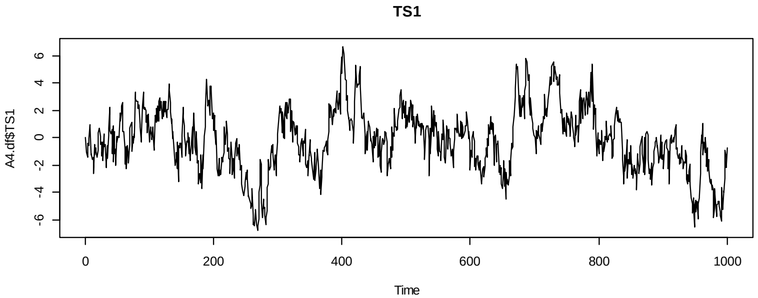
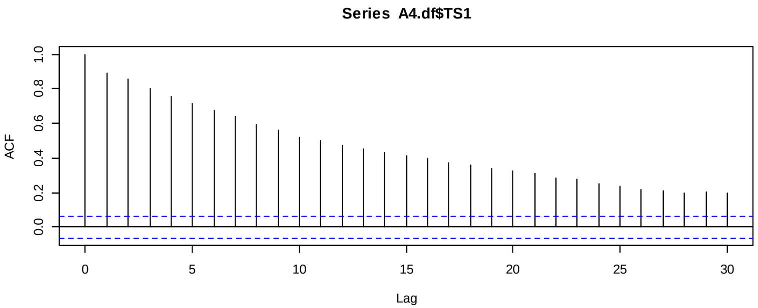
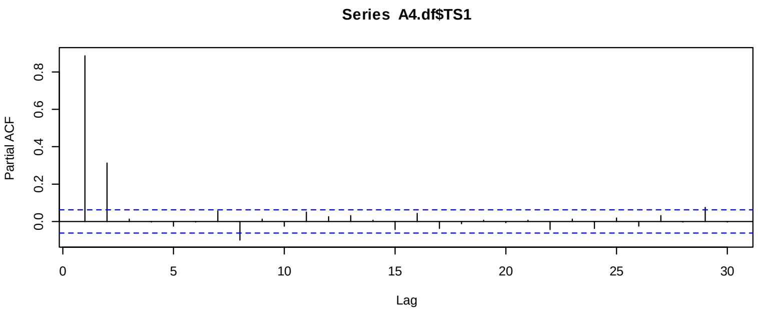
STATS 326
Applied Time Series
ASSIGNMENT FOUR
ANSWER GUIDE

<u>Question One</u>: TS1

```
> plot.ts(A4.df$TS1,main="TS1")
> acf(A4.df$TS1)
> pacf(A4.df$TS1)
```







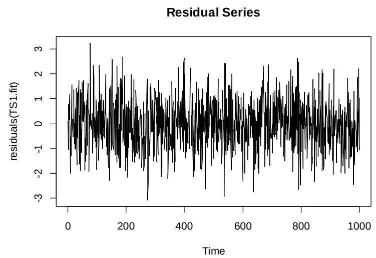
$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \varepsilon_t$$

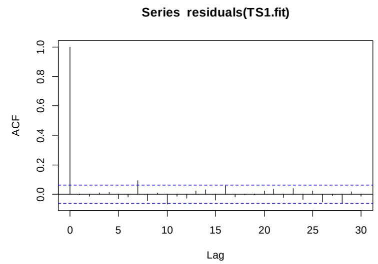
The plot of the series shows clustering indicating positive autocorrelation. The acf shows decay while the pacf shows cut-off at lag 2. This suggests an AR(2) is the most suitable model.

```
> TS1.fit = arima(A4.df$TS1,order=c(2,0,0))
> TS1.fit

Call:
arima(x = A4.df$TS1, order = c(2, 0, 0))

Coefficients:
         ar1     ar2  intercept
      0.6092  0.3133    -0.1816
s.e.  0.0300  0.0300     0.4016

sigma^2 estimated as 0.9991: log likelihood = -1419.38, aic = 2846.75
```

$$y_t = 0.6092 y_{t-1} + 0.3133 y_{t-2} + \varepsilon_t$$

```
> plot.ts(residuals(TS1.fit),main="Residual Series")
> acf(residuals(TS1.fit))
```





The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows lag 7 is slightly significant, but nothing to really worry about.

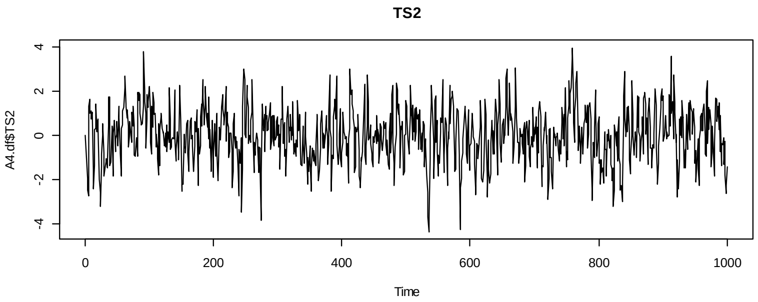
Other models tried:

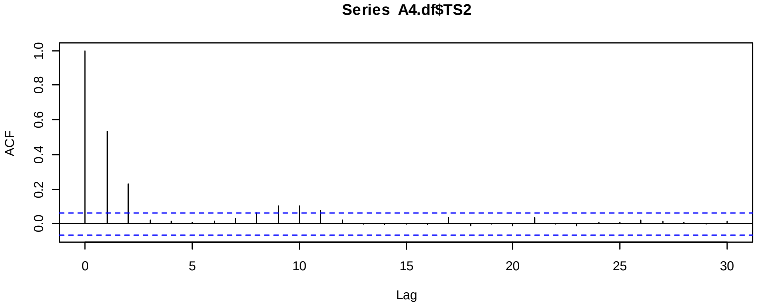
AR(3)	AIC = 2848.6	3rd AR term not significant
ARMA(1,1)	AIC = 2855.59	
ARMA(2,1)	AIC = 2848.6	1st MA term not significant
ARMA(1,2)	AIC = 2849.02	

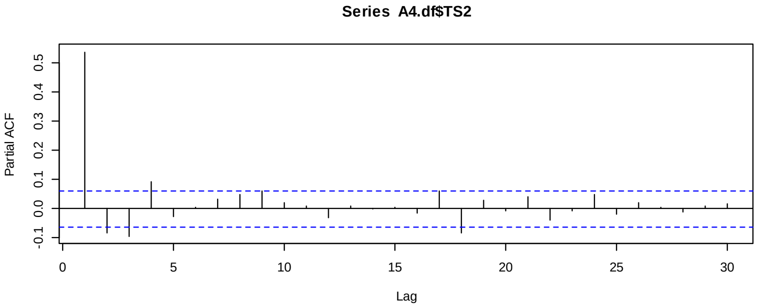
The AR(2) model had the smallest AIC and all terms were significant.

<u>Question Two</u>: TS2

```
> plot.ts(A4.df$TS2,main="TS2")
> acf(A4.df$TS2)
> pacf(A4.df$TS2)
```







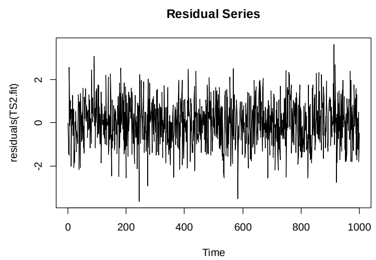
$$y_t = \varepsilon_t + \alpha_1 \varepsilon_{t-1} + \alpha_2 \varepsilon_{t-2}$$

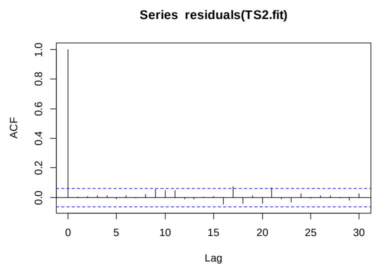
The plot of the series shows little in the way of a pattern. The acf shows cut-off at lag 2 and the pacf shows decay (or persistence). This suggests a MA(2) is the most suitable model.

```
> TS2.fit = arima(A4.df$TS2,order=c(0,0,2))
> TS2.fit

Call:
arima(x = A4.df$TS2, order = c(0, 0, 2))

Coefficients:
         ma1     ma2  intercept
      0.5817  0.3013    -0.0011
s.e.  0.0302  0.0295     0.0614

sigma^2 estimated as 1.063:  log likelihood = -1449.87,  aic = 2907.74
```

$$y_t = \varepsilon_t + 0.5817\varepsilon_{t-1} + 0.3013\varepsilon_{t-2}$$

```
> plot.ts(residuals(TS2.fit),main="Residual Series")
> acf(residuals(TS2.fit))
```





The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows no significant lags.

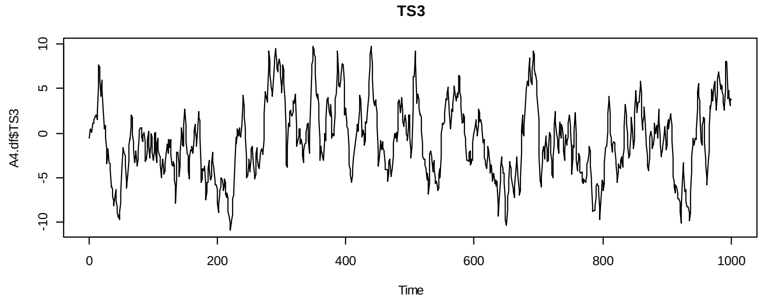
Other models tried:

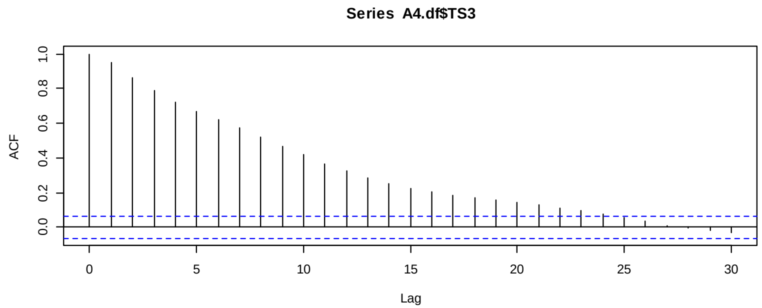
MA(3) AIC = 2909.69 3^{rd} MA term not significant

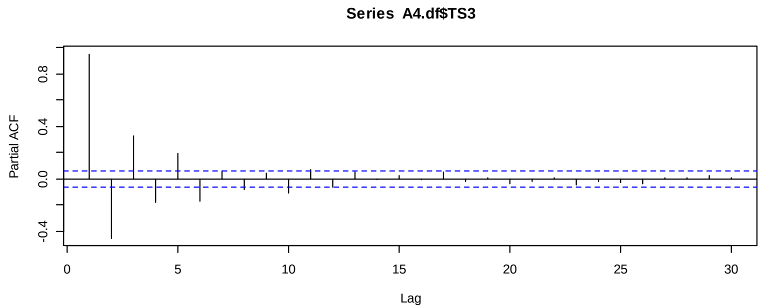
The MA(2) model had the smallest AIC and all terms were significant

<u>Question Three</u>: TS3

```
> plot.ts(A4.df$TS3,main="TS3")
> acf(A4.df$TS3)
> pacf(A4.df$TS3)
```







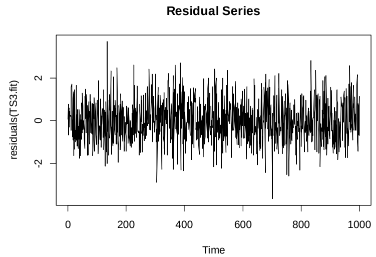
$$y_t = \rho_1 y_{t-1} + \varepsilon_t + \alpha_1 \varepsilon_{t-1}$$

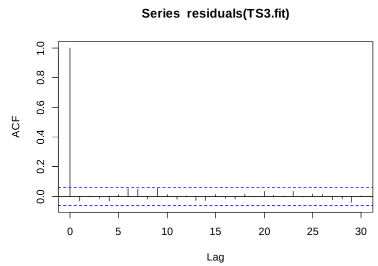
The plot of the series shows clustering indicating positive autocorrelation. The acf shows decay and the pacf also shows decay. This suggests an ARMA(p,q) is the appropriate model, but as we have no indication from the plots of the order we begin with an ARMA(1,1).

```
> TS3.fit = arima(A4.df$TS3,order=c(1,0,1))
> TS3.fit

Call:
arima(x = A4.df$TS3, order = c(1, 0, 1))

Coefficients:
         ar1     ma1  intercept
      0.9022  0.8602    -0.7937
s.e.  0.0136  0.0174     0.5825

sigma^2 estimated as 0.9557:  log likelihood = -1398.36,  aic = 2804.72
```

$$y_t = 0.9022 y_{t-1} + \varepsilon_t + 0.8602 \varepsilon_{t-1}$$

```
> plot.ts(residuals(TS3.fit),main="Residual Series")
> acf(residuals(TS3.fit))
```





The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows no significant lags.

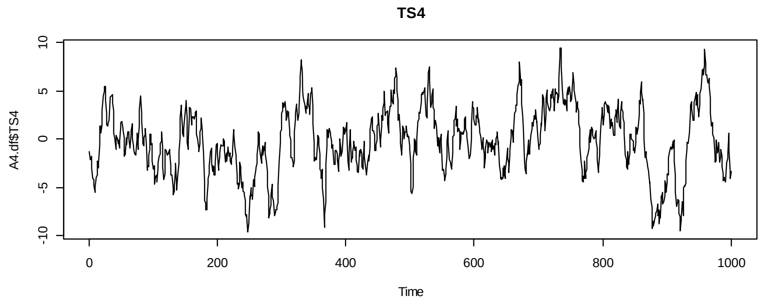
Other models tried:

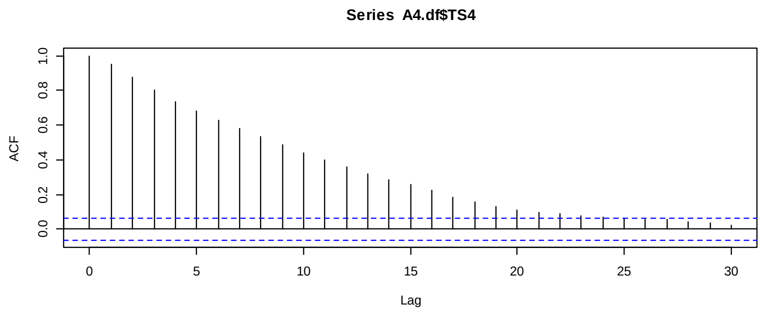
ARMA(2,1)	AIC = 2805.13	2nd AR term not significant
ARMA(1,2)	AIC = 2805.11	2nd MA term not significant

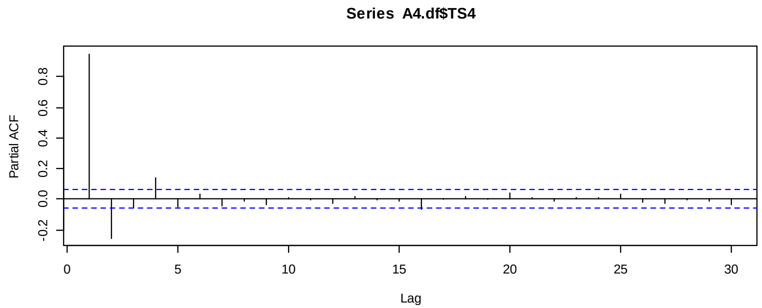
The ARMA(1,1) model had the smallest AIC and all terms were significant.

<u>Question Four</u>: TS4

```
> plot.ts(A4.df$TS4,main="TS4")
> acf(A4.df$TS4)
> pacf(A4.df$TS4)
```







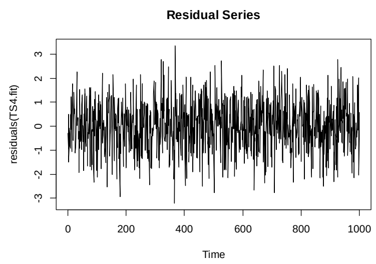
$$y_t = \rho_1 y_{t-1} + \varepsilon_t + \alpha_1 \varepsilon_{t-1}$$

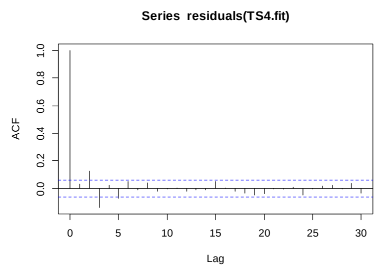
The plot of the series shows clustering indicating positive autocorrelation. The acf shows decay and the pacf also shows decay (or persistence). This suggests an ARMA(p,q) is the appropriate model, but as we have no indication from the plots of the order we begin with an ARMA(1,1).

```
> TS4.fit = arima(A4.df$TS4,order=c(1,0,1))
> TS4.fit

Call:
arima(x = A4.df$TS4, order = c(1, 0, 1))

Coefficients:
         ar1     ma1  intercept
      0.9314  0.2108    -0.2332
s.e.  0.0118  0.0274     0.5765

sigma^2 estimated as 1.095: log likelihood = -1465.63, aic = 2939.26
```

$$y_t = 0.9314 y_{t-1} + \varepsilon_t + 0.2108 \varepsilon_{t-1}$$

```
> plot.ts(residuals(TS4.fit),main="Residual Series")
> acf(residuals(TS4.fit))
```





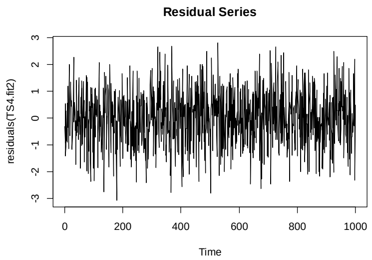
The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows 2 significant lags.

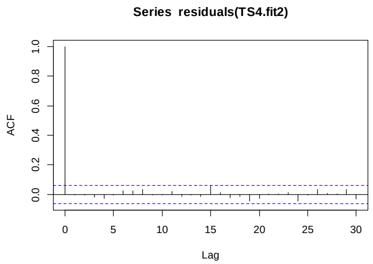
Better model:

```
> TS4.fit2 = arima(A4.df$TS4,order=c(2,0,2))
> TS4.fit2

Call:
arima(x = A4.df$TS4, order = c(2, 0, 2))

Coefficients:
         ar1     ar2     ma1     ma2  intercept
      0.3326  0.5374  0.8566  0.2958    -0.2254
s.e.  0.1548  0.1474  0.1508  0.0335     0.5316

sigma^2 estimated as 1.054: log likelihood = -1446.4, aic = 2904.8
```

$$y_t = 0.3326 y_{t-1} + 0.5374 y_{t-2} + \varepsilon_t + 0.8566 \varepsilon_{t-1} + 0.2958 \varepsilon_{t-2}$$

```
> plot.ts(residuals(TS4.fit2),main="Residual Series")
> acf(residuals(TS4.fit2))
```

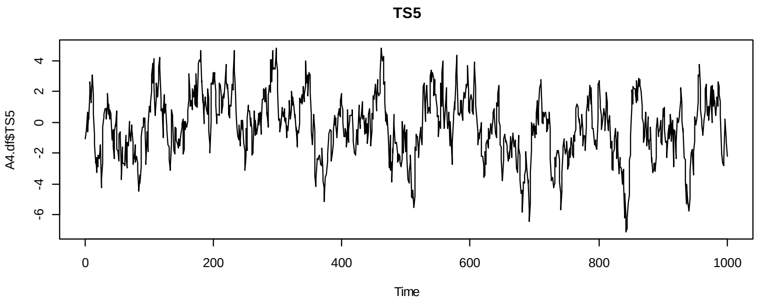


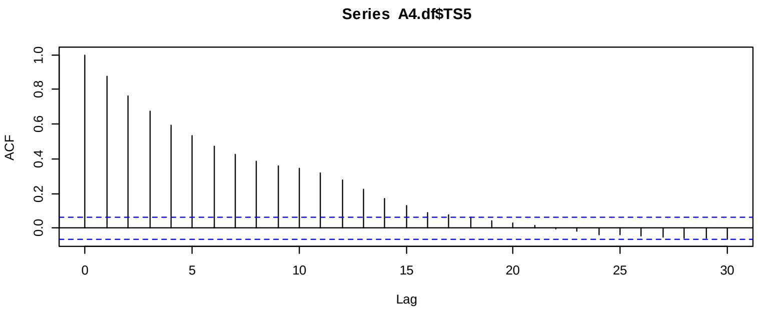


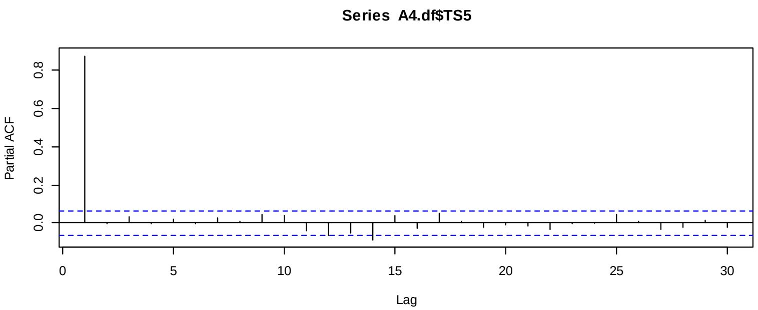
The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows no significant lags.

Question Five: TS5

```
> plot.ts(A4.df$TS5,main="TS5")
> acf(A4.df$TS5)
> pacf(A4.df$TS5)
```







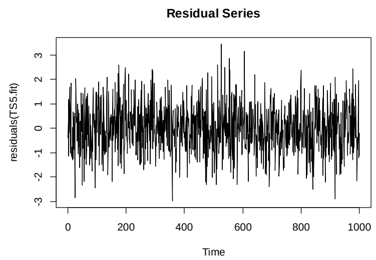
$$y_t = \rho_1 y_{t-1} + \varepsilon_t$$

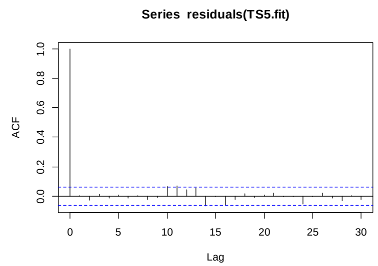
The plot of the series shows clustering indicating positive autocorrelation. The acf shows decay while the pacf shows cut-off at lag 1. This suggests an AR(1) is the most suitable model.

```
> TS5.fit = arima(A4.df$TS5,order=c(1,0,0))
> TS5.fit

Call:
arima(x = A4.df$TS5, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.8752    -0.2975
s.e.  0.0152     0.2488

sigma^2 estimated as 0.9779:  log likelihood = -1408.5,  aic = 2822.99
```

$$y_t = 0.8752 y_{t-1} + \varepsilon_t$$

```
> plot.ts(residuals(TS5.fit),main="Residual Series")
> acf(residuals(TS5.fit))
```





The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows 3 slightly significant lags (10,11 and 14) but they can be ignored.

Other models tried:

AR(2)	AIC = 2848.98	2^{nd} AR term not significant
ARMA(1,1)	AIC = 2824.98	MA term not significant

The AR(1) model had the smallest AIC and the AR term was significant.