ORIE 5380, CS 5277: Optimization Methods
Fall 2019, Project
Due December 13, 12:00 pm
Cargo Operations of Express Air

Problem Overview

Express Air operates an aircraft fleet to run a cargo business among three airports, airports A, B and C. Each day, a certain amount of cargo that needs to be delivered between each origin-destination airport arrives into the system. The cargo operator for Express Air considers the amount of cargo that was not delivered the day before for each origin-destination pair and the amount of cargo that arrives on the current day and decides what portion of the cargo should be delivered. Naturally, the amount of cargo that can be delivered between an origin-destination pair depends on the number of aircraft at the origin. In addition to carrying cargo, the cargo operator has the option of repositioning aircraft empty between the airports. Repositioning aircraft becomes useful when there is imbalance between the amounts of inbound and outbound cargo from a particular location. The important point is the dynamic nature of the cargo operations. In particular, assuming that the travel time between each origin-destination pair is a full day, if the carrier carries a certain amount of cargo from airport A to B during day t, then the aircraft that is used for this purpose during day t becomes available at airport B at the beginning of day t+1.

Data

The airports in the system are far enough that it is reasonable to assume that the time to travel between each origin-destination airport is a full day. For example, the aircraft that move from airport A to B on a Monday morning are available at airport B on the following Tuesday morning. Express Air has 1200 aircraft at its disposal. After talking to the cargo operator, you realize that you need to carefully keep track of the dynamics of the aircraft between the airports. For example, on a Monday morning, we may have 300 aircraft available at airport A. We can use 100 of them to move cargo from airport A to B and 70 of them to move cargo from airport A to C. For the remaining 130 aircraft, we can reposition 80 aircraft empty from airport A to B and let 50 aircraft remain at location A. As a result of these decisions, the aircraft that move from airport A to B full and empty becomes available at airport B the following morning. Thus, these decisions bring a total of 180 aircraft into airport B.

There is a consistent weekly pattern for the cargo that needs to be delivered. Table 1 shows the amount of cargo arriving into the system on each day of the week that needs to be delivered between each origin-destination airport. All of the quantities in the table are in full aircraft loads. Each aircraft can carry one aircraft load – no more, no less. For example, to carry 25 aircraft loads of cargo, you need 25 aircraft. Going over the figures in Table 1, on each Tuesday morning, 25 aircraft loads of cargo that needs to be delivered from airport B to C arrives into the system. If there was a certain amount of cargo that needed to be delivered on Monday from airport B to C but could not be delivered due to insufficient cargo carrying capacity at airport B on Monday, then these 25 aircraft loads add onto that amount. Given that the cargo that arrives into the system has a consistent weekly cycle, the cargo operator for Express Air asked you to find a repeatable weekly movement cycle for the aircraft. In particular, the number of aircraft that moves into an airport by Friday evening should be what is available at the same airport Monday morning. In this way, you can obtain a movement cycle for the aircraft that can be repeated every week.

Origin-destination \ Day	Monday	Tuesday	Wednesday	Thursday	Friday
A-B	100	200	100	400	300
A-C	50	50	50	50	50
B-A	25	25	25	25	25
B-C	25	25	25	25	25
C-A	40	40	40	40	40
C-B	400	200	300	200	400

Table 1: Amounts of cargo (in aircraft loads) arriving into the system on each day that need to be carried between each origin-destination airport

Cost Components

All of the cargo given in Table 1 must be carried from its origin airport to its destination, though the cargo that arrives into the system on a particular day does not have to be carried on the same day. Express Air has the option of letting the cargo sit on the ground a few days, if this option is beneficial from a cost perspective. Since all of the cargo in Table 1 must be delivered, the total amount of loaded movements that the aircraft have to do is fixed, which implies that we can ignore the revenue from the delivered cargo and the cost of loaded movements – both of these quantities are fixed and cannot be improved with optimization.

Figure 1 gives the cost of empty repositioning one aircraft between each origin-destination airport. So, one critical cost component to consider is the cost of empty repositioning movements. Also, there has to be aircraft at the origin airport to be able to carry cargo between a certain origin-destination airport and the amount of cargo that can be carried naturally depends on the number of available aircraft at the origin airport. If there is not enough aircraft at the origin airport, then cargo can be held on the ground for a few days. The cost of holding one full aircraft load of cargo on the ground is 10 per day. Thus, another cost component to consider is the cost of cargo that is held on the ground. After some deliberation, you decided that minimizing the total empty repositioning cost and cargo holding cost is a reasonable objective.

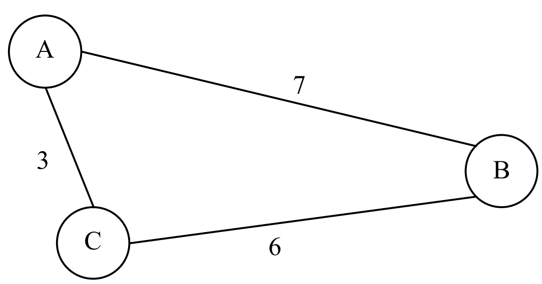

Figure 1: Empty repositioning costs among different airports

Objectives and Deliverables

The goal is to develop an optimization model that finds a weekly aircraft movement cycle with minimum total cost, while ensuring that all cargo is delivered from its origin to its destination. In addition to this goal, you are expected to investigate the managerial implications of your model. For example, possible questions that could be answered with your model are the benefits of increasing the fleet size or the effects of having more cargo to carry over different origin-destination pairs. You should also explore answers to other similar questions that can be answered by your model. You will present your findings in a professional report. You should organize your report as you see fit, but consider having an executive summary, problem overview, data description, an overview of your optimization model, mathematical details of your optimization model, convincing argument that your proposed schedule is sensible, the analysis of your results and the findings. Your report should not exceed 8 pages in 11-point font and single-spaced pages. You can include extra details in the appendix, which is not included in the page limit. Make sure that your writing is high-quality. The quality of your writing, as much as the correctness of your model, will count. Follow professional writing norms. Number each section, figure and table. Keep a healthy balance between providing the intuition and the mathematical details for your model. You should solve your optimization model by calling Gurobi from a programming language of your choice. In particular, you should not be using Gurobi as a standalone linear programming solver. You can submit your project report just like submitting a homework assignment. Your report should be one pdf document formatted to print, but make sure to include your code in your submission.

Useful Ideas

It is important to keep track of the flow balance of the aircraft. Specifically, if a certain number of aircraft is used between a particular origin-destination airport during day t, then these aircraft become available at the destination airport at the beginning of day t+1 – irrespective of whether the aircraft move between the origin-destination airport empty or loaded.

Since Express Air needs a weekly cycle to manage its fleet, the total number of aircraft that flies into a particular airport or stays at the airport at the end of a week should be equal to the total number of aircraft that leaves this airport or stays at the airport at the beginning of the next week.

You will also need flow balance constraints for the cargo that needs to move between different origin-destination airports. For example, the amount of cargo that needs to be delivered from airport A to B on a Tuesday morning is equal to the amount of cargo that needed to be delivered from airport A and B on the Monday morning before, plus the amount of cargo that arrived into the system on Monday that needed to be delivered from airport A and B, minus the amount of cargo that is actually delivered from airport A to B on Monday.

So, you will need flow balance constraints to keep the flow balance for aircraft between the airports, as well as the amount of cargo that needs to be delivered between each origin-destination airport.