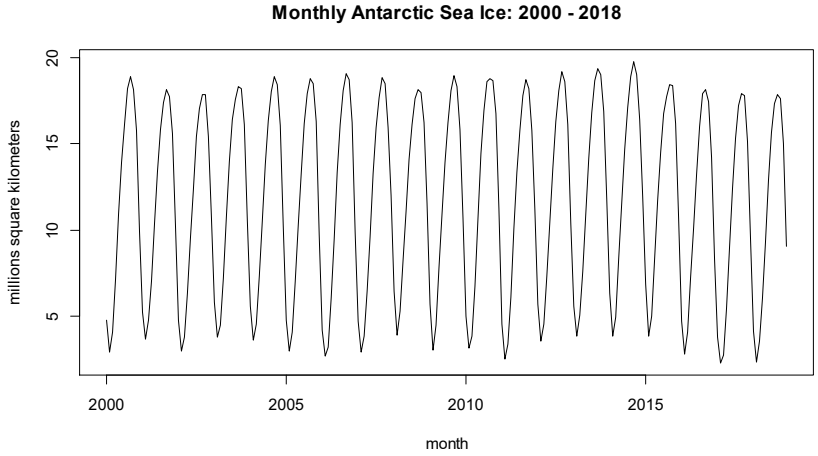
Department of Statistics
STATS 326: Applied Time Series
Summer Semester, 2020
Test 2

Appendix

Data: These data are monthly measurements of the area of sea ice (in millions of square kilometres) in the Antarctic Ocean between 2000 and 2018. We also have the values for the first 3 months of 2019.

```
> Ice.ts = ts(Ice.df$Ice[1:228],frequency=12,start=2000)

Year  Month Ice
2000  1      4.749
2000  2      2.907
2000  3      4.086
…
2018  10    17.655
2018  11    15.010
2018  12     9.033
```

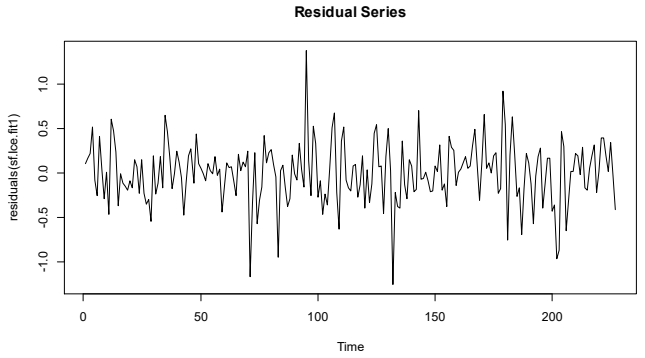
```
> plot.ts(Ice.ts,xlab="month",ylab="millions square kilometers",
   main="Monthly Antarctic Sea Ice: 2000 - 2018")
```



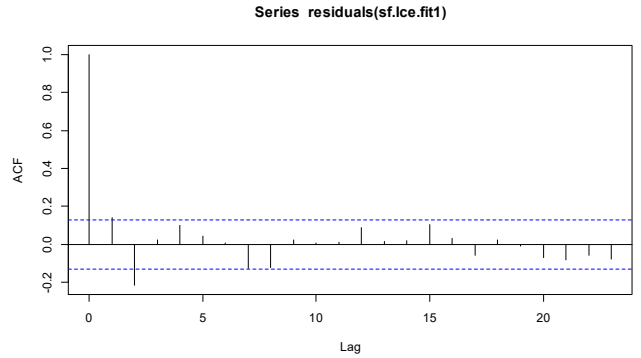

```
> actual
Jan 2019 Feb 2019 Mar 2019
   3.831    2.656    3.164
```


Seasonal Factor Model:

```
> Time = 1:228

> Month = factor(rep(1:12,19))

> Time.break = c(rep(0,185),Time[186:228]-Time[186])
> Time.break
  [1]  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 [25]  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 [49]  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 [73]  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
 [97]  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
[121]  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
[145]  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
[169]  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  1  2  3  4  5  6
[193]  7  8  9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
[217] 31 32 33 34 35 36 37 38 39 40 41 42

> sf.Ice.fit1 = lm(Ice.ts[-1]~Time[-1]+Time.break[-1]+Month[-1]+
   Ice.ts[-228])

> plot.ts(residuals(sf.Ice.fit1),main="Residual Series")
```




```
> acf(residuals(sf.Ice.fit1))
```



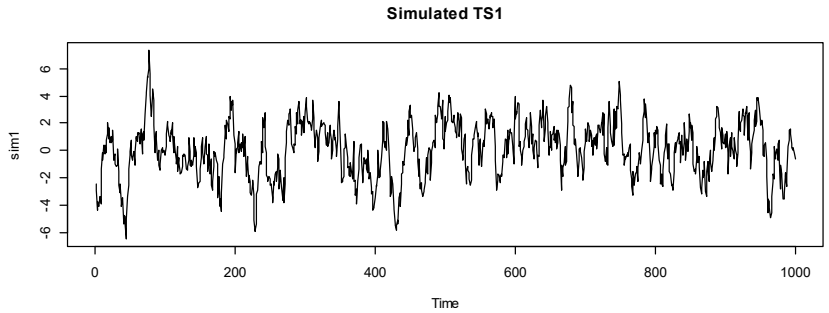
```
> summary(sf.Ice.fit1)

Call:
lm(formula = Ice.ts[-1] ~ Time[-1] + Time.break[-1] + Month[-1] +
    Ice.ts[-228])

Residuals:
     Min       1Q   Median       3Q      Max
-1.25460 -0.19714  0.02134  0.19434  1.37931

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)    -2.8807641  0.4757399  -6.055 6.29e-09 ***
Time[-1]        0.0009187  0.0004919   1.868 0.063183 .
Time.break[-1] -0.0134772  0.0038094  -3.538 0.000496 ***
Month[-1]2      2.0865614  0.2713194   7.690 5.43e-13 ***
Month[-1]3      4.5973477  0.3553030  12.939  < 2e-16 ***
Month[-1]4      6.6990413  0.3125032  21.437  < 2e-16 ***
Month[-1]5      7.8797133  0.1976488  39.867  < 2e-16 ***
Month[-1]6      8.5835764  0.1159246  74.044  < 2e-16 ***
Month[-1]7      8.7152266  0.1806313  48.249  < 2e-16 ***
Month[-1]8      8.4919778  0.2808867  30.233  < 2e-16 ***
Month[-1]9      7.9445210  0.3539011  22.448  < 2e-16 ***
Month[-1]10     7.0022192  0.3866507  18.110  < 2e-16 ***
Month[-1]11     4.9090470  0.3706908  13.243  < 2e-16 ***
Month[-1]12     1.3277019  0.2703557   4.911 1.81e-06 ***
Ice.ts[-228]    0.7563822  0.0454494  16.642  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.3515 on 212 degrees of freedom
Multiple R-squared:  0.9963,    Adjusted R-squared:  0.996
F-statistic:  4056 on 14 and 212 DF,  p-value: < 2.2e-16

> sf.pred
Jan 2019 Feb 2019 Mar 2019
******** ******** 2.482470
```

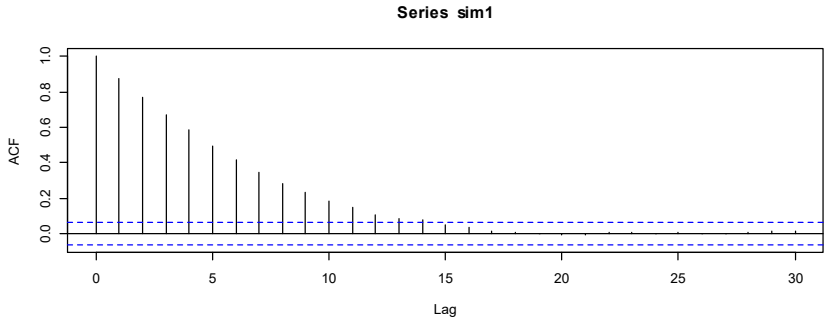

Full Harmonic Model:

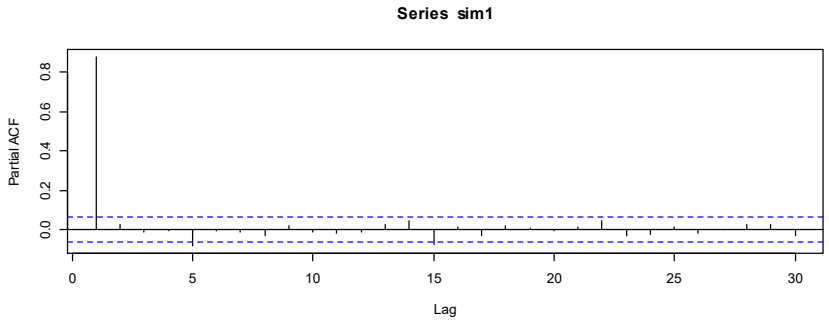
```
> c1 = cos(2*pi*Time*(1/12))
> s1 = sin(2*pi*Time*(1/12))
> c2 = cos(2*pi*Time*(2/12))
> s2 = sin(2*pi*Time*(2/12))
> c3 = cos(2*pi*Time*(3/12))
> s3 = sin(2*pi*Time*(3/12))
> c4 = cos(2*pi*Time*(4/12))
> s4 = sin(2*pi*Time*(4/12))
> c5 = cos(2*pi*Time*(5/12))
> s5 = sin(2*pi*Time*(5/12))
> c6 = cos(2*pi*Time*(6/12))

> fh.Ice.fit1 = lm(Ice.ts[-1]~Time[-1]+Time.break[-1]+c1[-1]+s1[-1]+
   c2[-1]+s2[-1]+c3[-1]+s3[-1]+c4[-1]+s4[-1]+c5[-1]+s5[-1]+c6[-1]+
   Ice.ts[-228])
> summary(fh.Ice.fit1)

Call:
lm(formula = Ice.ts[-1] ~ Time[-1] + Time.break[-1] + c1[-1] +
    s1[-1] + c2[-1] + s2[-1] + c3[-1] + s3[-1] + c4[-1] + s4[-1] +
    c5[-1] + s5[-1] + c6[-1] + Ice.ts[-228])

Residuals:
     Min       1Q   Median       3Q      Max
-1.25460 -0.19714  0.02134  0.19434  1.37931

Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)     2.8056470  0.5231811   5.363 2.14e-07 ***
Time[-1]        0.0009187  0.0004919   1.868 0.063183 .
Time.break[-1] -0.0134772  0.0038094  -3.538 0.000496 ***
c1[-1]         -3.4045456  0.1140914 -29.841  < 2e-16 ***
s1[-1]         -2.0048778  0.3351692  -5.982 9.28e-09 ***
c2[-1]         -0.6697495  0.0682078  -9.819  < 2e-16 ***
s2[-1]         -1.0386899  0.0373859 -27.783  < 2e-16 ***
c3[-1]         -0.1922727  0.0365173  -5.265 3.43e-07 ***
s3[-1]         -0.3995645  0.0332283 -12.025  < 2e-16 ***
c4[-1]         -0.0731244  0.0332145  -2.202 0.028774 *
s4[-1]         -0.1372380  0.0331052  -4.146 4.90e-05 ***
c5[-1]         -0.0311189  0.0330624  -0.941 0.347664
s5[-1]         -0.0682734  0.0329924  -2.069 0.039724 *
c6[-1]          0.0121019  0.0233343   0.519 0.604560
Ice.ts[-228]    0.7563822  0.0454494  16.642  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.3515 on 212 degrees of freedom
Multiple R-squared:  0.9963,    Adjusted R-squared:  0.996
F-statistic:  4056 on 14 and 212 DF,  p-value: < 2.2e-16
```

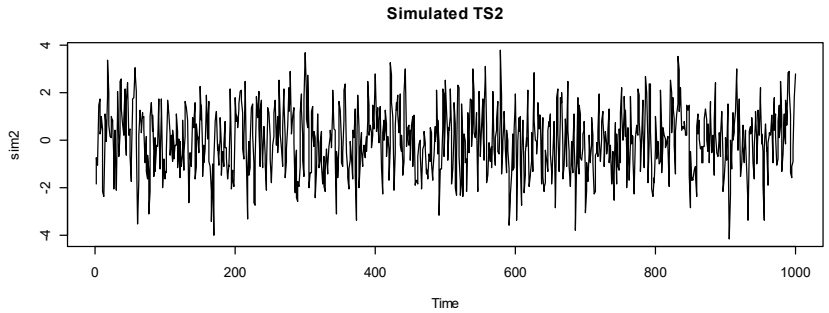
Model Type / Month	Jan 2019	Feb 2019	Mar 2019		**RMSEP**
Significant Pairs	3.600	1.532	2.494		0.768
Significant Harmonics	3.567	1.524	2.487		0.777

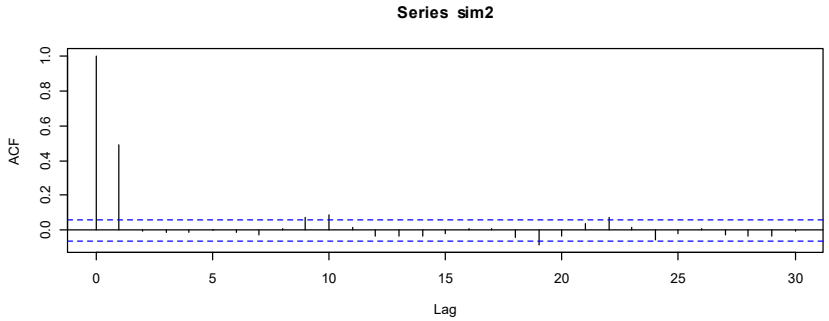
Simulated Stationary Time Series:

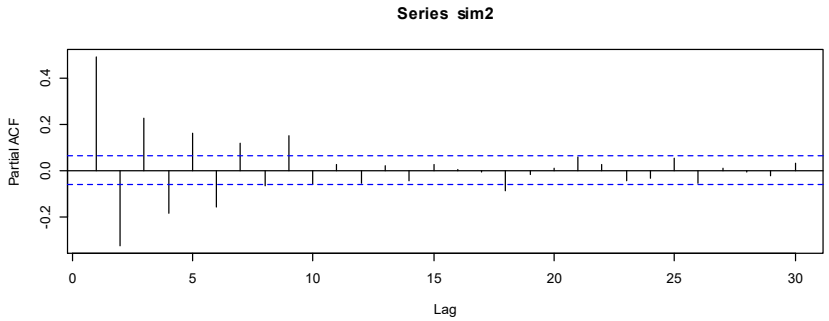


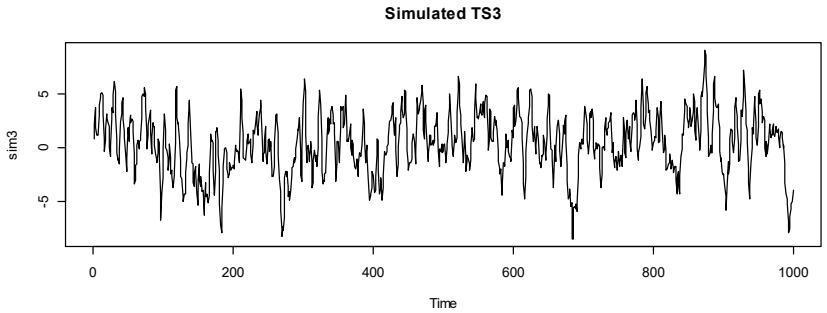


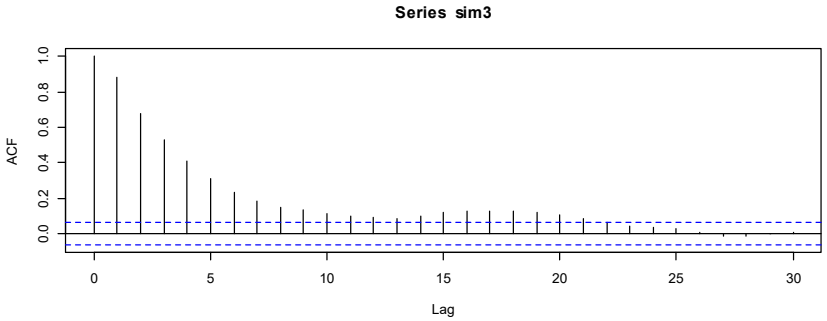


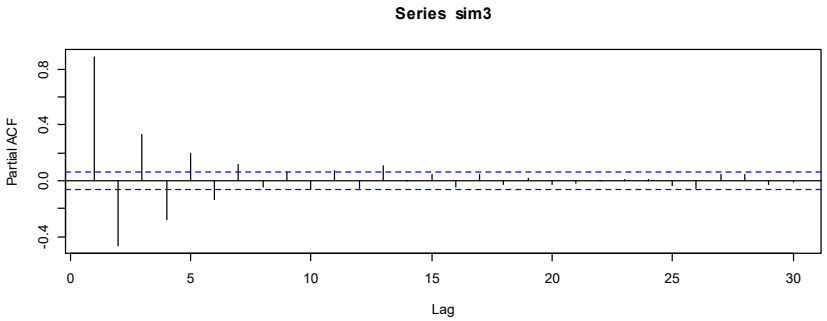


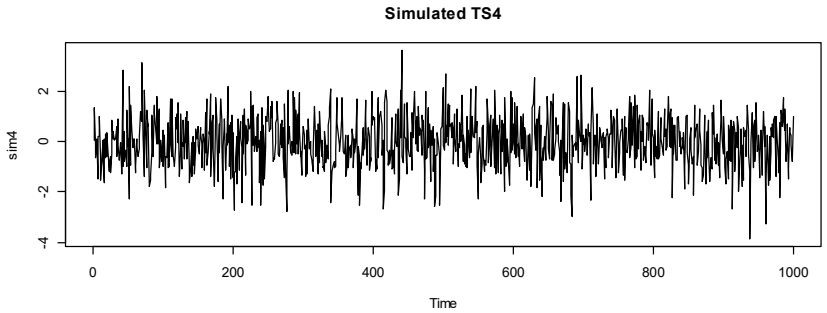


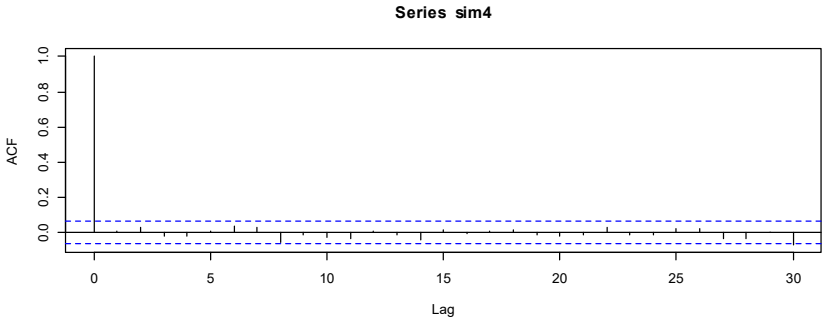


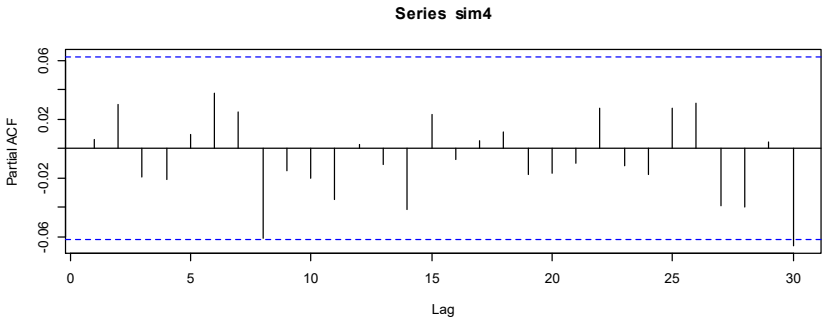

Simulated TS3
sim3
Time


Series sim3
ACF
Lag


Series sim3
Partial ACF
Lag


Simulated TS4
sim4
Time


Series sim4
ACF
Lag


Series sim4
Partial ACF
Lag