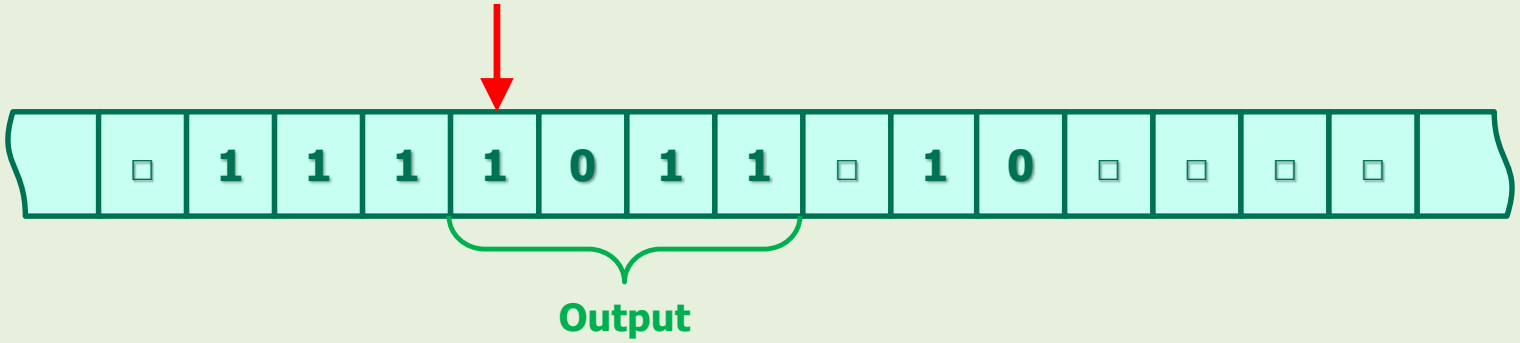
- We'll follow how JFLAP shows the output of TMs when we run them in transducer mode.

- Let's explain it through an example.

How JFLAP Shows TMs Outputs in Transducer Mode

Example 13

- When the TM halts in an accepting state, if the content of the tape is what the following schematic shows , ...



- ... what would be the output?
- Output = 1 0 1 1

- So, the output starts from the symbol at which the head is pointing, to the right, until the first blank.

Turing Computable Functions

Definition

- A function is said to be "Turing-computable" (or just "computable") if there exists a TM that implements it.

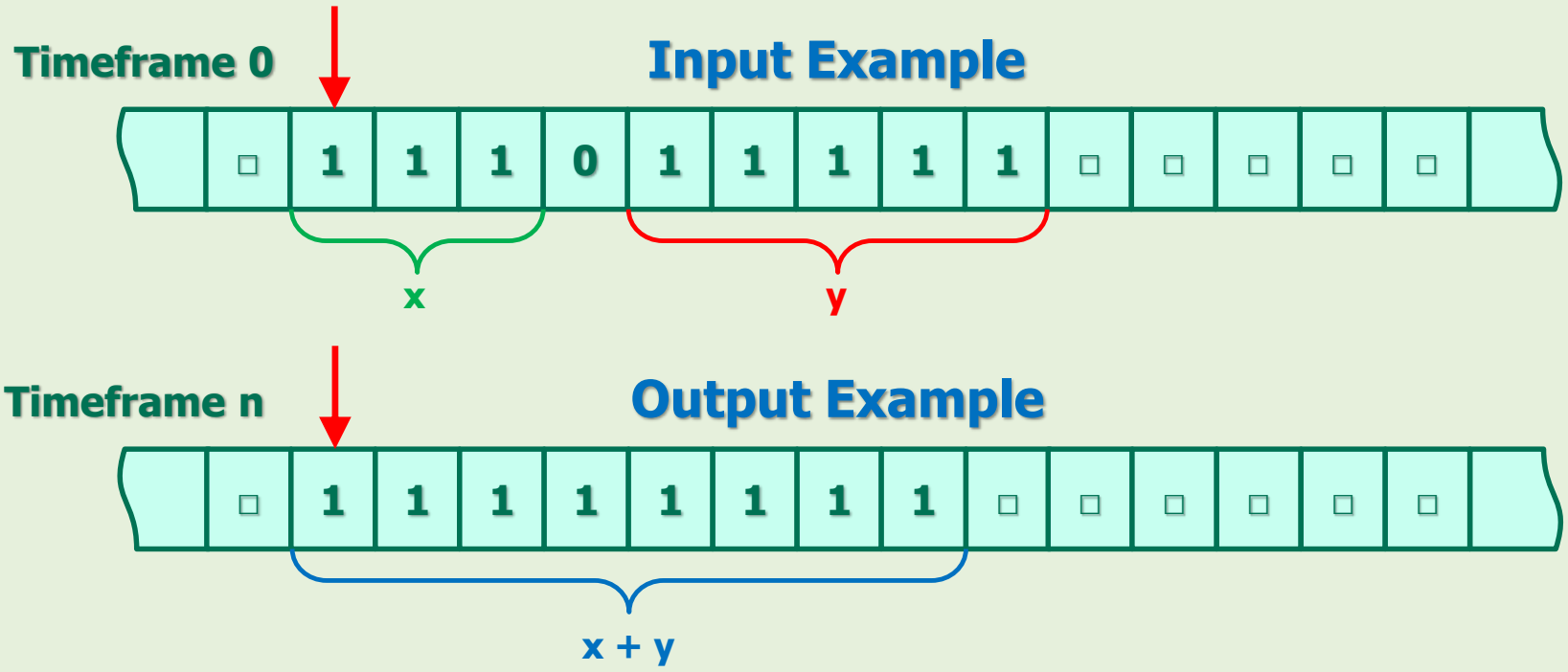
- It has been proved that all common mathematical functions, no matter how complicated, are Turing-computable.

- Let's take some practical examples (basic operations of computers) such as:
  - adding numbers
  - copying strings
  - simple comparisons

TMs as Transducers

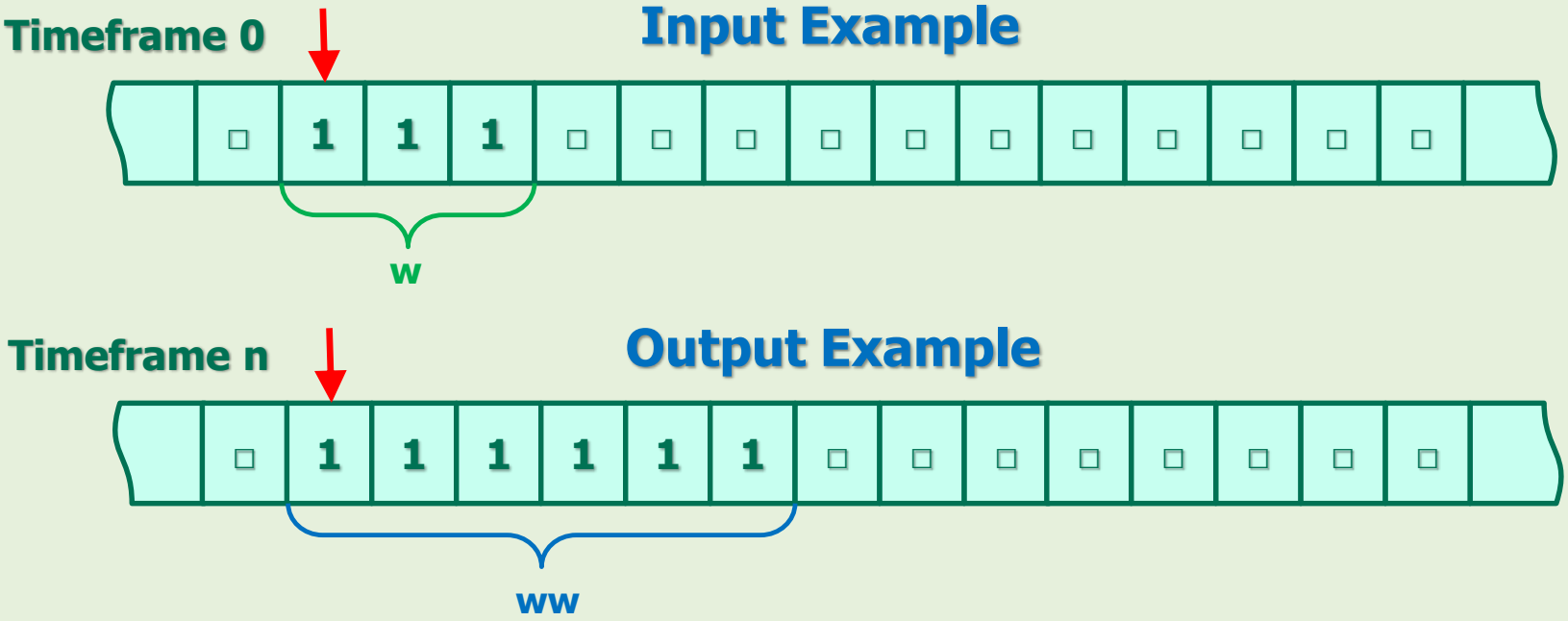
Example 14

- Given two non-zero positive integers x and y, represented in unary notation, separated by a single 0 (zero).
- Design a TM that computes x + y.



Homework

- Given a string $w \in L = \{1\}^+$.
- Design a TM that writes a copy of w at the end of the w.



- At the end of the execution, we'll have "ww" on the tape.

Homework

- Given two non-zero positive integers x and y (x goes first), represented in unary notation separated by a single 0 (zero).

- Design a TM that halts in an accepting state if $x \geq y$, otherwise it halts in a non-accepting state.

Combining TMs

Combining TMs

- How to combine TMs to solve more complex problems?

Example 15

- Given two non-zero positive integers x and y, represented in unary notation, and separated by a zero.
- Design a TM to compute the following function:

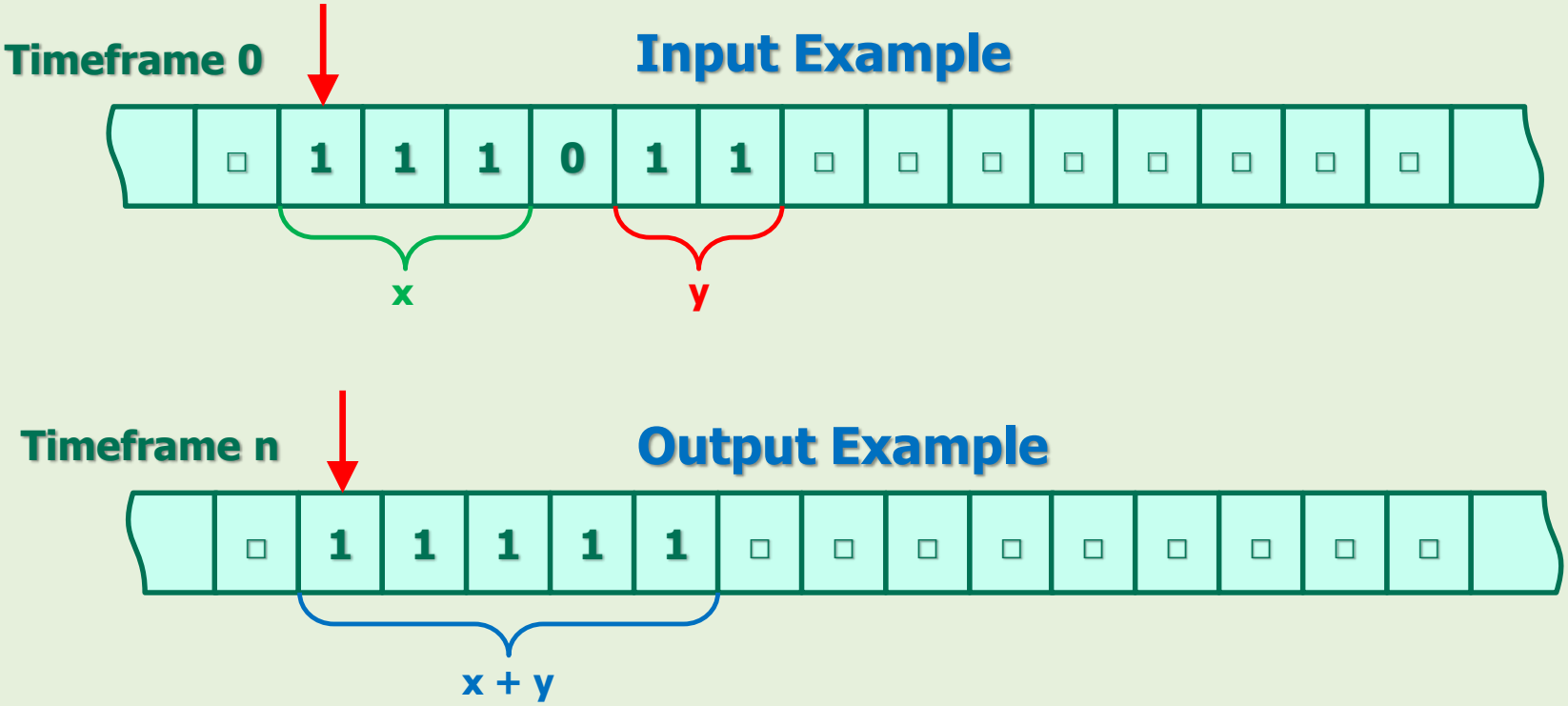
$$f(x, y) = \begin{cases} x + y & \text{if } x \geq y \\ 0 & \text{if } x < y \end{cases}$$

- Let's visualize what would be the output in different situations ...

TM as Transducer

Example 15: When x ≥ y

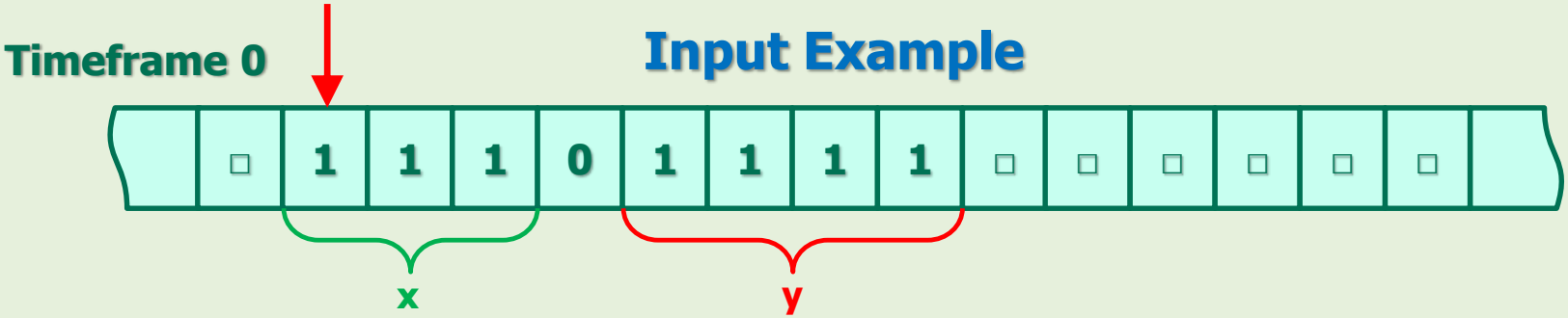
$$f(x, y) = \begin{cases} x + y & \text{if } x \geq y \\ 0 & \text{if } x < y \end{cases}$$

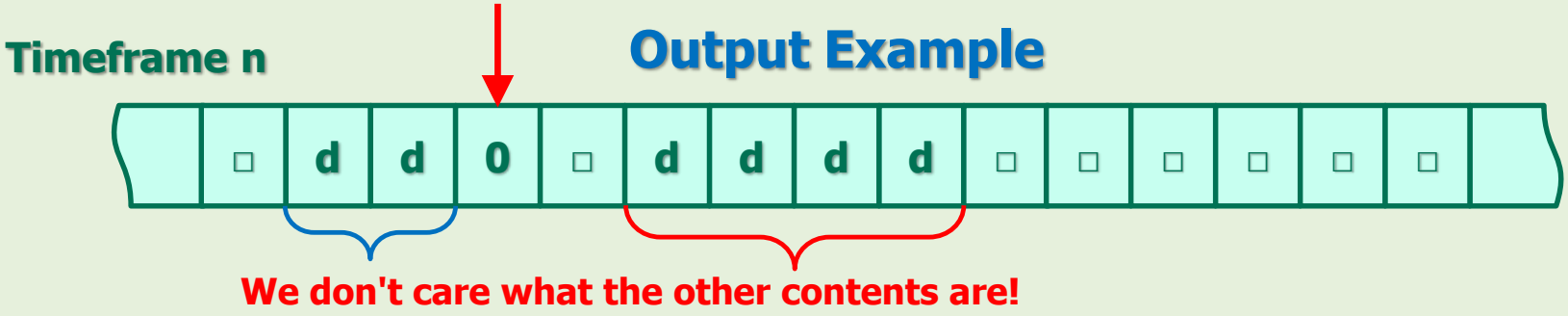


TM as Transducer

Example 15: When x < y

$$f(x, y) = \begin{cases} x + y & \text{if } x \geq y \\ 0 & \text{if } x < y \end{cases}$$

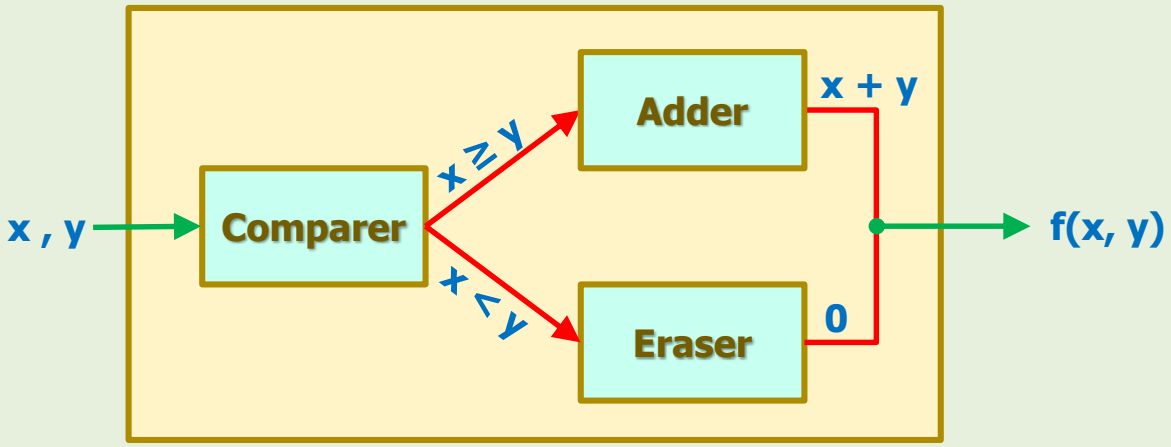




Combining TMs

Example 15 (cont'd)

- To implement this function, we would need the following routines:
  - a comparer
  - an adder
  - an eraser
- A comparer compares x and y ...
  - If $x \geq y$, then it activates the adder to compute x + y.
  - Else (i.e.: $x < y$), it activates the eraser to show zero.



Combining TMs

Example 15 (cont'd)

- This example shows that we can decompose any complex problem into simpler routines and implement each routine with a TM.

- Do the rest of this example as homework!

Homework

- Given two positive integers x and y, represented in unary notation, and separated by a zero.
- Design a TM to compute x * y.

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Other Models of TMs

Lecture 17 -2
Day 20/31

CS 154
Formal Languages and Computability
Spring 2018

Introduction

- We saw how adding memory could increase the computing power.
- Now the question is ...
- ... to make more powerful machines, can we add more memory, or different kind of memories?
- For example:
  - two or more stacks,
  - a queue and a stack,
  - two or more tapes
  - or ...
- How far can we go?
- What would be the most powerful automata?

Objective of This Lecture

- We are going to check some of those questions.
- We'll add some extra capabilities to the standard TMs.

- By any changes, we introduce a new class of automata.
- Then, the very first question would be:
  - Is this new class more powerful than the standard TMs?

- We've already saw how we compare the classes of automata by simulation.

TMs with Stay-Option

TMs with Stay-Option

- In standard TMs, the head can move "left (L)" or "right (R)".
- Now, we add a new movement to the head, ...
  - Stay (S), or no move

- Everything else would remain exactly the same as standard TMs.

Formal Definition

$$M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$$

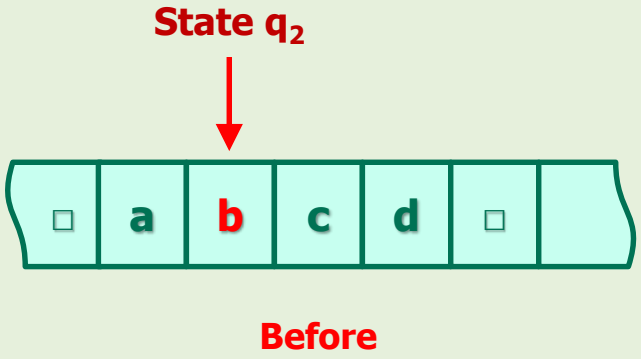
- Where:
  - ... (same as standard TM)
  - $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$

TMs with Stay-Option

Example 1

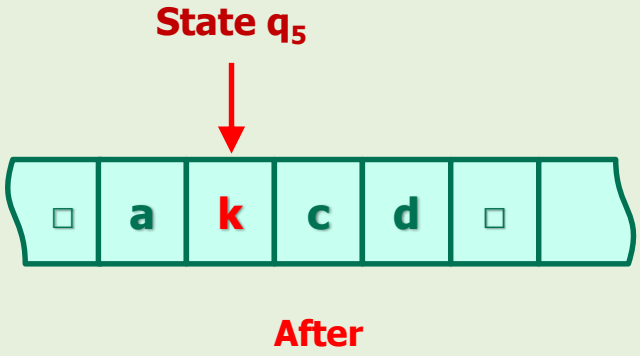
- Consider the following transition:

$\delta(q_2, b) = (q_5, k, S)$



Before

- The figures show the content of the tape and the position of the read-write-head before and after the transition.



After

Is this new class more powerful than standard TMs?

Theorem

- The class of TMs with stay-option is equivalent to the class of standard TMs.
- We need to prove two things:
  1. TMs with stay-option simulate standard TMs.
  2. Standard TMs simulate TMs with stay-option.

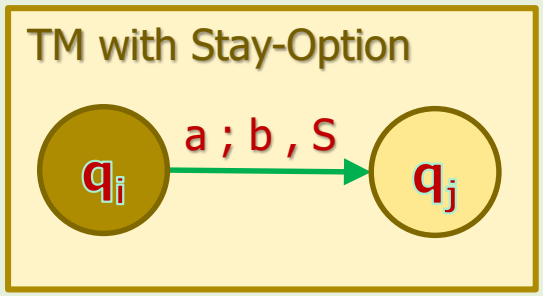
Proof of 1

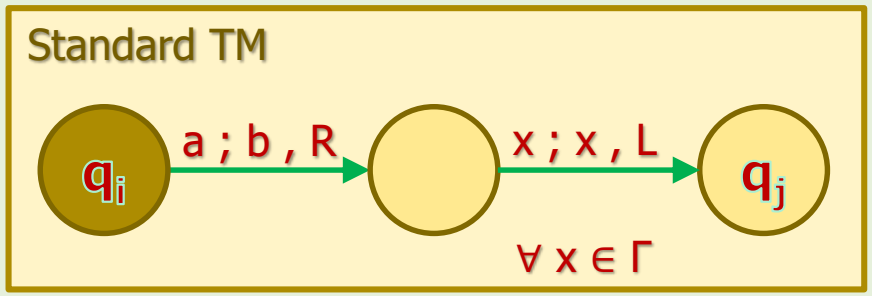
- Let's assume we've constructed a standard TM for an arbitrary language L.
- Can we always construct a TM with stay-option for L? How?
- Yes, just don't use "S" option!

Is this new class more powerful than standard TMs?

Proof of 2

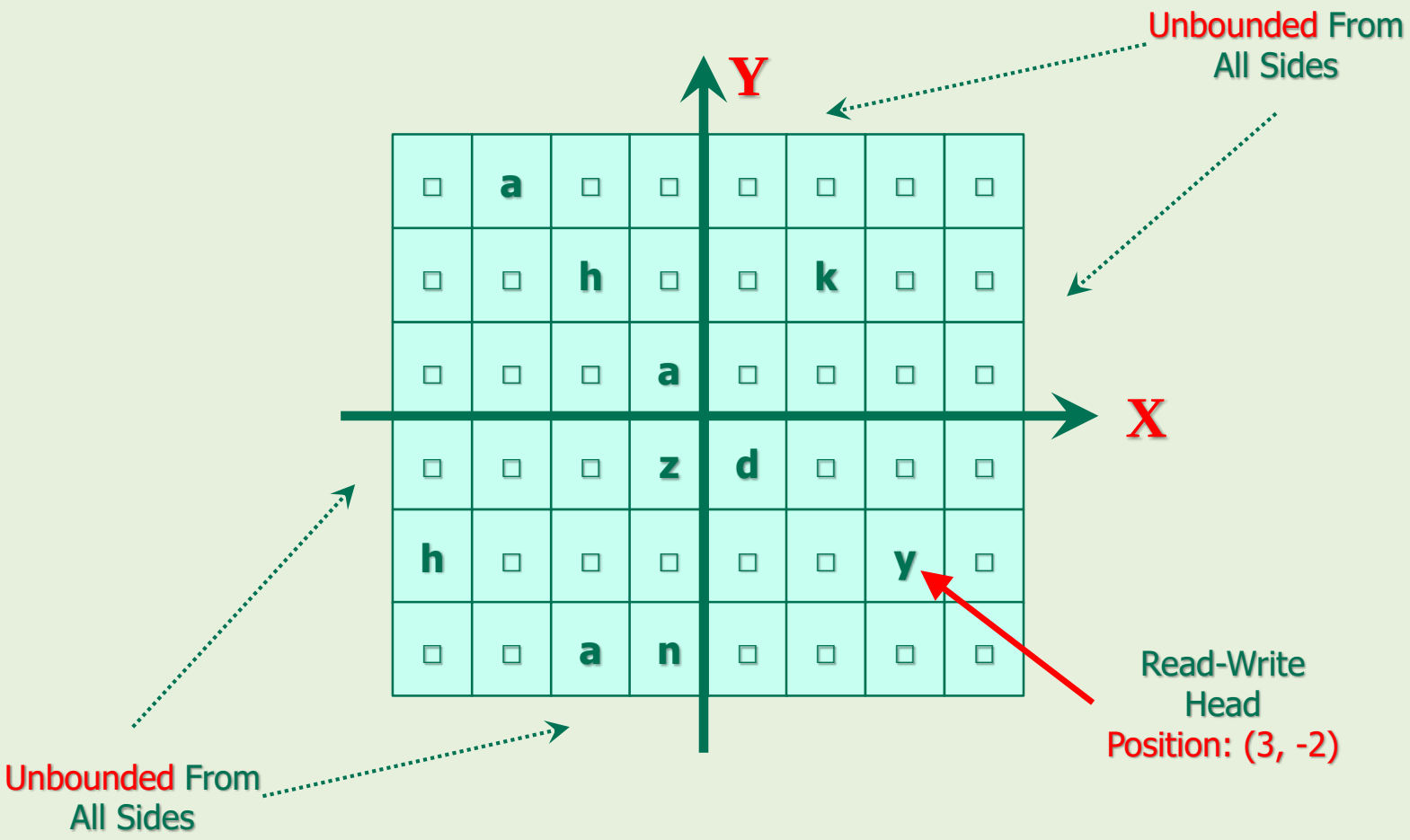
- Let's assume we've constructed a TM with stay-option for an arbitrary language L.
- Can we construct a standard TM for L? How?
- The only difference is those transitions that use "S" option.
- So, we just need to simulate "no move" for those transitions.
- That can be simulated by two moves of the head: "left, then right" (or "right, then left") as the following figures show.





Multidimensional-Tape TMs

Multidimensional-Tape TMs



Multidimensional-Tape TMs

Definition

- A multidimensional-tape TMs M is defined by the septuple:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$$

- Where:
  - ... (same as standard TM elements)

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, U, D\}$$

- The new movements: U=Up and D=Down

Is this new class more powerful than standard TMs?

Theorem

- The class of multidimensional-tape TMs is equivalent to the class of standard TMs.
- We need to prove two things:
  1. Multidimensional-tape TMs simulate standard TMs.
  2. Standard TMs simulate multidimensional-tape TMs.

Proof

1. Multidimensional-tape TMs simulate standard TMs.

- This step is trivial because if we just use one row of the tape, then we have standard TM.

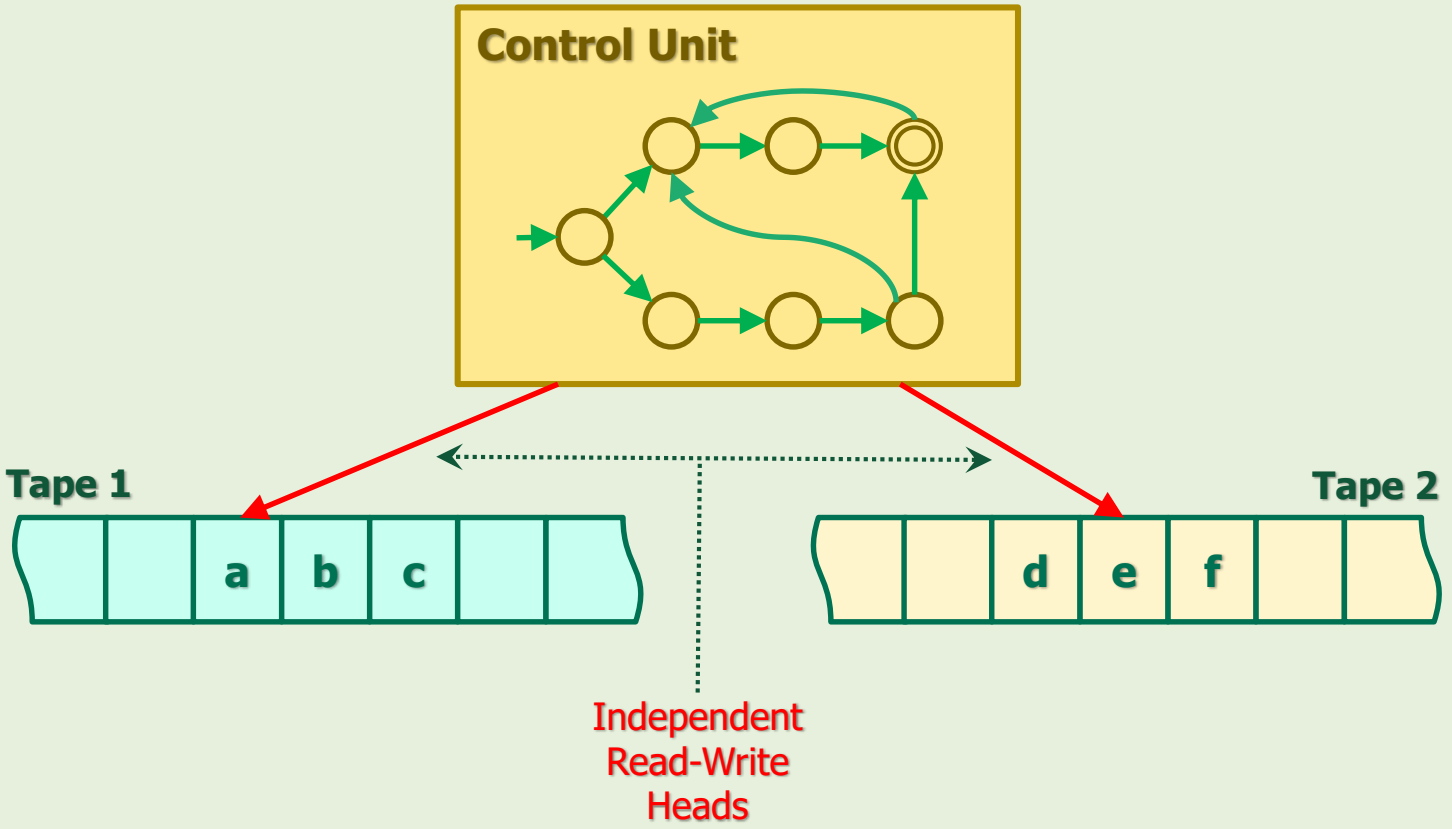
2. Standard TMs simulate multidimensional-tape TMs.

- Prove this as exercise!

Multi-Tape TMs

Multi-Tape TMs

- We can add additional tapes with independent read-write head to the standard TMs.
- For example, a double-tape TM looks like this:



Multi-Tape TMs

Formal Definition

- An n-tape TM M is defined by the septuple:

$$M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$$

- Where:
  - ... (same as standard TM elements)

$$\delta: Q \times \Gamma^n \rightarrow Q \times \Gamma^n \times \{L, R\}^n$$

- Note that: $\Gamma^n = \Gamma \times \Gamma \times \ldots \times \Gamma$ (Cartesian product)
- Therefore, the result is an n-tuple.

Formal Definition of Multi-Tape TMs

- **Example 2**

- This is a transition of a double-tape TM.
  - We separate the labels of different tapes with "|".
- The transition condition is both inputs:

  input symbol of tape 1 = 'a'

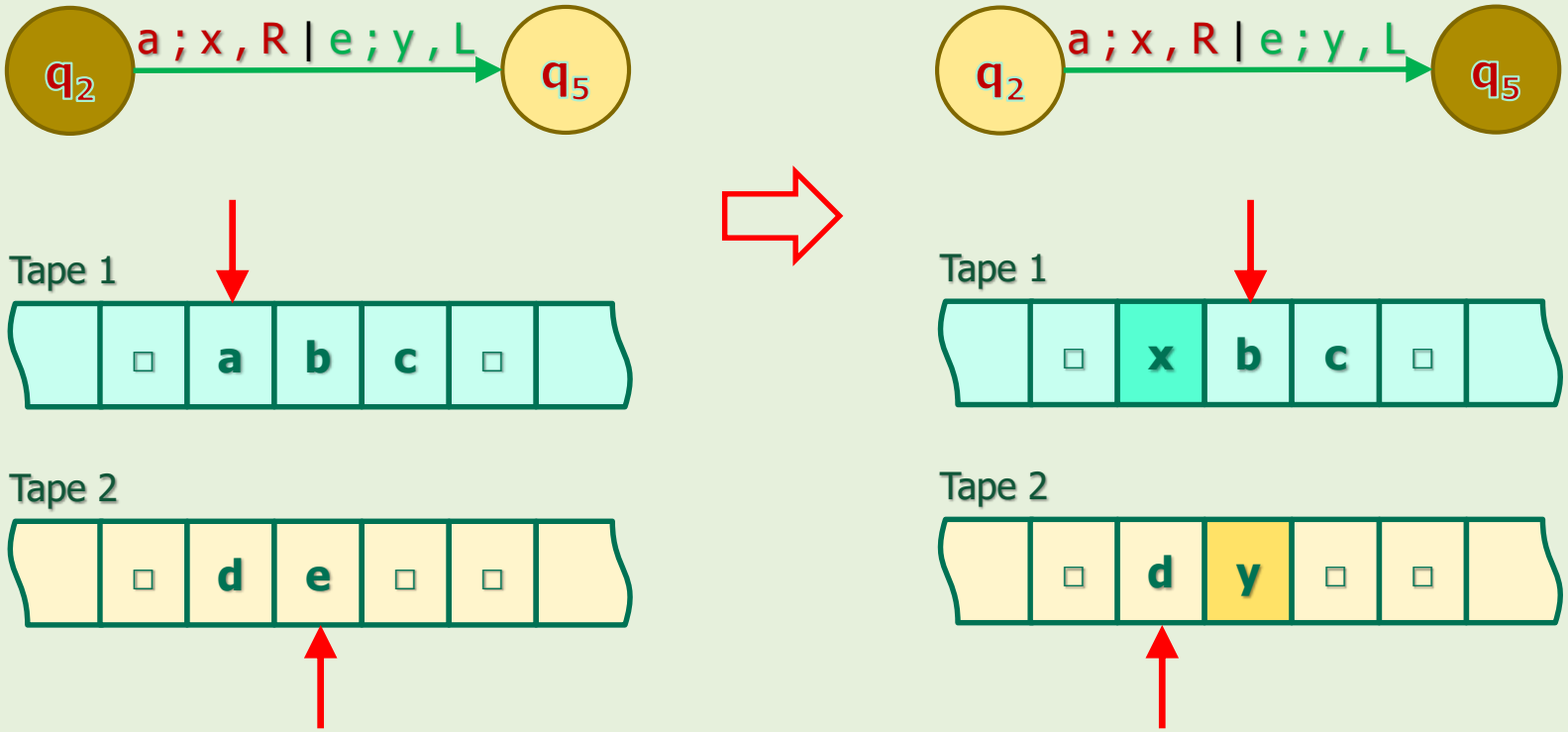
  AND

  input symbol of tape 2 = 'e'.

- The sub-rule looks like this: $\delta(q_2, a, e) = (q_5, x, y, R, L)$

Formal Definition of Multi-Tape TMs

- **Example 2 (cont'd)** $\delta(q_2, a, e) = (q_5, x, y, R, L)$

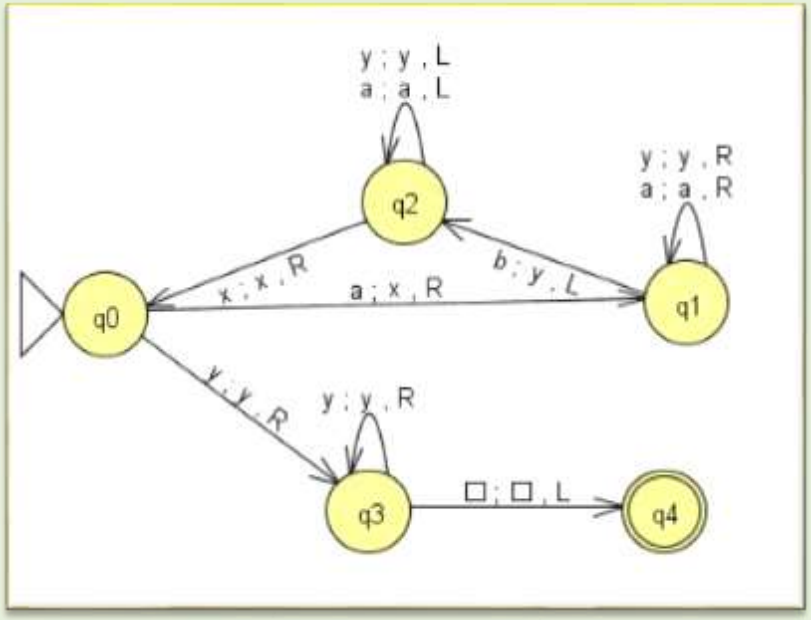


Before **After**

Multi-Tape TMs Example

Example 3

- Design a double-tape TM for accepting the language $L = \{a^n b^n : n \geq 1\}$ over $\Sigma = \{a, b\}$.
- Assume that the input is written on the tape 1 and for simplicity, we can use "stay-option".
- Recall that we had designed a standard TM for L before as the figure shows.



Multi-Tape TMs Example

Example 3 (cont'd)

Strategy

- Read a's from tape 1 and write them on tape 2.
- When sensed the first 'b' on tape 1, match b's with the a's on tape 2.
- If all match, then accept,
- otherwise, reject.

- Do double-tape TMs facilitate our programming?

Homework

- Design a double-tape TM for accepting the following language:

  $L = \{a^n b^n c^n : n \geq 1\}$ over $\Sigma = \{a, b, c\}$

- Assume that the input is written on the tape 1 and for simplicity, you can use "stay-option".

Is this new class more powerful than standard TMs?

Theorem

- The class of multi-tape TMs is equivalent to the class of standard TMs.
- We need to prove two things:
  1. Multi-tape TMs simulate standard TMs.
  2. Standard TMs simulate multi-tape TMs.

Proof

1. Multi-tape TMs simulate standard TMs.
- This step is trivial because if we just use one tape, then we have standard TM.

2. Standard TMs simulate multi-tape TMs.

Nice Videos

1. Turing machines explained visually
   https://www.youtube.com/watch?v=-ZS_zFg4w5k
2. A Turing machine – Overview
   https://www.youtube.com/watch?v=E3keLeMwfHY

References

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