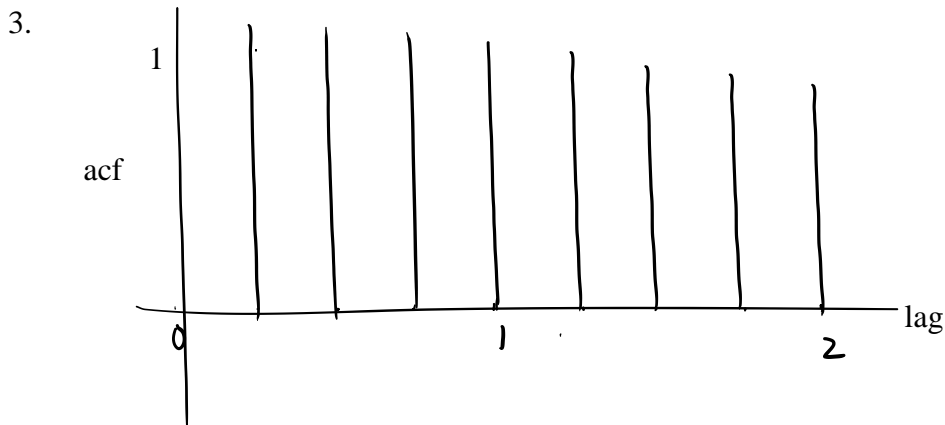


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1. Because when we plot the acf of the raw data, it might have other patterns like trend, cycle, and seasonality. These patterns together with autocorrelation will appear in the acf plot, which makes it harder to detect autocorrelation. When we plot the Residual Series, if we have modelled all the other patterns (trend, cycle, seasonality) successfully, it will just show autocorrelation (if present). From there, it is easier to see if we can model the autocorrelation successfully by fitting a lagged response.
2. When the residual series shows no clustering or oscillations and the plot of acf shows no significant lags (other than lag0, which is always = 1).



4. $\pm \frac{1.96}{\sqrt{625}} = \pm 0.0784$

5.

$$(1 - B)(1 - B^4)^2 y_t = \varepsilon_t$$

$$(1 - B)(1 - 2B^4 + B^8) y_t = \varepsilon_t$$

$$(1 - B - 2B^4 + 2B^5 + B^8 - B^9) y_t = \varepsilon_t$$

$$y_t - y_{t-1} - 2y_{t-4} + 2y_{t-5} + y_{t-8} - y_{t-9} = \varepsilon_t$$

$$y_t = \varepsilon_t + y_{t-1} + 2y_{t-4} + 2y_{t-5} - y_{t-8} + y_{t-9}$$

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6. The plot of monthly Atmospheric N₂O appears to have a positive increasing linear trend with reasonably constant seasonal components. Although the seasonality is not so obvious around year 2014. The linear trend seems to be very steady and the global N₂O has increased from 316 to 332 ppm, with an increase of approximately 1 ppm per year.

7.

$$2019.7.pred = 331.7998 + 0.0787 * 1 + 0.2211 = 332.0996 \text{ ppm}$$

$$2019.8.pred = 331.7998 + 0.0787 * 2 + 0.1832 = 332.1404 \text{ ppm}$$

$$2019.9.pred = 331.7998 + 0.0787 * 3 + 0.1209 = 332.1568 \text{ ppm}$$

$$2019.10.pred = 331.7998 + 0.0787 * 4 + 0.0192 = 332.1338 \text{ ppm}$$

8. a. We can see clear positive increasing linear trend with a possible break – change in slope somewhere around 2007. The slope after that became slightly steeper.
- b. There is still very slight clustering in the Residual Series, but it seems to come from a normal distribution with mean 0 and reasonably constant variance. The plot of acf showed only lag1 is slightly significant. It is very small autocorrelation so we can ignore it. Check residual in the summary, we can see the absolute value of min (-0.1668) is very similar to the max (0.1572), same for 1Q (-0.0402) and 3Q (0.0407), and the median is very close to 0 (0.0009). The residual standard error is very small as well (0.05969). We can be confident that the residual is normally distributed. So the assumptions are satisfied.

9.

$$\begin{aligned} 2019.7.sa.pred &= 32.04 + 0.006196 \times 223 + 0.001899 \times (223 - 80) \\ &\quad + 0.8987 \times (331.7 - (-0.12769267)) = 331.9068 \text{ ppm} \end{aligned}$$

$$2019.7.pred = 331.9068 - 0.18337078 = 331.7234 \text{ ppm}$$

$$\begin{aligned} 2019.8.sa.pred &= 32.04 + 0.006196 \times 224 + 0.001899 \times (224 - 80) \\ &\quad + 0.8987 \times 331.9068 = 331.986 \text{ ppm} \end{aligned}$$

$$2019.8.pred = 331.986 - 0.18823189 = 331.7978 \text{ ppm}$$

$$\begin{aligned} 2019.9.sa.pred &= 32.04 + 0.006196 \times 225 + 0.001899 \times (225 - 80) \\ &\quad + 0.8987 \times 331.986 = 332.0653 \text{ ppm} \end{aligned}$$

$$2019.9.pred = 332.0653 - 0.13198189 = 331.9333 \text{ ppm}$$

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$$\begin{aligned} 2019.10.sa.pred &= 32.04 + 0.006196 \times 226 + 0.001899 \times (226 - 80) \\ &\quad + 0.8987 \times 332.0653 = 332.1446 \text{ ppm} \end{aligned}$$

$$2019.10.pred = 332.1446 - 0.04263004 = 332.102 \text{ ppm}$$

$$\begin{aligned} RMSEP &= \sqrt{\frac{1}{4} \{ (2019.7.pred - actual)^2 + (2019.8.pred - actual)^2 + \\ &\quad (2019.9.pred - actual)^2 + (2019.10.pred - actual)^2 \}} \\ &= \sqrt{\frac{1}{4} \{ (331.7234 - 331.9)^2 + (331.7978 - 331.9)^2 + \\ &\quad (331.9333 - 331.9)^2 + (332.102 - 332.1)^2 \}} \\ &= 0.1033747 \text{ ppm} \end{aligned}$$

10. Moving Average Seasonally Adjusted model is better since it has smaller RMSEP (0.1033747 ppm) than the Holt-winters exponential smoothing model (0.1197699 ppm).