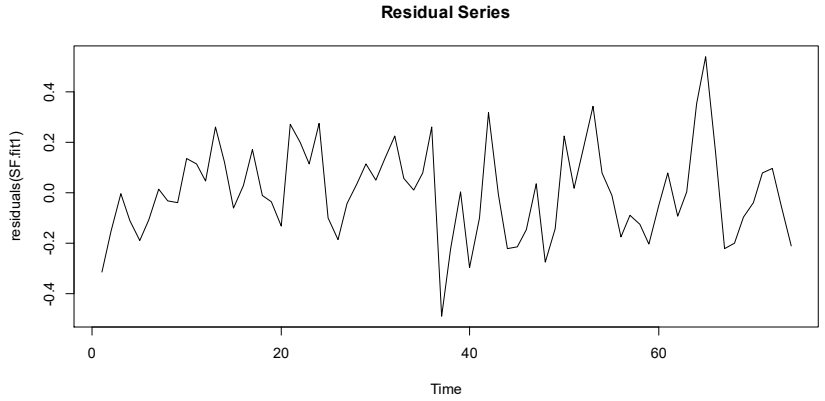
STATS 326
Applied Time Series
ASSIGNMENT THREE
R & MARKING GUIDE

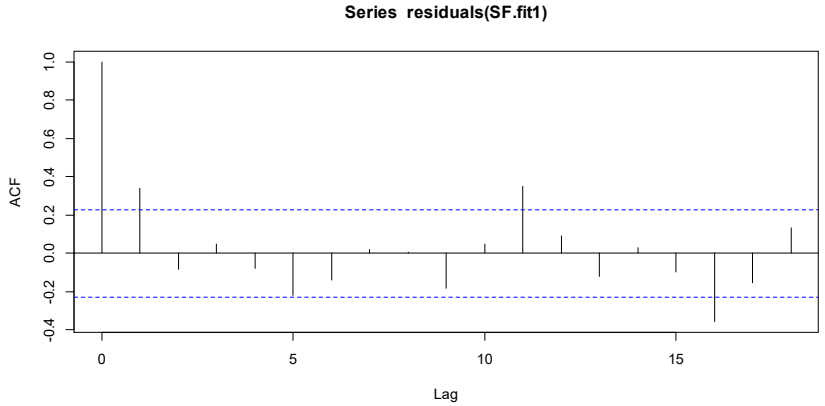
Question One: (20 marks)

Seasonal Factor:

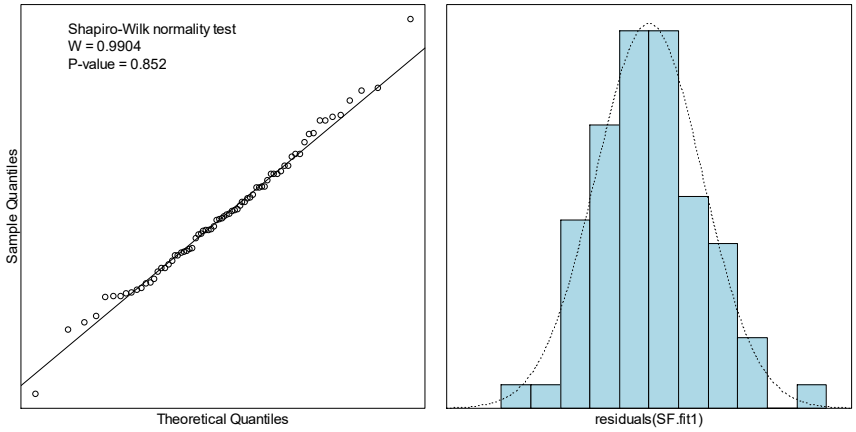
```
> red.Quarter = factor(c(rep(1:4,18),(1:3)))
> SF.fit1 = lm(red.CO2.ts[-1]~red.Time[-1]+red.Time.break[-1]+
   red.Quarter[-1]+red.CO2.ts[-75])

> plot.ts(residuals(SF.fit1),main="Residual Series")
```




```
> acf(residuals(SF.fit1))
```




```
> normcheck(residuals(SF.fit1),shapiro.wilk=T)
```




```
> summary(SF.fit1)

Call:
lm(formula = red.CO2.ts[-1] ~ red.Time[-1] + red.Time.break[-1] +
    red.Quarter[-1] + red.CO2.ts[-75])

Residuals:
     Min       1Q   Median       3Q      Max
-0.48990 -0.12209 -0.00581  0.11306  0.53995

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)        108.82449   29.39775   3.702 0.000435 ***
red.Time[-1]         0.14548    0.03860   3.769 0.000349 ***
red.Time.break[-1]   0.04182    0.01196   3.496 0.000843 ***
red.Quarter[-1]2     0.43876    0.07593   5.778 2.14e-07 ***
red.Quarter[-1]3     1.14763    0.07477  15.348  < 2e-16 ***
red.Quarter[-1]4     0.41510    0.06543   6.344 2.22e-08 ***
red.CO2.ts[-75]      0.70187    0.08043   8.727 1.18e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1889 on 67 degrees of freedom
Multiple R-squared:  0.9997,    Adjusted R-squared:  0.9997
F-statistic: 4.282e+04 on 6 and 67 DF,  p-value: < 2.2e-16
```

```
> t.76.pred = SF.fit1$coef[1]+SF.fit1$coef[2]*76+
  SF.fit1$coef[3]*26+SF.fit1$coef[6]+SF.fit1$coef[7]*red.CO2.ts[75]
> t.76.pred
(Intercept)
   406.0347

> t.77.pred = SF.fit1$coef[1]+SF.fit1$coef[2]*77+
  SF.fit1$coef[3]*27+SF.fit1$coef[7]*t.76.pred
> t.77.pred
(Intercept)
   406.1401

> t.78.pred = SF.fit1$coef[1]+SF.fit1$coef[2]*78+
  SF.fit1$coef[3]*28+SF.fit1$coef[4]+SF.fit1$coef[7]*t.77.pred
> t.78.pred
(Intercept)
   406.8401

> t.79.pred = SF.fit1$coef[1]+SF.fit1$coef[2]*79+
  SF.fit1$coef[3]*29+SF.fit1$coef[5]+SF.fit1$coef[7]*t.78.pred
> t.79.pred
(Intercept)
   408.2276

> SF.pred = c(t.76.pred,t.77.pred,t.78.pred,t.79.pred)
> names(SF.pred) = c("2018.4","2019.1","2019.2","2019.3")
> SF.pred
  2018.4   2019.1   2019.2   2019.3
406.0347 406.1401 406.8401 408.2276

> SF.RMSEP = sqrt(1/4*sum((actual-SF.pred)^2))
> SF.RMSEP
[1] 0.2384888
```

Question Two: (25 marks)

Full Harmonic:

```
> c1 = cos(2*pi*red.Time*(1/4))
> s1 = sin(2*pi*red.Time*(1/4))
> c2 = cos(2*pi*red.Time*(2/4))

> FH.fit1 = lm(red.CO2.ts[-1]~red.Time[-1]+red.Time.break[-1]+c1[-1]+
  s1[-1]+c2[-1]+red.CO2.ts[-75])

> summary(FH.fit1)

Call:
lm(formula = red.CO2.ts[-1] ~ red.Time[-1] + red.Time.break[-1] +
    c1[-1] + s1[-1] + c2[-1] + red.CO2.ts[-75])

Residuals:
     Min       1Q   Median       3Q      Max
-0.48990 -0.12209 -0.00581  0.11306  0.53995

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)        109.32486   29.38088   3.721 0.000408 ***
red.Time[-1]         0.14548    0.03860   3.769 0.000349 ***
red.Time.break[-1]   0.04182    0.01196   3.496 0.000843 ***
c1[-1]              -0.01183    0.04370  -0.271 0.787404
s1[-1]              -0.57381    0.03739 -15.348 < 2e-16 ***
c2[-1]              -0.07344    0.02232  -3.290 0.001597 **
red.CO2.ts[-75]      0.70187    0.08043   8.727 1.18e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1889 on 67 degrees of freedom
Multiple R-squared:  0.9997,    Adjusted R-squared:  0.9997
F-statistic: 4.282e+04 on 6 and 67 DF,  p-value: < 2.2e-16
```

```
> t.76.pred = FH.fit1$coef[1]+FH.fit1$coef[2]*76+FH.fit1$coef[3]*26+
  FH.fit1$coef[4]*cos(2*pi*76*(1/4))+ FH.fit1$coef[5]*sin(2*pi*76*(1/4))+
  FH.fit1$coef[6]*cos(2*pi*76*(2/4))+ FH.fit1$coef[7]*red.CO2.ts[75]
> t.76.pred
(Intercept)
   406.0347
> t.77.pred = FH.fit1$coef[1]+FH.fit1$coef[2]*77+FH.fit1$coef[3]*27+
  FH.fit1$coef[4]*cos(2*pi*77*(1/4))+ FH.fit1$coef[5]*sin(2*pi*77*(1/4))+
  FH.fit1$coef[6]*cos(2*pi*77*(2/4))+ FH.fit1$coef[7]*t.76.pred
> t.77.pred
(Intercept)
   406.1401

> t.78.pred = FH.fit1$coef[1]+FH.fit1$coef[2]*78+FH.fit1$coef[3]*28+
  FH.fit1$coef[4]*cos(2*pi*78*(1/4))+ FH.fit1$coef[5]*sin(2*pi*78*(1/4))+
  FH.fit1$coef[6]*cos(2*pi*78*(2/4))+ FH.fit1$coef[7]*t.77.pred
> t.78.pred
(Intercept)
   406.8401

> t.79.pred = FH.fit1$coef[1]+FH.fit1$coef[2]*79+FH.fit1$coef[3]*29+
  FH.fit1$coef[4]*cos(2*pi*79*(1/4))+ FH.fit1$coef[5]*sin(2*pi*79*(1/4))+
  FH.fit1$coef[6]*cos(2*pi*79*(2/4))+ FH.fit1$coef[7]*t.78.pred
> t.79.pred
(Intercept)
   408.2276

> FH.pred = c(t.76.pred,t.77.pred,t.78.pred,t.79.pred)
> names(FH.pred) = c("2018.4","2019.1","2019.2","2019.3")
> FH.pred
  2018.4   2019.1   2019.2   2019.3
406.0347 406.1401 406.8401 408.2276

> FH.RMSEP = sqrt(1/4*sum((actual-FH.pred)^2))
> FH.RMSEP
[1] 0.2384888
```

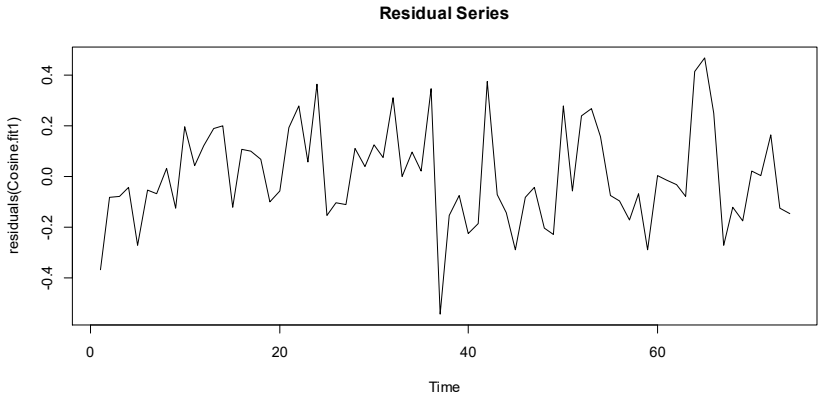
Reduced Harmonic:

```
> RH.fit1 = lm(red.CO2.ts[-1]~red.Time[-1]+red.Time.break[-1]+s1[-1]+
  c2[-1]+red.CO2.ts[-75])

> summary(RH.fit1)

Call:
lm(formula = red.CO2.ts[-1] ~ red.Time[-1] + red.Time.break[-1] +
    s1[-1] + c2[-1] + red.CO2.ts[-75])

Residuals:
      Min        1Q    Median        3Q       Max
-0.47624 -0.12632 -0.00989  0.11022  0.54821

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)       114.918225  20.749762   5.538 5.34e-07 ***
red.Time[-1]        0.152817   0.027317   5.594 4.29e-07 ***
red.Time.break[-1]  0.043931   0.008992   4.885 6.58e-06 ***
s1[-1]             -0.569849   0.034167 -16.678  < 2e-16 ***
c2[-1]             -0.072608   0.021955  -3.307  0.00151 **
red.CO2.ts[-75]     0.686561   0.056803  12.087  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1876 on 68 degrees of freedom
Multiple R-squared:  0.9997,    Adjusted R-squared:  0.9997
F-statistic: 5.209e+04 on 5 and 68 DF,  p-value: < 2.2e-16

> t.76.pred = RH.fit1$coef[1]+RH.fit1$coef[2]*76+RH.fit1$coef[3]*26+
  RH.fit1$coef[4]*sin(2*pi*76*(1/4))+RH.fit1$coef[5]*cos(2*pi*76*(2/4))+
  RH.fit1$coef[6]*red.CO2.ts[75]
> t.76.pred
(Intercept)
   406.0437

> t.77.pred = RH.fit1$coef[1]+RH.fit1$coef[2]*77+RH.fit1$coef[3]*27+
  RH.fit1$coef[4]*sin(2*pi*77*(1/4))+RH.fit1$coef[5]*cos(2*pi*77*(2/4))+
  RH.fit1$coef[6]*t.76.pred
> t.77.pred
(Intercept)
   406.1479

> t.78.pred = RH.fit1$coef[1]+RH.fit1$coef[2]*78+RH.fit1$coef[3]*28+
  RH.fit1$coef[4]*sin(2*pi*78*(1/4))+RH.fit1$coef[5]*cos(2*pi*78*(2/4))+
  RH.fit1$coef[6]*t.77.pred
> t.78.pred
(Intercept)
   406.8408

> t.79.pred = RH.fit1$coef[1]+RH.fit1$coef[2]*79+RH.fit1$coef[3]*29+
  RH.fit1$coef[4]*sin(2*pi*79*(1/4))+RH.fit1$coef[5]*cos(2*pi*79*(2/4))+
  RH.fit1$coef[6]*t.78.pred
> t.79.pred
(Intercept)
   408.2284
```

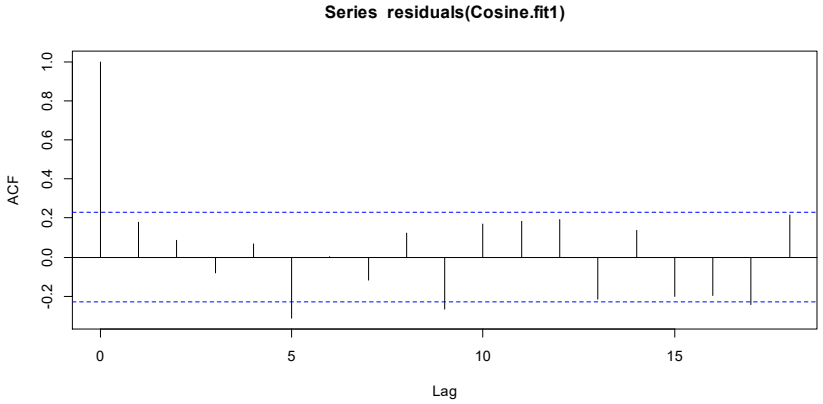
```
> RH.pred = c(t.76.pred,t.77.pred,t.78.pred,t.79.pred)
> names(RH.pred) = c("2018.4","2019.1","2019.2","2019.3")
> RH.pred
  2018.4   2019.1   2019.2   2019.3
406.0437 406.1479 406.8408 408.2284

> RH.RMSEP = sqrt(1/4*sum((actual-RH.pred)^2))
> RH.RMSEP
[1] 0.2438801
```

Cosine Harmonic:

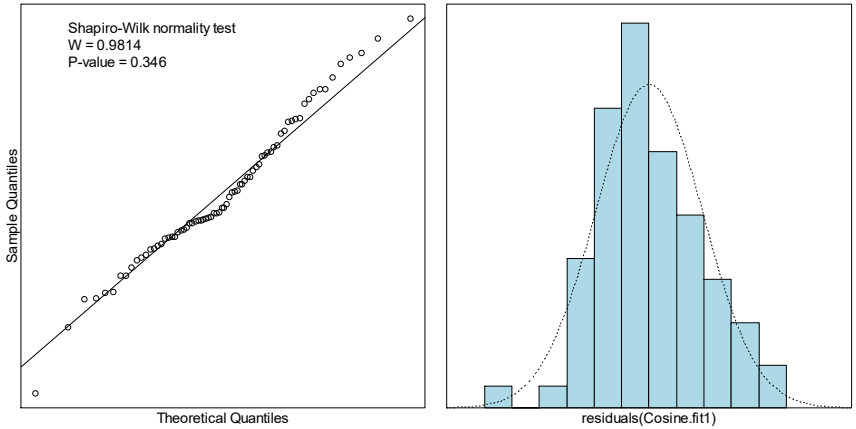
```
> Seasonal = cos((2*pi*(red.Time-3))/4)

> Cosine.fit1 = lm(red.CO2.ts[-1]~red.Time[-1]+red.Time.break[-1]+
  Seasonal[-1]+red.CO2.ts[-75])

> plot.ts(residuals(Cosine.fit1),main="Residual Series")
```




```
> acf(residuals(Cosine.fit1))
```



```
> normcheck(residuals(Cosine.fit1),shapiro.wilk=T)
```




```
> summary(Cosine.fit1)

Call:
lm(formula = red.CO2.ts[-1] ~ red.Time[-1] + red.Time.break[-1] +
    Seasonal[-1] + red.CO2.ts[-75])

Residuals:
     Min       1Q   Median       3Q      Max
-0.54471 -0.12082 -0.04226  0.11895  0.46973

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)       122.68788   22.05090   5.564 4.67e-07 ***
red.Time[-1]        0.16307    0.02903   5.618 3.78e-07 ***
red.Time.break[-1]  0.04691    0.00957   4.902 6.05e-06 ***
Seasonal[-1]        0.56627    0.03653  15.503  < 2e-16 ***
red.CO2.ts[-75]     0.66529    0.06037  11.021  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2006 on 69 degrees of freedom
Multiple R-squared:  0.9997,    Adjusted R-squared:  0.9997
F-statistic: 5.692e+04 on 4 and 69 DF,  p-value: < 2.2e-16
```

```
> t76.pred = Cosine.fit1$coef[1]+Cosine.fit1$coef[2]*76+
Cosine.fit1$coef[3]*26+Cosine.fit1$coef[4]*cos((2*pi*(76-3))/4)+
Cosine.fit1$coef[5]*red.CO2.ts[75]
> t76.pred
(Intercept)
   406.1144

> t77.pred = Cosine.fit1$coef[1]+Cosine.fit1$coef[2]*77+
Cosine.fit1$coef[3]*27+Cosine.fit1$coef[4]*cos((2*pi*(77-3))/4)+
Cosine.fit1$coef[5]*t76.pred
> t77.pred
(Intercept)
   406.1269

> t78.pred = Cosine.fit1$coef[1]+Cosine.fit1$coef[2]*78+
Cosine.fit1$coef[3]*28+Cosine.fit1$coef[4]*cos((2*pi*(78-3))/4)+
Cosine.fit1$coef[5]*t77.pred
> t78.pred
(Intercept)
   406.9115

> t79.pred = Cosine.fit1$coef[1]+Cosine.fit1$coef[2]*79+
Cosine.fit1$coef[3]*29+Cosine.fit1$coef[4]*cos((2*pi*(79-3))/4)+
Cosine.fit1$coef[5]*t78.pred
> t79.pred
(Intercept)
   408.2097

> Cosine.pred = c(t.76.pred,t.77.pred,t.78.pred,t.79.pred)
> names(Cosine.pred) = c("2018.4","2019.1","2019.2","2019.3")
> Cosine.pred
  2018.4   2019.1   2019.2   2019.3
406.1144 406.1269 406.9115 408.2097

> Cosine.RMSEP = sqrt(1/4*sum((actual-Cosine.pred)^2))
> Cosine.RMSEP
[1] 0.264877
```

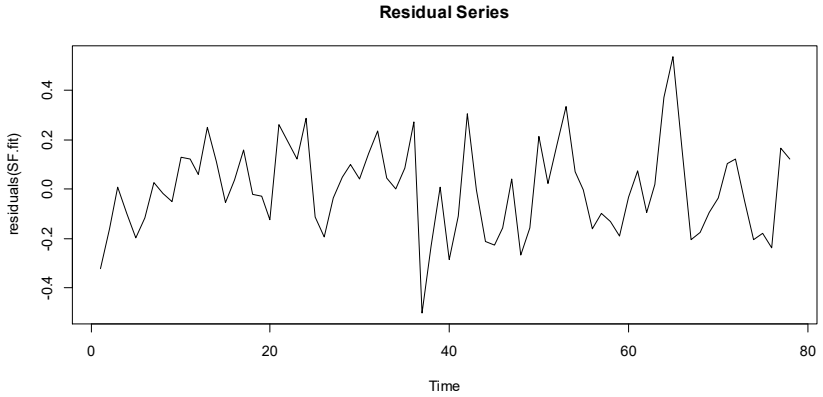
<u>Question Three</u>: (30 marks)

Tech Notes for Seasonal Factor model (**OR** Full Harmonic model)

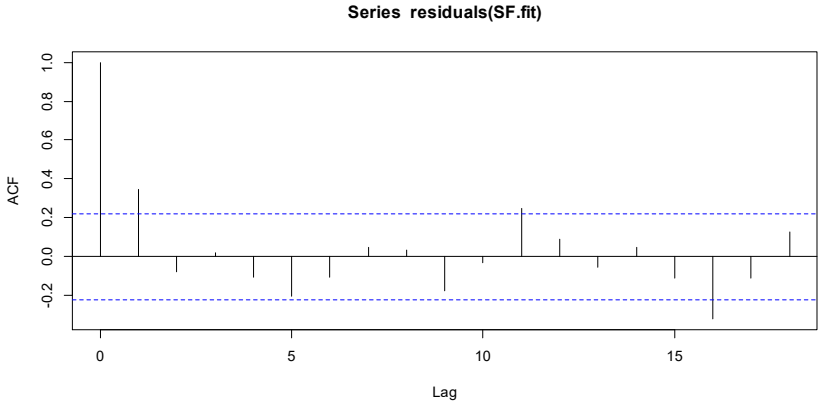
Question Four: (20 marks)

Seasonal Factor (Full):

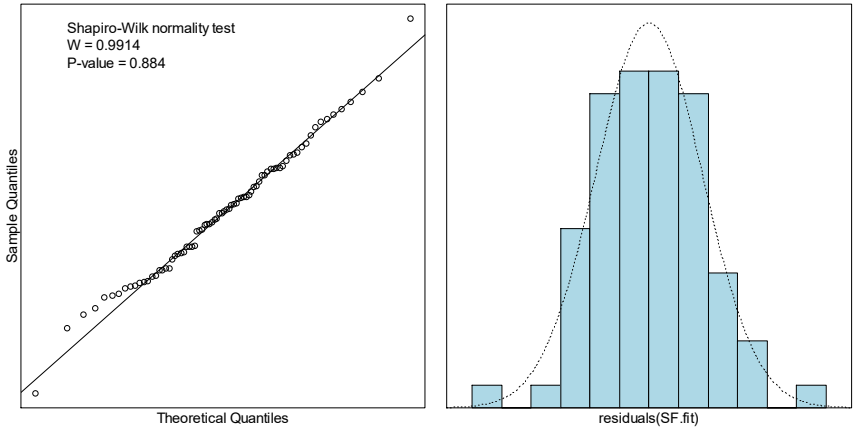
```
> full.Quarter = factor(c(rep(1:4,19),(1:3)))

> SF.fit = lm(full.CO2.ts[-1]~full.Time[-1]+full.Time.break[-1]+
   full.Quarter[-1]+full.CO2.ts[-79])

> plot.ts(residuals(SF.fit),main="Residual Series")
```




```
> acf(residuals(SF.fit))
```




```
> normcheck(residuals(SF.fit),shapiro.wilk=T)
```




```
> summary(SF.fit)

Call:
lm(formula = full.CO2.ts[-1] ~ full.Time[-1] + full.Time.break[-1] +
    full.Quarter[-1] + full.CO2.ts[-79])

Residuals:
      Min        1Q    Median        3Q       Max
-0.50285 -0.12867 -0.00066  0.12102  0.53787

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)         108.36491   28.78665   3.764 0.000341 ***
full.Time[-1]         0.14500    0.03785   3.831 0.000273 ***
full.Time.break[-1]   0.04082    0.01114   3.665 0.000474 ***
full.Quarter[-1]2     0.46150    0.07438   6.205 3.25e-08 ***
full.Quarter[-1]3     1.16834    0.07242  16.132  < 2e-16 ***
full.Quarter[-1]4     0.41786    0.06380   6.549 7.79e-09 ***
full.CO2.ts[-79]      0.70309    0.07876   8.927 3.20e-13 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.1888 on 71 degrees of freedom
Multiple R-squared:  0.9998,    Adjusted R-squared:  0.9997
F-statistic: 5.127e+04 on 6 and 71 DF,  p-value: < 2.2e-16
```

```
> t.80.pred = SF.fit$coef[1]+SF.fit$coef[2]*80+
   SF.fit$coef[3]*30+SF.fit$coef[6]+SF.fit$coef[7]*full.CO2.ts[79]
> t.80.pred
(Intercept)
   408.6447

> t.81.pred = SF.fit$coef[1]+SF.fit$coef[2]*81+
   SF.fit$coef[3]*31+SF.fit$coef[7]*t.80.pred
> t.81.pred
(Intercept)
   408.6901

> t.82.pred = SF.fit$coef[1]+SF.fit$coef[2]*82+
   SF.fit$coef[3]*32+SF.fit$coef[4]+SF.fit$coef[7]*t.81.pred
> t.82.pred
(Intercept)
   409.3694

> t.83.pred = SF.fit$coef[1]+SF.fit$coef[2]*83+
   SF.fit$coef[3]*33+SF.fit$coef[5]+SF.fit$coef[7]*t.82.pred
> t.83.pred
(Intercept)
   410.7396

> SF.Full.pred = c(t.80.pred,t.81.pred,t.82.pred,t.83.pred)
> names(SF.Full.pred) = c("2019.4","2020.1","2020.2","2020.3")
> SF.Full.pred
  2019.4   2020.1   2020.2   2020.3
408.6447 408.6901 409.3694 410.7396
```