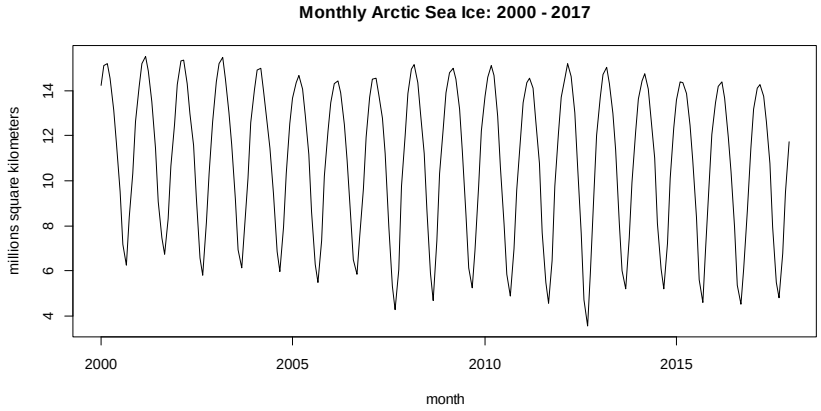
Department of Statistics
STATS 326: Applied Time Series
First Semester, 2019
Test 2

Appendix

Data: These data are monthly measurements of the area of sea ice (in millions of square kilometres) in the Arctic Ocean between 2000 and 2017.

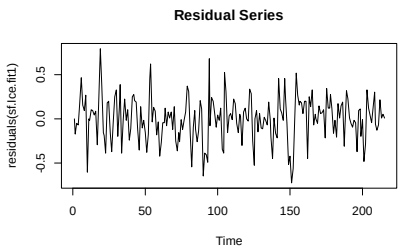
```
> Ice.ts = ts(Ice.df$Ice[1:216],frequency=12,start=2000)

> plot.ts(Ice.ts,xlab="month",ylab="millions square kilometers",
   main="Monthly Arctic Sea Ice: 2000 - 2017")
```

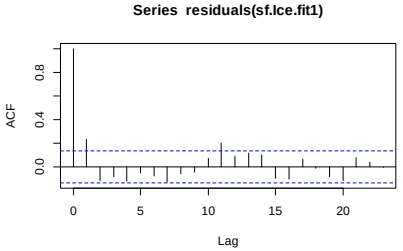



```
> Ice.ts
       Jan   Feb   Mar   Apr   May   Jun   Jul   Aug   Sep   Oct   Nov   Dec
2000 14.22 15.14 15.22 14.56 13.15 11.67  9.51  7.17  6.25  8.38 10.32 12.64
2001 14.20 15.21 15.52 14.86 13.51 11.46  9.07  7.46  6.73  8.30 10.66 12.49
2002 14.27 15.34 15.35 14.30 12.97 11.58  9.27  6.60  5.83  8.16 10.34 12.61
……
2015 13.60 14.40 14.37 13.89 12.47 10.88  8.38  5.60  4.62  6.97  9.85 12.04
2016 13.46 14.20 14.40 13.68 11.92 10.41  7.94  5.37  4.53  6.08  8.66 11.46
2017 13.19 14.12 14.29 13.75 12.63 10.76  7.94  5.48  4.82  6.77  9.49 11.74



> actual
Jan 2018 Feb 2018 Mar 2018
   13.06    13.95    14.30
```


Seasonal Factor Model:

```
> Time = 1:216
> Month = factor(rep(1:12,18))

> sf.Ice.fit1 = lm(Ice.ts[-1]~Time[-1]+Month[-1]+Ice.ts[-216])

> plot.ts(residuals(sf.Ice.fit1),main="Residual Series")
> acf(residuals(sf.Ice.fit1))
```

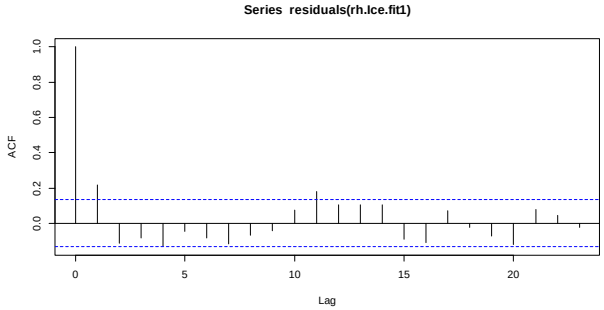





```
> summary(sf.Ice.fit1)

Call:
lm(formula = Ice.ts[-1] ~ Time[-1] + Month[-1] + Ice.ts[-216])

Residuals:
     Min       1Q   Median       3Q      Max
-0.72201 -0.14310  0.00296  0.14565  0.79068

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   4.9029817  0.6134707   7.992 1.02e-13 ***
Time[-1]     -0.0014629  0.0003866  -3.784 0.000203 ***
Month[-1]2   -0.2676245  0.1134801  -2.358 0.019317 *
Month[-1]3   -0.6930475  0.1443620  -4.801 3.08e-06 ***
Month[-1]4   -1.5272663  0.1531960  -9.969  < 2e-16 ***
Month[-1]5   -2.4283731  0.1283108 -18.926  < 2e-16 ***
Month[-1]6   -3.1061548  0.0909949 -34.135  < 2e-16 ***
Month[-1]7   -4.4726159  0.1010039 -44.282  < 2e-16 ***
Month[-1]8   -4.9045992  0.1971846 -24.873  < 2e-16 ***
Month[-1]9   -4.0323236  0.3026303 -13.324  < 2e-16 ***
Month[-1]10  -1.3896055  0.3429503  -4.052 7.26e-05 ***
Month[-1]11  -0.1496230  0.2524389  -0.593 0.554042
Month[-1]12   0.1333355  0.1385333   0.962 0.336965
Ice.ts[-216]  0.7387206  0.0475299  15.542  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2553 on 201 degrees of freedom
Multiple R-squared:  0.9945,    Adjusted R-squared:  0.9942
F-statistic:  2816 on 13 and 201 DF,  p-value: < 2.2e-16
```

Reduced Full Harmonic Model:

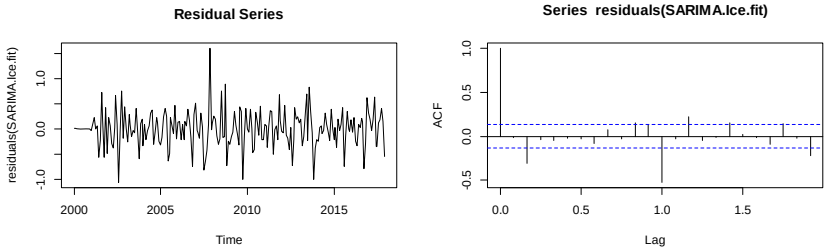
```
> rh.Ice.fit1 = lm(Ice.ts[-1]~Time[-1]+c1[-1]+s1[-1]+c2[-1]+s2[-1]+c3[-1]+
   s3[-1]+s4[-1]+s5[-1]+c6[-1]+Ice.ts[-216])
> summary(rh.Ice.fit1)

Call:
lm(formula = Ice.ts[-1] ~ Time[-1] + c1[-1] + s1[-1] + c2[-1] +
    s2[-1] + c3[-1] + s3[-1] + s4[-1] + s5[-1] + c6[-1] + Ice.ts[-216])

Residuals:
     Min       1Q   Median       3Q      Max
-0.69322 -0.15007  0.00171  0.15820  0.81926

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)   3.0009198  0.5487722   5.468 1.33e-07 ***
Time[-1]     -0.0014655  0.0003875  -3.782 0.000204 ***
c1[-1]        1.9117644  0.0843874  22.655  < 2e-16 ***
s1[-1]        1.3889757  0.2039938   6.809 1.09e-10 ***
c2[-1]        0.3784793  0.0353818  10.697  < 2e-16 ***
s2[-1]       -0.5993008  0.0371362 -16.138  < 2e-16 ***
c3[-1]       -0.2558322  0.0272982  -9.372  < 2e-16 ***
s3[-1]       -0.1913209  0.0249967  -7.654 7.75e-13 ***
s4[-1]        0.0517074  0.0247737   2.087 0.038119 *
s5[-1]        0.0897371  0.0248050   3.618 0.000375 ***
c6[-1]        0.0597086  0.0175310   3.406 0.000795 ***
Ice.ts[-216]  0.7386265  0.0476347  15.506  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2559 on 203 degrees of freedom
Multiple R-squared:  0.9945,    Adjusted R-squared:  0.9942
F-statistic:  3312 on 11 and 203 DF,  p-value: < 2.2e-16

> acf(residuals(rh.Ice.fit1))
```



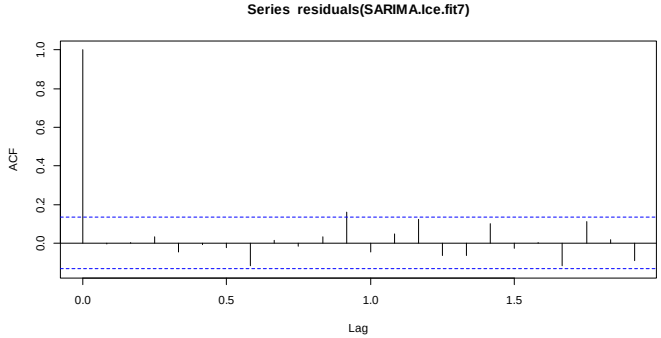

```
> rh.pred
Jan 2018 Feb 2018 Mar 2018
13.21336 14.08475 14.31520
```


SARIMA Model:

```
> SARIMA.Ice.fit = arima(Ice.ts,order=c(0,1,0),
   seasonal=list(order=c(0,1,0),period=12))
> plot.ts(residuals(SARIMA.Ice.fit),main="Residual Series")
> acf(residuals(SARIMA.Ice.fit))
```

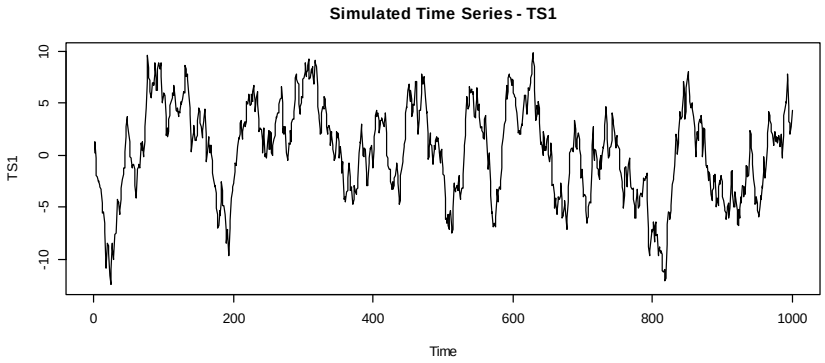


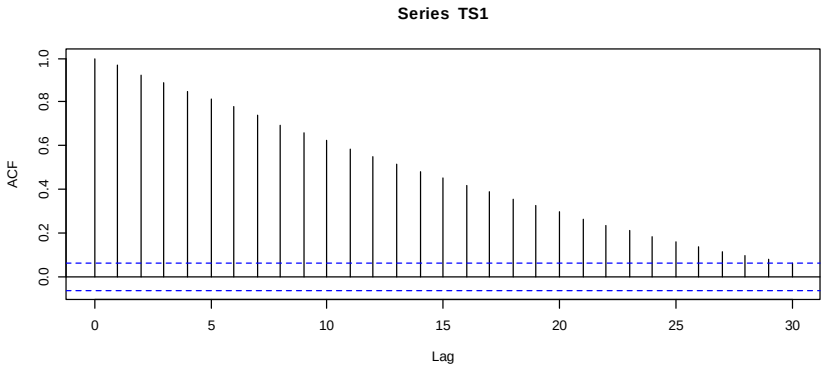

```
> SARIMA.Ice.fit7 = arima(Ice.ts,order=c(1,1,2),
   seasonal=list(order=c(0,1,1),period=12))


> acf(residuals(SARIMA.Ice.fit7))
```

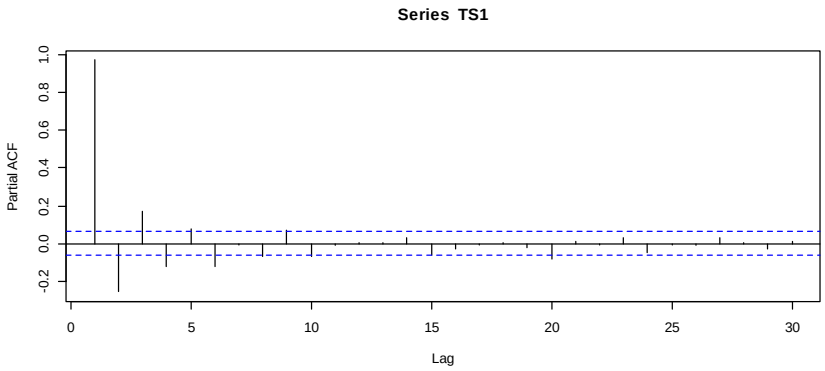


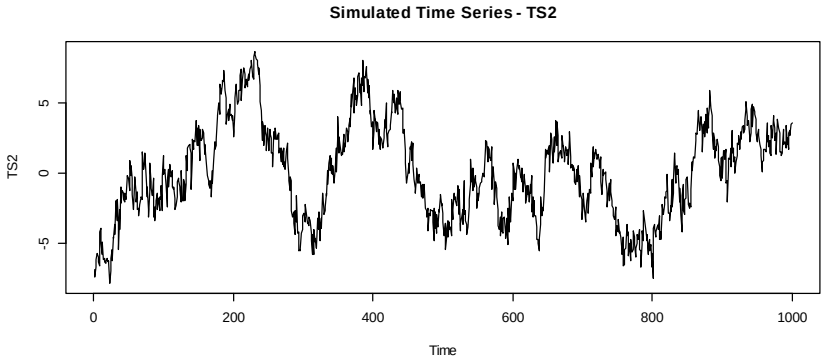

```
> SARIMA.pred
         Jan      Feb      Mar
2018 13.24054 14.10785 14.33729


> SARIMA.RMSEP
[1] 0.1401202
```

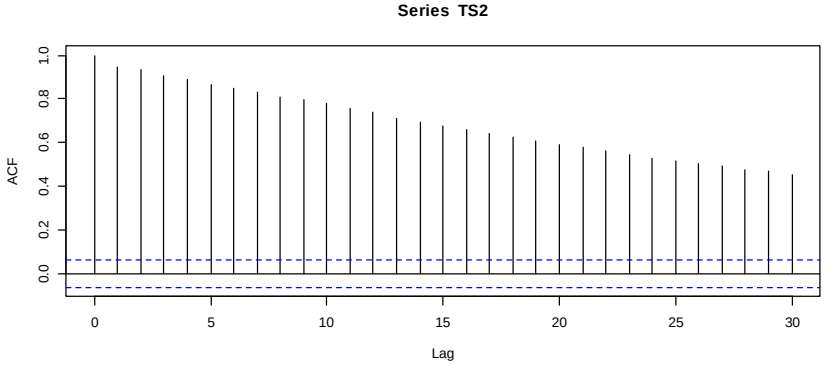
Simulated Stationary Time Series:

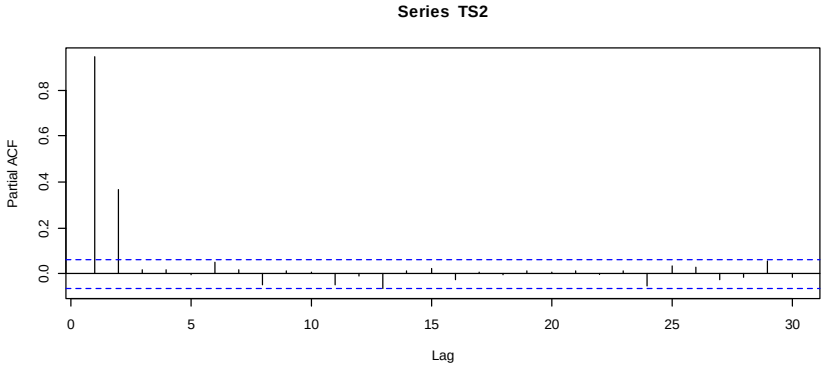


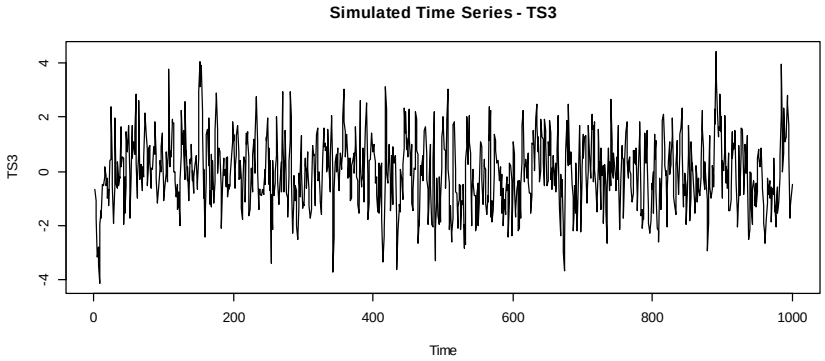


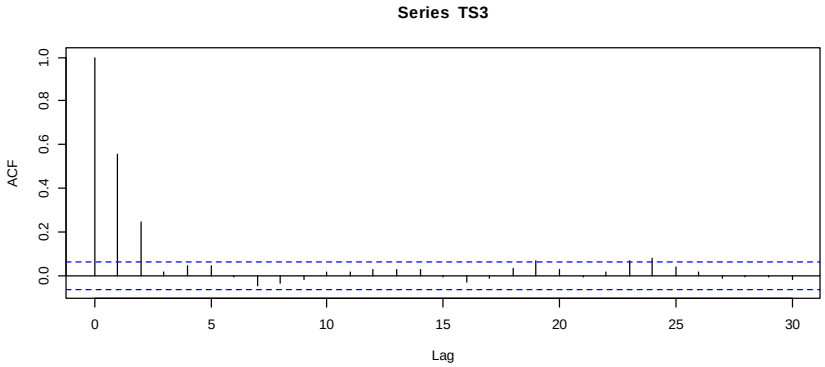


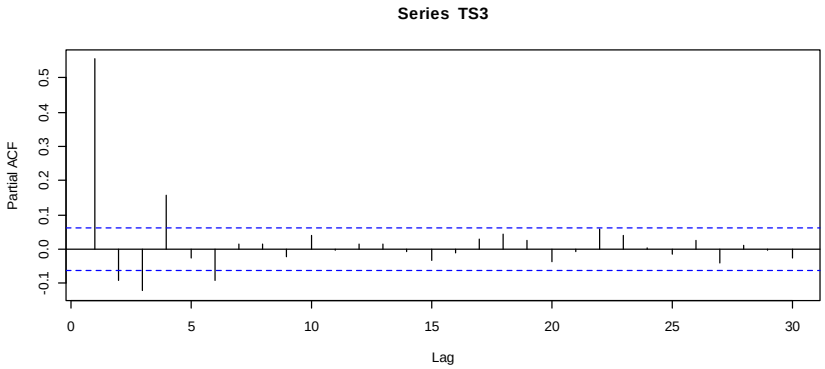


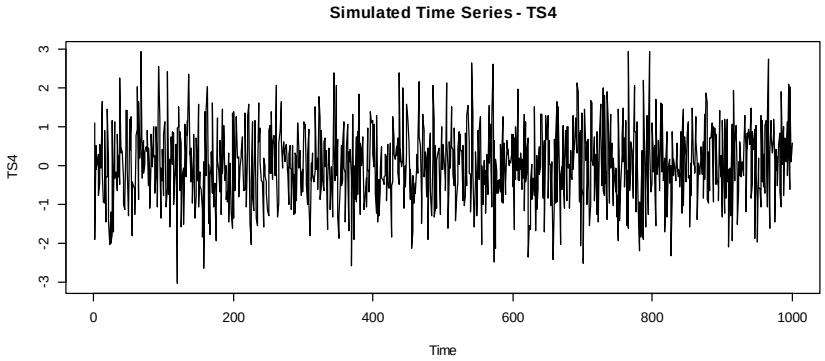


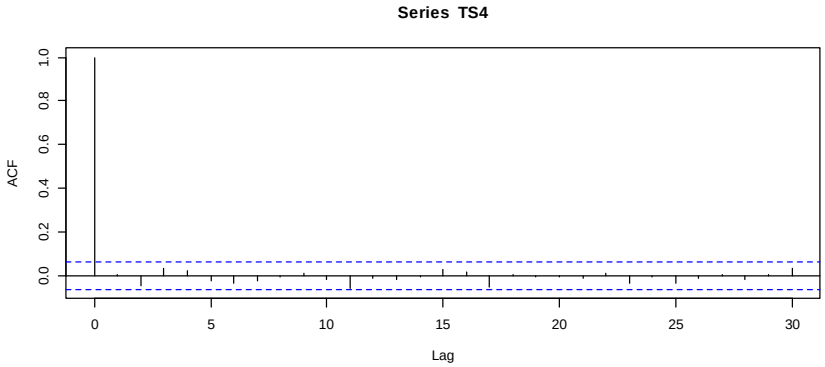


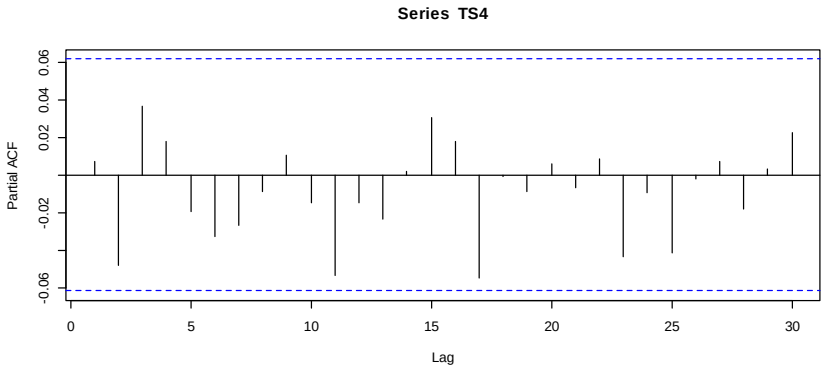

Simulated Time Series - TS3
TS3
Time


Series TS3
ACF
Lag


Series TS3
Partial ACF
Lag


Simulated Time Series - TS4
TS4
Time


Series TS4
ACF
Lag


Series TS4
Partial ACF
Lag