# Implicit self-adjusting computation for CostIt

## Internship Defense


Zoe Paraskevopoulou[1,2]
Advisor: Deepak Garg[2]

[1]ENS Cachan [2]Max Planck Institute for Software Systems


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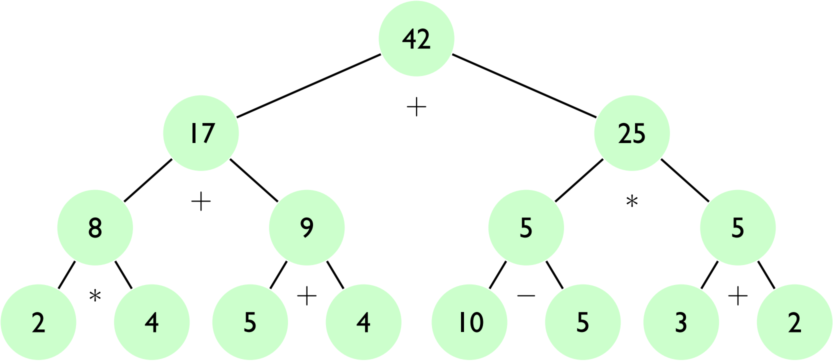
# Self Adjusting Computation

- An evaluation mechanism that recomputes *only* the parts of the output that depend on inputs that have changed between runs
- *Change propagation* (CP) : the process of updating the parts of the output that depend on changed data
- *Implicit self-adjusting computation*: The program responds automatically to changes in its inputs without any manual effort from the programmer
- Often results in asymptotic speedup

# Change Propagation by Example



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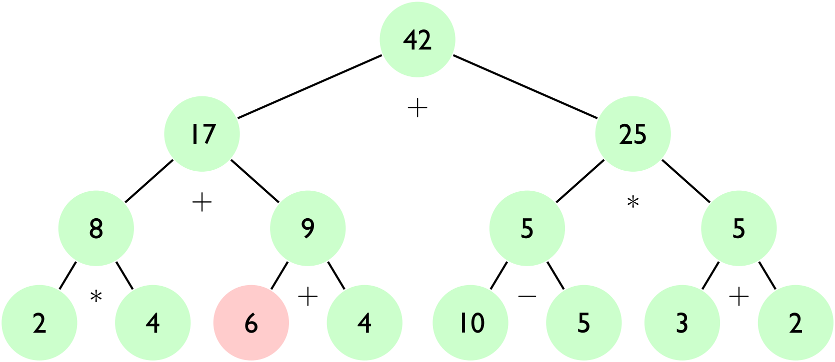

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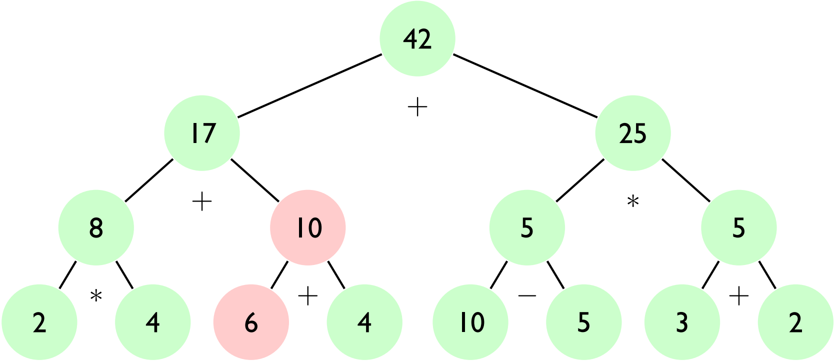

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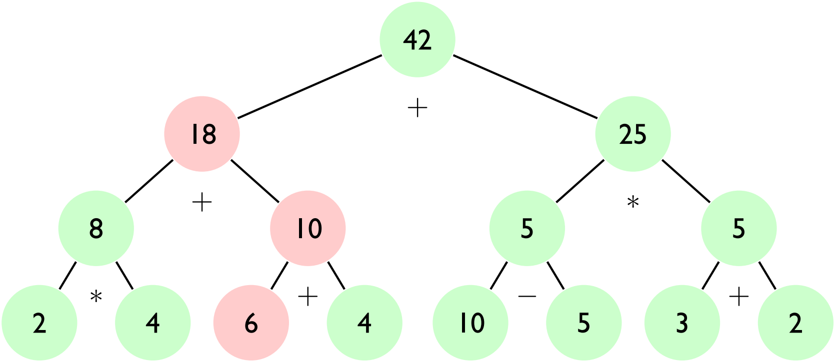

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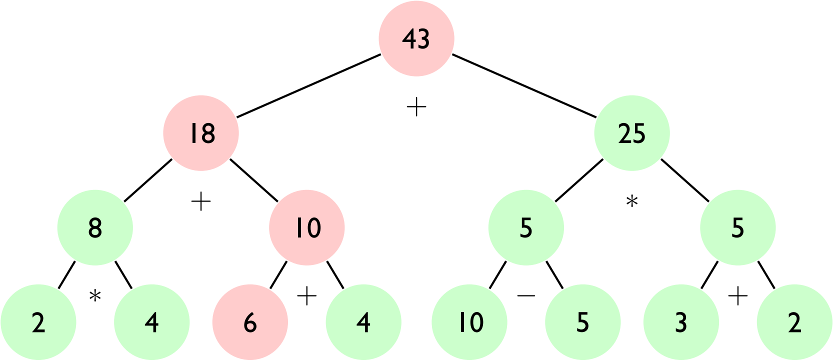

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- When $\epsilon = \mathbb{S}$ then $\kappa$ is the upper bound of CP
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- Types have changeability annotations $\tau^{\mu}$
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- Index refinement types (in the style of DML)
- Lists : $\texttt{list}\,[n]^{\alpha}\ \tau$
  - A vector of $n$ elements from which at most $\alpha$ can change

# Running Example: `map` (typing)

$$\texttt{map} : (\tau_1 \xrightarrow{\mathbb{C}(\kappa)} \tau_2)^{\square} \xrightarrow{\mathbb{S}(0)} \texttt{list}\,[n]^{\alpha}\ \tau_1 \xrightarrow{\mathbb{S}(\kappa\cdot\alpha)} \texttt{list}\,[n]^{\alpha}\ \tau_2$$

- If `f` executes from-scratch with cost $k$ and `l` has $n$ elements of which at most $\alpha$ can change then `map f l` propagates changes with cost at most $\alpha \cdot \kappa$

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- If `f` executes from-scratch with cost $k$ and `l` has $n$ elements of which at most $\alpha$ can change then `map f l` propagates changes with cost at most $\alpha \cdot \kappa$
- Intuition: we need to recompute and update in place only the elements of the list that can change

# Soundness (this internship)

- **Idea:** Translate a CostIt program to a self-adjusting program and show that the actual cost is no more that the cost derived by the type system

$$\begin{tikzcd}
\vdash^{\kappa}_{\epsilon} e : \tau \arrow[rr, "\text{evaluates to}"] \arrow[d, "\text{translates to}"] & & v_1 \arrow[d, phantom, "\approx"] \\
\vdash \ulcorner e \urcorner : \|\tau\| \arrow[rr, "\text{evaluates in } c \text{ steps to}", "\text{if } \epsilon = \mathbb{C} \text{ then } c \leq \kappa"'] & & v_2
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(a) First run

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(b) Incremental run after input changes

Figure: Schematic representation of the basic properties of the translation

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- Two modes: $\epsilon = \mathbb{C}$ and $\epsilon = \mathbb{S}$
- The code generated in $\mathbb{C}$ mode will be executed from scratch during CP
- The code generated in $\mathbb{S}$ mode is self-adjusting
  - During this mode we record the computations that need to be re-executed during CP

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# Similarity Relation

$$v_s \approx^{\tau}_{\sigma} v_t$$

- $v_s$ is the source value, $v_t$ is the target value
- $\sigma$ is the store in the target
- Changeable values are references in the target (stored in $\sigma$)
- For *stable values*, $v_s$ and $v_t$ should coincide
- For *changeable values*, $v_t$ should be a location and $v_s$ should coincide with the value of this location in the store.

$$(3, 42) \approx^{\mathtt{int}^{\mathbb{S}} \times \mathtt{int}^{\mathbb{C}}}_{[l \mapsto 42]} (3, l)$$

# Soundness, $\mathbb{C}$ mode

## Theorem

*Assume that*

$$\vdash^{\kappa}_{\mathbb{C}} e : \tau \hookrightarrow \ulcorner e \urcorner$$

*Then there exist $v'_s$, $v'_t$, $\sigma'$, $j$ and $c$, such that*

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(4) $v'_s \approx^{\tau}_{\sigma'} v'_t$

# Two-way similarity relation

$$(v_i,\ v_c) \approx^{\tau}_{(\sigma_i,\ \sigma_c)} v_t$$

- $v_i$ is the initial source value, $v_c$ is the source value after changes
- $v_t$ is the target value that stores changeable values in references
- $\sigma_i$ is the initial target store, $\sigma_c$ is the target store holding changed values
- For *stable values*, $v_i$, $v_c$ and $v_t$ should coincide under the two stores
- For *changeable values*, $v_i$ should be similar to $v_t$ under $\sigma_i$ and $v_c$ should be similar to $v_t$ under $\sigma_c$

$$((3, 42),\ (3, 43)) \approx^{\mathtt{int}^{\mathbb{S}} \times \mathtt{int}^{\mathbb{C}}}_{[l \mapsto 42],\ [l \mapsto 43]} (3, l)$$

# Soundness, $\mathbb{S}$ mode

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*Assume that*

$$\cdot;\cdot;x:\tau' \vdash^{\kappa}_{\mathbb{S}} e:\tau \hookrightarrow \ulcorner e \urcorner$$

$$(v_i,\ v_c) \approx^{\tau'}_{(\sigma_i,\ \sigma_c)} v_t$$

*Then if* $[x \mapsto v_i]e \Downarrow v_i',\ j$ *then there exist* $v_c'$, $v_t'$, $\sigma_f$, $\sigma_f'$, $Q$, $j$ *and* $c$, *such that*

# Soundness, $\mathbb{S}$ mode

## Theorem

*Assume that*

$$\cdot;\cdot;x:\tau' \vdash^{\kappa}_{\mathbb{S}} e:\tau \hookrightarrow \ulcorner e \urcorner$$

$$(v_i,\ v_c) \approx^{\tau'}_{(\sigma_i,\ \sigma_c)} v_t$$

*Then if* $[x \mapsto v_i]e \Downarrow v'_i,\ j$ *then there exist* $v'_c$, $v'_t$, $\sigma_f$, $\sigma'_f$, $Q$, $j$ *and* $c$, *such that*

(1) $[x \mapsto v_c]e \Downarrow v'_c,\ j'$

# Soundness, $\mathbb{S}$ mode

## Theorem

*Assume that*

$$\cdot; \cdot; x : \tau' \vdash^{\kappa}_{\mathbb{S}} e : \tau \hookrightarrow \ulcorner e \urcorner$$

$$(v_i,\ v_c) \approx^{\tau'}_{(\sigma_i,\ \sigma_c)} v_t$$

*Then if* $[x \mapsto v_i]e \Downarrow v_i',\ j$ *then there exist* $v_c'$*,* $v_t'$*,* $\sigma_f$*,* $\sigma_f'$*,* $Q$*,* $j$ *and* $c$*, such that*

(1) $[x \mapsto v_c]e \Downarrow v_c',\ j'$

(2) $[x \mapsto v_t]\ulcorner e \urcorner,\ \sigma_i \Downarrow v_t',\ \sigma_f,\ Q,\ c$

# Soundness, $\mathbb{S}$ mode

## Theorem

*Assume that*

$$\cdot;\cdot;x:\tau' \vdash^{\kappa}_{\mathbb{S}} e:\tau \hookrightarrow \ulcorner e \urcorner$$

$$(v_i,\ v_c) \approx^{\tau'}_{(\sigma_i,\ \sigma_c)} v_t$$

*Then if* $[x \mapsto v_i]e \Downarrow v_i',\ j$ *then there exist* $v_c'$, $v_t'$, $\sigma_f$, $\sigma_f'$, $Q$, $j$ *and* $c$, *such that*

(1) $[x \mapsto v_c]e \Downarrow v_c',\ j'$

(2) $[x \mapsto v_t]\ulcorner e \urcorner,\ \sigma_i \Downarrow v_t',\ \sigma_f,\ Q,\ c$

(3) $Q,\ \sigma_f[\sigma_c] \rightsquigarrow \sigma_f',\ c'$

# Soundness, $\mathbb{S}$ mode

## Theorem

*Assume that*

$$\cdot; \cdot; x : \tau' \vdash^{\kappa}_{\mathbb{S}} e : \tau \hookrightarrow \ulcorner e \urcorner$$

$$(v_i,\ v_c) \approx^{\tau'}_{(\sigma_i,\ \sigma_c)} v_t$$

*Then if* $[x \mapsto v_i]e \Downarrow v_i',\ j$ *then there exist* $v_c'$, $v_t'$, $\sigma_f$, $\sigma_f'$, $Q$, $j$ *and* $c$, *such that*

(1) $[x \mapsto v_c]e \Downarrow v_c',\ j'$

(2) $[x \mapsto v_t]\ulcorner e \urcorner,\ \sigma_i \Downarrow v_t',\ \sigma_f,\ Q,\ c$

(3) $Q,\ \sigma_f[\sigma_c] \rightsquigarrow \sigma_f',\ c'$

(4) $\models c' \mathrel{\dot{\leq}} \kappa$

# Soundness, $\mathbb{S}$ mode

## Theorem

*Assume that*

$$\cdot; \cdot; x : \tau' \vdash^{\kappa}_{\mathbb{S}} e : \tau \hookrightarrow \ulcorner e \urcorner$$

$$(v_i,\ v_c) \approx^{\tau'}_{(\sigma_i,\ \sigma_c)} v_t$$

*Then if* $[x \mapsto v_i]e \Downarrow v_i',\ j$ *then there exist* $v_c'$, $v_t'$, $\sigma_f$, $\sigma_f'$, $Q$, $j$ *and* $c$, *such that*

(1) $[x \mapsto v_c]e \Downarrow v_c',\ j'$

(2) $[x \mapsto v_t]\ulcorner e \urcorner,\ \sigma_i \Downarrow v_t',\ \sigma_f,\ Q,\ c$

(3) $Q,\ \sigma_f[\sigma_c] \rightsquigarrow \sigma_f',\ c'$

(4) $\models c' \,\dot{\leq}\, \kappa$

(5) $(v_i',\ v_c') \approx_{(\sigma_f,\ \sigma_f')} v_t'$

# Proof Method

- The soundness is proved using logical relations
- We construct two Kripke step-indexed relational models
- Two fundamental properties, one for each typing mode
- The soundness theorems are corollaries of the fundamental properties of the logical relations

# Summary

- Soundness proof for CostIt w.r.t. to concrete CP semantics
  - Older poof was w.r.t. an abstract semantics
- Designed a target language (saML) with infrastructure for CP
- Translated CostIt to saML
- Proved the correctness of the translation and the change propagation mechanism
- Proved that the cost derived by CostIt is a sound approximation of the actual cost (for both $\mathbb{C}$ and $\mathbb{S}$ modes)

# Future Work

- Devise a more efficient CP mechanism
- Mechanize the proof using a proof assistant
- Adapt CostIt to derive the cost for demand-driven self-adjusting computation
- Ongoing work: Implementation of the type system using bidirectional type checking (E. Çiçek and D. Garg)

# Thank You!

Questions?