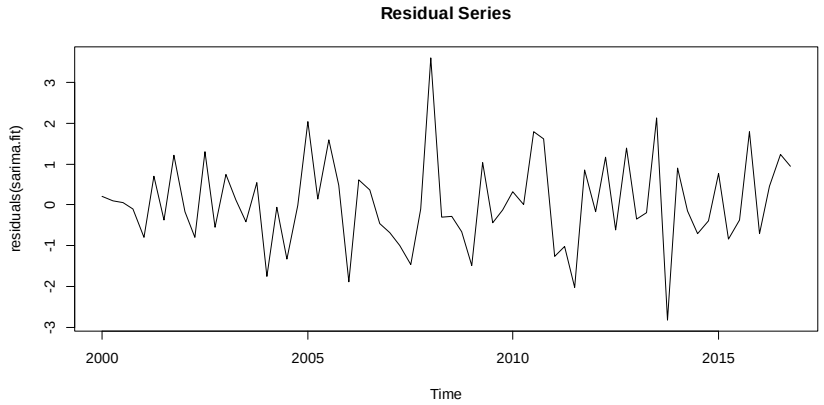
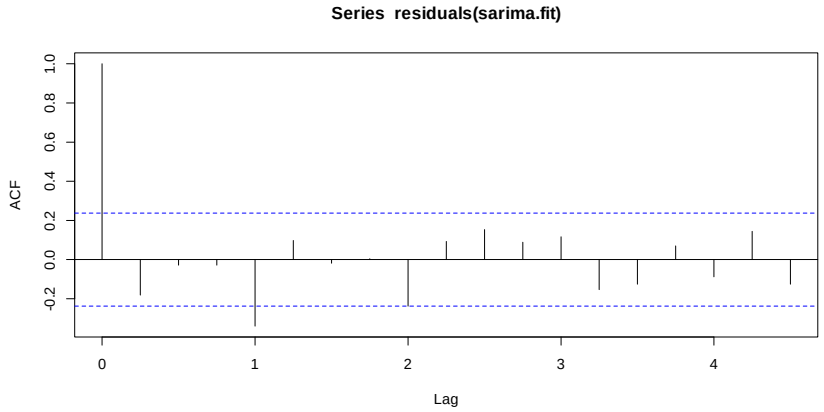
STATS 326
Applied Time Series
ASSIGNMENT FIVE
R CODE

Question One:

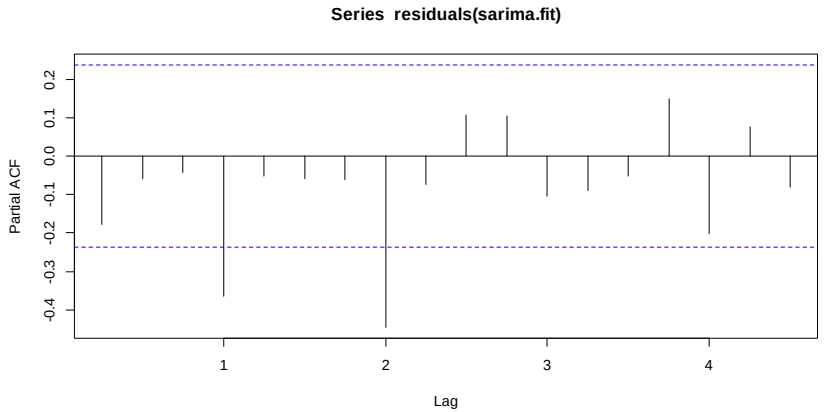
```
> sarima.fit = arima(red.CO2.ts,order=c(0,1,0),
   seasonal=list(order=c(0,1,0),period=4))
> plot.ts(residuals(sarima.fit),main="Residual Series")
```




```
> acf(residuals(sarima.fit))
```



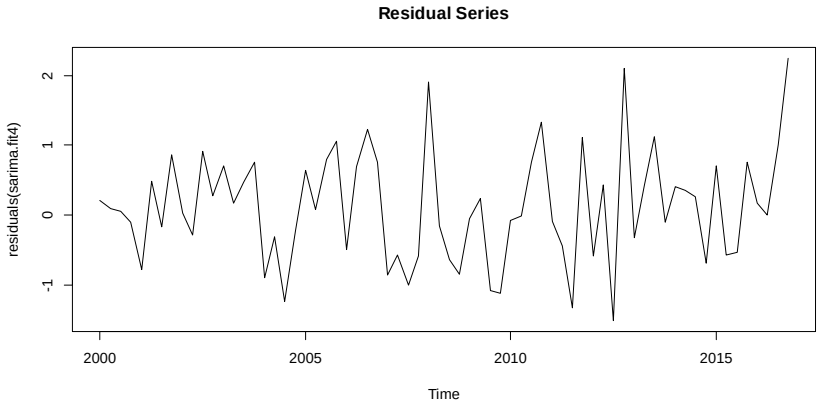

```
> pacf(residuals(sarima.fit))
```



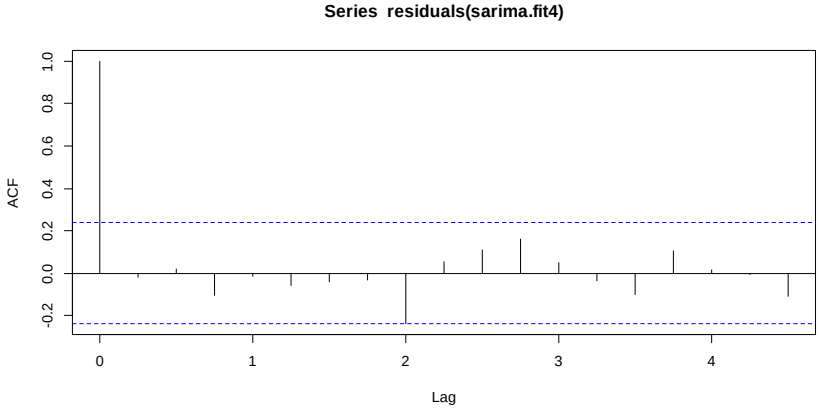

```
> sarima.fit1 = arima(red.CO2.ts,order=c(1,1,0),
   seasonal=list(order=c(1,1,0),period=4))
> sarima.fit1

Call:
arima(x = red.CO2.ts, order = c(1, 1, 0), seasonal = list(order =
c(1, 1, 0), period = 4))

Coefficients:
          ar1     sar1
      -0.1901  -0.3467
s.e.   0.1253   0.1172

sigma^2 estimated as 1.089:  log likelihood = -92.36,  aic = 190.71

> sarima.fit2 = arima(red.CO2.ts,order=c(0,1,1),
   seasonal=list(order=c(0,1,1),period=4))
> sarima.fit2

Call:
arima(x = red.CO2.ts, order = c(0, 1, 1), seasonal = list(order =
c(0, 1, 1), period = 4))

Coefficients:
          ma1     sma1
      -0.3633  -0.8166
s.e.   0.1762   0.0920

sigma^2 estimated as 0.763:  log likelihood = -83.15,  aic = 172.31
```

```
> sarima.fit3 = arima(red.CO2.ts,order=c(1,1,1),
   seasonal=list(order=c(1,1,1),period=4))
> sarima.fit3

Call:
arima(x = red.CO2.ts, order = c(1, 1, 1), seasonal = list(order =
c(1, 1, 1), period = 4))

Coefficients:
         ar1      ma1    sar1     sma1
      0.4909  -0.9244  0.0497  -0.7931
s.e.  0.1465   0.0969  0.1616   0.1119

sigma^2 estimated as 0.6863:  log likelihood = -80.64,  aic = 171.28

> sarima.fit5 = arima(red.CO2.ts,order=c(1,1,1),
   seasonal=list(order=c(0,1,2),period=4))
> sarima.fit5

Call:
arima(x = red.CO2.ts, order = c(1, 1, 1), seasonal = list(order =
c(0, 1, 2), period = 4))

Coefficients:
         ar1      ma1     sma1     sma2
      0.4987  -0.9285  -0.7067  -0.0801
s.e.  0.1454   0.0961   0.1845   0.1772

sigma^2 estimated as 0.685:  log likelihood = -80.59,  aic = 171.17

> sarima.fit6 = arima(red.CO2.ts,order=c(1,1,2),
seasonal=list(order=c(0,1,1),period=4))
> sarima.fit6

Call:
arima(x = red.CO2.ts, order = c(1, 1, 2), seasonal = list(order =
c(0, 1, 1),
    period = 4))

Coefficients:
         ar1      ma1      ma2     sma1
      0.4583  -0.8903  -0.0254  -0.7773
s.e.  0.2666   0.2804   0.2241   0.1026

sigma^2 estimated as 0.6872:  log likelihood = -80.68,  aic = 171.36

> sarima.fit7 = arima(red.CO2.ts,order=c(2,1,1),
   seasonal=list(order=c(0,1,1),period=4))
> sarima.fit7

Call:
arima(x = red.CO2.ts, order = c(2, 1, 1), seasonal = list(order =
c(0, 1, 1),
    period = 4))

Coefficients:
         ar1      ar2      ma1     sma1
      0.4871  -0.0194  -0.9169  -0.7772
s.e.  0.1510   0.1421   0.1014   0.1027

sigma^2 estimated as 0.6872:  log likelihood = -80.68,  aic = 171.36
```

```
> sarima.fit4 = arima(red.CO2.ts,order=c(1,1,1),
   seasonal=list(order=c(0,1,1),period=4))
> sarima.fit4

Call:
arima(x = red.CO2.ts, order = c(1, 1, 1), seasonal = list(order =
c(0, 1, 1), period = 4))

Coefficients:
         ar1      ma1     sma1
      0.4830  -0.9199  -0.7776
s.e.  0.1471   0.0977   0.1024

sigma^2 estimated as 0.6873:  log likelihood = -80.69,  aic = 169.37

> plot.ts(residuals(sarima.fit4),main="Residual Series")
```




```
> acf(residuals(sarima.fit4))
```



```
> SARIMA4.pred = predict(sarima.fit4,n.ahead=4)
> SARIMA4.pred
$pred
         Qtr1     Qtr2     Qtr3     Qtr4
2017 412.1403 411.4029 398.1207 408.2079

$se
          Qtr1      Qtr2      Qtr3      Qtr4
2017 0.8292973 0.9518559 0.9957232 1.0171728

> RMSEP.SARIMA4 = sqrt(1/4*sum((actual-SARIMA4.pred$pred)^2))
> RMSEP.SARIMA4
[1] 0.974876
```

Question Two:

```
> sarima.fit4.full = arima(full.CO2.ts,order=c(1,1,1),
   seasonal=list(order=c(0,1,1),period=4))
> sarima.fit4.full

Call:
arima(x = full.CO2.ts, order = c(1, 1, 1), seasonal = list(order =
c(0, 1, 1),
    period = 4))

Coefficients:
        ar1      ma1     sma1
      0.549  -0.9153  -0.7732
s.e.  0.139   0.0863   0.0940

sigma^2 estimated as 0.6855:  log likelihood = -85.34,  aic = 178.68

> SARIMA4.full.pred = predict(sarima.fit4.full,n.ahead=4)
> SARIMA4.full.pred
$pred
         Qtr1     Qtr2     Qtr3     Qtr4
2018 414.1692 413.7243 400.4962 410.6661

$se
          Qtr1      Qtr2      Qtr3      Qtr4
2018 0.8280516 0.9803490 1.0437764 1.0773616

> full.CO2.ts
       Qtr1   Qtr2   Qtr3   Qtr4
2000 375.33 375.21 362.52 370.42
……
2014 404.37 404.47 391.32 400.18
2015 406.55 405.82 392.31 402.96
2016 408.63 408.35 396.07 407.67
2017 413.75 412.42 398.47 408.44

> residuals(sarima.fit4.full)
              Qtr1           Qtr2          Qtr3          Qtr4
2000  0.2166968053  0.0968167496  0.0526563888 -0.0997203630
……
2014  0.3717980251  0.3002281272  0.2120476940 -0.7429045475
2015  0.7310728098 -0.6086097361 -0.5093977597  0.7965998392
2016  0.1372648171 -0.0279754929  0.9936960060  2.1601285641
2017  1.4076001116 -0.1055204121 -0.4018711569 -0.1136896868
```

$(1-\rho_1 \mathrm{B})(1-\mathrm{B})(1-\mathrm{B}^4)y_t = (1+\alpha_1 \mathrm{B})(1+\mathrm{A}_1\mathrm{B}^4)\varepsilon_t$

$(1-\rho_1 \mathrm{B})(1-\mathrm{B}-\mathrm{B}^4+\mathrm{B}^5)y_t = (1+\alpha_1 \mathrm{B}+\mathrm{A}_1\mathrm{B}^4+\alpha_1\mathrm{A}_1\mathrm{B}^5)\varepsilon_t$

$(1-\mathrm{B}-\mathrm{B}^4+\mathrm{B}^5-\rho_1\mathrm{B}+\rho_1\mathrm{B}^2+\rho_1\mathrm{B}^5-\rho_1\mathrm{B}^6)y_t = (1+\alpha_1\mathrm{B}+\mathrm{A}_1\mathrm{B}^4+\alpha_1\mathrm{A}_1\mathrm{B}^5)\varepsilon_t$

$y_t-(1+\rho_1)y_{t-1}+\rho_1 y_{t-2}-y_{t-4}+(1+\rho_1)y_{t-5}-\rho_1 y_{t-6} = \varepsilon_t+\alpha_1\varepsilon_{t-1}+\mathrm{A}_1\varepsilon_{t-4}+\alpha_1\mathrm{A}_1\varepsilon_{t-5}$

$y_t = (1+\rho_1)y_{t-1}-\rho_1 y_{t-2}+y_{t-4}-(1+\rho_1)y_{t-5}+\rho_1 y_{t-6}+\varepsilon_t+\alpha_1\varepsilon_{t-1}+\mathrm{A}_1\varepsilon_{t-4}+\alpha_1\mathrm{A}_1\varepsilon_{t-5}$

$y_t = 1.549y_{t-1}-0.549y_{t-2}+y_{t-4}-1.549y_{t-5}+0.549y_{t-6}+\varepsilon_t-0.9153\varepsilon_{t-1}-0.7732\varepsilon_{t-4}+0.70770996\varepsilon_{t-5}$

$y_{t+1} = 1.549y_t-0.549y_{t-1}+y_{t-3}-1.549y_{t-4}+0.549y_{t-5}+\varepsilon_{t+1}-0.9153\varepsilon_t-0.7732\varepsilon_{t-3}+0.70770996\varepsilon_{t-4}$

```
> (1.549*408.44)-(0.549*398.47)+413.75-
(1.549*407.67)+(0.549*396.07)+(0.9153*0.1136896868)-
(0.7732*1.407600116)+(0.70770996*2.1601285641)
[1] 414.1696
```

$y_{t+2} = 1.549y_{t+1}-0.549y_t+y_{t-2}-1.549y_{t-3}+0.549y_{t-4}+\varepsilon_{t+2}-0.9153\varepsilon_{t+1}-0.7732\varepsilon_{t-2}+0.70770996\varepsilon_{t-3}$

```
> (1.549*414.1696)-(0.549*408.44)+412.42-
(1.549*413.75)+(0.549*407.67)+(0.7732*0.1055204121)+
(0.70770996*1.407600116)
[1] 413.725
```

$y_{t+3} = 1.549y_{t+2}-0.549y_{t+1}+y_{t-1}-1.549y_{t-2}+0.549y_{t-3}+\varepsilon_{t+3}-0.9153\varepsilon_{t+2}-0.7732\varepsilon_{t-1}+0.70770996\varepsilon_{t-2}$

```
> (1.549*413.725)-(0.549*414.1696)+398.47-
(1.549*412.42)+(0.549*413.75)+(0.7732*0.4018711569)-
(0.70770996*0.1055204121)
[1] 400.4971
```

$y_{t+4} = 1.549y_{t+3}-0.549y_{t+2}+y_t-1.549y_{t-1}+0.549y_{t-2}+\varepsilon_{t+4}-0.9153\varepsilon_{t+3}-0.7732\varepsilon_t+0.70770996\varepsilon_{t-1}$

```
> (1.549*400.4971)-(0.549*413.725)+408.44-
(1.549*398.47)+(0.549*412.42)+(0.7732*0.1136896868)-
(0.70770996*0.4018711569)
[1] 410.667
```