Getting Serious about Sirius

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1 Introduction

Sirius, meaning "glowing" in greek, is a binary star system composed of a main sequence star (Sirius A) and a white dwarf (Sirius B). Sirius A, being the more massive and luminous of the two stars has some of the oldest observational recordings known to man. The Egyptian society centered its calendar upon the first appearance of Sirius A, while the Greeks feared it and thought it meant harsh summers. It wasn't until recently in history that Sirius B was even discovered. In 1844, Friedrich Bessel concluded from the orbital motion of Sirius A that there may have been a second star. By 1862 the existence of Sirius B was finally confirmed by Alvan Graham Clark when he observed it directly through a refracting telescope.

Although Sirius A is larger and more visible to the naked eye, Sirius B is the younger companion; Wiser from having advanced faster through its life cycle, and at one time far larger than Sirius A. It is the goal of this project to better understand Sirius B, the pup who in comparison to its younger and brighter partner has had little attention over the years.

2 Calculations

2.1 Mass

To better understand Sirius A and B we begin by first determining the mass of both stars. We relate their total mass to the ratio of their average orbital period cubed, ${\overline{R}_{orbit}}^3$, and the period squared in which it takes for the stars to complete one full orbit, ${T_{orbit}}^2$. Where $\overline{R}_{orbit} = 19.83AU$ and $T_{orbit} = 50.1yrs$

$$M_{tot} = \frac{M_A + M_B}{M_\odot} = \frac{{\overline{R}_{orbit}}^3}{{T_{orbit}}^2} = 3.107 \tag{1}$$

The determination of the center of mass will help us to relate the distance each star is from the center to their individual masses.

$$x_{cm} = \frac{M_A x_A + M_B x_B}{M_A + M_B} = M_B \cdot 6.383 \frac{AU}{M_\odot} \tag{2}$$

Equation 2 expresses that the center of mass may be found through the addition of each mass times its distance from the origin of our coordinate system, divided by total mass. The origin may be placed anywhere along the the coordinate system to simplify we have placed it at the center of Sirius A. By making $M_A x_A$ equal to zero, this simplification allows us to estimate simply with just M_B.

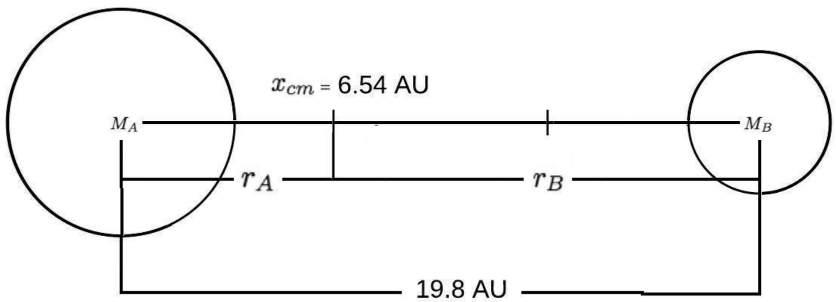

Figure 1: Figure showing important distances and their relation to the center of mass.

With a little deduction we can see from Figure 1 that $x_{cm} \approx 6.4AU$ is about $\frac{1}{3}$ away from Sirius A and $\frac{2}{3}$ away from Sirius B. Taking this knowledge, we use the ratio of their masses multiplied by their fractional distance away from x_{cm}.

$$M_A r_A = M_B r_B \tag{3}$$

$$\frac{M_A}{M_B} = \frac{r_B}{r_A} \tag{4}$$

$$M_A = 2M_B \tag{5}$$

We find that M_A is equal to $2M_B$. Returning back to their total masses in Equation 1, we are able to replace M_A with $2M_B$ and solve for the mass of Sirius B. Further algebra then returns the mass of Sirius A. We find that Sirius A has a mass of $2.071M_\odot$ and that Sirius B has a mass of $1.036M_\odot$.

2.2 Luminosity

Next we determine the luminosity of each star by looking at the ratio of their flux to the Sun's ($M_{Absolute\odot} = -26.5$). This ratio can then be used in Equation 7.

$$\frac{F_A}{F_\odot} = 10^{(M_\odot - \frac{M_A}{2.5})} \tag{6}$$

$$\frac{L_A}{L_\odot} = \frac{4\pi d_A^2 F_A}{4\pi d_\odot^2 F_\odot} \tag{7}$$

Where d represents the distance from earth (Values for d, along with absolute magnitudes, may be found in the Appendix). We find that the Luminosity of Sirius A is $28.825L_\odot$ and that Sirius B is equal to $.035L_\odot$.

2.3 Radius

To determine the radius of Sirius A, we use the mass-radius relation for stars of $M > 2M_\odot$.

$$R_A = M^{1/2} \tag{8}$$

To determine the radius of Sirius B we have to look at the star from a relativistic perspective. Due to degeneracy pressure, the typical mass-radius relationship for a white dwarf is $M = R^{\frac{1}{3}}$. Because Sirius B is closer to the Chandrasekhar mass limit of $1.4M_\odot$, we needed to make a better approximation that included the limiting mass [1].

$$R_B = 0.0126R_\odot(\frac{2}{\mu_e})^{\frac{5}{3}}(\frac{M}{M_\odot})^{-\frac{1}{3}}[1-(\frac{M}{M_{ch}})^{\frac{4}{3}}]^{\frac{1}{2}} \tag{9}$$

$$M_{ch} = 1.456M_\odot(\frac{2}{\mu_e})^2 \tag{10}$$

Where $\mu_e = 2$.

We found that the radius of Sirius A is $1.35R_\odot$ and the radius of Sirius B to be $.00753R_\odot$.

2.4 Temperature

Finally, armed with all the necessary values we were able to evaluate the temperature of both Sirius A and B with Equation 11. Sirius A was found to be $11,512.553K$ and the temperature of Sirius B equal to $28,775.025K$.

$$T = \sqrt[4]{\frac{L}{4\pi R^2\sigma}} \tag{11}$$

2.5 Bringing it all Together

To determine the nature of these stars it is also helpful to determine density. We can see that while the density of Sirius A is $7.47\frac{g}{cm}$, Sirius B is $2.156 \cdot 10^7\frac{g}{cm}$. Such a high density for Sirius B indicates that it is most likely a White Dwarf.

Originally, Sirius B was a B-type main sequence star with a mass estimated to be around $5M_\odot$. Sirius B is a white dwarf containing a core of carbon-oxygen that is wrapped in an outer most shell of hydrogen. As it aged through the red giant phase it is very likely that Sirius B is the cause for the metal enrichment of Sirius A.

Because Sirius B was originally very massive, it aged through the main sequence much quicker than its parter. Sirius B is about 228 Myrs, where as Sirius A is around 237.

2.6 Mass loss

We begin to find the mass loss by relating the luminosity of a star to its gravitational potential energy. solving for $\dot{M}$ we retrieve Equation 13.

$$L = \frac{1}{2}\dot{M}v_{esc}^2 = \frac{GM\dot{M}}{R} \tag{12}$$

$$\dot{M} = \frac{2L}{v_{esc}^2} = \frac{LR}{GM} \tag{13}$$

Plugging into Equation 13, the values for the rate of mass loss are found to be $3.779 \cdot 10^{19}\frac{g}{s} = 1.19 \cdot 10^{27}\frac{g}{yr}$ and $5.11 \cdot 10^{14}\frac{g}{s} = 1.61 \cdot 10^{22}\frac{g}{yr}$ from Sirius A and B respectively. With the values for mass loss determined, one may then solve for the lifetime of each star via Equation 14.

$$t_{loss} = \frac{M}{\dot{M}} \tag{14}$$

We found the lifetime of Sirius A equal to $1.63 \cdot 10^{14}s = 5.68 \cdot 10^6 yrs$ and the lifetime of Sirius B being $6.05 \cdot 10^{18}s = 1.91 \cdot 10^{11} yrs$.

Finally, we can solve for the amount of time it will take for the white dwarf to reach its maximum mass if we consider rate of accretion onto Sirius B equal to that of sirius A's rate of mass loss.

$$t = \frac{GM}{LR}(m_{CH}(t) - m_0) \tag{15}$$

Using Equation 15, we find that it will take $1.91 \cdot 10^{13}s = 6.05 \cdot 10^5 yrs$ for Sirius B to reach the Chandrasekhar mass limit of $1.4M_\odot$.

3 Conclusion

3.1 Mass

The calculated masses of 2.071 $M_\odot$ and 1.036 $M_\odot$ for Sirius A and B, were both within the acceptable range of deviation from standard values found in Table 2 of the Appendix.

3.2 Luminosity

Similarly to mass, the values in which we found for Sirius A and B were fairly close to the accepted values with Sirius A being on the larger than acceptable side. We found Sirius A to have a luminosity of 28.825 $L_\odot$ and Sirius B to be equal to 0.035 $L_\odot$.

3.3 Radius

Our values for radius were very close to the standard accepted values, especially that of Sirius B. We found that Sirius B was .0075 $R_\odot$, only 0.0003 $R_\odot$ away from the accepted value, but well within the allowed deviation of $\pm.03R_\odot$. It was also shown that Sirius A had a radius of 1.35 $R_\odot$, again close to the value found in Table 2 of the Appendix.

3.4 Temperature

The values in which we derived for Sirius A and B were a bit on the high side, more than the standard deviation would allow. Sirius A was found to have a temperature of 11,512.58, this value is a single magnitude off from the standard value. Sirius B's temperature is within the expected order of magnitude but not within the allowed deviation. We found Sirius B to have a temperature of 28,775.03K.

3.5 Mass Loss

We found from Equation 13 that Sirius A would accrete $1.19 \cdot 10^{27} \frac{g}{yr}$ onto its companion Sirius B. It was then found from Equation 15 that at this rate, Sirius B would have a life expectancy of $6.05 \cdot 10^5 yrs$ before reaching the Chandrasekhar mass limit of $1.4M_\odot$. This is a reasonable amount of time and well within the expected lifetime of $5.68 \cdot 10^6 yrs$ and $1.91 \cdot 10^{11} yrs$ for both Sirius A and B respectively.

Once Sirius B does reach the Chandrasekhar mass limit, which we expect it too via our calculations, it will end its life as a type 1A super nova.

4 Appendix

	Mass	Luminosity	Radius	Temperature (K)	Density ($\frac{g}{cm^3}$)	Age (yrs)	Magnitude$_{abs}$
Sirius A	2.071 $M_\odot$	28.825 $L_\odot$	1.35 $R_\odot$	11,512.58	7.74	2.36·10^8	-1.47
Sirius B	1.036 $M_\odot$	.035 $L_\odot$	.0075 $R_\odot$	28,775.03	2.16E+07	2.27·10^8	8.44

Table 1: All important values found and/or used for this paper

$d_a = 544,568AU$ and $d_\odot = 1AU$

	Mass	Luminosity	Radius	Temperature (K)
Sirius A	2.063 $\pm .023 M_\odot$	25.4 $L_\odot$	1.711 $R_\odot$	9,940
Sirius B	1.018 $\pm .011 M_\odot$	0.056 $L_\odot$	.00783 $\pm .03 R_\odot$	25,000 $\pm$200

Table 2: All standard accepted values in which our data was compared [2].

References

[1] Burrows, A. (2018). White Dwarfs (Degenerate Dwarfs). [ebook] New Jersey: Princeton University. Available at: `https://www.astro.princeton.edu/~burrows/classes/403/white.dwarfs.pdf` [Accessed 10 Dec. 2018].

[2] Wikipedia contributors. (2018). Sirius. Wikipedia, The Free Encyclopedia. Available at: `https://en.wikipedia.org/wiki/Sirius` [Accessed 10 Dec. 2018]