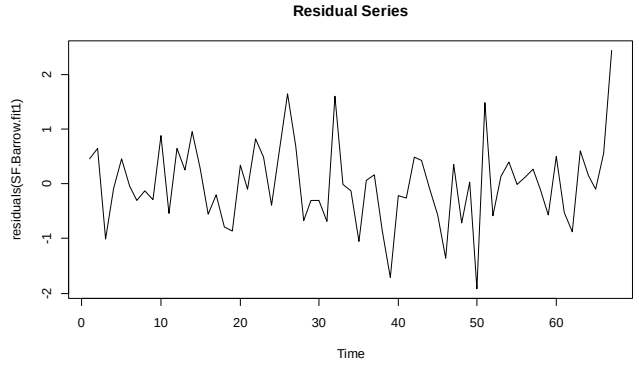
STATS 326
Applied Time Series
ASSIGNMENT THREE
R & MARKING GUIDE

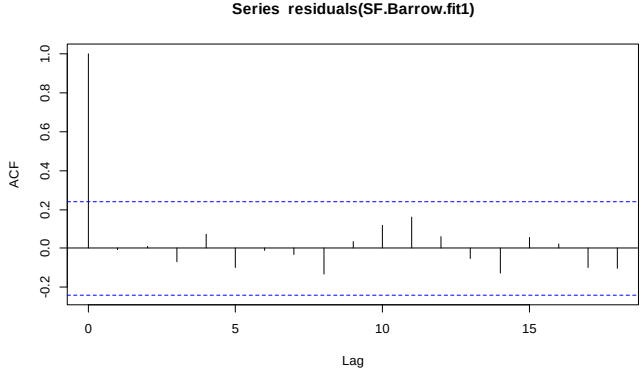
Question One: (20 marks)

Seasonal Factor:

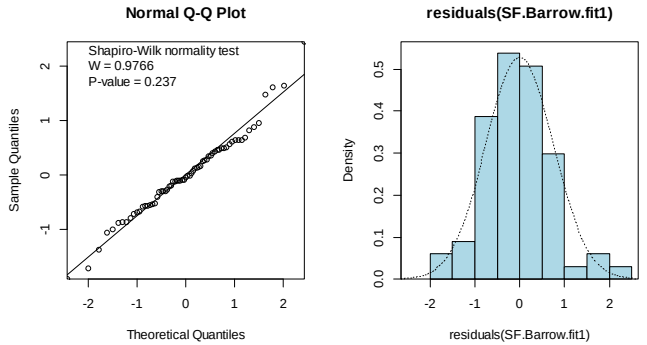
```
> Quarter = factor(rep(1:4,17))

> SF.Barrow.fit1 = lm(red.CO2.ts[-1]~Time[-1]+Quarter[-1]+red.CO2.ts[-68])
> plot.ts(residuals(SF.Barrow.fit1),main="Residual Series")
```




```
> acf(residuals(SF.Barrow.fit1))
```




```
> normcheck(residuals(SF.Barrow.fit1),shapiro.wilk=T)
```



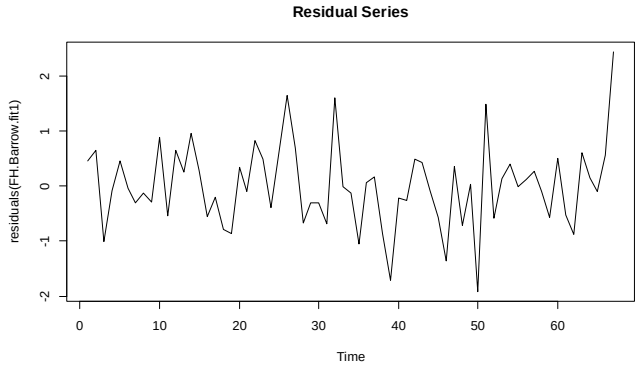

```
> summary(SF.Barrow.fit1)

Call:
lm(formula = red.CO2.ts[-1] ~ Time[-1] + Quarter[-1] + red.CO2.ts[-68])

Residuals:
     Min       1Q   Median       3Q      Max
-1.91405 -0.53160 -0.04121  0.45716  2.43353

Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)    203.99637   47.48575   4.296 6.35e-05 ***
Time[-1]         0.28656    0.06754   4.243 7.64e-05 ***
Quarter[-1]2    -2.92314    0.72227  -4.047 0.000148 ***
Quarter[-1]3   -16.03066    0.65394 -24.514  < 2e-16 ***
Quarter[-1]4    -0.92157    1.16188  -0.793 0.430753
red.CO2.ts[-68]  0.46122    0.12888   3.579 0.000684 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.7858 on 61 degrees of freedom
Multiple R-squared:  0.9958,    Adjusted R-squared:  0.9955
F-statistic:  2912 on 5 and 61 DF,  p-value: < 2.2e-16
```

```
> t.69.sf.pred = SF.Barrow.fit1$coef[1]+SF.Barrow.fit1$coef[2]*69+
   SF.Barrow.fit1$coef[6]*red.CO2.ts[68]
> t.69.sf.pred
(Intercept)
   411.7948
> t.70.sf.pred = SF.Barrow.fit1$coef[1]+SF.Barrow.fit1$coef[2]*70+
   SF.Barrow.fit1$coef[3]+SF.Barrow.fit1$coef[6]*t.69.sf.pred
> t.70.sf.pred
(Intercept)
   411.0606
> t.71.sf.pred = SF.Barrow.fit1$coef[1]+SF.Barrow.fit1$coef[2]*71+
   SF.Barrow.fit1$coef[4]+SF.Barrow.fit1$coef[6]*t.70.sf.pred
> t.71.sf.pred
(Intercept)
    397.901
> t.72.sf.pred = SF.Barrow.fit1$coef[1]+SF.Barrow.fit1$coef[2]*72+
   SF.Barrow.fit1$coef[5]+SF.Barrow.fit1$coef[6]*t.71.sf.pred
> t.72.sf.pred
(Intercept)
   407.2272

> SF.pred = c(t.69.sf.pred,t.70.sf.pred,t.71.sf.pred,t.72.sf.pred)
> names(SF.pred) = c("2017.1","2017.2","2017.3","2017.4")
> SF.pred
  2017.1   2017.2   2017.3   2017.4
411.7948 411.0606 397.9010 407.2272

> RMSEP.SF.Barrow = sqrt(1/4*sum((actual-SF.pred)^2))
> RMSEP.SF.Barrow
[1] 1.366159
```

Question Two: (25 marks)

Full Harmonic:

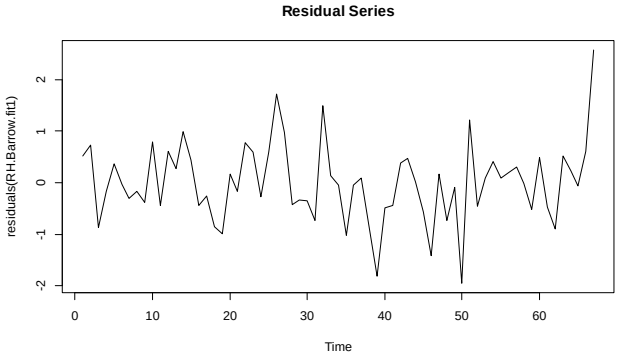
```
> c1 = cos(2*pi*Time*(1/4))
> s1 = sin(2*pi*Time*(1/4))
> c2 = cos(2*pi*Time*(2/4))
>
> FH.Barrow.fit1 = lm(red.CO2.ts[-1]~Time[-1]+c1[-1]+s1[-1]+c2[-1]+
   red.CO2.ts[-68])
> summary(FH.Barrow.fit1)

Call:
lm(formula = red.CO2.ts[-1] ~ Time[-1] + c1[-1] + s1[-1] + c2[-1] +
    red.CO2.ts[-68])

Residuals:
     Min       1Q   Median       3Q      Max
-1.91405 -0.53160 -0.04121  0.45716  2.43353

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)     199.02753   47.51872   4.188 9.19e-05 ***
Time[-1]          0.28656    0.06754   4.243 7.64e-05 ***
c1[-1]            1.00078    0.90884   1.101 0.275150
s1[-1]            8.01533    0.32697  24.514  < 2e-16 ***
c2[-1]            3.04649    0.28062  10.856 7.00e-16 ***
red.CO2.ts[-68]   0.46122    0.12888   3.579 0.000684 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.7858 on 61 degrees of freedom
Multiple R-squared:  0.9958,    Adjusted R-squared:  0.9955
F-statistic:  2912 on 5 and 61 DF,  p-value: < 2.2e-16

> plot.ts(residuals(FH.Barrow.fit1),main="Residual Series")
```



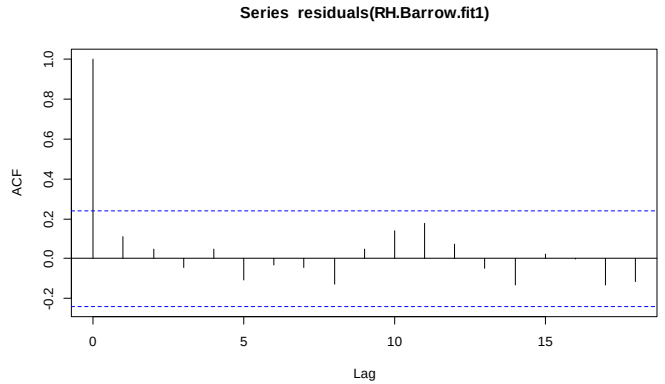
```
> t.69.fh.pred = FH.Barrow.fit1$coef[1]+FH.Barrow.fit1$coef[2]*69+
FH.Barrow.fit1$coef[3]*cos(2*pi*69*(1/4))+
FH.Barrow.fit1$coef[4]*sin(2*pi*69*(1/4))+
FH.Barrow.fit1$coef[5]*cos(2*pi*69*(2/4))+
FH.Barrow.fit1$coef[6]*red.CO2.ts[68]
> t.69.fh.pred
(Intercept)
   411.7948

> t.70.fh.pred = FH.Barrow.fit1$coef[1]+FH.Barrow.fit1$coef[2]*70+
FH.Barrow.fit1$coef[3]*cos(2*pi*70*(1/4))+
FH.Barrow.fit1$coef[4]*sin(2*pi*70*(1/4))+
FH.Barrow.fit1$coef[5]*cos(2*pi*70*(2/4))+
FH.Barrow.fit1$coef[6]*t.69.fh.pred
> t.70.fh.pred
(Intercept)
   411.0606

> t.71.fh.pred = FH.Barrow.fit1$coef[1]+FH.Barrow.fit1$coef[2]*71+
FH.Barrow.fit1$coef[3]*cos(2*pi*71*(1/4))+
FH.Barrow.fit1$coef[4]*sin(2*pi*71*(1/4))+
FH.Barrow.fit1$coef[5]*cos(2*pi*71*(2/4))+
FH.Barrow.fit1$coef[6]*t.70.fh.pred
> t.71.fh.pred
(Intercept)
    397.901

> t.72.fh.pred = FH.Barrow.fit1$coef[1]+FH.Barrow.fit1$coef[2]*72+
FH.Barrow.fit1$coef[3]*cos(2*pi*72*(1/4))+
FH.Barrow.fit1$coef[4]*sin(2*pi*72*(1/4))+
FH.Barrow.fit1$coef[5]*cos(2*pi*72*(2/4))+
FH.Barrow.fit1$coef[6]*t.71.fh.pred
> t.72.fh.pred
(Intercept)
   407.2272

> FH.pred = c(t.69.fh.pred,t.70.fh.pred,t.71.fh.pred,t.72.fh.pred)
> names(FH.pred) = c("2017.1","2017.2","2017.3","2017.4")
> FH.pred
  2017.1   2017.2   2017.3   2017.4
411.7948 411.0606 397.9010 407.2272

> RMSEP.FH.Barrow = sqrt(1/4*sum((actual-FH.pred)^2))
> RMSEP.FH.Barrow
[1] 1.366159
```

Reduced Full Harmonic:

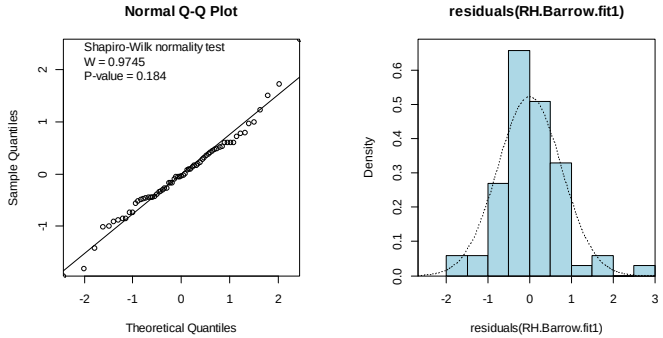
```
> RH.Barrow.fit1 = lm(red.CO2.ts[-1]~Time[-1]+s1[-1]+c2[-1]+
   red.CO2.ts[-68])
> summary(RH.Barrow.fit1)

Call:
lm(formula = red.CO2.ts[-1] ~ Time[-1] + s1[-1] + c2[-1] + red.CO2.ts[-68])

Residuals:
     Min       1Q   Median       3Q      Max
-1.94606 -0.44361 -0.03171  0.45547  2.57521

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)     250.77281    7.07232   35.46   <2e-16 ***
Time[-1]          0.35994    0.01102   32.67   <2e-16 ***
s1[-1]            7.69196    0.14402   53.41   <2e-16 ***
c2[-1]            2.75936    0.10388   26.56   <2e-16 ***
red.CO2.ts[-68]   0.32087    0.01916   16.75   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.7872 on 62 degrees of freedom
Multiple R-squared:  0.9957,    Adjusted R-squared:  0.9955
F-statistic:  3627 on 4 and 62 DF,  p-value: < 2.2e-16

> plot.ts(residuals(RH.Barrow.fit1),main="Residual Series")
```



```
> acf(residuals(RH.Barrow.fit1))
```




```
> normcheck(residuals(RH.Barrow.fit1),shapiro.wilk=T)
```




```
> t.69.rh.pred = RH.Barrow.fit1$coef[1]+RH.Barrow.fit1$coef[2]*69+
  RH.Barrow.fit1$coef[3]*sin(2*pi*69*(1/4))+
  RH.Barrow.fit1$coef[4]*cos(2*pi*69*(2/4))+
  RH.Barrow.fit1$coef[5]*red.CO2.ts[68]
> t.69.rh.pred
(Intercept)
   411.3501
> t.70.rh.pred = RH.Barrow.fit1$coef[1]+RH.Barrow.fit1$coef[2]*70+
  RH.Barrow.fit1$coef[3]*sin(2*pi*70*(1/4))+
  RH.Barrow.fit1$coef[4]*cos(2*pi*70*(2/4))+
  RH.Barrow.fit1$coef[5]*t.69.rh.pred
> t.70.rh.pred
(Intercept)
   410.7176
> t.71.rh.pred = RH.Barrow.fit1$coef[1]+RH.Barrow.fit1$coef[2]*71+
  RH.Barrow.fit1$coef[3]*sin(2*pi*71*(1/4))+
  RH.Barrow.fit1$coef[4]*cos(2*pi*71*(2/4))+
  RH.Barrow.fit1$coef[5]*t.70.rh.pred
> t.71.rh.pred
(Intercept)
   397.6639
> t.72.rh.pred = RH.Barrow.fit1$coef[1]+RH.Barrow.fit1$coef[2]*72+
  RH.Barrow.fit1$coef[3]*sin(2*pi*72*(1/4))+
  RH.Barrow.fit1$coef[4]*cos(2*pi*72*(2/4))+
  RH.Barrow.fit1$coef[5]*t.71.rh.pred
> t.72.rh.pred
(Intercept)
    407.046

> RH.pred = c(t.69.rh.pred,t.70.rh.pred,t.71.rh.pred,t.72.rh.pred)
> names(RH.pred) = c("2017.1","2017.2","2017.3","2017.4")
> RH.pred
  2017.1   2017.2   2017.3   2017.4
411.3501 410.7176 397.6639 407.0460

> RMSEP.RH.Barrow = sqrt(1/4*sum((actual-RH.pred)^2))
> RMSEP.RH.Barrow
[1] 1.677135
```

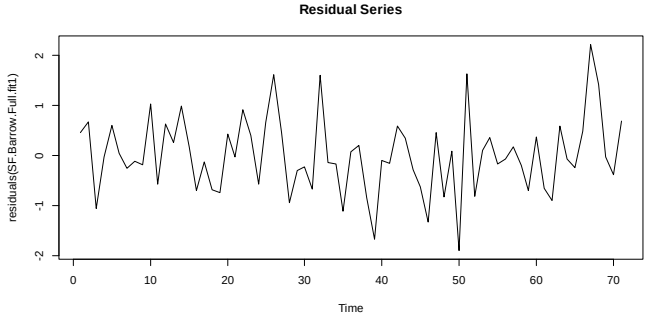
<u>Question Three</u>: (30 marks)

Tech Notes for Seasonal Factor model (**OR** Full Harmonic model)

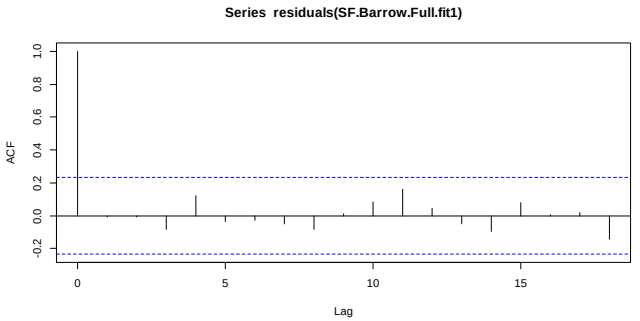
Question Four: (20 marks)

Seasonal Factor (Full):

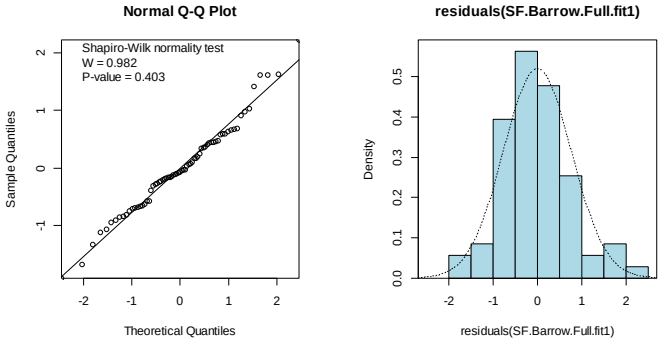
```
> Quarter.F = factor(rep(1:4,18))
> Time.F = 1:72

> SF.Barrow.Full.fit1 = lm(full.CO2.ts[-1]~Time.F[-1]+
   Quarter.F[-1]+full.CO2.ts[-72])
> summary(SF.Barrow.Full.fit1)

Call:
lm(formula = full.CO2.ts[-1] ~ Time.F[-1] + Quarter.F[-1] + full.CO2.ts[-
72])

Residuals:
     Min       1Q   Median       3Q      Max
-1.90255 -0.57295 -0.06903  0.45185  2.21890

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)    161.22208   37.53521   4.295 5.95e-05 ***
Time.F[-1]       0.22819    0.05463   4.177 9.00e-05 ***
Quarter.F[-1]2  -3.61378    0.59512  -6.072 7.27e-08 ***
Quarter.F[-1]3 -16.67694    0.53594 -31.117  < 2e-16 ***
Quarter.F[-1]4   0.04538    0.94576   0.048    0.962
full.CO2.ts[-72]   0.57734    0.10194   5.663 3.64e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.7952 on 65 degrees of freedom
Multiple R-squared:  0.9962,    Adjusted R-squared:  0.9959
F-statistic:  3404 on 5 and 65 DF,  p-value: < 2.2e-16

> plot.ts(residuals(SF.Barrow.Full.fit1),main="Residual Series")
```




```
> acf(residuals(SF.Barrow.Full.fit1))
```




```
> normcheck(residuals(SF.Barrow.Full.fit1),shapiro.wilk=T)
```



```
> t.73.sf.pred = SF.Barrow.Full.fit1$coef[1]+
   SF.Barrow.Full.fit1$coef[2]*73+
   SF.Barrow.Full.fit1$coef[6]*full.CO2.ts[72]
> t.73.sf.pred
(Intercept)
   413.6884

> t.74.sf.pred = SF.Barrow.Full.fit1$coef[1]+
   SF.Barrow.Full.fit1$coef[2]*74+SF.Barrow.Full.fit1$coef[3]*1+
   SF.Barrow.Full.fit1$coef[6]*t.73.sf.pred
> t.74.sf.pred
(Intercept)
   413.3328

> t.75.sf.pred = SF.Barrow.Full.fit1$coef[1]+
   SF.Barrow.Full.fit1$coef[2]*75+SF.Barrow.Full.fit1$coef[4]*1+
   SF.Barrow.Full.fit1$coef[6]*t.74.sf.pred
> t.75.sf.pred
(Intercept)
   400.2926

> t.76.sf.pred = SF.Barrow.Full.fit1$coef[1]+
   SF.Barrow.Full.fit1$coef[2]*76+SF.Barrow.Full.fit1$coef[5]*1+
   SF.Barrow.Full.fit1$coef[6]*t.75.sf.pred
> t.76.sf.pred
(Intercept)
   409.7145

> SF.Full.pred = c(t.73.sf.pred,t.74.sf.pred,t.75.sf.pred,t.76.sf.pred)
> names(SF.Full.pred) = c("2018.1","2018.2","2018.3","2018.4")
> SF.Full.pred
  2018.1   2018.2   2018.3   2018.4
413.6884 413.3328 400.2926 409.7145

> 413.6884-1.96*0.7952
[1] 412.1298
> 413.6884+1.96*0.7952
[1] 415.247

> 413.3328-1.96*0.7952*sqrt(1+0.57734^2)
[1] 411.5331
> 413.3328+1.96*0.7952*sqrt(1+0.57734^2)
[1] 415.1325

> 400.2926-1.96*0.7952*sqrt(1+0.57734^2+0.57734^4)
[1] 398.4194
> 400.2926+1.96*0.7952*sqrt(1+0.57734^2+0.57734^4)
[1] 402.1658

> 409.7145-1.96*0.7952*sqrt(1+0.57734^2+0.57734^4+0.57734^6)
[1] 407.8175
> 409.7145+1.96*0.7952*sqrt(1+0.57734^2+0.57734^4+0.57734^6)
[1] 411.6115

> 415.247-412.1298
[1] 3.1172
> 411.6115-407.8175
[1] 3.794
```