Final Exam Section 1 Fall 2015, Part B
Computer Science Department, SJSU
CS146: Data Structures and Algorithms
Instructor: Katerina Potika

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Section	Points
Question 1	______Out of 6
Question 2	______Out of 16
Question 3	______Out of 10
Question 4	______Out of 16
Total	______ Out of 48

Duration 1 hour. Closed Books. Good luck!

Question 1

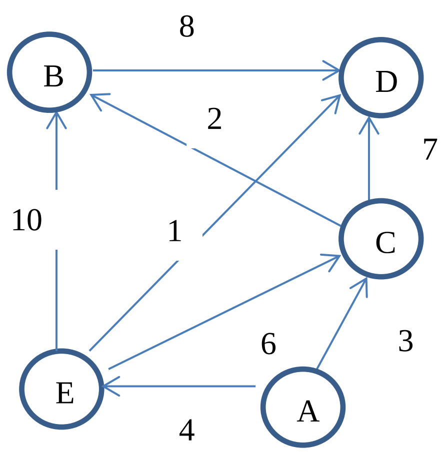
[6pts] Suppose that we define a different kind of graph, where we have weights on the vertices and not the edges. Does the shortest-paths problem make sense for this kind of graph? If so, give a precise and formal description of the problem and an application. If not, explain why not. Note we are not asking for an algorithm, just what the problem is or that it makes no sense.

Solution:

Yes, this problem makes sense: Given a starting vertex v find the lowest-cost path from v to every other vertex. The cost of a path is the sum of the weights of the vertices on the path.

Question 2

Use the following graph for this problem. Where needed and not determined by the algorithm, assume that any algorithm begins at node A.



a. Give both the adjacency matrix and adjacency list representations of this graph. (Specify which is which for full credit.)[3 pts/each]

Adjacency list:
Adj[A]->C->E
Adj[B]->D
Adj[C]->B->D
Adj[D]->null
Adj[E]->B->C->D

Adjacency matrix:

	A	B	C	D	E
A	0	0	1	0	1
B	0	0	0	1	0
C	0	1	0	1	0
D	0	0	0	0	0
E	0	1	1	1	0

b. Give a valid topological sorting of the nodes in the graph.[4pts]

Key:

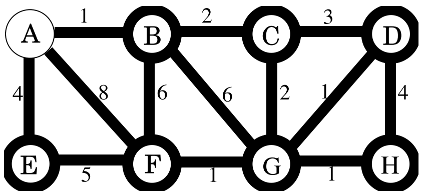
A E C B D

c. Step through Dijkstra's Algorithm to calculate the single source shortest path from A to every other vertex. You need to show your steps in a table for full credit.[6pts]

Set S	A	B	C	D	E
null 0	∞	∞	∞	∞	
A	-	∞	3	∞	4
C	-	5	-	10	4
E	-	5	-	10	-
D	-	5	-	5	-
B	-	-	-	5	-
D	-	-	-	-	-

Question 3

Suppose we want to find the minimum spanning tree of the following graph.



(a) [5pts] Run Prim's algorithm; whenever there is a choice of nodes, always use alphabetic ordering (e.g., start from node A). Fill the next table showing the intermediate values of the priority queue for each vertex.

A	B	C	D	E	F	G	H
0	∞	∞	∞	∞	∞	∞	∞
	1			4	8		
		2			6	6	
			3			2	
			1		1		1

(b) [5pts] Run Kruskal's algorithm on the same graph. Show how the disjoint-sets data structure looks at every intermediate stage (use connected components).

Answer: order weight

1: AB FG GH GD, 2: BC CG, 3: CD, 4: AE DH, 5: EF, 6: BF BG, 8: AF

Take up to AE

Question 4

Identify a Celebrity

We call a person a celebrity among a group of n people, if that person knows nobody but is known to everybody else. The problem is to identify a celebrity by only asking questions to people of the form: “Do you know him/her?”

How would you model this problem with a directed graph that is represented with an adjacency matrix? In particular, answer the following questions **[1pts/each]**:
(a) Who are the vertices of this graph?
(b) What is the meaning of a directed edge in this graph?
(c) What is a celebrity in our model (what property does it have)?
(d) What is the equivalent (in graph terms) to the defined question? (how would reformulate it)
(e) What do we want to minimize?
(f) Can a group have more than one celebrity?

Design an efficient algorithm to identify a celebrity or determine that the group has no such person. How many questions does this algorithm need in the worst case? Justify **[10pts]**.

Hint: Think about exactly what information a single edge provides after answering the question.

Answer:
1. (a) The vertices represent the people in the group.
(b) A directed edge indicates that the person represented by the source vertex “knows” the person represented by the destination vertex.
(c) A celebrity is an edge with in-degree n - 1 and out-degree zero.
(d) Does edge (v1-> v2) exist?
(e) The objective is to reduce the number of questions asked (matrix accesses) as a function of n.
(f) No. Since a celebrity must be known by everyone else in the group, and know no one, if one celebrity exists, another person could not be a celebrity because that person could not be known by the existing celebrity.
2. Begin with all n people as candidates to be a celebrity. Ask a random person, x if he knows random person y. If so, eliminate x as a candidate to be a celebrity. If not, eliminate y as a candidate to be a celebrity. Continue in this manner until, after n - 1 questions, there is only one candidate to be a celebrity, then check with each of the other n -1 people to confirm that they all know that person (at most n - 1 questions), and with the person to confirm that she does not know any of the other people (also at most n- 1 questions). Thus, it takes at most 3n - 3 questions to identify a celebrity if such a person exists (order O(n)).

SCRATCH PAPER