Radiation

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Abstract

For our main experiment we explored the interactions of gamma rays produce by Cesium-137(Cs^{137}) with several plates of lead. The plates of lead varied in thickness and were placed one at a time over our source, Cs^{137}. Over time, we placed a total of seven pieces of lead over the radioactive material. In doing this we gain a better understanding of how far the emitted gamma rays may penetrate through larger densities of matter. Prior to this aforementioned experiment, nine other preliminary experiments were executed with a similar hope of understanding the interaction of gamma rays with matter.

Introduction

Radiation and its applications in our daily lives is as numerous as it is substantial. From watching T.V. to sitting on the beach, every day we inevitably use and/or absorb many bits of radiation. It is all around us, both directly and indirectly helping people live their daily lives.

Radioactive decay is the process in which unstable nuclei emit either an alpha particle, beta particle or gamma ray in order to form a more stable nuclei. The process in which a nuclei decays is as spontaneous as it is uncontrolled. These uncontrolled and decaying nuclei are called the parent nuclei; while the daughter nuclei refers to the new element formed by the decay of the parent. We say that when an atom decays it becomes a new element because when it emits either a particle or gamma ray, the parent nuclei changes in mass and charge, losing protons and/or neutrons [1].

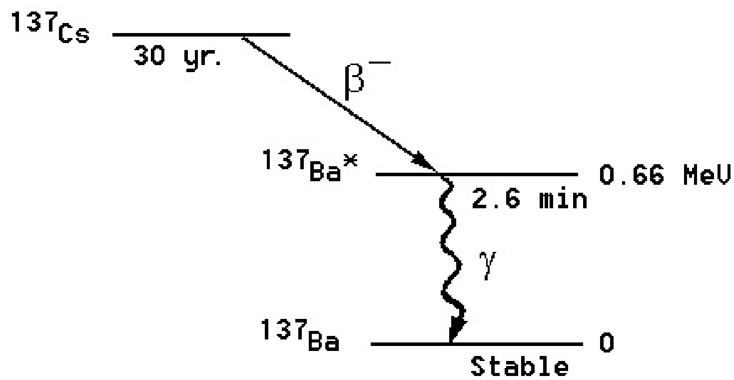

Figure 1: Decay process of Cs^{137} [1].

We conducted our main experiment with the purpose of better understanding in what way and how far gamma rays travel through varying sizes of lead. To do so we specifically used the radio-active isotope, Cs^{137} as the radiative source in which we intended to study. Pictured above in Figure 1 is the process known as Beta decay, in which Cs^{137} decays into its daughter nucleus, Barium^{137} (Br^{137}). As it decays it will simultaneously release a beta particle and a gamma ray with an energy energy .662 MeV. This emitted gamma ray will be the source of our curiosity, as we seek to learn more about its abilities as well as the properties in which it holds.

Apparatus and Procedure

For the majority of these experiments, there were four total instruments used to make measurements; give or take the oscilloscope depending on its necessity to the experiment. We used the Geiger-Miller tube to take advantage of the Ionizing effect of radiation so that we could send the corresponding pulses to the counter and oscilloscope. To run all these instruments together we of course had a power supply.

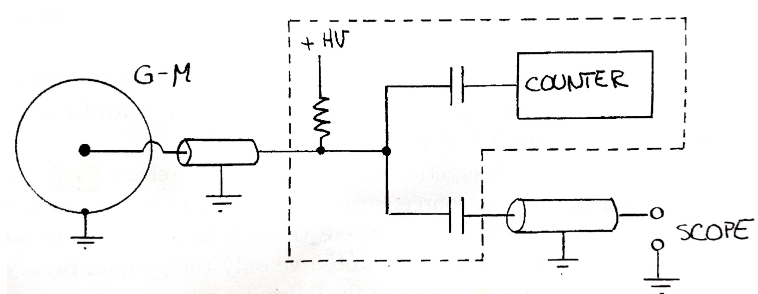

Figure 2: Depiction of how the Geiger-Muller tube is connected to the counter, power source and Oscilloscope. Everything inside the dotted line is within the Geiger-Mueller tube [2].

Preliminary Experiments

Plateau Region

For this experiment we used all the instruments described prior; The Geiger-Mueller tube connected to the counter, connected to the oscilloscope. First we started by placing our active side of the source, Cs^{137}, up and under the Geiger-Mueller tube. Our counter was connected from its SIGNAL output to the first channel of the oscilloscope. Starting the Geiger-Mueller tube at 400V, we slowly worked our way up to 880V in effort to identify a plateau region. Using the Oscilloscope, we were able to determine the the amplitude of our pulses while the counter enabled us to record how many radiative signals we received from the source. We measured data for both the count rate and amplitude to compare each data set with the voltage of the Geiger-Mueller tube in hope of finding the plateau region of the Geiger-Mueller tube.

Intrinsic Capacitance and Resistance

The objective of this experiment was to obtain intrinsic capacitance and resistance of the resistor and capacitor that are held within our SPECTECH counter box. Keeping the previous setup with our placement of the Cesium source, we set the Geiger-Mueller tube set to 880V. In order to make our measurements and determine the necessary intrinsic values, we needed to start by attaching a capacitor of known capacitance, $C_k = 2.7 \cdot 10^{-9}F$, in parallel across the input of the oscilloscope to the counter. Having done this we were then able to determine the amplitude and time constant. Next, we detached the capacitor leaving the input of our oscilloscope free. Similarly, we measured both the amplitude and time constant as we had before. Armed with these values we were able to solve for the resistance, capacitance and charge. Finally, to complete our experiment we attached the resistor to the input of the oscilloscope and observed its pulses in relation to those seen with and without the capacitor.

Background Counting Rate

For this experiment we intended to measure how many counts could obtain without our source. With the resistor still in place and the voltage set to 880V, we removed the source from our setup and let our counter measure the background for five minutes. To get a counting rate, as shown in Equation 1, we divided the number of counts measured by the five minutes that it took to receive those counts.

$$Counting\ Rate = \frac{\#\ of\ Counts}{Time} \tag{1}$$

Counting Statistics

This experiment was lead with the intention of obtaining a better understanding of the distribution of signals received by the counter from the radiative source over 1 second intervals. To start, the voltage was kept at 880V. Next we placed the Geiger-Mueller tube a far enough distance away from our source so that we would receive an average of five to ten counts per second. Once the ideal height was reached, we adjusted the counter so that it would stop counting after every second; we ran this this 100 times and recorded the measured counts for each interval.

Dead Time

The term, "dead time" is used to describe the time when the Geiger-Mueller tube has been fully saturated by photons. When this happens the Geiger-Mueller tube will no longer send signals to the counter. This experiment intends to explores this phenomenon. The voltage was kept and 880V and the Geiger-Mueller tube was placed directly onto the source so that we could receive a fair amount of counts. With the oscilloscope programmed so that persistence mode was set to infinity and the sweep rate to a few hundred microseconds, we were able to make our first estimate of the dead time. Observing the pulses shown on the oscilloscope, similar to that seen in Figure 3, we were able to measure the time between when the tube became saturated to when it regained its high voltage (Threshold Voltage, V_T).

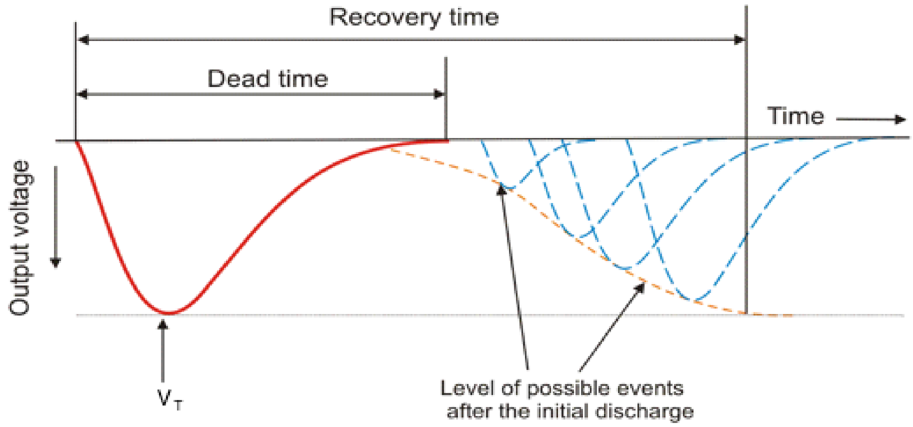

Figure 3: Dead Time [3]

Next, we made a more indirect estimate of the dead time. To do so, we started by placing the Geiger-Mueller tube further away at 64.65mm. Using two sources, both beta-ray side up, we took measurements of the sources separately and then together at the same time. Our measurements consisted of the count rates over three, three minute intervals for each data set. Armed with our measurements we were able to apply the count rates, n_a, n_b and n_{ab} to Equation 2

$$\tau_{dead} = 1 - [1 - \frac{n_{ab}(n_a + n_b - n_{ab})}{n_a n_b}]^{1/2} \tag{2}$$

Main Emperiment

Absorption of Gamma Rays

For the final experiment we explored the absorption of gamma radiation through varying widths of lead. For this experiment we did not use the Oscilloscope. To begin we placed the beta side face down and set our Geiger-Mueller tube 49.95mm away; just far enough so that the dead time was minimal. Before we measured count rates with lead, we had to determine the background rate by observing the number of counts without any lead. Once this value was determined we were able to measure the thickness of each lead slab and then place them on top of our source. For each varying thickness we let the counter run for 453 seconds after which we recorded each count and determined the count rate via Equation 1.

Results

Plateau Region

For this experiment we found that no significant number of counts were found between 400 and 740 volts. Count rate was determined via Equation 1.

Table 1: Raw data of G-M Voltage and the count rates as well as measured pulse amplitudes.

G-M Voltage (V)	Counts	Time (s)	Count Rate (s^{-1})	Amplitude (V)
400	0	30	0.000	0
420	0	30	0.000	0
440	0	30	0.000	0
460	0	30	0.000	0
480	0	30	0.000	0
500	0	30	0.000	0
520	0	30	0.000	0
540	0	30	0.000	0
560	0	30	0.000	0
580	1	30	0.033	0
600	0	30	0.000	0
620	1	30	0.033	0
640	0	30	0.000	96
660	4	30	0.133	136
680	0	30	0.000	224
700	0	30	0.000	288
720	0	30	0.000	392
740	4	30	0.133	464
760	3666	30	122.200	576
780	3755	30	125.167	664
800	3937	30	131.233	792
820	4056	30	135.200	928
840	4087	30	136.233	1040
860	4202	30	140.067	1160
880	4324	30	144.133	1460

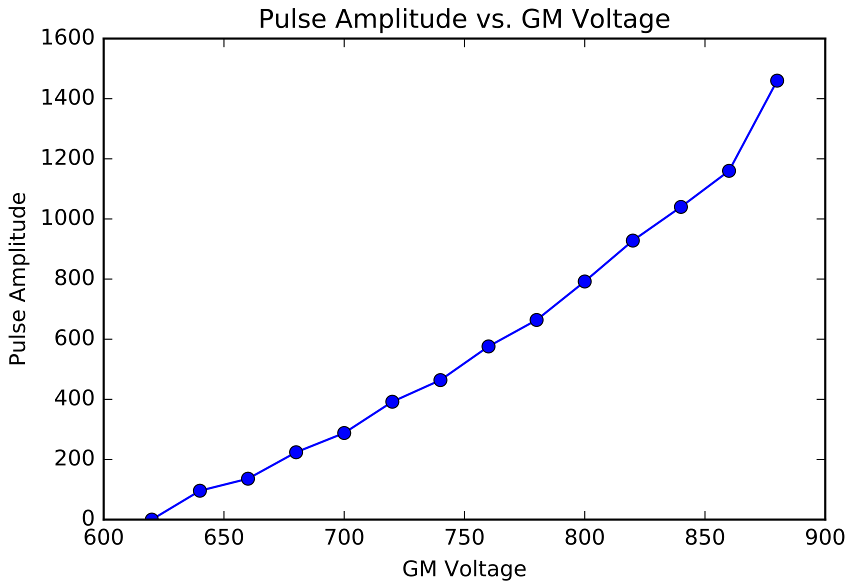

Figure 4: Here we plot the significant values of pulse amplitudes

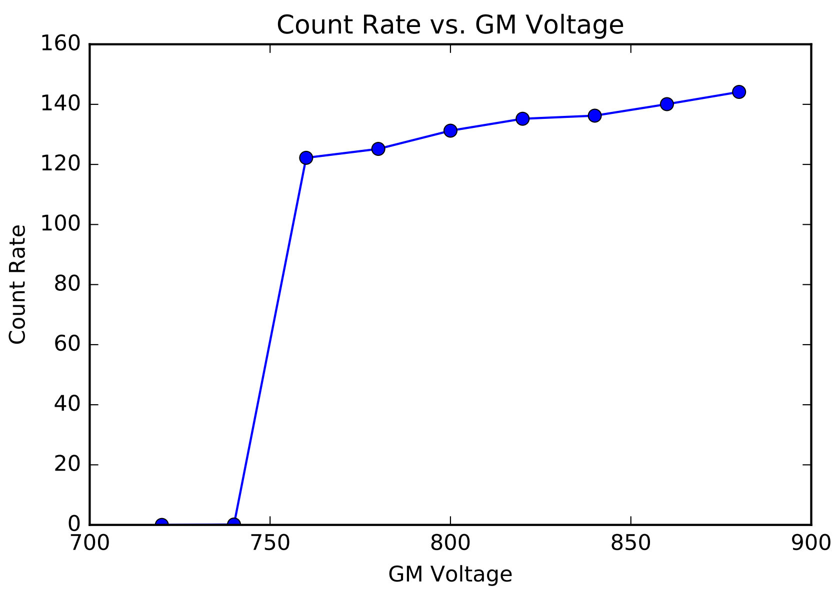

Figure 5: Here we plot the values of the count rates to see how they plateau.

It is clear from the count rate graph that we almost immediately begin to see the plateau as counts begin to be received. From 770 V to 880 V we are able to see this plateau region. For the rest of the experiments where the plateau voltage was required, our group determined 880V to be the appropriate value.

Intrinsic Capacitance and Resistance

for this experiment we were able to solve for the intrinsic capacitance, resistance and charge build on the anode via the following system of Equations 3 through 8.

$$V(t) = V_o e^{\frac{-t_{1/2}}{\tau}} \tag{3}$$

$$\tau = \frac{t_{\frac{1}{2}}}{ln2} \tag{4}$$

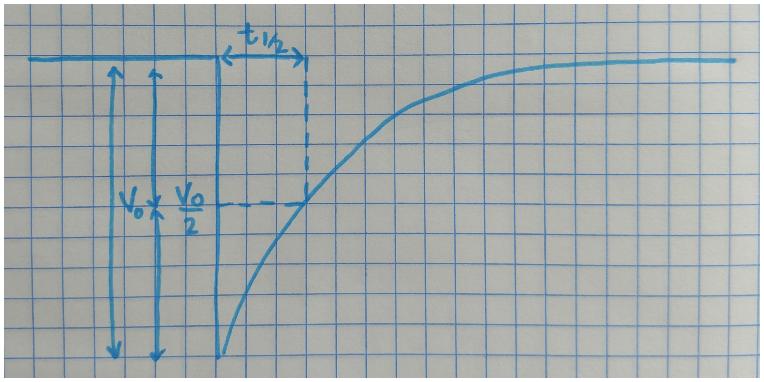

Figure 6: Depiction of the pulse as see by the Oscilloscope. Drawn by Z. Weber

Equation 4 can be used to solve for τ_c or τ_n. For this experiment, τ_c represents the time constant for when the capacitor is attached, τ_n when detached. $t_{1/2}$ is the time constant at half the amplitude of the pulse, as shown in Figure 6.

$$\tau_n = RC \tag{5}$$

$$\tau_C = R(C + C_k) \tag{6}$$

$$R = \frac{\tau_C - \tau_n}{C_k} \tag{7}$$

$$C = \frac{\tau_n}{R} \tag{8}$$

Once a value for capacitance was obtained we were able to utilize Equation 9 and solve for the charge Q collected by the anode.

$$Q = CV_o \tag{9}$$

Finally, to complete our experiment we attached the resistor to the input of the oscilloscope and observed its pulses in relation to those seen with and without the capacitor.

Table 2: Data taken representing amplitudes and time constants measured with and without the capacitor plugged into the oscilloscope.

Situation	Voltage (V)	$t_{\frac{1}{2}}$ (s)	$\frac{V_o}{2}(V)$	τ (s)	V_o (V)
Capacitor On	8.80E+02	1.04E-03	3.18E-01	1.50E-03	6.36E-01
Nothing On	8.80E+02	6.00E-04	8.00E-01	8.66E-04	1.60E+00

Using Equations 3 through 8 we found our value as well as the error for the intrinsic resistance to be, $2.35 \cdot 10^5 \pm 5.51 \cdot 10^{-14} \Omega$; for the intrinsic capacitance, $3.68 \cdot 10^{-9} \pm 1.67 \cdot 10^{-10} F$; and finally for the charge Q to be, $5.89 \cdot 10^{-9} \pm 3.35 \cdot 10^{-8} C$.

Table 3: Calculated values and errors for the intrinsic resistance, the intrinsic capacitance and charge held on the anode.

R (Ω)	σ_R (Ω)	C (F)	σ_C (F)	Q (C)	σ_Q (C)
2.35E+05	5.51E-14	3.68E-09	1.67E-10	5.89E-09	3.35E-08

Errors were determined via the derivative method:

$$\sigma_R = \sqrt{(\frac{\partial R}{\partial t_{C\frac{1}{2}}} \cdot \sigma_{t_{C\frac{1}{2}}})^2 + (\frac{\partial R}{\partial t_{\frac{1}{2}}} \cdot \sigma_{t_{\frac{1}{2}}})^2} \tag{10}$$

$$\sigma_C = \sqrt{(\frac{\partial C}{\partial t_{C\frac{1}{2}}} \cdot \sigma_{t_{C\frac{1}{2}}})^2 + (\frac{\partial C}{\partial t_{\frac{1}{2}}} \cdot \sigma_{t_{\frac{1}{2}}})^2} \tag{11}$$

$$\sigma_R = \sqrt{(\frac{\partial Q}{\partial V_o} \cdot \sigma_{V_o})^2 + (\frac{\partial Q}{\partial C} \cdot \sigma_C)} \tag{12}$$

The values of $\sigma_{t_{C\frac{1}{2}}}$, $\sigma_{t_{\frac{1}{2}}}$ and σ_{V_o} were all determined experimentally to be $\pm.004V$, $\pm.004V$ and $\pm\ 1 \cdot 10^{-5}s$ respectively.

Background Counting Rate

The count rate was determined via Equation 1. We found that its value was .742 s^{-1}

Table 4: Measured counts and the rates in which we received them

Time (s)	Counts	Count Rate (s^{-1})
300	213	0.710
300	231	0.770
300	224	0.747

Counting Statistics

The first table depicts the 1 second intervals in which the counts were taken.

Table 5: Raw data showing number of counts taken per one second intervals

Counts									
10	10	7	6	9	4	6	8	10	14
9	6	6	6	6	6	6	6	8	11
6	7	13	8	6	7	8	5	13	9
3	7	10	9	12	8	9	8	4	13
5	10	5	5	5	9	7	8	4	7
4	8	5	12	13	5	6	7	8	9
9	6	7	10	8	11	11	8	5	5
11	4	14	12	13	8	8	5	8	9
8	6	5	8	9	9	5	5	7	7
9	10	8	3	6	8	12	7	8	4

λ represents the sample mean $\overline{m}$ of the data given in Table 5, as well as the population variance, σ^2 (because we are dealing with a Poisson distribution). Equation 13 was used to determine λ.

$$\lambda = \frac{\sum counts}{100} \tag{13}$$

Table 6: Data representing calculated frequencies of the counts taken per one second intervals and their poisson distributions

m	$F(m)$	$P(m)$	$100 \cdot P(m)$
3.00	2.00	0.03	3.30
4.00	6.00	0.06	6.41
5.00	13.00	0.10	9.96
6.00	15.00	0.13	12.90
7.00	11.00	0.14	14.32
8.00	19.00	0.14	13.91
9.00	12.00	0.12	12.01
10.00	7.00	0.09	9.33
11.00	4.00	0.07	6.59
12.00	4.00	0.04	4.27
13.00	5.00	0.03	2.55
14.00	2.00	0.01	1.42

$F(m)$ is the frequency in which each count occurs with in the values of $m = 3$ and $m = 14$. This value was calculated via a function in google sheets. $P(m)$ is the Poisson value for each bin in the frequency distribution; multiplying it by the total number of counts gives us the $P(m)$ values that are appropriately scaled to our frequencies, $F(m)$.

$$P(m) = e^{-\lambda} \frac{\lambda^m}{m!} \tag{14}$$

$$\chi_m^2 = \frac{[F(m) - 100 \cdot P(m)]^2}{100 \cdot P(m)} \tag{15}$$

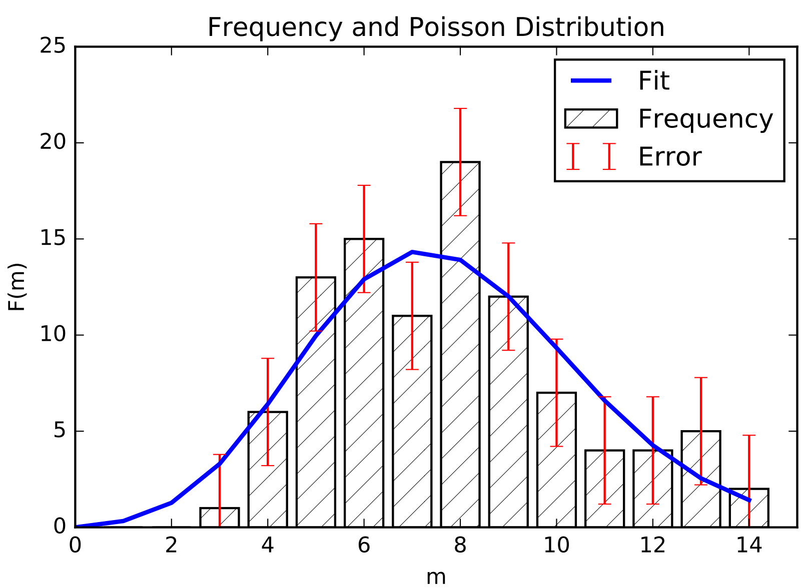

Figure 7: The error σ for this graph is just $\sqrt{\lambda}$, this is because λ is also σ^2 for a poisson distribution. $\sigma = 2.79$.

Table 7: Altered data of frequencies. We have combine the bins with smaller frequencies from Table 6; this enables a better χ^2 fit. Table also includes the calculated poisson distributions and the value found for χ^2 as well as our p-value.

m	$F(m)$	$P(m)$	$100 \cdot P(m)$	χ^2_m
<4	8.00	0.10	9.71	0.30
5	13.00	0.10	9.96	0.92
6	15.00	0.13	12.90	0.34
7	11.00	0.14	14.32	0.77
8	19.00	0.14	13.91	1.86
9	12.00	0.12	12.01	0.00
10	7.00	0.09	9.33	0.58
>10<13	8.00	0.11	10.86	0.75
13	5.00	0.03	2.55	2.35
14	2.00	0.01	1.42	0.24
χ^2			8.13	
$\frac{\chi^2}{v}$			1.02	
$p-value$			0.42	

$v = 8$, which is the degrees of freedom. The following Equations 16 and 17, can be used to determine the p-value. χ^2 represents the sum of all χ^2_m values. For this experiment we used an online calculator to evaluate our p-value.

$$f(\chi^2) = \frac{e^{-\frac{\chi^2}{2}}(\chi^2)^{\frac{v}{2}-1}}{2^{\frac{v}{2}}\Gamma\frac{v}{2}} \tag{16}$$

$$p = \int_{\chi^2_v}^{\infty} f(\chi^2)d\chi^2 \tag{17}$$

Our calculated p-value was .42, well enough within range to say that this is indeed a Poisson distribution.

Dead Time

Table 8: Data representing our first estimate that shows the observed voltage threshold and peak as well as the determined dead time.

V_T (V)	V_P (V)	$\tau_{DeadTime}$ (s)	$\sigma_{\tau_{DeadTime}}$ (s)
0.576	13.6	5.00E-04	1.00E-05

For our second estimate, τ was found via Equation 2. Using the values of n found in Table 9, we were able to solve for the sample variance, $\sigma^2_{\bar{n}}$ as well as the error of our estimated dead time.

$$\sigma^2_{\bar{n}} = \frac{\sum_{T=1}^{3}(n_T - \bar{n})^2}{n(n-1)} \tag{18}$$

$$\sigma_{\bar{\tau}} = \sqrt{(\frac{\partial\tau}{\partial n_a})^2 \cdot \sigma^2_{\bar{n}_a} + (\frac{\partial\tau}{\partial n_b})^2 \cdot \sigma^2_{\bar{n}_b} + (\frac{\partial\tau}{\partial n_{ab}})^2 \cdot \sigma^2_{\bar{n}_{ab}}} \tag{19}$$

Table 9: Second estimate, data of counts and the rate in which we received them for each source as well as both put together. Averages of these values were then used to calculate shown dead time and the error in which it was found.

	t (s)	counts (n)			$\bar{n}$	$\sigma^2_{\bar{n}}$
n_a	180	1587	1604	1632	1607.667	172.11
n_b	180	1171	1111	1094	1125.333	545.44
n_{ab}	180	2316	2270	2436	2340.667	2448.44
$\bar{\tau}_{DeadTime}$ (s)				1.08E-04		
$\sigma_{\bar{\tau}_{DeadTime}}$ (s)				1.457E-05		

We found the value of our dead time to be $\bar{\tau}_{DeadTime} = 1.08 \cdot 10^{-4} \pm 1.457 \cdot 10^{-5}$ s. This value is far lower than expected and does not coincide with our first estimate made with the Oscilloscope. It is determined that the reasoning behind this could have been due to our GM tube being held too far away from our sources, this would have made it so that a signifigant dead time effect was not maintained. Another source of error could have been in the movement between sources, which would have altered the amount of counts the GM tube received from one measurement to another. Finally, there also could have been a source facing the wrong side up; this may explain the mildly significant difference between counts of sources a and b. Had source b been facing down it would have only of exuded gamma rays, possibly explaining why there are fewer counts for those measurements.

Absorption of Gamma Rays

The counting rate for both σ_B and σ_R were found via Equation 20.

$$\sigma_R = \frac{\sqrt{N}}{t} \tag{20}$$

Table 10: Measurement of the background counting rate

t(s)	N (counts)	B $((s^{-1})$	σ_B^2
600	270	0.45	0.025

Equation 21 represents the Intensity of a gamma ray beam that is incident with an absorber as a function of time. The thickness of the absorber is represented by x and I_o is the incident intensity.

$$I(t) = I_o e^{-\mu x} \tag{21}$$

Table 11: Table showing measured thickness of each separate slab.

Slab	1	2	3	4	5	6	7
x (mm)	1.74	6.58	1.65	1.67	5.95	1.69	1.67

Table 12: Table representing the number of slabs placed over our source and the count rate as well as error for each set. For this table the background counting rate was subtracted from R, such that $R = R - B$.

Slabs Used	x_{total} (mm)	t(s)	N (counts)	R (s^{-1})	σ_R^2
1-7	20.94	453	1002	1.76	0.070
1-6	19.27	430	1001	1.88	0.074
1-5	17.59	363	1003	2.31	0.087
1-4	11.64	421	1990	4.28	0.106
1-3	9.97	420	2460	5.41	0.118
1-2	8.32	420	2758	6.12	0.125
1	1.74	420	5417	12.45	0.175
0	0.00	420	5740	13.22	0.180

Using a linear regression model in Python we were able to determine the value of μ, which is the Coefficient of Absorption, and I_o to be $\mu = .101 \pm .0025 cm^{-1}$ and $I_o = 2.65$.

$$ln(I) = ln(I_o) - \mu x \tag{22}$$

Armed with the values of μ, I_o and our varying values of x; we were able to use Equation 22 to plot a line of best fit. Our best fit line from Figure 8 represents how the Intensity of a gamma ray varies over larger thicknesses of the lead slabs.

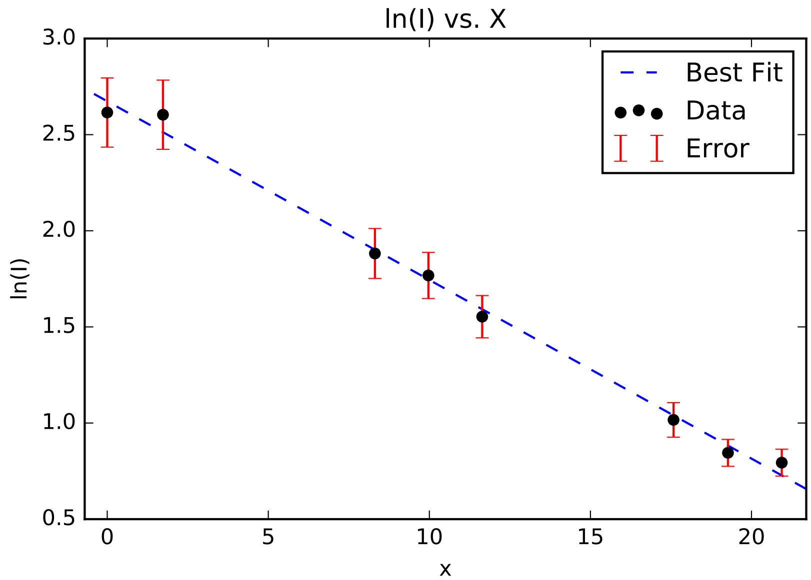

Figure 8: Linear model showing how the values of lead thickness correspond to the intensity of radiation

Next we aimed to determine whether or not our model agreed just as well without the subtraction of the background counting rate. to do so we used the χ^2 test.

Table 13: This table represents the values of R without the subtraction of the background rate.

Slabs Used	x_{total} (mm)	R (s^{-1})	σ_R^2
1-7	20.94	2.21	0.070
1-6	19.27	2.33	0.074
1-5	17.59	2.76	0.087
1-4	11.64	4.73	0.106
1-3	9.97	5.86	0.118
1-2	8.32	6.57	0.125
1	1.74	12.90	0.175
0	0.00	13.67	0.180

The χ^2 test resulted in a value $\chi^2 = 32.78$ and a $p - value = 2.9 \cdot 10^{-5}$. Because the p-value is so small we can say that the probability is small of our data being well described by anything other than a linear function.

Next we illustrate the dependence that the Absorption Coefficient μ has upon the gamma ray energy. Making a plot of the theoretical values from Table 14 and our own value of μ, we were able to show that our measurement falls within the range of excepted values for μ.

Table 14: Given values used to plot Photon Energy versus μ [2].

Photon Energy (MeV)	.4086	.5108	.6811	1.022	1.362	2.043
mu	2.24	1.68	1.16	.771	.620	.499

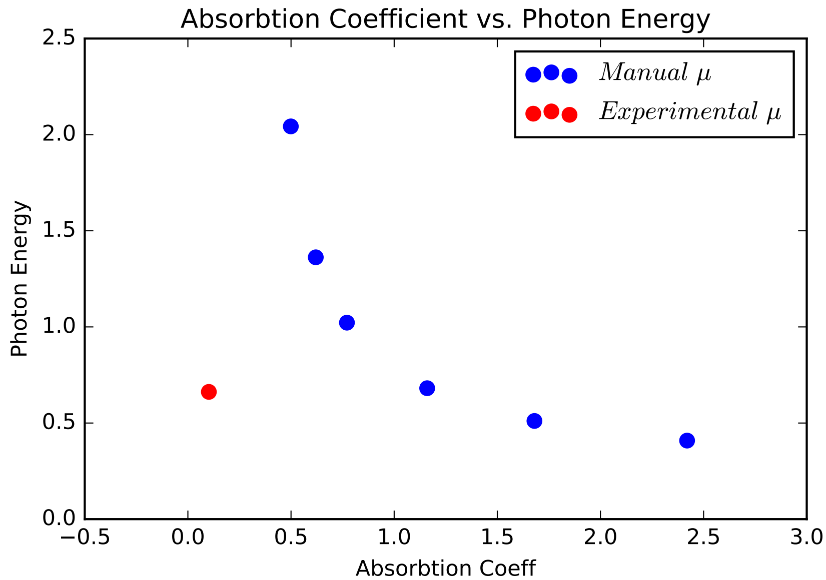

Figure 9: Model showing $\mu_{experimental}$ plotted versus the photon energy in which Cs^{137} emits a gamma ray, 662MeV. This point is shown with data for acceptable theoretical values of μ and their corresponding photon energies.

Conclusion

The experiments we have reported were done with the purpose of having a better understanding of gamma radiation and it's interactions with the matter around us.

We started by finding the GM voltage, this was to be the applied voltage to all experiments there after. To determine this voltage we took measurements of count rate and pulse amplitudes versus their corresponding GM voltages, this was so that we could visualize where these graphs would plateau. The plateau regions of these graphs was then found to be at $880V$.

Next we set out to determine the what the intrinsic resistance, capacitance and charge held on the capacitor of the capacitor was of these elements held within the counter. These values were found to be: $2.35 \cdot 10^5 \pm 5.51 \cdot 10^{-14}\Omega$ for the resistor; $3.68 \cdot 10^{-9} \pm 1.67 \cdot 10^{-10}F$ for the capacitor; and finally $5.89 \cdot 10^{-9} \pm 3.35 \cdot 10^{-8}C$ for the charge Q.

Next the average counting rate was determined by measuring the number of counts per five minute intervals. This value was found to be .742 s^{-1}.

Continuing on with our investigation of counting rate, we attempted to prove that the probability distribution of one-hundred, one second counts, take the form of a Poisson distribution. With a value of $\chi = 8.3$ and a $p - value = .42$, we were able to prove our hypothesis true, showing that these values determined our data as having a high probability of being drawn from a Poisson distribution.

For the next experiment we made two separate measurements of the dead time. We found that our second measurement was not extremely accurate, nor did it coincide very well with our first estimate of $\tau_{DeadTime} = 5.00 \cdot 10^{-4} \pm 1 \cdot 10^{-5}$ s. Our second, seemingly inaccurate estimate was $\tau_{DeadTime} = 1.08 \cdot 10^{-4} \pm 1.46 \cdot 10^{-5}$. This non-correlating data was determined to be fault of either the position of our sources, or the distance in which the GM tube had been placed.

For the primary experiment we sought to better understand the attenuation of gamma rays through lead slabs. We found that our measured data is linear and to a good fit. Using the method of least squares, we determined a value of $\chi^2 = 32.78$ and a significantly small $p - value = 2.9 \cdot 10^{-5}$. This small p-value tells us that the data is an extremely good fit to our linear model. We were also able to show our value for μ was in range with the theoretical model, our value being $\mu_{Experimental} = .101 \pm .0025cm^{-1}$ nearest to the theoretical value of $\mu_{Theoretical} = .102cm^{-1}$

References

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