Outlier

Section Summary | Introduction to Statistics
Discrete Random Variables

Introduction to Random Variables + Probability Distributions

1. Mapping
    a. Mapping is a way of converting the possibilities contained in a sample space to real numbers
    b. Given a sample space, S, and a random variable, X, we say that X maps every element of S to a real number
    c. You plug the elements of S into X and X outputs a number, i.e. X(S)
2. Discrete Random Variables
    a. A discrete random variable is a variable that can only take on a finite number of values or a countably infinite sequence of values
    b. The sets X = {0.5, 3.23, 10, -1} and Y = {0, 1, 2, 3…} are both examples of discrete random variables
    c. A discrete distribution is the discrete random variable and its corresponding probabilities
    d. A random variable has a discrete distribution if that random variable is a discrete random variable according to the definition in the bullet point a.
3. Probability Mass Function (PMF)
    a. A function that maps an element of a random variable to its associated probability
    b. PMF = f(x) = Pr(X = x) for all possible values of x
4. Cumulative Probabilities
    a. We can use summations to figure out the probability of one of multiple events occurring
    b. Say X, a discrete random variable, is the set {0, 1, 2, 3}
    c. We can find the probability of X being at most 2 by summing up the probabilities of each independent element: $Pr(X \leq 2) = Pr(0) + Pr(1) + Pr(2)$
    d. This summation can be done for any accumulation of events, i.e. Pr(1 < X < 4) = Pr(2) + Pr(3)
    e. The sum of all probabilities of all possible events for a discrete random variable must be 1

f. This fact can be used to calculate a cumulative probability in another way
g. Instead of adding up all the desired probabilities, you could subtract the undesired probabilities from 1
h. $$Pr(X \leq 2) = 1 - Pr(3)$$
i. This could be useful for discrete random variables with many elements
j. If X ranges from 1 to 100 and you want to know the probability of X taking on a value less than or equal to 98, you don't have to add up all 98 probabilities. You could instead just calculate 1 - Pr(100) - Pr(99)

Expectations of Discrete Random Variables

1. Expected Value
    a. The expected value is the weighted average of all possible values where the weights are the probabilities of each value
    b. Written as E[X] or μ
    c. $$\mu = \sum_{all\,x} x \cdot Pr(x)$$
    d. Over many trials, the average of all trials will approach the expected value
2. Variance
    a. V[X] = $\sigma^2 = E[X^2] - (E[X])^2 = E[X^2] - \mu^2$
    b. $$E[X^2] = \sum_{all\,x} x^2 \cdot Pr(x)$$
3. Standard Deviation
    a. $$\sigma = \sqrt{V[X]} = \sqrt{E[X^2] - \mu^2}$$
    b. The standard deviation is on the same order as the data, making it usually more useful than the variance

Outlier

Binomial Distribution

1. Bernoulli Experiment
    a. A random experiment with only 2 possible outcomes
    b. Success or Failure
    c. The probability of success is the same for each time you try the experiment
    d. The discrete random variable, X, can only take on the values 0 or 1
2. Binomial Distribution
    a. Sum of all successes over independent Bernoulli experiments
    b. There are two parameters in this distribution
    c. n: number of trials
    d. p: probability of success
3. Probability Mass Function (PMF) of a Binomial Distribution
    a. f(x) = Pr(X = x) = $\binom{n}{x} p^x (1-p)^{n-}$
    b. This is the probability of x successes in n trials
    c. n = number of trials
    d. x = number of successes (observed value of X) = 0, 1, 2, …, n
    e. n - x = number failures
    f. p = probability of success
    g. 1 - p = probability of failure
    h. $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ = number of possible ways to get x successes in n trials
    i. $\binom{n}{x}$ is pronounced "n choose x"
    j. $\binom{n}{x} \neq \binom{n}{x}$
4. Mean, Variance, and Standard Deviation of the Binomial Distribution
    a. Mean: $\mu = np$
    b. Variance: $\sigma^2 = np(1-p)$

c. Standard Deviation: $\sigma = \sqrt{np(1-p)}$