Final Exam Section 2 Fall 2015, Part B
Computer Science Department, SJSU
CS146: Data Structures and Algorithms
Instructor: Katerina Potika

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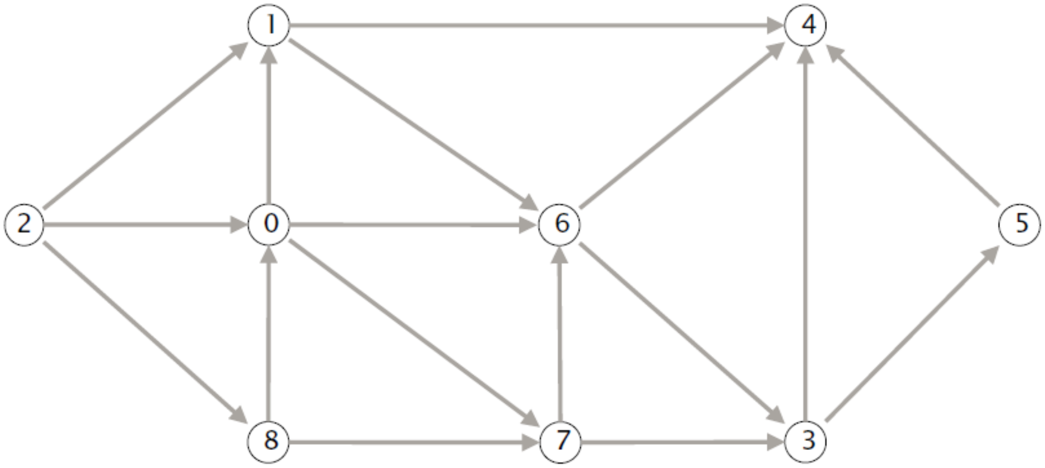
Section	Points
Question 1	______Out of 20
Question 2	______Out of 20
Question 3	______Out of 20
Question 4	______Out of 10
Total	______ Out of 90

Duration 1 hour. Closed Books. Good luck!

Question 1

Consider the following acyclic digraph. Assume the adjacency lists are in sorted order: for example, when iterating through the edges pointing from 0, consider the edge 0 → 1 before 0 → 6 or 0 → 7.



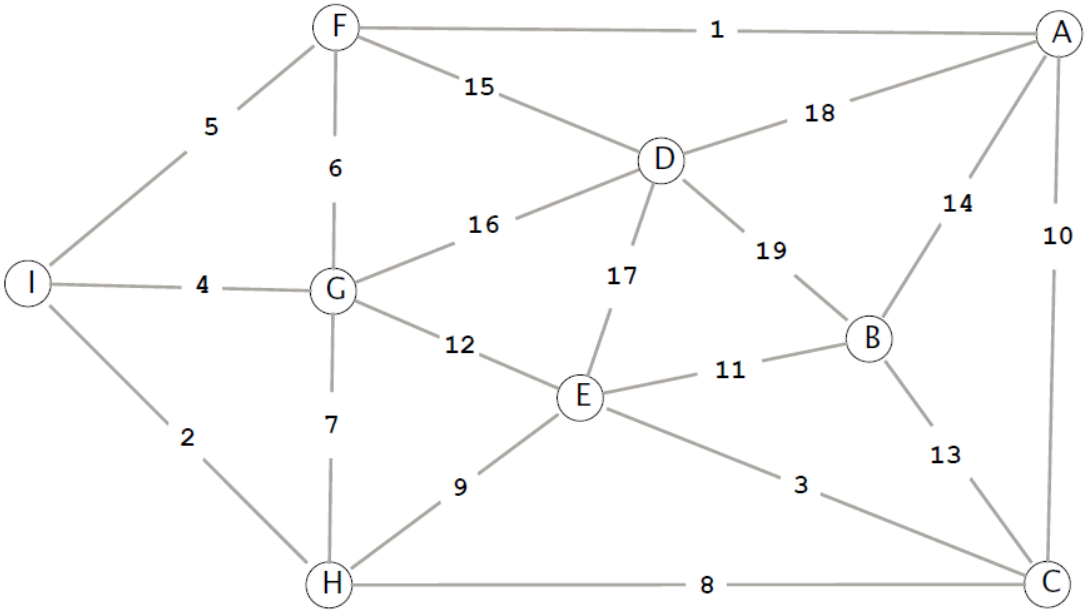
(a) Compute the topological order by running the DFS-based algorithm. Give the topological sorting and

(b) Run breadth-first search on the digraph, starting from vertex 2. List the vertices in the order in which they are dequeued from the FIFO queue.

Answer:
(a) 2 8 0 7 1 6 3 5 4 (b) 2 0 1 8 6 4 7 3 5

Question 2

Consider the following edge-weighted graph with 9 vertices and 19 edges. Note that the edge weights are distinct integers between 1 and 19.



a) Complete the sequence of edges in the MST in the order that Kruskal's algorithm includes them (by specifying their edge weights). 1 ____ ____ ____ ____ ____ ____ ____ ____

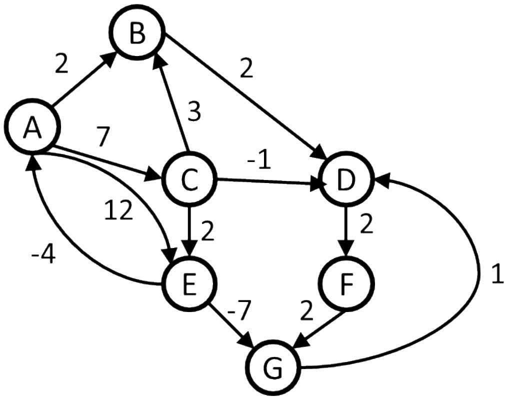
b) Complete the sequence of edges in the MST in the order that Prim's algorithm includes them (by specifying their edge weights).

Answer:

Minimum spanning trees. (a) 1 2 3 4 5 8 11 15 (b) 1 5 2 4 8 3 11 15 The starting vertex must be either A or F (but it doesn't matter which).

Question 3

Consider the following directed, weighted graph:



a. [10pts] Even though the graph has negative weight edges, step through Dijkstra's algorithm to calculate supposedly shortest paths from A to every other vertex. Show your steps in the table below. Set S includes all converged vertices. Also list the vertices in the order which you marked them known.

S	A	B	C	D	E	F	G

b. [6pts] Dijkstra's algorithm found the wrong path to some of the vertices. For just the vertices where the wrong path was computed, indicate both the path that was computed and the correct path.

Computed path to G is A,B,D,F,G but shortest path is A,C,E,G. Computed path to D is A,B,D but shortest path is A,C,E,G,D. Computed path to F is A,B,D,F but shortest path is A,C,E,G,D,F.

c. [4pts] What single edge could be removed from the graph such that Dijkstra's algorithm would happen to compute correct answers for all vertices in the remaining graph?

The edge from E to G

Question 4

Shortest directed cycle. Given a directed graph with V vertices and E edges, design an efficient algorithm to find a directed cycle with the minimum number of edges (or report that the graph is acyclic). Your answer will be graded on correctness, efficiency, clarity, and succinctness. For full credit, your algorithm should run in O(EV) time and use O(E + V) space. Assume V ≤ E ≤ V^2 . (a) Describe your algorithm in the space below.

Answer:
(a) The critical observation is that the shortest directed cycle is a shortest path (number of edges) from s to v, plus a single edge v→s. For each vertex s: * Use BFS to compute shortest path from s to each other vertex. * For each edge v->s entering s, consider cycle formed by shortest path from s to v (if the path exists) plus the edge v->s. Return shortest overall cycle. (b) The running time is O(EV). The single-source shortest path computation from s takes O(E +V) time per using BFS. Finding all edges entering s takes O(E +V) time by scanning all edges (though a better way is to compute the reverse graph at once and access the adjacency lists). We must do this for each vertex s. Thus, the overall running time is O(EV). (c) The memory usage is O(E + V). BFS uses O(V) extra memory and we only need to run one at a time. (A less efficient solution is to compute a V -by-V table containing the shortest path from v to w for every v and w. This uses O(V 2) memory.)

SCRATCH PAPER