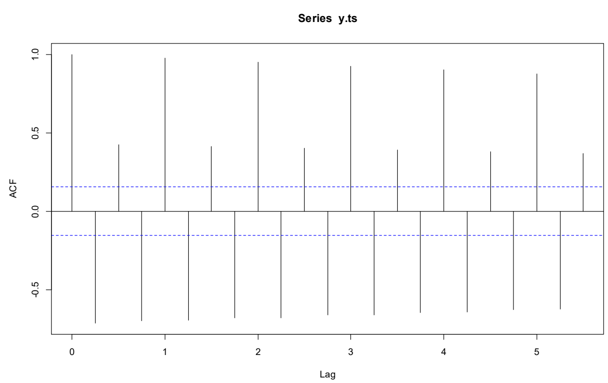
Department of Statistics
STATS 326: Applied Time Series
First Semester, 2019
Test 1 – Answer Guide
Total Marks = 100

1. Stationary Time Series have constant mean and constant variance. They may also have some type of autocorrelation pattern.
Non-stationary Time Series have trend and/or cycle and/or seasonality. They may also have an autocorrelation pattern.
Both types of series have a random component.
(10 marks)

2. Dependence on the past (autocorrelation or serial correlation) is a pattern that runs through a Time Series variable where observations are related (similar in value) to previous observations. It is a pattern that needs to be modelled
   - so the assumptions of our model, $\varepsilon_t \sim iidN(0, \sigma^2)$ are satisfied, and
   - our predictions using the model are reliable.

(10 marks)

3.



(10 marks)

4. $\pm \frac{1.96}{\sqrt{169}} = \pm\, 0.15076923$
(5 marks)

5.
$$(1-\mathrm{B})(1-\mathrm{B}^{12})y_t = \varepsilon_t$$
$$\Rightarrow (1-\mathrm{B}-\mathrm{B}^{12}+\mathrm{B}^{13})y_t = \varepsilon_t$$
$$\Rightarrow y_t - y_{t-1} - y_{t-12} + y_{t-13} = \varepsilon_t$$
$$\Rightarrow y_t = y_{t-1} + y_{t-12} - y_{t-13} + \varepsilon_t \quad where \quad \varepsilon_t \sim iidN(0, \sigma^2)$$
(10 marks)

6. The Monthly Artic Sea Ice series has a slight decreasing trend and a reasonably constant seasonal component. The seasonal peak appears to be in March with the seasonal trough in September. The seasonal component is slightly larger in 2007, 2008 and 2012. There will most likely be an autocorrelation pattern in the Residual Series from a model of these data.
(10 marks)

7. Jan 18:
10.043514876 + 0.009411422*1 + 3.043538037
= 13.09646433 (= 13.10 million square kilometres)
Feb 18:
10.043514876 + 0.009411422*2 + 3.717208561
= 13.77954628 (= 13.78 million square kilometres)
Mar 18:
10.043514876 + 0.009411422*3 + 3.845767189
= 13.91751633 (= 13.92 million square kilometres)

HW RMSEP:
sqrt(1/3 * [(13.06 – 13.09646433)^2 +(13.95 – 13.77954628)^2 +
(14.30 – 13.91751633)^2]) = 0.242678027 (= 0.24 million square kilometres)
(15 marks)

8a. The plot shows that the trend and random components have very similar ranges. The range of the seasonal component is about 10 and the range of the data is about 12 million square kilometres.
(5 marks)

8b. The plot of the Residual Series shows reasonably constant variation but there appears to be slight clustering. The acf of the Residual Series shows 2 significant lags (lag 1 and lag 11) which are both positive and too large to ignore. The 5-number summary of the Residual Series has similar absolute values for the min and max and also for the 1st and 3rd quartiles with a median close to 0 indicating the residuals are normally distributed. The autocorrelation in the Residual Series means the assumptions are not satisfied.
(10 marks)

9. Feb 79:
2.998559 – 0.001460 * 218 + 0.738814 * (13.26479 – 2.8855419) = 10.34861281
10.34861281 + 3.7698393 = 14.11845211 (= 14.12 million square kilometres)

STL SA RMSEP:
sqrt(1/3 * [(13.06 – 13.26479)^2 + (13.95 – 14.11845211)^2 + (14.30 – 14.32184)^2])
= 0.153614281 (= 0.15 million square kilometres)
(10 marks)

10. The Seasonal Trend Lowess model is the best predicting model as it has the lowest RMSEP (0.15 compared to HW at 0.24 million square kilometres).
(**<u>Bonus</u>:** However, since the independence assumption of the STL model is not satisfied, HW is the best predicting model as it has no assumptions to satisfy.)
(5 marks + **5 marks**)