Decision Tree

LING 572

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Sunburn Example

Name	**Hair**	**Height**	**Weight**	**Lotion**	**Result**
Sarah	Blonde	Average	Light	No	Burn
Dana	Blonde	Tall	Average	Yes	None
Alex	Brown	Short	Average	Yes	None
Annie	Blonde	Short	Average	No	Burn
Emily	Red	Average	Heavy	No	Burn
Pete	Brown	Tall	Heavy	No	None
John	Brown	Average	Heavy	No	None
Katie	Blonde	Short	Light	Yes	None

Learning about Sunburn

- Goal:
  - Train on labelled examples
  - Predict Burn/None for new instances

- Solution??
  - Exact match: same features, same output
    - Problem: N*3*3*3*2 feature combinations, which could be much worse when there are thousands or even millions of features.

  - Same label as ‘most similar’
    - Problem: What’s close? Which features matter? Many match on two features but differ on result.

DT highlight

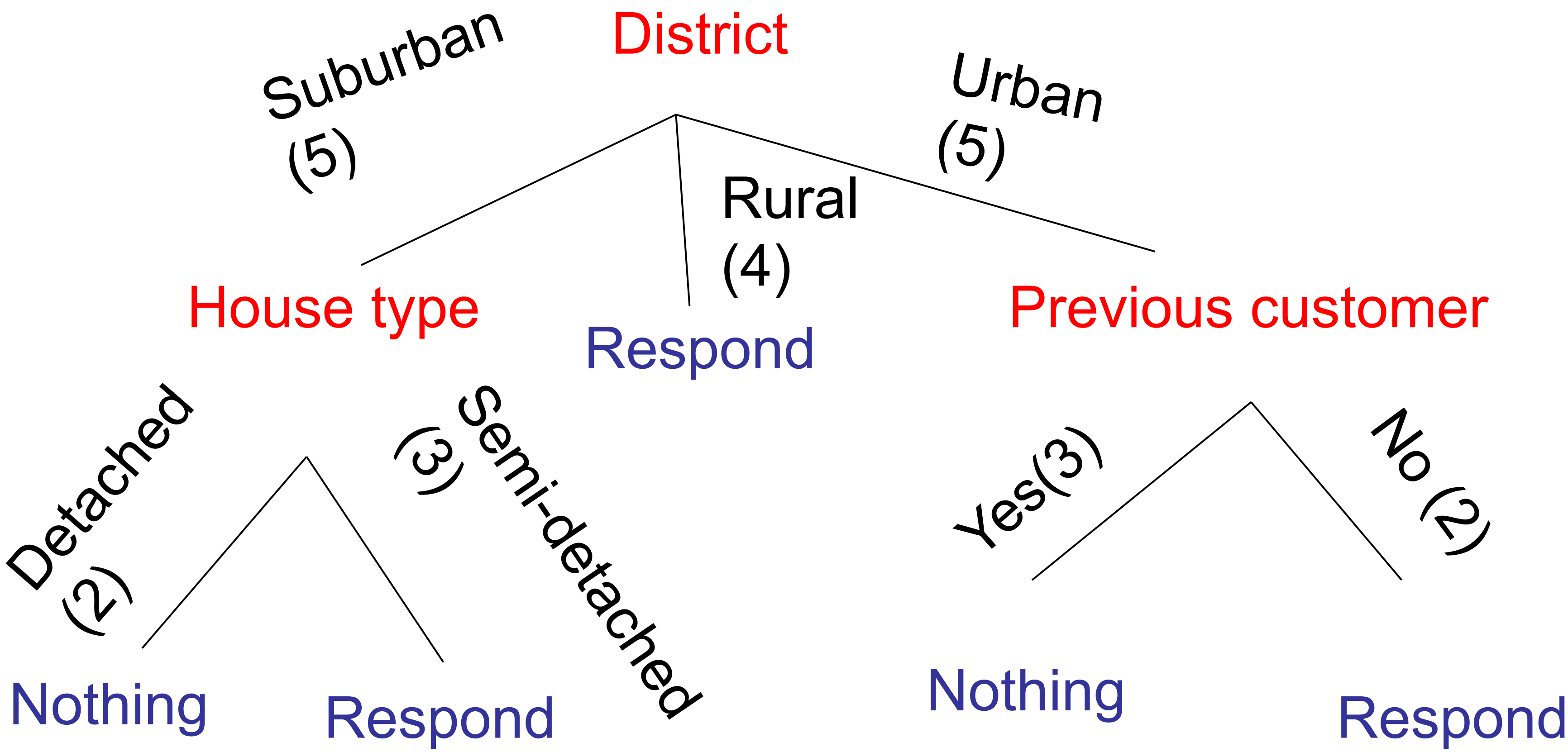
- Training stage: build a tree (aka decision tree) using a greedy algorithm:
  – Each node represents a test.
  – Training instances are split at each node.
  – The set of samples at a leaf node indicates decision

- Test stage:
  – Route NEW instance through tree to leaf based on feature tests
  – Assign same value as samples at leaf

Where should we send Ads?

District	House type	Income	Previous Customer	Outcome (target)
Suburban	Detached	High	No	Nothing
Suburban	Semi-detached	High	Yes	Respond
Rural	Semi-detached	Low	No	Respond
Urban	Detached	Low	Yes	Nothing
…				

Decision tree



Decision tree representation

- Each internal node is a test:
  - Theoretically, a node can test multiple features
  - In general, a node tests exactly one feature
- Each branch corresponds to test results
  - A branch corresponds to a feature value or a range of feature values
- Each leaf node assigns
  - a class: decision tree
  - a real value: regression tree

What’s the best decision tree?

- “Best”: We need a bias (e.g., prefer the “smallest” tree):
  - Smallest depth?
  - Fewest nodes?
  - Most accurate on unseen data?

- Occam's Razor: we prefer the simplest hypothesis that fits the data.

➔ Find a decision tree that is as small as possible and fits the data

Finding a smallest decision tree

- The space of decision trees is too big for systemic search for a smallest decision tree.

- Solution: greedy algorithm
  - At each node, pick test using ‘best’ feature

  - Split into subsets based on outcomes of feature test

  - Repeat process until stopping criterion

Basic algorithm: top-down induction

1. Find the “best” feature, A, and assign A as the decision feature for the node

2. For each value (or a range of values) of A, create a new branch, and divide up training examples

3. Repeat the process 1-2 until the gain is small enough

➔ Effectively creates set of rectangular regions
Repeatedly draws lines in different axes

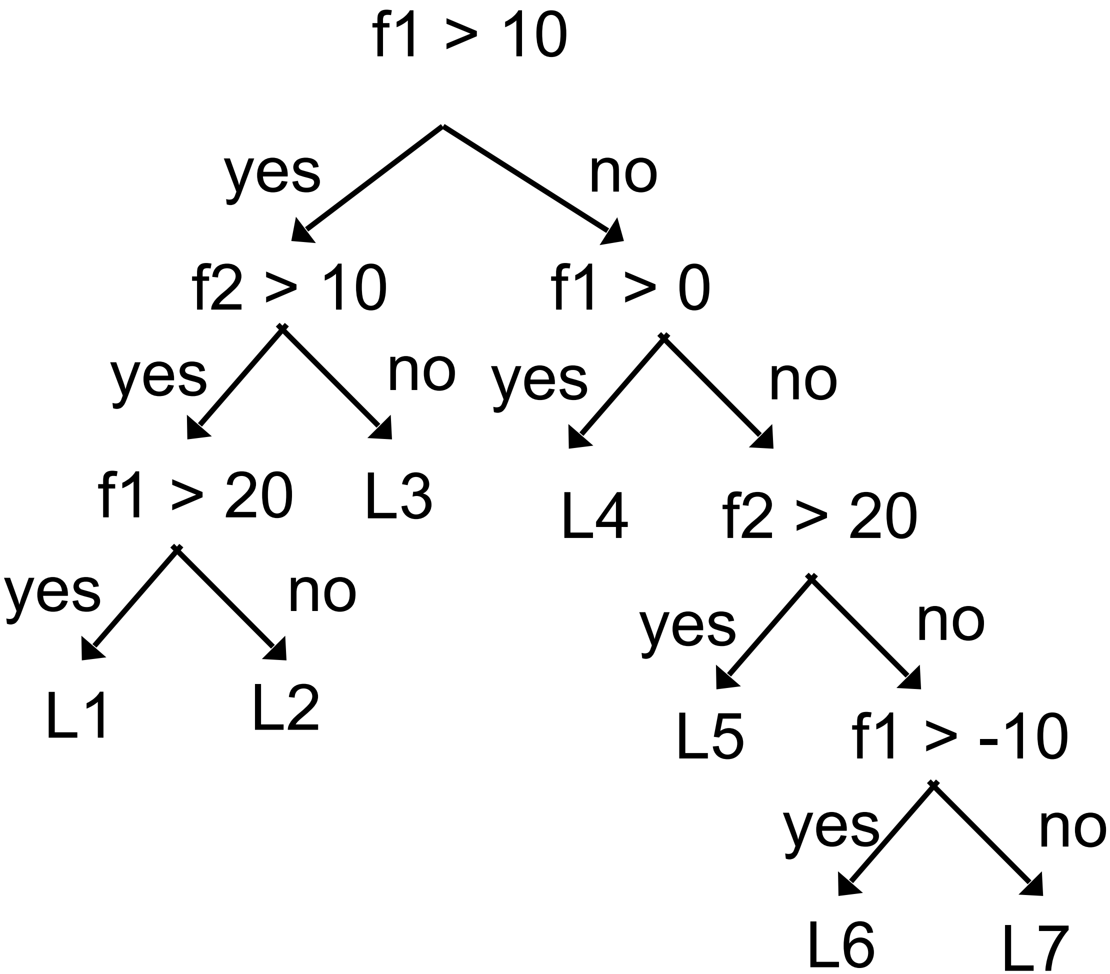
Features in DT

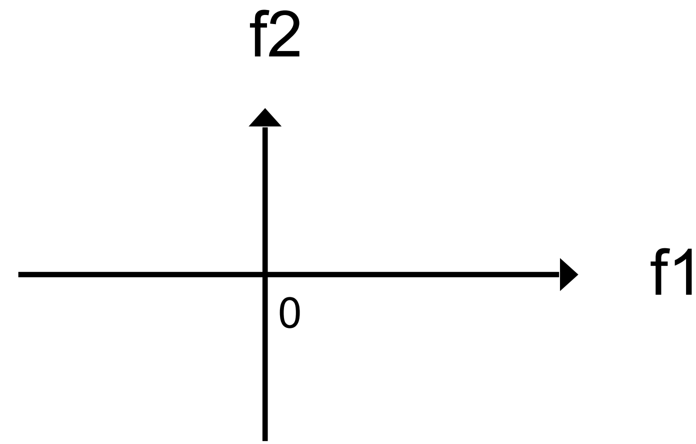
- Pros: Only features with high gains are used as tests when building DT

  ➔ irrelevant features are ignored

- Cons: Features are assumed to be independent

  ➔ if one wants to capture group effect, one must model that explicitly (e.g., creating tests that look at feature combinations)





-20 <= f1 <= 30

-10 <= f2 <= 30

Major issues

Q1: Choosing best feature: what quality measure to use?

Q2: Determining when to stop splitting: avoid overfitting

Q3: Handling features with continuous values

Q1: What quality measure

- Information gain
- Gain Ratio
- χ^2
- Mutual information
- ….

Entropy of a training set

- S is a sample of training examples
- Entropy is one way of measuring the impurity of S
- $P(c_i)$ is the proportion of examples in S whose category is c_i.

$$H(S)=-\sum_i p(c_i) \log p(c_i)$$

Information gain

- InfoGain(Y | X): We must transmit Y. How many bits on average would it save us if both ends of the line knew X?

- Definition:

  $$\text{InfoGain}(Y \mid X) = H(Y) - H(Y|X)$$

- Also written as InfoGain (Y, X)

Information Gain

- InfoGain(S, A): expected reduction in entropy due to knowing A.

$$
\begin{aligned}
InfoGain(S, A) &= H(S) - H(S \mid A) \\
&= H(S) - \sum_{a} p(A = a) H(S \mid A = a) \\
&= H(S) - \boxed{\sum_{a \in Values(A)} \frac{|S_a|}{|S|} H(S_a)}
\end{aligned}
$$

Average Entropy

- Choose the A with the max information gain.

(a.k.a. choose the A with the min average entropy)

$H(S) = -(9/14 * \log 9/14 + 5/14 * \log 5/14)$

S=[9+,5-]
H=0.940

Income

High: [3+,4-] H=0.985

Low: [6+,1-] H=0.592

S=[9+,5-]
H=0.940

PrevCustom

Yes: [6+,2-] H=0.811

No: [3+,3-] H=1.00

InfoGain (S, Income)
=0.940-(7/14)*0.985-(7/14)*0.592
=0.151

InfoGain(S, PrevCustom)
=0.940-(8/14)*0.811-(6/14)*1.0
=0.048

Other quality measures

- Problem of information gain:
  - Information Gain prefers attributes with many values.

- An alternative: Gain Ratio

$$GainRatio(S, A) = \frac{InfoGain(S, A)}{SplitInfo(S, A)}$$

$$SplitInfo(S, A) = H_S(A) = -\sum_{a \in Values(A)} \frac{|S_a|}{|S|} \log_2 \frac{|S_a|}{|S|}$$

Where S_a is subset of S for which A has value a.

Q2: Avoiding overfitting

- Overfitting occurs when the model fits the training data too well:
  - The model characterizes too much detail or noise in our training data.
  - Why is this bad?
    - Harms generalization
    - Fits training data well, fits new data badly

- Consider error of hypothesis h over
  - Training data: ErrorTrain(h)
  - Entire distribution D of data: ErrorD(h)

- A hypothesis h overfits training data if there is an alternative hypothesis h’, such that
  - ErrorTrain(h) < ErrorTrain(h’), and
  - ErrorD(h) > errorD(h’)

How to avoiding overfitting

- Strategies:
  - Early stopping: e.g., stop when
    - InfoGain < threshold
    - Size of examples in a node < threshold
    - Depth of the tree > threshold
  - Post-pruning
    - Grow full tree, and then remove branches

- Which is better?
  - Unclear, both are used.
  - For some applications, post-pruning better

Post-pruning

- Split data into training and validation sets
- Do until further pruning is harmful:
  - Evaluate impact on validation set of pruning each possible node (plus those below it)
  - Greedily remove the ones that don't improve the performance on validation set

➔Produces a smaller tree with good performance

Performance measure

- Accuracy:
  - on validation data
  - K-fold cross validation

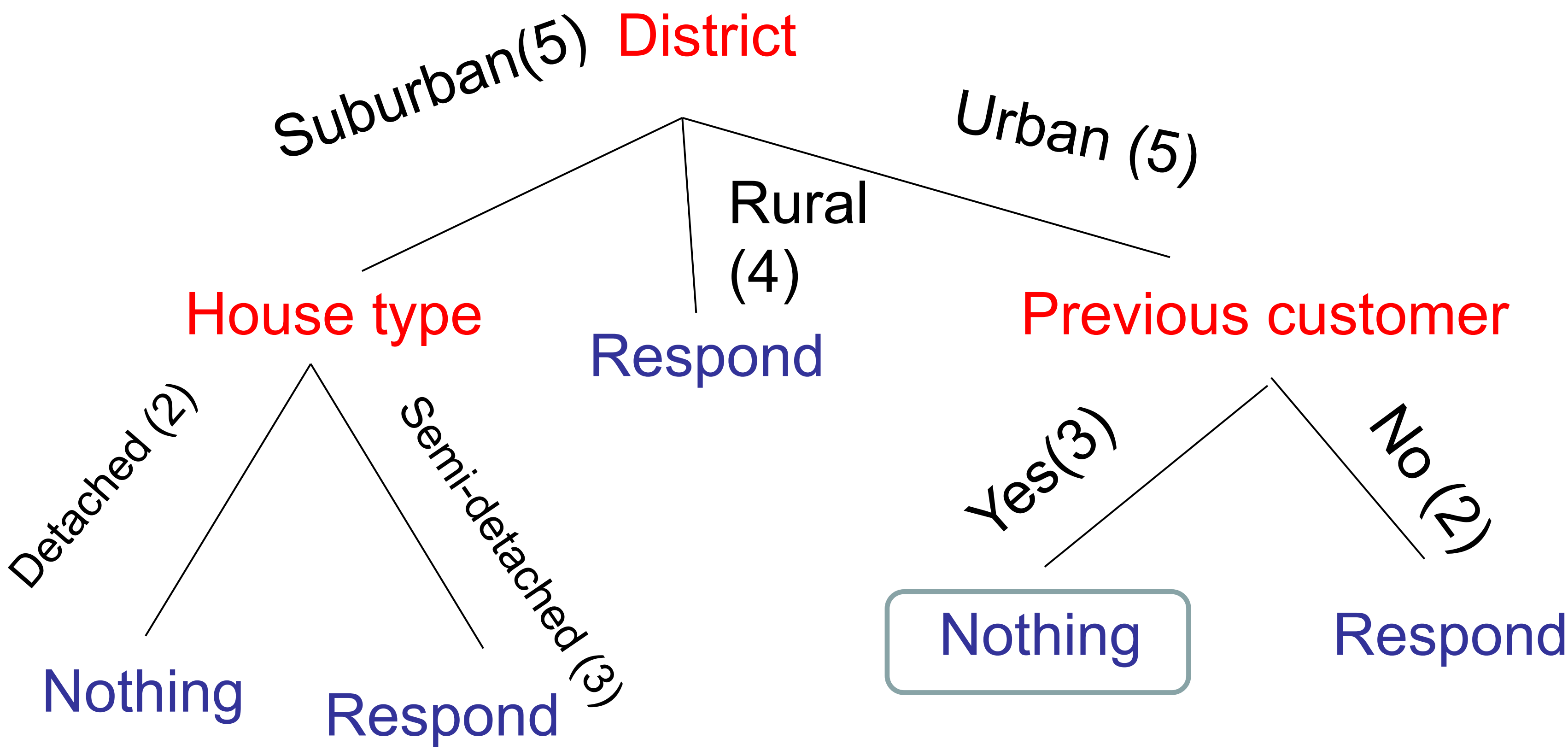
- Misclassification cost: Sometimes more accuracy is desired for some classes than others.

- Minimum description length (MDL):
  - Favor good accuracy on compact model
  - MDL = model_size(tree) + errors(tree)

Rule post-pruning

- Convert the tree to an equivalent set of rules
- Prune each rule independently of others
- Sort final rules into a desired sequence for use
- Perhaps most frequently used method (e.g., C4.5)

Decision tree ➔ rules



If District==Urban && PrevCustomer==Yes then Nothing

Q3: handling numeric features

- Different types of features need different tests:
  - Binary: Test branches on true/false
  - Discrete: Branches for each discrete value
  - Continuous feature ➔ discrete feature

- Example
  - Original attribute: Temperature = 82.5
  - New attribute: (temperature > 72.3) = true, false

➔ Question: how to choose split points?

Choosing split points for a continuous attribute

- Sort the examples according to the values of the continuous attribute.

- Identify adjacent examples that differ in their target labels and attribute values
  ➔ a set of candidate split points

- Calculate the gain for each split point and choose the one with the highest gain.

Summary of Major issues

Q1: Choosing best attribute: different quality measures.

Q2: Determining when to stop splitting: stop earlier or post-pruning

Q3: Handling continuous attributes: find the breakpoints

Other issues

Q4: Handling training data with missing feature values

Q5: Handing features with different costs

- Ex: features are medical test results

Q6: Dealing with the target being a continuous value

Q4: Unknown attribute values

Possible solutions:

- Assume an attribute can take the value “blank”.
- Assign most common value of A among training data at node n.
- Assign most common value of A among training data at node n which have the same target class.
- Assign prob p_i to each possible value v_i of A
  - Assign a fraction (p_i) of example to each descendant in tree
  - This method is used in C4.5.

Q5: Attributes with cost

- Ex: Medical diagnosis (e.g., blood test) has a cost
- Question: how to learn a consistent tree with low expected cost?
- One approach: replace info gain with
  - Tan and Schlimmer (1990)

$$\frac{Gain^2(S,A)}{Cost(A)}$$

Q6: Dealing with continuous target attribute ➔ Regression tree

- A variant of decision trees
- Estimation problem: approximate real-valued functions: e.g., the crime rate
- A leaf node is marked with a real value or a linear function: e.g., the mean of the target values of the examples at the node.
- Measure of impurity: e.g., variance, standard deviation, …

Summary

- Basic case:
  - Discrete input attributes
  - Discrete target attribute
  - No missing attribute values
  - Same cost for all tests and all kinds of misclassification.

- Extended cases:
  - Continuous attributes
  - Real-valued target attribute
  - Some examples miss some attribute values
  - Some tests are more expensive than others.

Strengths of decision tree

- Simplicity (conceptual)
- Robust to irrelevant features
- Efficiency at testing time
- Interpretability: Ability to generate understandable rules
- Ability to handle both continuous and discrete attributes.

Weaknesses of decision tree

- Efficiency at training: sorting, calculating gain, etc.
- Poor feature combination
- Theoretical validity: greedy algorithm, no global optimization
- Predication accuracy: trouble with non-rectangular regions
- Stability: not stable
- Sparse data problem: split data at each node.

Addressing the weaknesses

- Used in classifier ensemble algorithms:
  - Bagging: sample the training data m times, build a classifier for each sample, and then let the m classifiers vote on a test instance.
  - Boosting: build one classifier at a time, based on the results of the current ensemble
- Decision tree stump: one-level DT

Common algorithms

- ID3
- C4.5
- CART

More in “additional slides”

Additional slides

Common algorithms

- ID3
- C4.5
- CART

ID3

- Proposed by Quinlan (so is C4.5)
- Can handle basic cases: discrete attributes, no missing information, etc.
- Information gain as quality measure

C4.5

- An extension of ID3:
  - Several quality measures
  - Incomplete information (missing attribute values)
  - Numerical (continuous) attributes
  - Pruning of decision trees
  - Rule derivation
  - Random mood and batch mood

CART

- CART (classification and regression tree)
- Proposed by Breiman et. al. (1984)
- Constant numerical values in leaves
- Variance as measure of impurity