MFMC toy problem write up

1 Toy problem

In this model we consider the diffusion problem

$$\begin{aligned} &-\nabla \cdot (a_1(x,\xi_1)\nabla u(x)) = f_1(x), && x \in (0,1),\\ &-\nabla \cdot (a_2(x,\xi_2)\nabla v(x)) = f_2(x), && x \in (1,2),\\ &u(x) = g(x), && x = 0,\\ &v(x) = g(x), && x = 2,\\ &u(x) = v(x) && x = 1,\\ &a_1(x)\partial_\nu u(x) + a_2(x)\partial_\nu v(x) = 0, && x = 1. \end{aligned} \tag{1}$$

The diffusion coefficients and source terms for the toy problem are

$$a_1 = 1 + \xi_1, \quad a_2 = 10(1 + \xi_2), \quad f_1 = -2 - 3x^2, \quad f_2 = 1 - 6x.$$

2 Code implementation

This code employs a high-fidelity model with 5,121 grid points and eight low-fidelity models generated from a sparse grid, aligning with the same setup as the plasma problem.

1. `script_BuildSurrogforToy.m`: Builds surrogate models, which are later used for dynamic sampling.
2. `script_test_covar.m`: Handles dynamic sampling.
   - modify line 22 `tol` to adjust the stopping criterion threshold.
   - The model selection process (lines 195–238) computes parameters for the most recent and second most recent updates.
   - The stopping criterion is located around line 240—this is where adjustments may be needed.
   - All results are stored in `Result_test_covar`.
3. `FEM_solver.m`: Implements the finite element method to solve a 1D Poisson equation, with two parameters in the diffusion coefficient. (parameter dimension $d = 2$).
4. `Surrog_Eval.m`: Evaluates the surrogate models, serving as low-fidelity models.
5. `load_hfm_mesh_n_com_mesh_toy.m`: Loads the high-fidelity model mesh and a common mesh for interpolating solutions from low-fidelity models.
6. `load_lfm_toy.m`: Loads surrogates for low-fidelity models.
7. `MFMC_model_selection_exhausted.m`: Handles model selection using the exhausted method.
8. `MFMC_model_selection_backtrack.m`: Handles model selection using the backtrack method.
9. `L2_inprod_toy.m`: Computes the L2 inner product of two functions.
10. `interp2grid_toy.m`: Interpolates a function from a coarse mesh to a fine mesh.
11. `plot_n_print.m`: Generates and plots tables from the dynamic sampling results.