Midterm

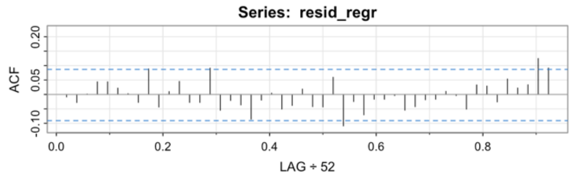
1. **Read and review Section 3.4. Do Problem 3.10.**
   a.

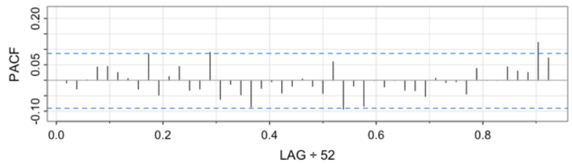
```
Coefficients:
     1       2
0.4286  0.4418

Intercept: 11.45 (2.394)

Order selected 2  sigma^2 estimated as  32.32
> regr$asy.se.coef
$x.mean
[1] 2.393673

$ar
[1] 0.03979433 0.03976163
```





in Example 2-2. M_t denotes cardiovascular mortality
T_t denotes temperature.
P_t denotes the particulate levels.

X_t represents the cardiovascular mortality series.
Therefore, based on the results of AR(2) using linear regression

		SE.	t.
ϕ_1 :	0.4286	0.0398	10.77
ϕ_2 :	0.4418.	0.0398.	11.10
ϕ_0 :	11.45.	2.394.	4.78.

e.g. $t = \frac{0.4286}{0.0398} = 10.77 > 1.96$

mean : 2.394. σ^2 : 32.32 , $n = 508$

$\therefore \hat{X_t} = 11.45 + 0.4286 X_{t-1} + 0.4418 X_{t-2}$

Since all t-statistics exceed the critical value of 1.96, we reject the null hypothesis that the coefficients are zero, which means all prior lagged values have a highly statistically significant on the current value.

Also, the plots of residuals show that almost all ACF and PACF fall in the 95% significance boundaries, which means the remaining errors are close to white noise.

b.

Horizon <int>	Forecast <dbl>	Lower_95 <dbl>	Upper_95 <dbl>
1	87.59986	76.45777	98.74196
2	86.76349	74.64117	98.88581
3	87.33714	73.35431	101.31997
4	87.21350	72.33079	102.09621

m represents weeks to forecast.

m	X^n_{n+m}	95% CI
1	87.5999	76.4578 , 98.7420
2	86.7635	74.6412 , 98.8858
3	87.3371	73.3543 , 101.3200
4.	87.2135	72.3308 , 102.0962

2. **Read Section 3.5 and Example 3.27, do Problem 3.17.**

   a.

```
        [,1]           [,2]
Method  "OLS (linear)" "Yule-Walker"
phi_1   "0.4285906"    "0.4339481"
phi_2   "0.4417874"    "0.4375768"
Sigma2  "32.31749"     "32.84056"
SE_phi1 "0.03979433"   "0.04001303"
SE_phi2 "0.03976163"   "0.04001303"
```

   b. The standard errors from both methods are highly similar in this question. This confirms the expectations from large-sample theory. Since the sample size (n=508) is quite large, the standard errors derived from OLS and those from the asymptotic distribution used by Yule-Walker converge.

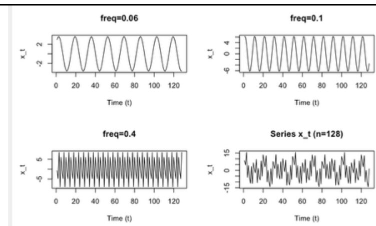
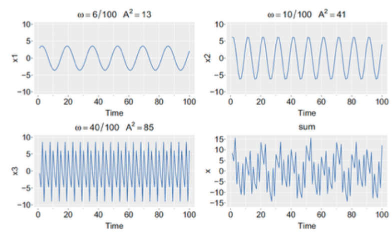
3. **Follow Example 3.30 to find the MLE for the AR(2) model in fitting the cardiovascular mortality series (cmort).**

```
> regmle = ar.mle(cmort, order=2)
> regmle$x.mean
[1] 88.6993
> regmle$ar
[1] 0.4300667 0.4424522
> sqrt(diag(regmle$asy.var.coef))
[1] 0.03972621 0.03972621
> regmle$var.pred
[1] 32.37144
```

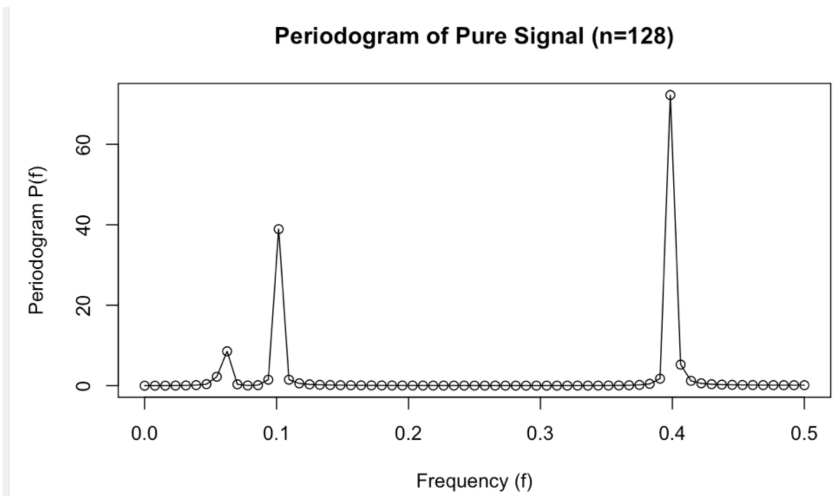
$\hat{X}t = 88.6993 + 0.4301(x_{t-1} - 88.6993) + 0.4425(x_{t-2} - 88.6993)$

4. Problem 4.2.

a.

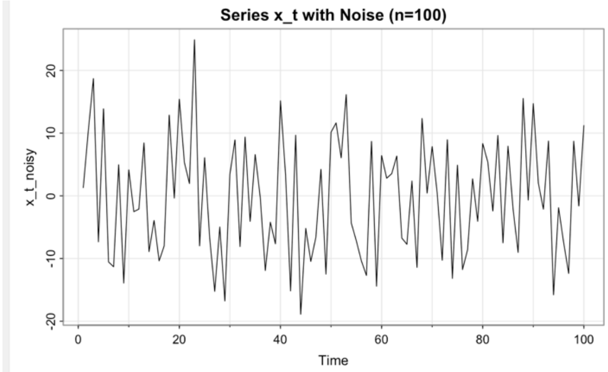
	Current question	Example question
Sample size	128	100
Frequencies	0.06; 0.1; 0.4	0.06; 0.1; 0.4
Calculation	Highly composite (2^7), may be computed quickly using the fast Fourier transform	Not highly composite
		 Fig. 4.2. Periodic components and their sum as described in Example 4.1

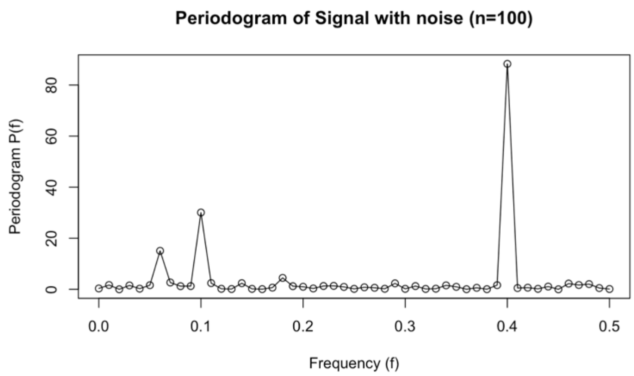
b.



The periodogram perfectly identifies and separates the three true underlying frequencies (0.06, 0.1, and 0.4) used to generate the series. Since there is no noise added, it is concentrated solely at these frequency points, resulting in a clean and flat periodogram. The contribution for f=0.4 is the largest.

c.





Unlike the pure signal, which has a flat baseline, this plot (noise added) shows a fluctuate background. For the periodogram, it still has three peaks (f=0.06, 0.1, and 0.4), which means the contributions are still focused on these frequency points. However, when comparing the height of three peaks, it seems that more contribution has shifted from f=0.1 to f=0.4, which may be caused by the difference in noise sensitivity (f=0.1 peak is more robust).