A Study of Laminar Deflagrations Within O,C and O,Ne White Dwarfs

A thesis presented for the degree of

Bachelors of Science in Physics(Astrophysics)

by

Zoë Weber

Department of Astronomy and Astrophysics

University of California Santa Cruz

February 19, 2020

Abstract

The research presented in this paper looks to advance the understanding of how propagating flame fronts through the core of a white dwarf will affect the stars evolutionary fate. The stars of this study are white dwarfs that have accreted enough mass to reignite nuclear fusion. More specifically, focus was placed on white dwarfs containing C,O and O,Ne within their core.

To conduct this research, one-dimensional simulations were run through a stellar evolutionary code called MESA (Modules for Experiments in Stellar Astrophysics). The purpose of using MESA was to run through varying isotope libraries to determine the ways in which the code itself could be improved for accuracy. The simulation was run through values of 21 isotopes, 125 isotopes and 204 isotopes. For each run, MESA would produce a flame velocity for the laminar front. To determine accuracy, a comparison was made between the flame velocity found by MESA, Timmes & Woosley (1992) as well as those from the order-of-magnitude calculations.

It was found that for libraries of lower isotope levels MESA was very accurate; whereas for higher isotope libraries, MESA needed to improve on not only speed, but also on accuracy.

Acknowledgements

David Smith, for picking up the broken pieces and helping me put them back together. I appreciate him for not letting me be anything less than what he knows I can be. Thank you for never giving up on me. He's an extremely hard worker who has helped so much through this process, there is not enough thanks in this world I could give to this man.

I'd like to also acknowledge Josiah Schwab. Josiah not only wrote the code used in this research but was also always able to answer my questions in a timely manner, even when it was a bit complicated to do so.

Jeltema Tesla, an amazing scientist in the Astrophysics department at UCSC. Her astounding presence in the department and constant encouragement inspired me for the entirety of my bachelors studies.

Lastly, thank you to anyone who knew me through the process of not just writing this thesis, but for the entire time I spent studying for my bachelors. I whined, I expressed excitement. I cried and laughed. All the anxiety followed by success couldn't have happened without an army of support. Thank you, from the bottom of my heart. I wouldn't be here without the people who continued to believe in me, even when I did not.

Contents

1 Introduction

1.1 Purpose

White dwarfs are cold compact stars that, at one time, had been shining hot and bright. This paper will be focusing on stars around the mass of $7M_\odot$; In other words, intermediate mass stars. As these stars age, they become red supergiants. If a supergiant is massive enough, it will eventually be destroyed via a supernovae. On the other hand, lower initial mass red supergiants will evolve into a white dwarfs. A white dwarf is a star that is no longer burning its fuel. In most cases these stars will spend the remainder of their life cooling down, eventually fading into darkness. In the rare case that a white dwarf is able to reignite nuclear fusion within its core, the stars evolutionary fate can change drastically.

This paper will be a study of how white dwarfs of specific chemical compositions are effected by the re-ignition of nuclear fusion within their cores. Under the right circumstances this re-ignition will cause subsonic[1] flames to be sent through the core of the star. The purpose of this paper will be to simulate laminar flame fronts[2] and to improve the code through which they run. It is with these improvements that this paper will hope to give the scientific community a better understanding of how flame deflagrations[3] effect a white dwarf's evolution and even more specifically, its death.

[1] slower than the speed of sound.

[2] The case where the flame is smooth, orderly and being advanced by conduction.

[3] Combustion moving at subsonic speeds via conduction.

1.2 Stellar Evolution

1.2.1 Protostar

Stellar evolution begins within the interstellar Medium[4], where giant molecular clouds are the birthplace of stars. As simulated by Richard & Larson (1996), these gas clouds begin to collapse under gravity creating smaller denser regions that typically give way to clusters of protostars. These protostars form when the contraction from the gravitational potential energy begins to balance with the kinetic energy of the gas particles. The more these gas clouds contract under gravity, the greater the kinetic energy of the particles become. If a protostar is of large enough mass ($> 0.08M_{\odot}$) this process will eventually ignite nuclear fusion of hydrogen and a main sequence[5] star is born (Garrison, 2001).

1.2.2 Main Sequence

Stars that are under ($< 0.08M_{\odot}$) never reach temperatures great enough to begin nuclear fusion, these stars are called brown dwarfs and are not a part of the main sequence. Stars that reach core temperatures of 10 million degrees kelvin are able to fuse hydrogen into helium; these are the stars that comprise the main sequence. This and the following reactions for more massive stars are studied by Bethe (1939). Stars on the main sequence are grouped in categories based on their initial mass. Stars $< 1.5M_{\odot}$ primarily fuse hydrogen atoms together in a series of stages to form helium, a sequence called the proton–proton chain. Those with an initial mass greater than $1.5M_{\odot}$ have a nuclear fusion process that mainly uses atoms of carbon, nitrogen and oxygen as intermediaries in the CNO[6] cycle that produces

[4] The space between stars, composed mostly of hydrogen
[5] Stars the fuse hydrogen in their cores to create helium.
[6] Catalytic cycle with a six-stage sequence of reactions to create hydrogen from helium.

helium from hydrogen atoms. Stars that begin with a mass of greater than $2M_{\odot}$ undergo convection in their core regions, which acts to stir up the newly created helium and maintain the proportion of fuel needed for fusion to occur.

The amount of time a star spends on the main sequence is a function of its initial mass. The larger a star is at birth, the shorter its overall lifetime on the main sequence. A star that is of greater mass must burn hydrogen faster in order to maintain hydrostatic equilibrium[7](Garrison, 2001).

Regardless of size, all stars that begin on the main sequence will eventually deviate once their source of fuel (hydrogen) is completely depleted. It is also here, off the main sequence, that one finds white dwarfs.

1.3 White Dwarfs

White dwarfs are the remnants of either a red supergiant or red giant star. Red supergiant stars were unable to produce core temperatures hot enough to fuse beyond the neon burning stages, whereas red giant stars do not make it past the carbon burning stages (Marigo & Girardi, 2007). Both of these stars eventually shed their outer layers creating a planetary nebula that leaves behind a core of either carbon and oxygen or a core of oxygen and neon; this core is what makes a white dwarf.

White dwarfs are cold, compact stars that no longer fuse hydrogen. Because these stars no longer have nuclear fusion to support themselves, they instead do so by electron degeneracy. Electron degeneracy is the result of the Pauli exclusion principle (Pauli, 1950) which states

[7]Hydrostatic equilibrium is the balance between the pressure produced by gravity onto the star and the pressure outward created by the kinetic energy of the particles inside the star.

that two electrons may not share the same energy state while simultaneously sharing the same spin state. When the lowest energy level is filled, electrons are forced into higher levels. When electrons jump to higher energy levels they have more kinetic energy and move faster; this increase in particle speed is what creates the pressure needed for a white dwarf to withstand the force of gravity. Electrons are unable to exceed velocities greater than the speed of light. If the mass of a white dwarf is large enough, the contraction from gravity will cause electrons to move into such a small volume that they begin to fill their highest possible energy states. It is at these levels that electrons begin nearing velocities towards the speed of light. Once the electrons have reached such speeds, they can no longer create the pressure needed to withstand the force of gravity. The mass in which these velocities are induced is called the Chandrasekhar limit, as derived by Chandrasekhar (1931). The Chandrasekhar limit is considered to be $1.4M_{\odot}$; it is the largest mass a white dwarf may attain before it collapses upon itself.

Unless a white dwarf reignites nuclear fusion in it's core, it will simply continue to cool down until it can no longer be seen; this is the fate and death of the majority of white dwarfs. On the other hand, if a white dwarf can reignite nuclear fusion again, then it can drastically change its cold dark fate. When a white dwarf pairs up with another star can say that it is in binary. It is thought that flame deflagrations may only occur inside a white dwarf that is apart of a binary system. In these systems, mass may be supplied by the companion onto the white dwarf. The addition of matter from a less massive object onto another is called accretion [8]. A unique trait of white dwarfs is that the more massive they are, the smaller in size they become. This relationship is called the mass-radius relation and was first

[8] Addition of matter to one object from another less massive object via gravitational attraction.

derived by Chandrasekhar (1935). It was found that the radius of a white dwarf is inversely proportional to the cube root of its mass (Chandrasekhar, 1935). White dwarfs in a binary that have mass accreted onto them will experience an increase in pressure. This increase in pressure arises as a result of accumulating matter being squeezed into an increasingly smaller space, as given by the mass-radius relationship. As the pressure increases electrons within the core begin speeding up, thus heating the star. Eventually, if this process continues, the star may become hot enough to reignite nuclear fusion in its core. This re-ignition of nuclear fusion is what will then lead to the study of this paper, flame deflagrations. Lastly, it is important to note that the companion must be a smaller white dwarf or a non-degenerate star in order for there to be enough mass available for accretion to reignite nuclear fusion.

1.3.1 Deflagration

For an accreting C,O white dwarf, these deflagrations occur once the convection within the star causes a hot spot to ignite, this in turn creates what is called a runaway flame; of which are thoroughly studied by Timmes & Woosley (1992). For a O,Ne White dwarf, electron capture is happening within the core. This process heats up the ions which then also lead to the ignition of a flame. In both cases these flames are considered paper thin. It is the speed of these deflagrations that are of great interest; their speeds are what have great cause and effect on the stars evolution, namely whether or not it can maintain hydrostatic equilibrium through the tumultuous deflagrations (Timmes & Woosley, 1992). If the flames from a C,O white dwarf proceed faster than the stars ability to stabilize, the white dwarf will explode as a type 1a supernova. On the other hand flames in an O,Ne star race to release energy at the rate of energy absorption from electron capture, at the same time cooling from neutrinos takes place. If the deflagration is too slow then the neutrinos will cause the star to lose energy and pressure at such a rate that the star will collapse into a neutron star. In short, it is a race. If electron capture wins, the deflagration has the ability to cause the star to go super nova; If the neutrinos win, the white dwarf becomes a neutron star.

1.3.2 Detonation

There are cases in which these deflagrations may turn into detonations[9], but this happens farther away from the core and only in white dwarfs that are nearing the Chandrasekhar mass. While detonation is important to the evolution of some white dwarfs, it does not apply to the scope of this research which focuses on deflagrations at the immediate re-ignition of nuclear fusion within the core of a white dwarf.

1.4 MESA

To study these flames directly, a stellar evolution code named MESA (Modules for Experiments in Stellar Astrophysics) was used. MESA is a suite of open source libraries used to preform computational stellar astrophysics. MESA is used by over 1000 scientist worldwide for its wide range of capabilities. A few of MESA's operation include being able to simulate mass-loss, diffusion and gravitational settling for single stars. MESA is also used for studying accretion, tides and angular momentum transfer for binary systems. All of MESA's capabilities can be used at any stage of sellar evolution, either on or off the main sequence. More information about MESA and its uses can be found in the instrument papers: Paxton

[9]A type of combustion front that moves at supersonic speeds. This type of shock front moves at such fast speeds that it does not allow for material to expand before being burned, therefore detonation heats by what is called shock compression(Hillebrandt & Niemeyer, 2000).

et al. (2011), Paxton et al. (2013), Paxton et al. (2015), Paxton et al. (2018) and Paxton et al. (2019).

For the purpose of this paper MESA was implemented to model weak reaction rates that heat up the core of white dwarfs. These reactions in turn cause flame deflagrations within the star. This code was first developed and used in MESA by Schwab et al. (2015). We will be following the work Schwab et al. (2015) closely, with the only difference being an addition of higher order isotope networks[10]. MESA is found to be highly accurate when using smaller isotope libraries [11]. When increasing the number of isotopes [12] the computation time is increased and the accuracy of the code decreases. Further improvement of the MESA code in regards to its handling of larger isotope libraries would greatly improve the study of these white dwarf models. Further explanation of how we used MESA will be given in more detail in Section 2.2.

2 Study

2.1 Order of Magnitude Calculations

To determine the effect these burning fronts have on white dwarfs one must know the speed in which the flames travel. The speed of propagating flames for a C,O white dwarf, if moving fast enough, will cause a stratification of hot light matter and cold dense matter. This stratification creates turbulence and instability that can lead to a thermonuclear explosion. For a O,Ne white dwarf, if the speed of the burning front is greater then that of which

[10] Isotope networks are profiles held within the MESA code that we can vary to simulate the number of burning elements within a white dwarf.

[11] For the simulations in this research, the 21 isotope and 125 isotope libraries were used.

[12] Accuracy depleted when using the 204 isotope library.

electron capture is happening, then the star will also explode as a type 1a super nova. When the reverse is true, if the flame speed is less than that of which electron capture is taking place, than the star will collapse into a neutron star.

Equation 1 is the equation used to determine the speed of the laminar fronts. This equation reflects the nuclear burning timescale and is equivalent the characteristic energy per unit mass over the nuclear generation rate.

$$\tau_{burn} = \frac{E}{\dot{E}} = \frac{C_p \Delta T}{\epsilon_{nuc}} \tag{1}$$

In Equation 2 the diffusion timescale is equivalent to the flame width squared over the thermal diffusion coefficient, D. The thermal diffusion coefficient is represented by the mean free path (λ) of the electrons that are traveling (for these purposes) at the speed of light[13], c.

$$\tau_{diff} = \frac{\delta^2}{D} = \frac{\delta^2}{\lambda c} \tag{2}$$

If the thermal timescale is much greater than that of the diffusion timescale, the flames will diffuse instead of propagate through the star (Timmes & Woosley, 1992), thus the thermal diffusion timescale and the nuclear burning timescale must be nearly equivalent. This condition is then implemented to solve for the width (δ) of the propagating fronts by setting Equations 1 and 2 equal to one another.

$$\frac{E}{\dot{E}} = \frac{\delta^2}{D} \rightarrow \delta = \sqrt{D\frac{E}{\dot{E}}} \tag{3}$$

[13]Electrons traveling near the speed of light tell us that we are looking at the star in relativistic terms.

With the knowledge that velocity has units of length over time, the flame width divided by the speed in which the front is burning material. This is shown in Equation 4 and is order of magnitude estimate needed for the speed of the propagating flames.

$$v_{cond} = \frac{\delta}{\tau_{burn}} = \frac{\sqrt{D\frac{E}{\dot{E}}}}{\frac{E}{\dot{E}}} = \sqrt{\frac{D\dot{E}}{E}} \tag{4}$$

Rewriting this in terms of the mean free path gives v_{cond} as seen bellow in Equation 5.

$$v_{cond} = \sqrt{\frac{\lambda c\dot{E}}{E}} \tag{5}$$

2.2 MESA Implementation

2.2.1 What is MESA

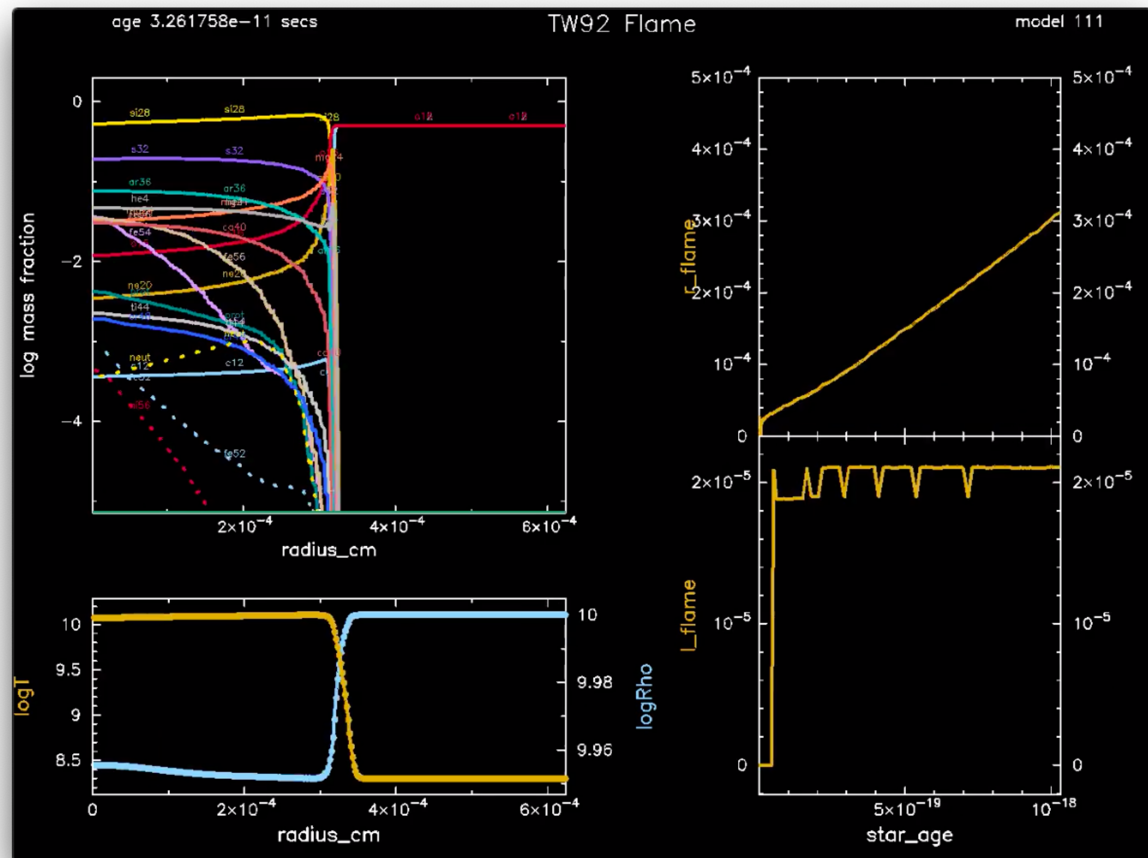
MESA is a one-dimensional code that follows a Lagrangian[14] approach as a way to track the nuclear reaction rates and multiple other important variables related to the particles temperature, mass fraction, velocity and density.

2.2.2 Implementation

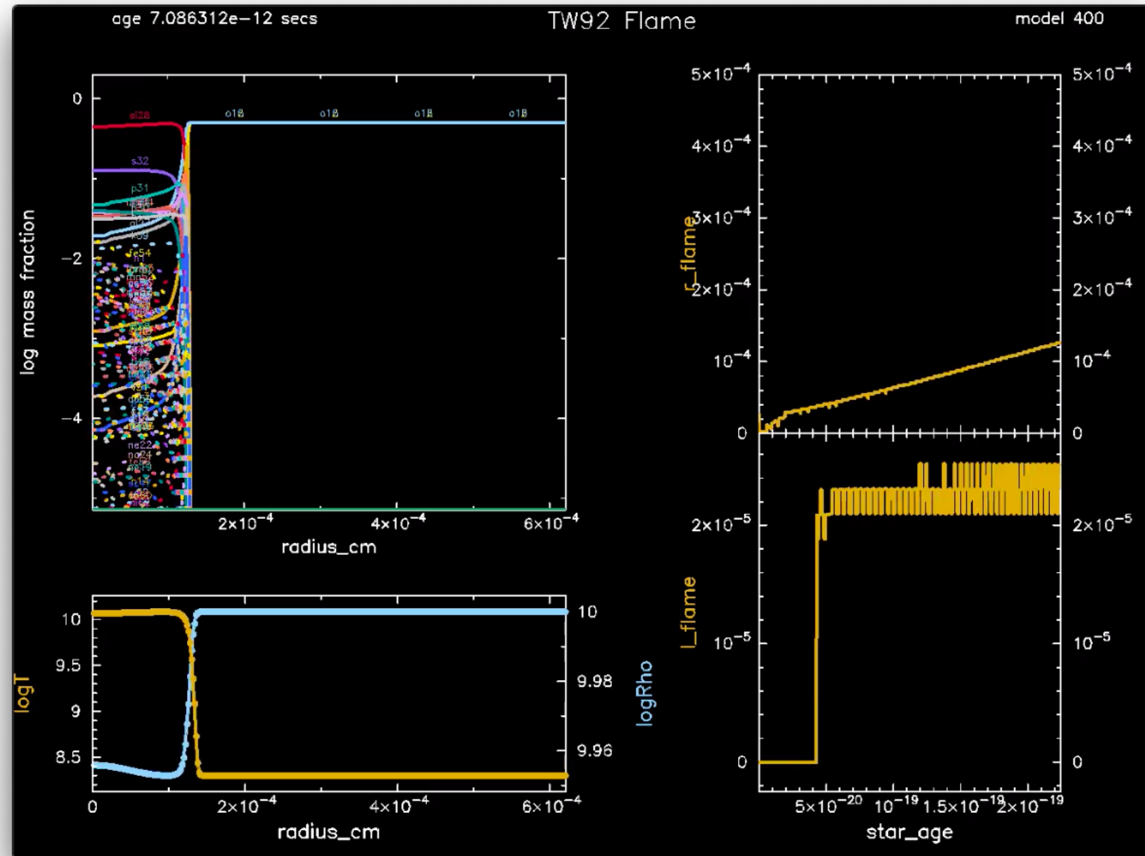
To understand a star's evolution is to know what elements it is burning. MESA's capabilities of calculating weak reaction rates was used to determine what and how much of each specific element is being burned within the core of a white dwarf. The specifics as to how MESA calculates this can be found in the MESA Instrument Paper III (Paxton et al., 2013). These reaction rates are particularly important for understanding how electron cap-

[14]Method of analyzing fluid flow by tracking a particle's movement. This approach is normally used under the assumption that there are many particles.

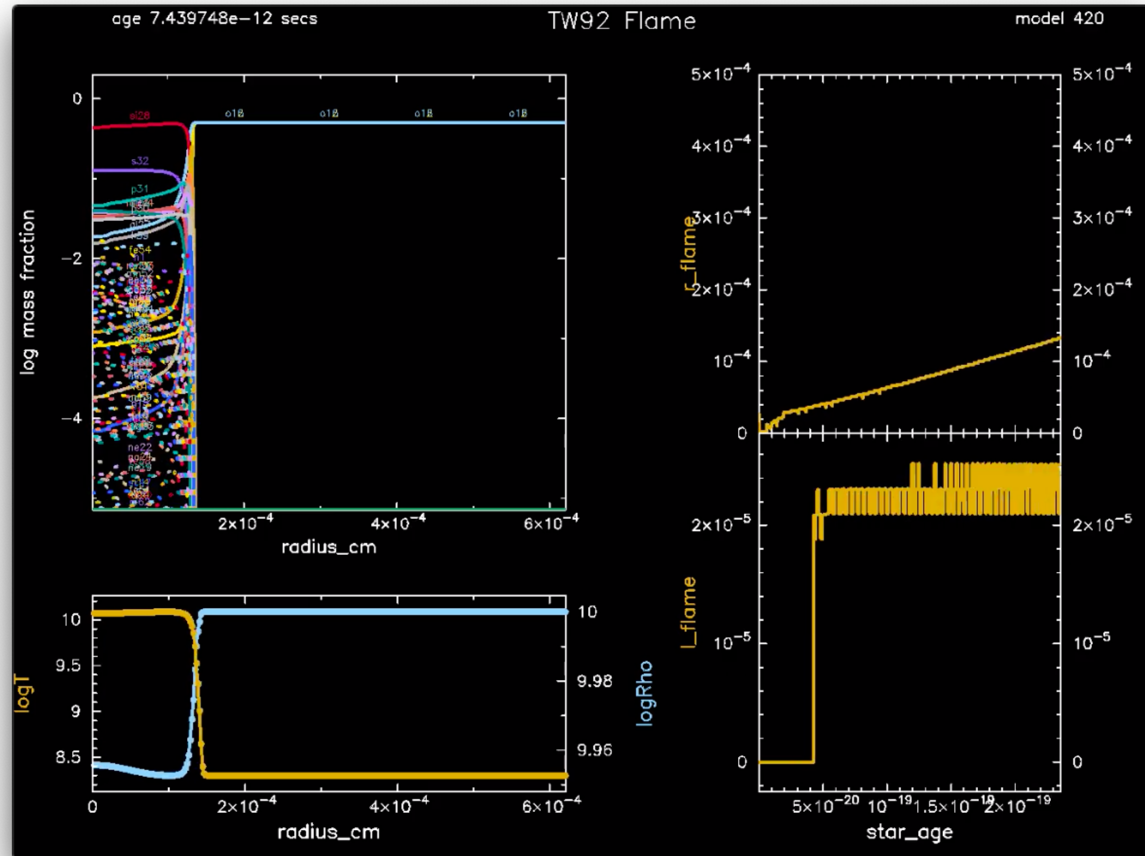
ture affects deflagration and eventually the overall evolution of C,O and O,Ne white dwarfs. A visualization of the nuclear reactions and how other important calculated variables change over time are produced by MESA and are given as an example of the work done in Figures 1.



(a) 21 isotopes



(b) 125 isotopes.



(c) 204 isotopes

Figure 1: Snapshots taken of the MESA module at the end of each run. The upper left corner of each sub-figure represents the mass fractions of isotopes, which vary as the propagating flame burns through material. It can be seen that in each figure the red line is the only element ahead of the burning front. In Figure 1a this red line is O^{12}, for Figures 1b and 1c this element is Si^{28}.

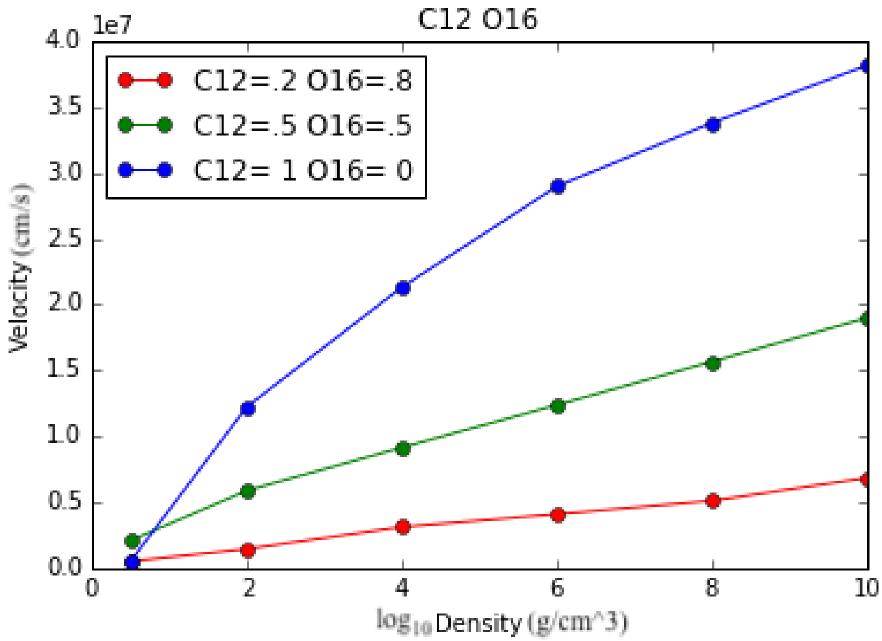
The other parameters that MESA provides are important for both the evolution of O,Ne and C,O white dwarfs. Such Parameters include how core density and core temperature change over time, which can be seen directly in the lower left corners of the MESA modules found in Figures 1. MESA also calculates the width and radius of the flame propagations seen in the previously mentioned figures. These last two variables were particularly important by informing whether or not the deflagrating flame diffused.

For this study, MESA was given the initial parameters as listed below in Table 1

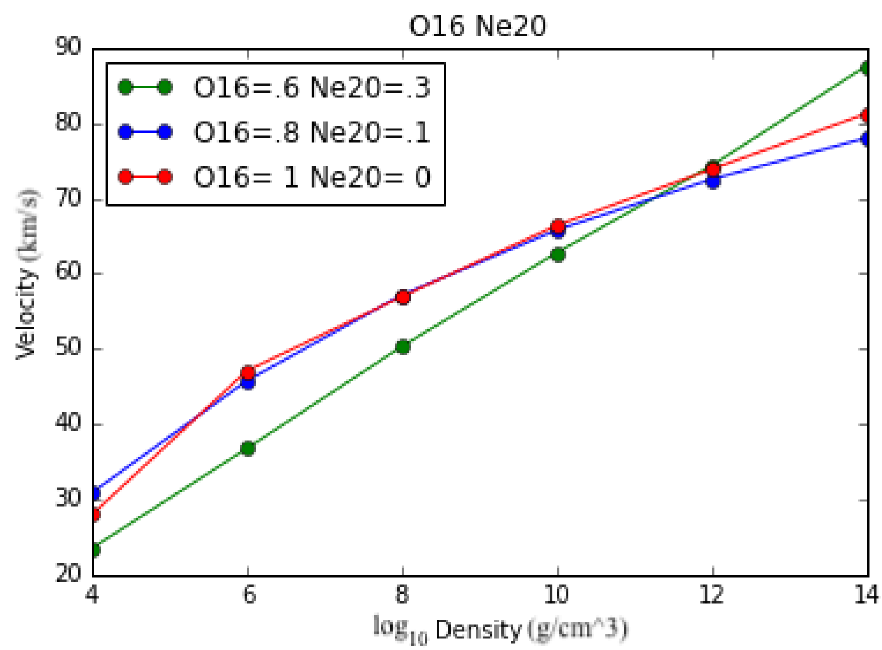
Parameter	Value
ρ_c	$1 \cdot 10^{10}\ gcm^{-3}$
M_c	$1000\ g$
T_c	$2 \cdot 10^{8}\ K$
T_{BR}	$5 \cdot 10^{9}\ K$
δ_{BR}	$1 \cdot 10^{-4}\ cm$

Table 1: Initial conditions input into MESA. T_{BR} and δ_{BR} are the initial temperature and size of the burning region where as ρ_c, M_c and T_c are the initial density, mass and temperature of the core.

Focus was given to the core of a white dwarf, where fusion is re-ignited. In order for a flame to ignite and then propagate the core temperature and density must be precise. For this study MESA was given an initial core density of $\rho_c = 1 \cdot 10^{10} gcm^{-3}$ and an initial core temperature of $T_c = 2 \cdot 10^{8} K$ with a core mass of $M_c = 1000g$. Within the core, a small burning region was created for MESA to set off the initial flame from. This burning region had a size of $\delta_{BR} = 1 \cdot 10^{-4}\ cm$ with an initial temperature of $T_{BR} = 5 \cdot 10^{9}\ K$. The simulation was run using three different isotope networks: $A = 21$, $A = 125$ and $A = 204$. Varying the isotope networks was done with the intention of testing MESA's capabilities and to hopefully obtain data of higher accuracy. With these initial parameters the initial mass fractions of C,O as well as those of O,Ne white dwarfs were then varied. The adjustment of these initial mass fractions help to understand how the flame deflagration speed is affected by the chemical abundance within the white dwarfs core. For both C,O and O,Ne stars, a run was done for each isotope network with three separate modified mass fractions. These abundances and how they affect velocity can be found bellow in Figure 2.



(a) C,O



(b) O,Ne

Figure 2: Plots showing how velocity of flames change with differing initial abundances. Run with an isotope network of A=21.

MESA provides a calculated flame width, which was then used to compare with the 2.1 calculations. This was done as a way to double check MESA's accuracy with the derived values. This data can be found within the history.data files as well as each separate profile.data file [15]. These parameters were taken along with the same time-step in which MESA calculated the same variables (such as flame width) to determine a flame velocity via the order of magnitude calculations. Lastly, a comparison was made between the order of magnitude estimate and the velocity MESA calculated at the equivalent time-step in which the data was pulled from; The order of magnitude estimate is the theoretical velocity and can be seen compared to MESA's experimental velocity bellow in Figure 3 for a white dwarf having a core mass fraction of $C^{12} = 0.5$ and $O^{16} = 0.5$.

[15] These are the files produced by each run of the simulation. These files contain all the data MESA has either calculated or used as a constant for said run.

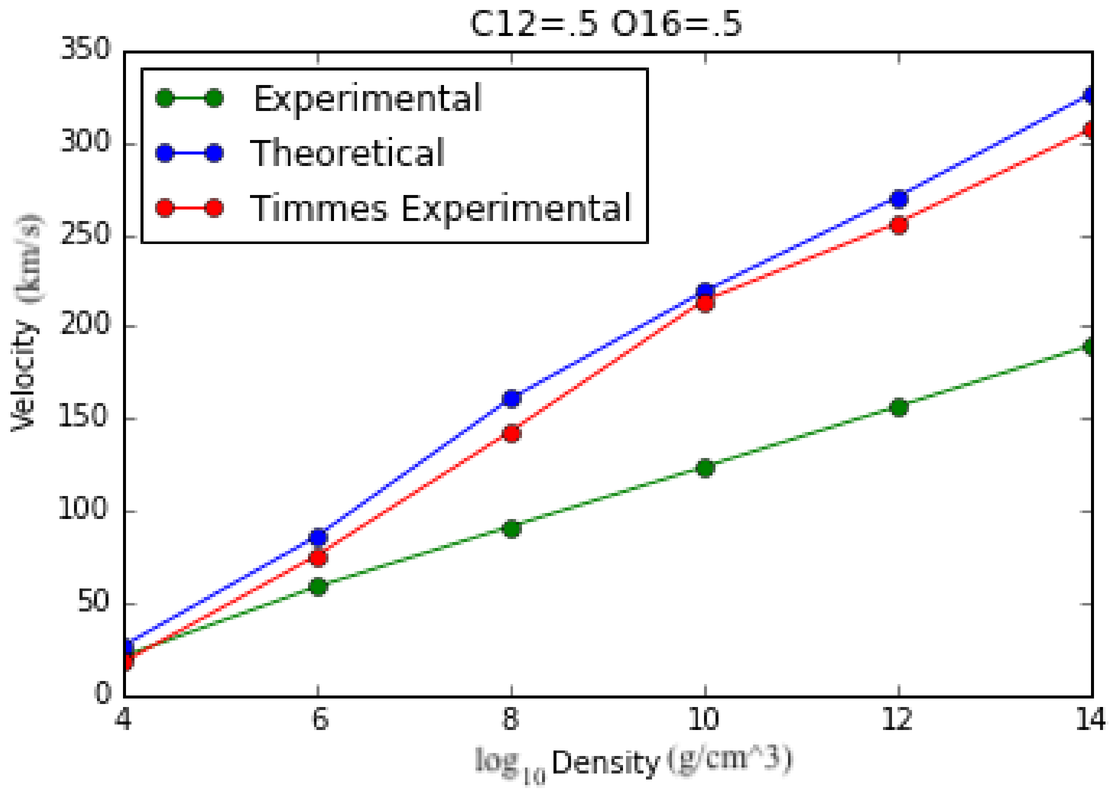


Figure 3: Our experiment on MESA versus Timmes & Woosley (1992) and the accepted theoretical values. Run with an isotope network of A=21.

It is important to note that prior MESA simulations like those done by Schwab et al. (2015) are run just until thermal runaway heats up the core to temperatures great enough for the fusion of either carbon in the core of an C,O white dwarfs or, for the case of a O,Ne white dwarf, temperatures great enough for the fusion of oxygen. Unlike the calculations done in this paper, prior studies on laminar fronts with MESA were given these temperature restrictions due to the incredibly small width of the deflagrations. These propagating flames have a width that is on the order of $\delta = 10^{-5} cm$, which is far too small for MESA to properly resolve when studying the entire star. As stated in Schwab et al. (2015), this is appropriate because it was at specific core temperatures that one could conclude the conditions as being just right for thermal runaway to release a deflagration similar to those shown in the calculations done by Timmes & Woosley (1992). A Major difference about the MESA simulation used in this paper is that it follows the laminar front, this is done through the restriction of a burning region. The introduction of these capabilities enables MESA to resolve the flame width and find its velocity. The simulation discontinues when the laminar front has reached a halfway point; this is also different to prior studies which only run until the core temperature becomes high enough to assume a flame deflagration will take place.

3 Results

It can be seen from Figure 2a that the greater the mass fraction of C^{12} the higher the velocity becomes at larger densities. This is because the temperatures required to burn carbon are lower than that required to burn oxygen. Deflagrations from a white dwarf with higher mass fractions of C^{12} are then able to not only happen sooner but heat up faster with increasing density. These higher values are crucial and in turn cause the deflagrations to propagate at higher velocities.

In Figure 2b the velocity for each differing mass fraction vary less from one another due to the balance between the heating from the thermonuclear energy release and the rate at which that energy can be transported by thermal conduction. Over all, the material in a O,Ne white dwarf is denser than that of a C,O white dwarf. This higher density is because of the stars advanced burning stage's; the advancement in burning stages creates heavier elements that in turn compress (as proven mass-radius relation of $R \approx M^{-\frac{1}{3}}$). With this said, it is noted by Schwab et al. (2015) that due to Rayleigh-Taylor instability[16] the conductive flames within the core of an O,Ne white dwarf remain relatively stable as the newer denser material pushes back on the older lighter material, similar to the result shown in Figure 2b.

The temperature differences of the core per mass fraction remain fairly steady, this is due to neutrino cooling balancing the heating done from electron capture. This stability means that there is very little effect on the density at which oxygen ignition occurs and thus little effect on how fast the flames propagate.

Figure 3 compares the results of the flame speed MESA calculate versus the theoretical order of magnitude estimates found in 2.1 and with the velocities found by Timmes & Woosley (1992). It was found that the theoretical results agree very well with Timmes & Woosley (1992) and those of MESA's agree well on a linear scale. It is assumed that the difference in agreement from MESA's calculations is related to the scale of information within its nuclear network. The program itself is made to run as efficiently as possible to calculate the reactions "on the fly" (Paxton et al., 2013). This efficiency can translate to some loss of accuracy in order to run within a shorter amount of time.

4 Conclusion

This research has provided significant evidence toward the ability of laminar fronts to propagate through the core of a white dwarf via the use of the stellar evolution code, MESA. With this study we have enabled the further development of future studies to understand

[16]Described as the instability caused between two fluids of different densities, this instability is created when the denser fluid is being compressed by the less dense one.

how these propagating flames disrupt hydrostatic equilibrium for accreting white dwarfs consisting of either a C,O core or a O,Ne core.

From Figures 1a to 1c it can be seen that at larger isotopes the program runs for a shorter amount of time. The reason for this is the fact the MESA is not as well equipped to run so many nuclear reaction at one time. To further improve the accuracy of these models, it would be helpful to increase MESA's abilities when handling such large isotope networks.

Progress could also be made on the size of the region in which MESA is able to resolve these flames. Work done by Schwab et al. (2015) is run as a study of the entire core of a white dwarf while ours focuses in on a smaller burning region. If MESA were able to resolve the width of these laminar fronts for larger boundaries it would allow for the combination of prior studies and this one. By doing this it would help to increase the understanding of how these propagating flames truly affect the white dwarfs ability to maintain hydrostatic equilibrium.

Relating this work in a competitive sense, it cannot be said that the one-dimensional model run in MESA is more accurate than those run by Timmes & Woosley (1992). Further investigation into MESA's capabilities of calculating weak reaction rates with a higher number of isotopes would help to resolve whether the information MESA provides can further the advancement of these one-dimensional models beyond those done by Timmes & Woosley (1992).

A focus was placed on one-dimensional models to maintain a spotlight the laminar speed[17] of propagating flames. Meanwhile, it can be said that these one-dimensional models are

[17] Flame fronts are always laminar in one-dimension because there is no turbulence. In two and three-dimensions you can only have a laminar speed if there is no turbulence, which is not usually the case for these propagating flame models.

complementary to similarly condition two and three-dimensional models. The reasoning for this is because when these models are run with added dimensions the surface area of these flames increase and thus the amount that can be burned at one time rises. All this causes turbulence to kick in and speed up the advancement of the flame front. The turbulence creates a flame front that no longer has a laminar velocity, thus making the laminar speed un-relatable to the propagation speed. One-dimesional models like the ones provided in this paper can provide the necessary information about laminar speed that two and three dimensional models otherwise would not have.

The research done here has been helpful with its additional understanding of how MESA handles larger isotope libraries. Based on the research provided in this paper, further improvements can made to MESA. It is hoped that these improvements will in turn progress the field of knowledge around flame delegations within white dwarfs. Continued research simulating the effects of these propagating flames will give astronomers a better idea of the more catastrophic ways (such as a supernova) in which a white dwarf's life may end. With that said, it is hoped that the work done here can be used for the greater good by helping other researchers who are interested in using MESA.

References

Bethe, H. (1939). Energy Production in Stars. *Physical Review*, 55, 434.

Chamulak, D., Brown, E., & Timmes, F. (2007). The Laminar Flame Speedup By Ne Enrichment in White Dwarf Supernovae. *The Astrophysical Journal*, 655, 93.

Chandrasekhar, S. (1931”). The Maximum Mass of Ideal White Dwarfs. *America Astronomical Society*, 74, 82.

Chandrasekhar, S. (1935”). The Highly Collapsed Configurations of Stellar Mass. *Royal Astronomical Society*, 95, 460.

Garrison, R. (2001). *Stellar Evolution*. Nature Publishing Group, first edition.

Hillebrandt, W. & Niemeyer, J. (2000). Type 1a Supernova Explosion Models. *Annual Review of Astronomy and Astrophysics*, 38, 1.

Marigo, P. & Girardi, L. (2007). Evolution of Asymptotic Giant Branch Stars. *Astronomy and Astrophysics*, 469, 435.

Pauli, W. (1950). On the Connection Between Spin and Statistics. *Progress of Theoretical Physics*, 5, 526.

Paxton, B., Bildsten, L., Dotter, A., Herwig, F., Lesaffre, P., & Timmes, F. (2011). Modules for Experiments in Stellar Astrophysics (MESA). , 192, 3.

Paxton, B., Cantiello, M., Arras, P., Bildsten, L., Brown, E. F., Dotter, A., Mankovich, C., Montgomery, M. H., Stello, D., Timmes, F. X., & Townsend, R. (2013). Modules for Experiments in Stellar Astrophysics (MESA): Planets, Oscillations, Rotation, and Massive Stars. , 208, 4.

Paxton, B., Marchant, P., Schwab, J., Bauer, E. B., Bildsten, L., Cantiello, M., Dessart, L., Farmer, R., Hu, H., Langer, N., Townsend, R. H. D., Townsley, D. M., & Timmes, F. X. (2015). Modules for Experiments in Stellar Astrophysics (MESA): Binaries, Pulsations, and Explosions. , 220, 15.

Paxton, B., Schwab, J., Bauer, E. B., Bildsten, L., Blinnikov, S., Duffell, P., Farmer, R., Goldberg, J. A., Marchant, P., Sorokina, E., Thoul, A., Townsend, R. H. D., & Timmes, F. X. (2018). Modules for Experiments in Stellar Astrophysics (MESA): Convective Boundaries, Element Diffusion, and Massive Star Explosions. , 234, 34.

Paxton, B., Smolec, R., Schwab, J., Gautschy, A., Bildsten, L., Cantiello, M., Dotter, A., Farmer, R., Goldberg, J. A., Jermyn, A. S., Kanbur, S. M., Marchant, P., Thoul, A., Townsend, R. H. D., Wolf, W. M., Zhang, M., & Timmes, F. X. (2019). Modules for Experiments in Stellar Astrophysics (MESA): Pulsating Variable Stars, Rotation, Convective Boundaries, and Energy Conservation. , 243(1), 10.

Richard, B. & Larson (1996). Numerical Calculations of the Dynamics of a Collapsing Proto-Star. *Royal Astronomical Society*, 145, 272.

Schonberg, M. & Chandrasekhar, S. (1942). On the Evolution of The Main-Sequence Stars. *The Astrophysical Journal*, 96, 161.

Schwab, J., Quataert, E., & Bildsten, E. (2015). Thermal runaway during the Evolution of ONeMg cores towards accretion-induced collapse. *Monthly Notices of the Royal Astronomical Society*, 453, 2.

Timmes, F. & Woosley, S. (1992). The Conductive Propagation of Nuclear Flames. I. Degenerate C+O and O+Ne+Mg White Dwarfs. *The Astrophysical Journal*, 396, 650.