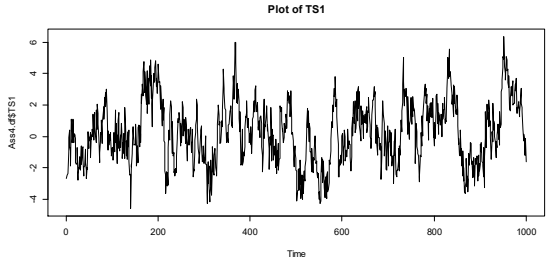
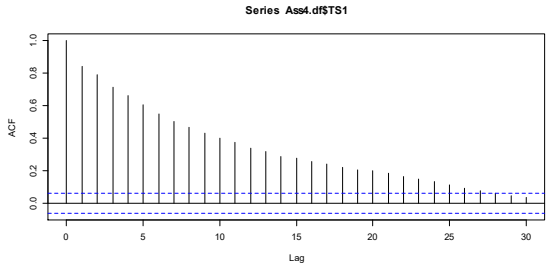
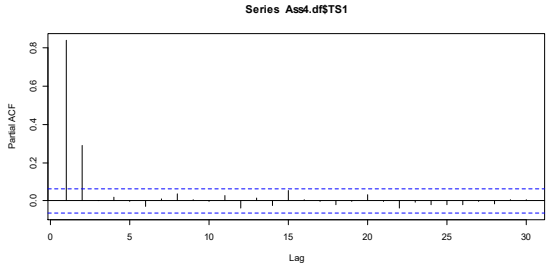
STATS 326
Applied Time Series
ASSIGNMENT FOUR
Answer Guide

Question One: TS1

```
> plot.ts(Ass4.df$TS1,main="Plot of TS1")
> acf(Ass4.df$TS1)
> pacf(Ass4.df$TS1)
```







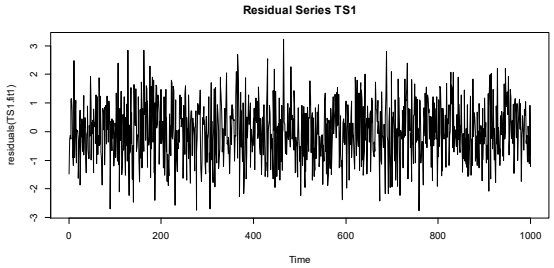
The acf shows slow (exponential) decay while the pacf cuts off suddenly at lag 2. This suggests an AR(2) model is the most appropriate for TS1.

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \varepsilon_t$$

```
> TS1.fit1 = arima(Ass4.df$TS1,order=c(2,0,0))
> TS1.fit1

Call:
arima(x = Ass4.df$TS1, order = c(2, 0, 0))

Coefficients:
         ar1     ar2  intercept
      0.5958  0.2928     0.2106
s.e.  0.0302  0.0303     0.2821

sigma^2 estimated as 1.008: log likelihood = -1423.72, aic = 2855.44
```

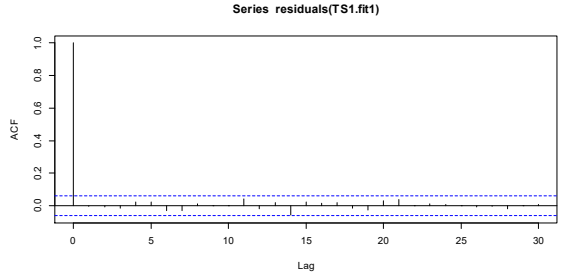
$$\hat{y}_t = 0.5958 y_{t-1} + 0.2928 y_{t-2}$$

```
> plot.ts(residuals(TS1.fit1),main="Residual Series TS1")
```



The residuals appear to be centred at 0 with constant variance.

```
> acf(residuals(TS1.fit1))
```



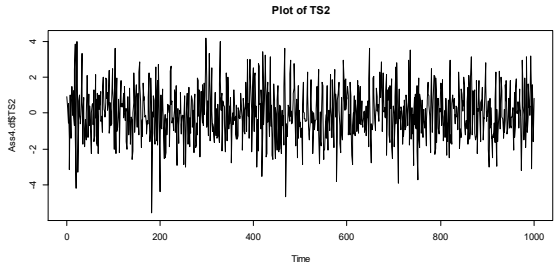
The acf shows no significant lags. The Residual Series appears to be a White Noise series.

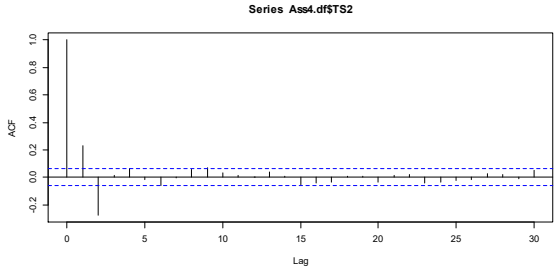
No better model was found:

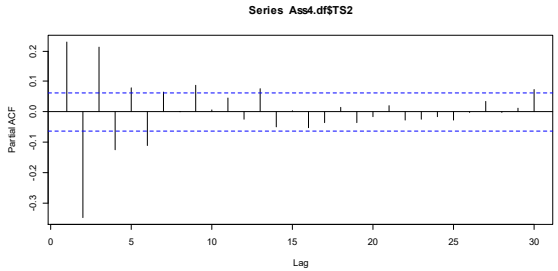
AR(3)	AIC = 2857.43	3rd AR term not significant
ARMA(1,1)	AIC = 2863.19	All terms significant
ARMA(2,1)	AIC = 2857.43	MA term not significant
ARMA(1,2)	AIC = 2858.79	All terms significant

Question Two: TS2

```
> plot.ts(Ass4.df$TS2,main="Plot of TS2")
> acf(Ass4.df$TS2)
> pacf(Ass4.df$TS2)
```







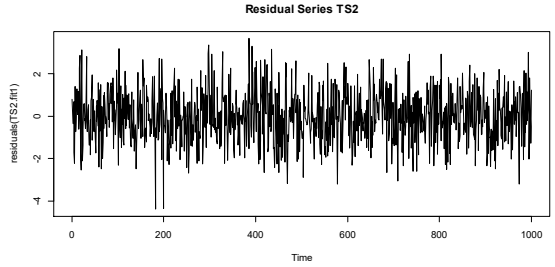
The acf cuts-off at lag 2 and the pacf shows decay (or persistence). This suggests an MA(2) model is most appropriate for TS2.

$$y_t = \varepsilon_t + \alpha_1\varepsilon_{t-1} + \alpha_2\varepsilon_{t-2}$$

```
> TS2.fit1 = arima(Ass4.df$TS2,order=c(0,0,2))
> TS2.fit1

Call:
arima(x = Ass4.df$TS2, order = c(0, 0, 2))

Coefficients:
         ma1      ma2  intercept
      0.4377  -0.311    -0.0086
s.e.  0.0302   0.030     0.0433

sigma^2 estimated as 1.475:  log likelihood = -1613.7,  aic = 3235.4
```

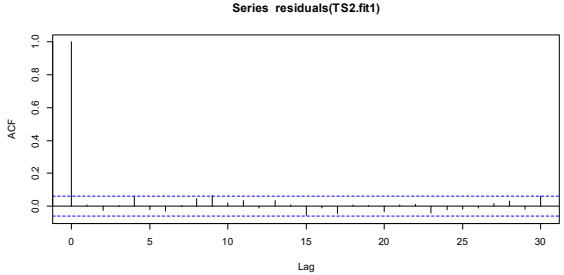
$$\hat{y}_t = \varepsilon_t + 0.4377\varepsilon_{t-1} - 0.311\varepsilon_{t-2}$$

```
> plot.ts(residuals(TS2.fit1),main="Residual Series TS2")
```



The residuals appear to be centred at 0 with constant variance.

```
> acf(residuals(TS2.fit1))
```



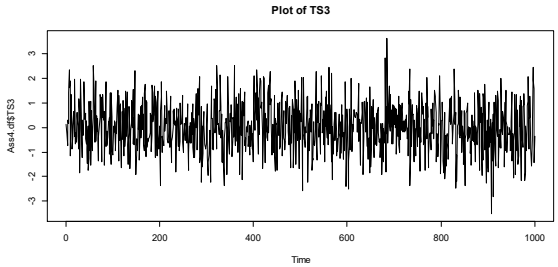
The acf shows no significant lags. The Residual Series appears to be a White Noise series.

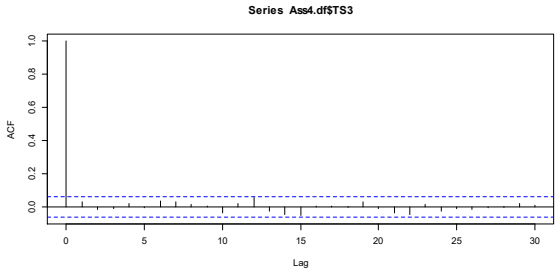
No better model was found:

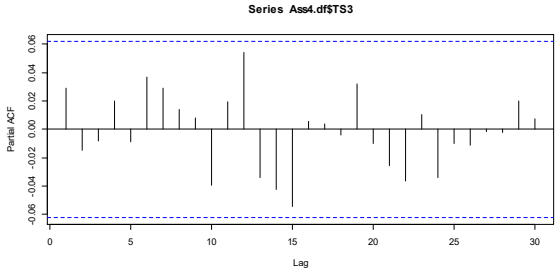
MA(3) AIC = 3236.86 3rd MA term not significant

Question Three: TS3

```
> plot.ts(Ass4.df$TS3,main="Plot of TS3")
> acf(Ass4.df$TS3)
> pacf(Ass4.df$TS3)
```







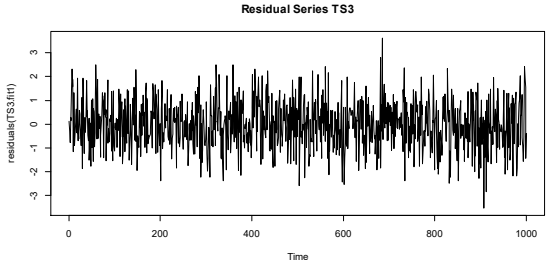
The acf and the pacf show no significant lags. This suggests a WN model is the most appropriate for TS3.

$$y_t = \varepsilon_t$$

```
> TS3.fit1 = arima(Ass4.df$TS3,order=c(0,0,0))
> TS3.fit1

Call:
arima(x = Ass4.df$TS3, order = c(0, 0, 0))

Coefficients:
      intercept
         0.0211
s.e.     0.0316

sigma^2 estimated as 0.9961: log likelihood = -1417, aic = 2838
```

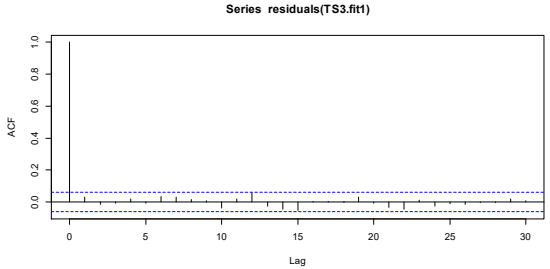
$$\hat{y}_t = \varepsilon_t$$

```
> plot.ts(residuals(TS3.fit1),main="Residual Series TS3")
```



The residuals appear to be centred at 0 with constant variance.

```
> acf(residuals(TS3.fit1))
```



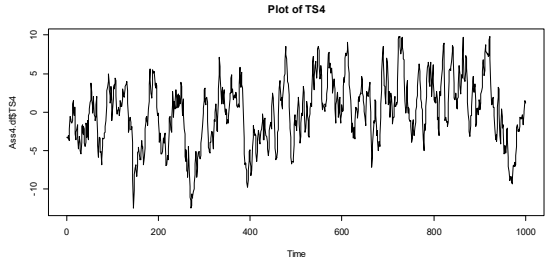
The acf shows no significant autocorrelations. The Residual Series appears to be a White Noise series.

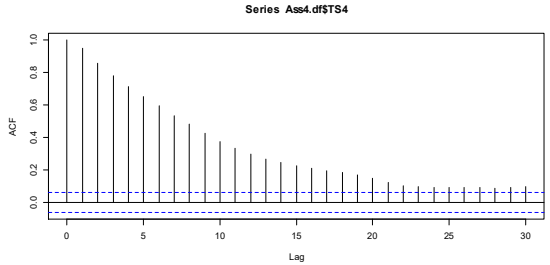
No better model was found:

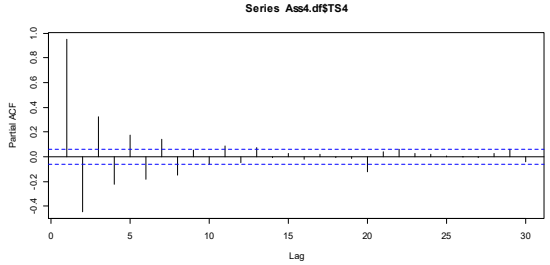
AR(1)	AIC = 2839.17	AR term not significant
MA(1)	AIC = 2839.14	MA term not significant

Question Four: TS4

```
> plot.ts(Ass4.df$TS4,main="Plot of TS4")
> acf(Ass4.df$TS4)
> pacf(Ass4.df$TS4)
```







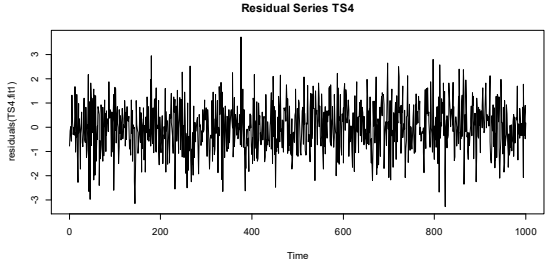
The acf and the pacf decay (exponentially) with the pacf oscillating in sign. This suggests an ARMA (p,q) model is most appropriate for TS4 but as we have no indication of the order we will start with an ARMA(1,1).

$$y_t = \rho_1 y_{t-1} + \varepsilon_t + \alpha_1 \varepsilon_{t-1}$$

```
> TS4.fit1 = arima(Ass4.df$TS4,order=c(1,0,1))
> TS4.fit1

Call:
arima(x = Ass4.df$TS4, order = c(1, 0, 1))

Coefficients:
         ar1     ma1  intercept
      0.8974  0.9121    -0.0147
s.e.  0.0139  0.0128     0.5786

sigma^2 estimated as 0.9828: log likelihood = -1412.55, aic = 2833.11
```

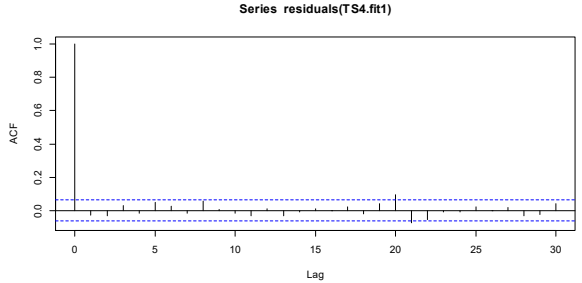
$$\hat{y}_t = 0.8974 y_{t-1} + \varepsilon_t + 0.9121 \varepsilon_{t-1}$$

```
> plot.ts(residuals(TS4.fit1),main="Residual Series TS4")
```



The residuals appear to be centred at 0 with constant variance.

```
> acf(residuals(TS4.fit1))
```



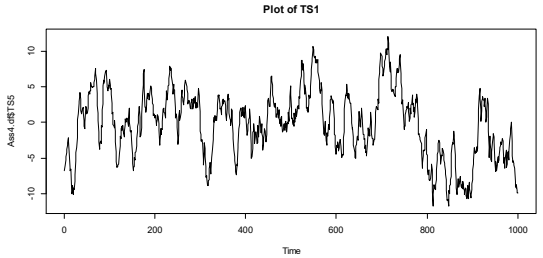
The acf shows 1 weakly significant autocorrelation for lag 20, but it should be of no concern. The Residual Series appears to be White Noise.

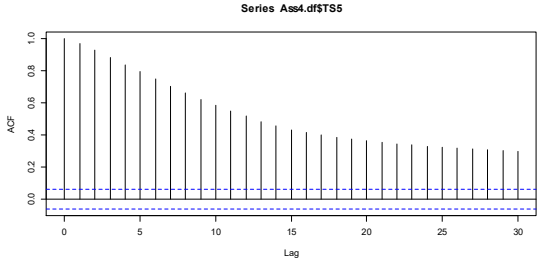
No better model was found:

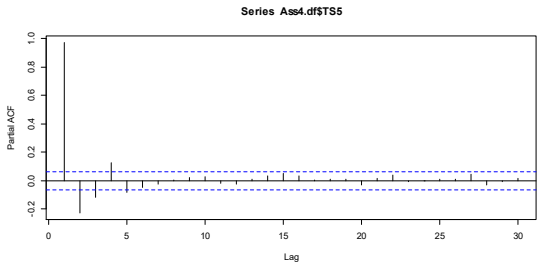
ARMA(2,1)	AIC = 2833.86	2nd AR term not significant
ARMA(1,2)	AIC = 2833.73	2nd MA term not significant

Question Five: TS5

```
> plot.ts(Ass4.df$TS5,main="Plot of TS1")
> acf(Ass4.df$TS5)
> pacf(Ass4.df$TS5)
```







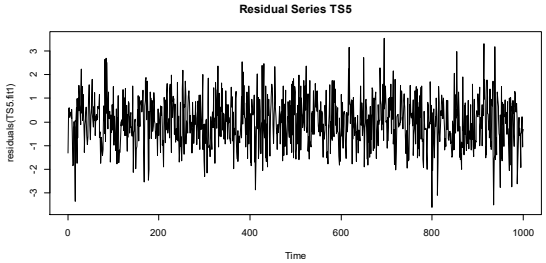
The acf shows slow (exponential) decay and the pacf also shows slow decay (or persistence). This suggests an ARMA(p,q) model is most appropriate for TS5. As we have no indication of the order, we start with an ARMA(1,1).

$$y_t = \rho_1 y_{t-1} + \varepsilon_t + \alpha_1 \varepsilon_{t-1}$$

```
> TS5.fit1 = arima(Ass4.df$TS5,order=c(1,0,1))
Warning message:
In arima(Ass4.df$TS5, order = c(1, 0, 1)) :
  possible convergence problem: optim gave code = 1
> TS5.fit1

Call:
arima(x = Ass4.df$TS5, order = c(1, 0, 1))

Coefficients:
         ar1     ma1  intercept
      0.9674  0.1876    -0.6895
s.e.  0.0082  0.0260     1.1571

sigma^2 estimated as 1.063: log likelihood = -1450.83, aic = 2909.67
```

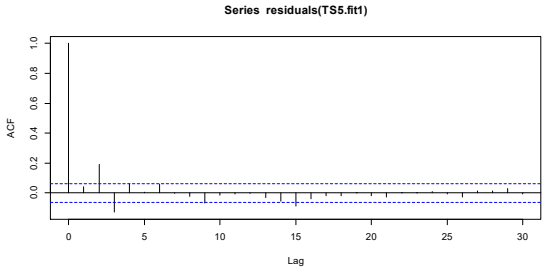
$$\hat{y}_t = 0.9674 y_{t-1} + \varepsilon_t + 0.1876 \varepsilon_{t-1}$$

```
> plot.ts(residuals(TS5.fit1),main="Residual Series TS5")
```



The residuals appear to be centred at 0 with constant variance.

```
> acf(residuals(TS5.fit1))
```



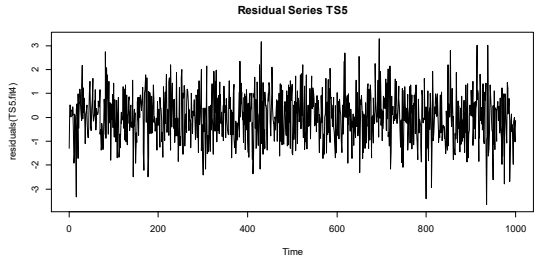
The acf shows several weakly significant autocorrelations for lag 15 but this is probably of little concern. However there are also significant autocorrelations for lags (2 and 3). The Residual Series does not appear to be a White Noise Series.

A better model was found:

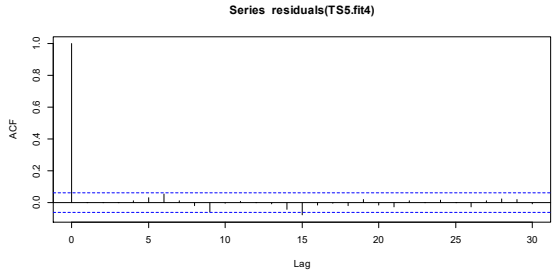
```
> TS5.fit4 = arima(Ass4.df$TS5,order=c(2,0,2))
> TS5.fit4

Call:
arima(x = Ass4.df$TS5, order = c(2, 0, 2))

Coefficients:
         ar1     ar2     ma1     ma2  intercept
      0.5840  0.3552  0.6361  0.3235    -0.5989
s.e.  0.1063  0.1040  0.1014  0.0325     0.9989

sigma^2 estimated as 0.9954: log likelihood = -1418.31, aic = 2848.63
```

$$\hat{y}_t = 0.5840y_{t-1} + 0.3552y_{t-2} + \varepsilon_t + 0.6361\varepsilon_{t-1} + 0.3235\varepsilon_{t-2}$$

```
> plot.ts(residuals(TS5.fit4),main="Residual Series TS5")
```




```
> acf(residuals(TS5.fit4))
```



The ARMA(2,2) has lower AIC = 2848.63. The Residual Series looks like White Noise and the plot of the autocorrelation function confirms this as there are no significant autocorrelations. Lag 15 is very weakly significant, but is of no concern.