Low Frequency Impedance

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Abstract

In the following paper we discuss how the measurement of low frequency impedance helps to determine the circuitry elements held within three separate mystery boxes: Box F, E and B. By creating plots of the angular frequency versus impedance, we were able to determine that Box F contained an RC circuit in parallel. The elements in this circuit were found to have the values of R = 2247.73 Ω and C = 10.13 nF. Similarly, the graph of impedance versus angular frequency for Box E led us to conclude that it was made up of an RLC circuit in series; for which the values of each element were calculated to be R = 310 Ω, L = 8.948 mH and C = 1.39 nF. Finally, we found that Box B consisted of a single inductor with a calculated value of L = $6.830 \cdot 10^{-3}$ mH.

Introduction

We define electrical impedance to be the hindrance of flowing current through a circuitry element. Impedance is represented as a complex number and has units of Ohms (Ω). The word impedance was unheard of until 1886, when Oliver Heaviside coined the term; it wasn't until 1893 that Arthur Kennelly properly represented impedance with complex numbers [1]. For a double terminal network we can define impedance in he following way:

$$Z = R + iX \tag{1}$$

In Equation 1, R represents resistance as the real part of impedance; while X, the reactance, represent the imaginary part. The reactance serves as an important part of impedance that relates to a specific equation for both capacitance and inductance.

Impedance of an AC circuit has both magnitude and phase, while a DC circuits impedance has no phase (or more appropriately, a phase angle equal to zero). When the voltage output is sinusoidal, we relate the magnitude of impedance to be the rate of voltage amplitude versus current amplitude. Meanwhile, the phase of impedance can be described by the difference in phase between the current and voltage. For this paper, both DC and AC current are extremely important in the analysis of our data.

For our experiment we hypothesized that it would be possible to use impedance as a way to determine what kind of circuit elements were contained inside of a mystery box. These elements could have been a capacitor, resistor or inductor either in series or parallel with other elements. We also had the possibility of finding a circuit with a single element. To determine what a circuit contained, we first looked to measure DC resistance of the mystery box standing alone; this helped us make some initial conclusions about what may, or may not, have been inside of our mystery box. Next, using AC current, we took measurements of phase, frequency, resistance of our substitution box, and the voltage across both the resistor box and the mystery box. With all of these values determined, we created plots of the angular frequency versus impedance. These plots are what helped us to draw our final conclusion as to what circuitry elements were contained inside of each box.

Procedure

To begin this experiment, we started by choosing the three boxes that we wanted to take measurements of. Each of our three boxes contained varying linear circuit elements; with our measurements we hoped to determine exactly what these elements were.

Before setting on the circuit seen in Figure 1, we used a multimeter to measure the resistance of each box. By doing this we were able to make some early conclusions about what each box may or may not contain.

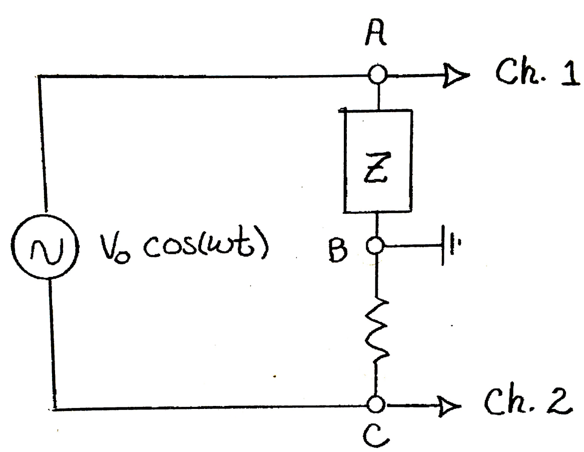

Figure 1: Diagram representing how a single mystery box was connected to our other instruments [2].

Next, we constructed our circuit as seen in Figure 1. V_o represents our function generator, which was wired in series with our mystery box (labeled Z in Figure 1) as well as the resistance substitution box. Our mystery box was then wired to the resistance box as well as channel one of our oscilloscope. Finally, the resistance box was connected to channel two of the oscilloscope.

With everything properly connected, we were then eligible to begin taking measurements. The function generator enabled us to set and determined the frequency (f) for each data point measured. Using our oscilloscope, we were able to visualize the electrical sinusoidal waves that were being emitted from the resistance substitution box as well as the mystery box. These visualizations helped us to adjust the resistance substitution box so that the amplitude of its sinusoidal wave matched that of the mystery box. With this adjustment done, we were then able to determine the phase difference as well as voltage of both boxes (V_1 being the voltage across the mystery box and V_2 being that across the resistor). And finally, we used our multimeter to directly measure the resistance of our substitution box, which gave us our value for R_T.

Using Equation 2, we were able to calculate the current; this determination then enabled us to use Equation 3 to find the impedance of each data point.

$$I = \frac{V_2}{R_T} \tag{2}$$

$$Z = \frac{V_!}{I} \tag{3}$$

When appropriate, we were also able to solve for the admittance via Equation 4.

$$Y = \frac{1}{Z} \tag{4}$$

Results

Box F

Before taking any AC measurements, we used our multimeter to measure the DC voltage of the mystery box; we found the value to be R = 2338 Ω. This allowed us to make the assumption that an inductor in parallel or a capacitor in series would be highly improbable. See Table 1 in Appendix.

After gathering all the data, we were able to calculate the angular frequency via Equation 5 and plot that versus the inductance; as seen in Figure 2.

$$\omega = 2\pi f \tag{5}$$

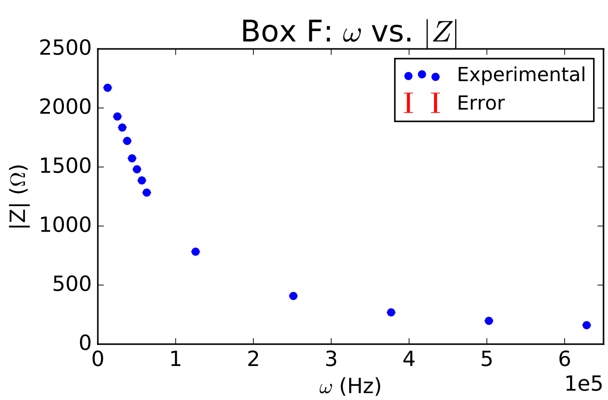

Figure 2: Plot depicting the curve of impedance versus angular frequency for Box F. Curve proves similar to that of RC circuitry in parallel.

The plot in Figure 2 is reminiscent of an RC circuit wired in Parallel. We then went on to determine the admittance via Equation 4 and then plot the square of those values versus the square of the angular frequency; seen in Figure 3.

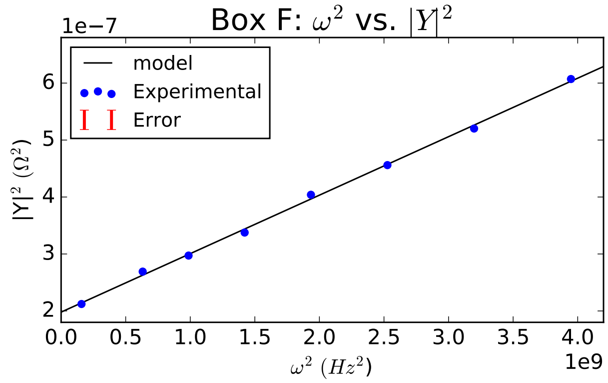

Figure 3: Plot of admittance versus Angular frequency and the line of best fit used to determine element values.

The line of best fit in Figure 3 helped us to determine both the capacitance and resistance via its slope and intercept. This can be seen from Equation 6, where capacitance squared is the slope and resistance squared is the intercept.

$$|Y|^2 = (\omega C)^2 + \frac{1}{R^2} \tag{6}$$

Armed with these values we were able to determine the capacitance to be C = 10.13 nF and the resistance equal to R = 2247.73 Ω. For one last check, we created a plot of frequency versus phase. To determine the phase we used equation 7:

$$\theta = \arctan\left(-\omega RC\right) \tag{7}$$

At low frequencies the capacitors impedance is so high that current is far more unlikely to flow through it; thus the resistor is in charge and the phase angle goes to zero. At higher frequencies the capacitors impedance becomes much smaller, now creating a flow of current. At these high frequencies the capacitor takes over while the phase angle goes to -90^o, as seen below in Figure 4.

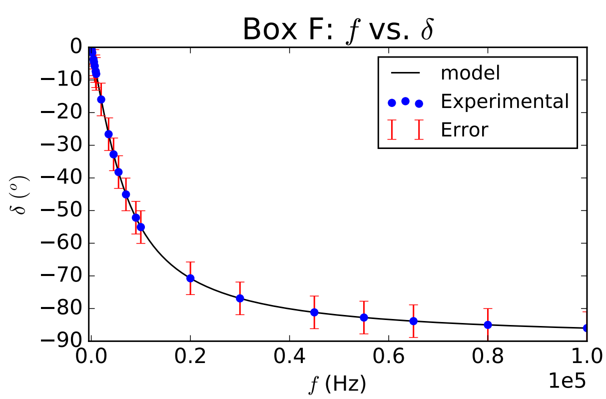

Figure 4: Frequency versus phase diagram of Box F with errors being estimated via our visual measurements on the oscilloscope.

Box E

When using the multimeter to determine the boxes resistance, we found that resistance was far to high to be measurable by the multimeter. This suggested to us that the box had a possibility of containing a capacitor either alone or in series with other elements. See Table 2 in Appendix.

When we produced the plot of frequency versus impedance, we found our graph resembled that of an RLC circuit in series: as shown in Figure 5.

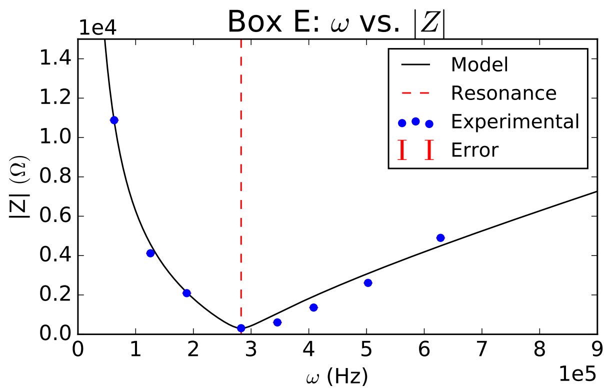

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Figure 5: Plot of the angular frequency versus impedance for Box E. Plot depicting values for a frequency of 7000Hz+ due to large impedance of capacitor. This large impedance of the capacitor causes current to flow through the oscilloscope rather than through the capacitor itself, making data prior to frequencies above 7000 Hz not as accurate for this specific plot.

To be able to further determine the values of the elements within Box E, we created a graph of the angular frequency versus admittance, as seen in Figure 6. The peak of this graph represents where the impedance is at a minimum and admittance is at a maximum. At this point we also find the resistance to be R = $Z_{min} = Y_{max}^{-1} = 310\ \Omega$.

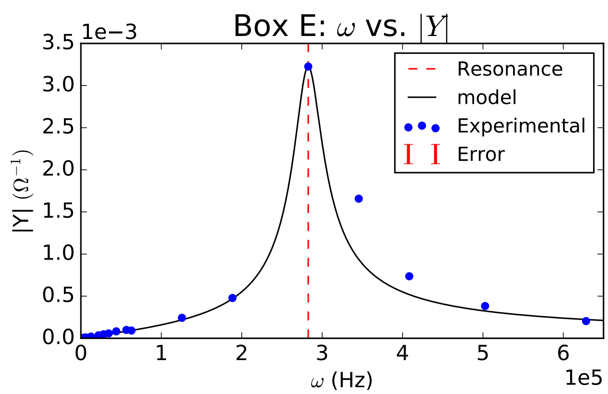

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Figure 6: Angular frequency versus admittance for box E, also showing where resonance frequency lies.

At the peak admittance, we determine the value for the resonant frequency via equation 8, which we found to be $\omega = 282.74 \cdot 10^3$ Hz.

$$\omega_o = \frac{1}{\sqrt{LC}} \tag{8}$$

Following the procedure given in Brown [2], we were able to solve for the inductance and capacitance using Equations 9 through 11.

$$|Z|^2 = R^2 + \left(wL - \frac{1}{wC}\right)^2 \tag{9}$$

$$|Z| = \alpha R \tag{10}$$

$$\omega_b - \omega_a = \frac{R}{L}\sqrt{\alpha^2 - 1} \tag{11}$$

Solving the system of equations, we found inductance and capacitance in the form of Equation 12 and 13:

$$L = \frac{\sqrt{|Z|^2 - R^2}}{\omega_b - \omega_a} \tag{12}$$

$$C = \frac{1}{L{w_o}^2} \tag{13}$$

With this we were able to determine the values for inductance and capacitance to be 8.948 mH and 1.39 nF respectively. Lastly, using Equation 14, we created a plot of frequency versus phase.

$$\theta = \arctan\frac{\omega L - \frac{1}{\omega C}}{R} \tag{14}$$

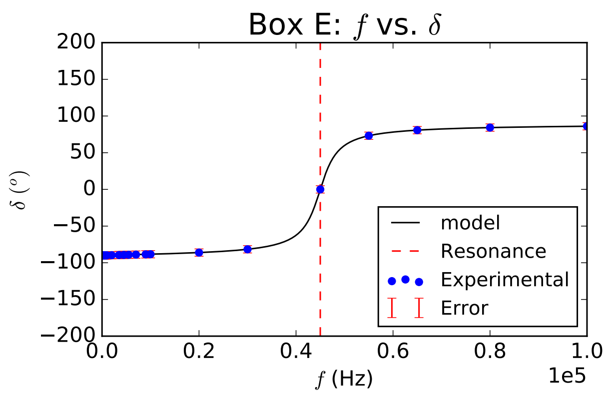

Figure 7: Frequency versus phase diagram of Box E with errors being estimated via our visual measurements on the oscilloscope.

Figure 7 emanates well with out prior graphs. As expected, we can see that the resonance occurs when phase is equal to 0^o; which is also when the capacitance and inductance are equal. The equivalency of both L and C sets the numerator of Equation 14 equal to zero, making the phase angle zero as well.

Box B

For Box E we measured the DC resistance to be 53.8 Ω; small enough that an inductor in parallel or alone is still plausible, but that a capacitor in series or alone is not. See Table 3 in Appendix.We then produced a plot of the angular frequency versus impedance via Equations 3 and 5.

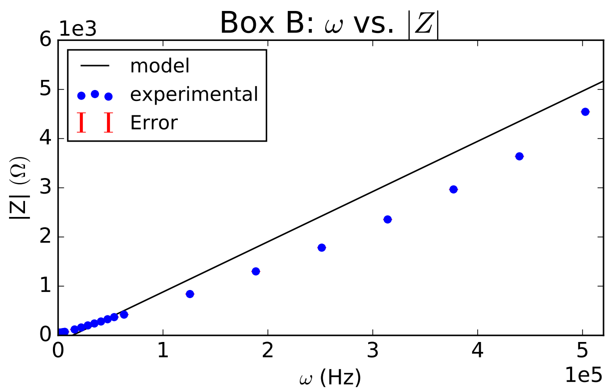

Figure 8: Pot of angular frequency versus admittance for Box B.

The Plot in Figure 8 is reminiscent of that for an inductor standing alone. To determine the inductance of our inductor we plot the values of ω^2 versus $|Z|^2$

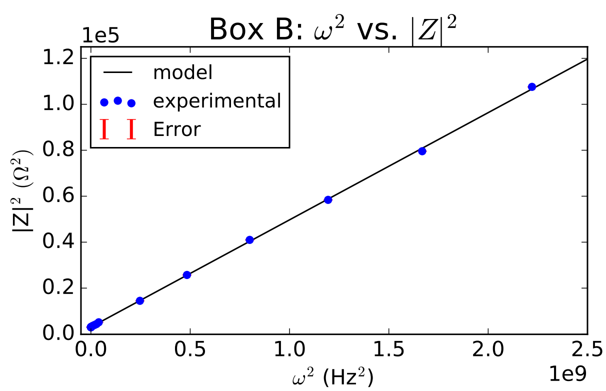

Figure 9: Pot of angular frequency squared versus admittance squared for Box B. This plot was used to obtain a better fit. Also noted, that due to reasons unknown, the inductor in Box B starts to follow a nonlinear trend when measuring towards higher frequencies. Due to the circumstances, only frequencies bellow 10000 were plot in this figure.

With this knowledge, we could then determine the value for the inductance by relating it to the slope of our best fit line via the Equations 15 and 16. The intercept of this line is then the intrinsic resistance of the inductor.

$$Z = i\omega L \tag{15}$$

$$|Z|^2 = \omega^2 L^2 \tag{16}$$

We were then able to determine the inductance to be $L = 6.830 \cdot 10^{-3}$mH and the intrinsic resistance of the inductor to be $L_R = 55.45\Omega$ for Box B. Lastly, for further visual proof, a plot of frequency versus phase was created via Equation 17.

$$\theta = \arctan(\omega L) \tag{17}$$

Figure 10 makes visual sense due to the large plateau that we see after a high enough frequency has been reached. At this frequency the phase angle is equal to 90^o, as it should be for an inductor standing alone. The vertical line can be explained by the fact that at lower frequencies the impedance and intrinsic resistance of our inductor were high enough that current preferred to not flow through it.

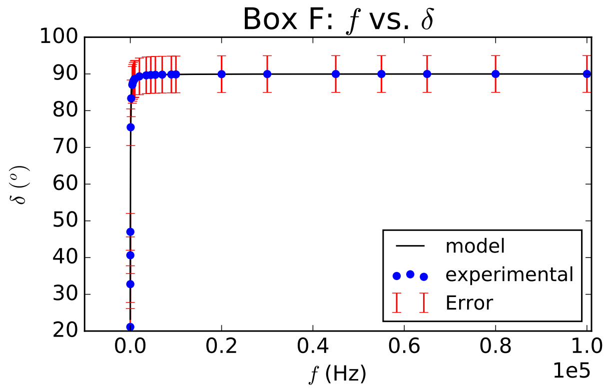

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Figure 10: Frequency versus phase diagram of Box B with errors being estimated via our visual measurements on the oscilloscope.

Error Analysis

In the previous graphs of angular frequency versus either impedance or admittance, the errors for the latter two quantities were solved for in the following way:

$$Z_{err} = \frac{V_{1err} R_{err}}{V_{2err}} = \frac{1}{Y_{err}} \tag{18}$$

Where the values of V_{1err} , R_{err} and V_{2err} were estimated visually via the readings on our oscilloscope and multimeter. It was found that we had a general error of $Z = \pm\ .5\ \Omega$ and $Y = \pm\ 2\ \Omega^{-1}$.

Conclusion

In conclusion, we determined Box F to contain a capacitor and resistor in parallel; Box E to contain a resistor, capacitor and inductor in series; and Box B to contain a single inductor.

The resistor in Box F had a value of R = 2247.73 Ω, while he capacitor had a value of C = 10.13 nF. The line of best fit that we used to determine this value agreed extremely well with our experimental values, only proving further that Box F contained an RC circuit in parallel. The resistor in Box E had a value of R = 310 Ω, the capacitor had a value of C = 1.39 nF and the inductor with L = 8.948 mH. Finally, the single inductor in Box B had a value of $L = 6.830 \cdot 10^{-3}$ mH; while both Box's fit great with their model.

References

[1] Wikipedia Contributors. (2018). *Electrical Impedance.* [online] https://en.wikipedia.org/wiki/Electrical_impedance [Accessed 29 Aug. 2018].

[2] Brown, G. (2018). *Physics 133 Lab Manual.* Santa Cruz: University of California, Santa Cruz, pp.1-12.

Appendix

Table 1: Measured values for Box F

$f\ (Hz)$	$V_1\ (mV)$	$V_2\ (mV)$	$\delta\ (^\circ)$	$R_t\ (\Omega)$
20	204	212	0	2400
40	200	212	0	2400
60	200	205	0.5	2339
80	200	212	0	2400
100	200	210	0	2400
200	200	214	0	2400
400	204	210	0	2348
600	202	212	1	2300
800	206	210	5	2288
1000	204	214	8	2300
2000	204	218	14	2320
4000	214	222	30	2000
5000	214	224	35	1920
6000	218	228	40	1800
7000	228	232	43	1601
8000	232	235	48	1500
9000	228	235	50	1429
10000	230	238	53	1328
20000	265	260	68	768
40000	272	280	75	420
60000	284	284	80	269
80000	280	284	82	200
100000	284	284	83	160.5

Table 2: Measured values for Box E

f (Hz)	V_1 (mV)	V_2 (mV)	δ $(°)$	R_t (Ω)
9	7.4	6	17	3700000
15	6.8	6.2	12.2	7010000
20	5.88	6.68	31	10010000
25	6.02	5.68	22	11010000
90	5.12	6.4	33	11110000
200	5.76	6.4	56	1000000
450	6.72	6.8	68	240000
550	6.64	6.64	72.6	190000
700	6.8	6.8	75	140000
900	6.72	6.96	74	110000
1000	6.88	6.96	75.6	100000
2000	7.04	7.04	79.1	50000
3500	7.04	7.28	81	29000
4500	7.2	7.2	80	21000
5500	7.28	7.2	80.9	17000
7000	7.28	7.2	81.7	12000
9000	7.28	7.2	81.8	10000
10000	7.2	7.28	78	11000
20000	7.2	7.52	80.9	4300
30000	7.36	7.36	82.3	2090
45000	6.72	6.72	69.6	310
55000	7.44	7.52	82	610
65000	7.92	8.16	90.9	1400
80000	8.83	8.96	103	2650
100000	11	11	100.19	4900

Table 3: Measured values for Box B

f (Hz)	V_1 (mV)	V_2 (mV)	δ $(°)$	R_t (Ω)
10	5.44	5.36	0	54
30	5.44	5.44	0.825	54.3
50	5.52	5.36	0.725	54.2
80	5.44	5.36	2.07	54.2
100	5.44	5.44	3.6	55.2
300	5.36	5.44	14	58.2
500	5.44	5.44	23.3	60.2
700	5.52	5.6	30.8	64.2
850	5.6	5.68	34.4	67.2
1000	5.68	5.76	38.9	72.7
2500	6.56	6.4	61	117.4
3500	6.8	6.8	67	160.4
4500	7	6.88	72	199
5500	7.2	7.12	76	239
6500	7.28	7.2	76	279
7500	7.28	7.28	78	328
8500	7.28	7.44	79	379
10000	7.36	7.68	79	440
20000	7.84	7.84	84	840
30000	7.92	8.16	88	1340
40000	8.24	8.32	90	1800
50000	8.48	8.64	95	2401
60000	8.96	9.06	100	3000
70000	9.6	9.76	106	3700
80000	10.3	10.2	113	4500
90000	11.7	112	122	5900
100000	14	14.2	132	8030