MFMC toy problem write up

1 Toy problem

In this model we consider the diffusion problem (1) with non-overlapping domain D_1 and D_2 with lipschitz boundary Γ_1 and Γ_2, both D_1 and D_2 share boundary Γ_2, denote $D = D_1 \cup D_2$.

$$\begin{aligned}
&-\nabla \cdot (a_1(x, y, \xi_1)\nabla u) = f_1(x, y), && \text{in } D_1,\\
&u = g(x, y), && \text{on } \Gamma_1,\\
\\
&-\nabla \cdot (a_2(x, y, \xi_2)\nabla v) = f_2(x, y), && \text{in } D_2,\\
&a_1\partial_\nu u + a_2\partial_\nu v = 0, && \text{on } \Gamma_2,\\
&u = v, && \text{on } \Gamma_2.
\end{aligned} \tag{1}$$

The Model problem (1) is equivalent to the following problem

$$\begin{aligned}
&-\nabla \cdot (d(x, y, \xi)\nabla u) = f(x, y), && \text{in } D,\\
&u = g(x, y), && \text{on } \Gamma_1,
\end{aligned} \tag{2}$$

where $d(x, y, \xi) = \begin{cases} a_1(x, y, \xi_1) & x \in D_1 \\ a_2(x, y, \xi_2) & x \in D_2 \end{cases}$ and $f(x, y) = \begin{cases} f_1(x, y) & x \in D_1 \\ f_2(x, y) & x \in D_2 \end{cases}$.

The diffusion coefficients and source terms for the toy problem are

$$a_1 = 1 + \xi_1, \quad a_2 = 10(1 + \xi_2), \quad f_1 = -2 - 3x^2, \quad f_2 = 1 - 6x.$$

Domain $D = [0, 2]$, $D_1 = [0, 1]$.

2 Code implementation

This code employs a high-fidelity model with 5,121 grid points and eight low-fidelity models generated from a sparse grid, aligning with the same setup as the plasma problem.

1. `script_BuildSurrogforToy.m`: Builds surrogate models, which are later used for dynamic sampling.

2. `script_test_covar.m`: Handles dynamic sampling.

   - modify line 22 `tol` to adjust the stopping criterion threshold.
   - The model selection process (lines 195–238) computes parameters for the most recent and second most recent updates.
   - The stopping criterion is located around line 240—this is where adjustments may be needed.
   - All results are stored in `Result_test_covar`.

3. `FEM_solver.m`: Implements the finite element method to solve a 1D Poisson equation, with two parameters in the diffusion coefficient. (parameter dimension $d = 2$).

4. `Surrog_Eval.m`: Evaluates the surrogate models, serving as low-fidelity models.

5. `load_hfm_mesh_n_com_mesh_toy.m`: Loads the high-fidelity model mesh and a common mesh for interpolating solutions from low-fidelity models.
6. `load_lfm_toy.m`: Loads surrogates for low-fidelity models.
7. `MFMC_model_selection_exhausted.m`: Handles model selection.
8. `L2_inprod_toy.m`: Computes the L2 inner product of two functions.
9. `interp2grid_toy.m`: Interpolates a function from a coarse mesh to a fine mesh.
10. `plot_n_print.m`: Generates and plots tables from the dynamic sampling results.