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BIOS 740 Hwk3

Hand in the following problems given in TSA5.

Problems 2.3, 2.6,

Reading Assignment: Problems 2.4 and 2.5.

Acknowledgement:
1. Plots were generated by R
2. “key differences summarized” table was generated by Gemini

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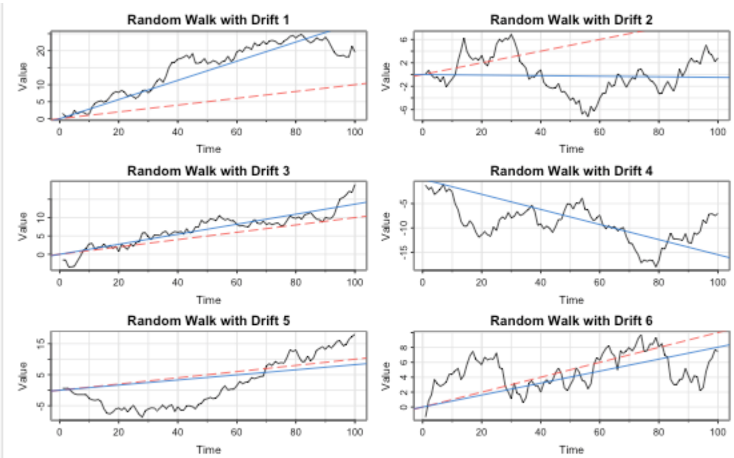
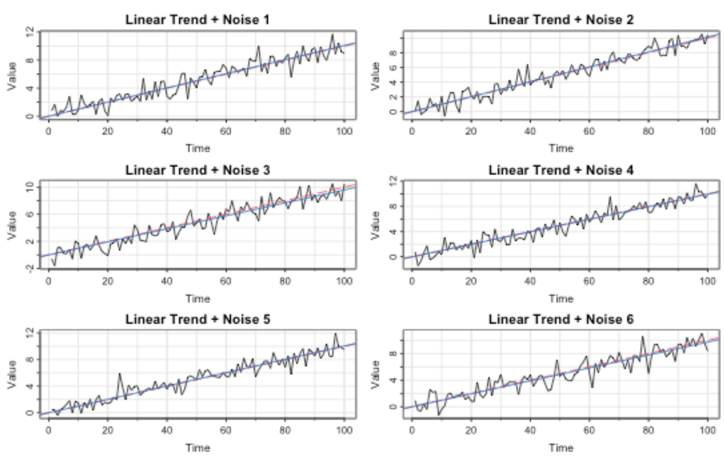
2.3

a) Random walk with drift



b) Linear trend plus noise



c) The trend of random walk with drift in part a) is stochastic and the process is non-stationary. There is a general trend of upward and downward over time. Also, the true mean (red line) and the fitted line (blue line) are disconnected with each other. So, the result of future influenced by the past value, but the random steps mean that the exact path is not perfectly predictable. However, the trend of linear plus noise in part b) is deterministic, which means a fixed slope. The true mean (red line) and the fitted line (blue line) are aligning with each other. and it is clear to predict the future value based on the established linear trend.

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Key Differences Summarized

Feature	Random Walk with Drift	Linear + Noise
Trend Nature	Stochastic/Random: Influenced by past values and random steps	Deterministic: A fixed, pre-determined line
Underlying Process	Sum of previous value, a constant drift, and random noise	A constant slope plus white noise
Predictability	Less predictable due to the random component	Predictable underlying trend, though individual points are random
Confidence Intervals	Widen over time as the random component accumulates	Do not widen significantly beyond the inherent noise

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2.6

a)

a) $x_t = \beta_0 + \beta_1 t + w_t$. , w_t with zero mean and variances σ_w^2

$E(x_t) = E(\beta_0 + \beta_1 t + w_t)$
$= \beta_0 + \beta_1 t + \underbrace{E(w_t)}_{=0}$
$= \beta_0 + \beta_1 t$

$\therefore$ x_t is nonstationary since the results rely on time and not constant.

b)

b) $\nabla x_t = x_t - x_{t-1}$
$= \beta_0 + \beta_1 t + w_t - \{\beta_0 + \beta_1 (t-1) + w_{t-1}\}$
$= \beta_1 t + w_t - \beta_1 (t-1) - w_{t-1}$

$= w_t - w_{t-1} + \beta_1$

$E(\nabla x_t) = E(w_t - w_{t-1} + \beta_1)$
$= \beta_1 + \underbrace{E(w_t)}_{0} - \underbrace{E(w_{t-1})}_{0}$
$= \beta_1$ (constant)

$Cov(A, B) = E[(A - E(A))(B - E(B))]$

$Cov(\nabla x_{t+h}, \nabla x_t) = Cov[(\beta_1 + w_{t+h} - w_{t+h-1}), (\beta_1 + w_t - w_{t-1})]$

$= Cov(w_{t+h} - w_{t+h-1}, w_t - w_{t-1})$

$=$ { $h = 0$, $= Cov(\nabla x_t, \nabla x_t) = Var(\nabla x_t) = Var(w_t - w_{t-1})$

$= Var(w_t) + Var(w_{t-1})$

$= \sigma_w^2 + \sigma_w^2 = 2\sigma_w^2$

$Cov(A+B, C+D) = Cov(A, C) + Cov(A, D) + Cov(B, C) + Cov(B, D)$

$|h| = 1$, $= Cov(\nabla x_{t+1}, \nabla x_t) = Cov(w_{t+1} - w_t, w_t - w_{t-1})$

$= \underbrace{Cov(w_{t+1}, w_t)}_{0} - \underbrace{Cov(w_{t+1}, w_{t-1})}_{0} - Cov(w_t, w_t) + \underbrace{Cov(w_t, w_{t-1})}_{0}$

$= -Cov(w_t, w_t)$ $Cov(A, A) = Var(A)$

$= -Var(w_t)$

$= -\sigma_w^2$

$|h| > 1$ no overlapping. $= 0$.

$\therefore$ The first difference series is stationary.

c)

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c) W_t is replaced by Y_t, μ_Y $\Rightarrow$ $X_t = \beta_0 + \beta_1 t + Y_t$.

general stationary process

$$\nabla X_t = X_t - X_{t-1}$$
$$= (\beta_0 + \beta_1 t + Y_t) - \{\beta_0 + \beta_1 (t-1) + Y_{t-1}\}$$
$$= \beta_1 + Y_t - Y_{t-1}$$

$$E(\nabla X_t) = E(\beta_1 + Y_t - Y_{t-1})$$
$$= \beta_1 + E(Y_t) - E(Y_{t-1})$$
$$= \beta_1 + \mu_Y - \mu_Y$$
$$= \beta_1 \rightarrow \text{is constant}$$

$$\mathrm{Cov}(\nabla X_{t+h}, \nabla X_t) = \mathrm{Cov}(Y_{t+h} - Y_{t+h-1}, Y_t - Y_{t-1})$$
$$= \mathrm{Cov}(Y_{t+h}, Y_t) - \mathrm{Cov}(Y_{t+h}, Y_{t-1}) - \mathrm{Cov}(Y_{t+h-1}, Y_t) + \mathrm{Cov}(Y_{t+h-1}, Y_{t-1})$$

$t+h-t=h$ $\quad$ $t+h-(t-1)=h+1$ $\quad$ $t+h-1-t=h-1$ $\quad$ $t+h-1-(t-1)=h$

$$= \gamma_Y(h) - \gamma_Y(h+1) - \gamma_Y(h-1) + \gamma_Y(h)$$
$$= 2\gamma_Y(h) - \gamma_Y(h+1) - \gamma_Y(h-1) \rightarrow \text{not depend on } t. \text{ but only time lag } h.$$

$\therefore$ it is stationary

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