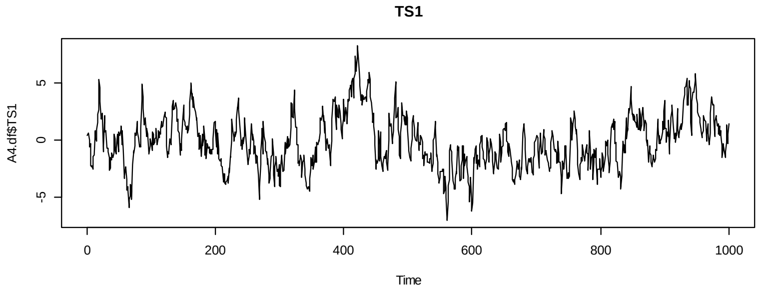
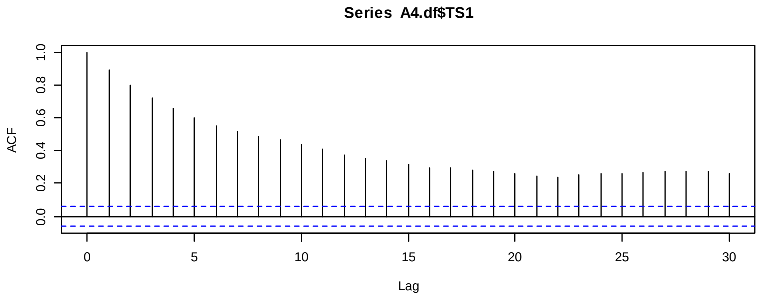
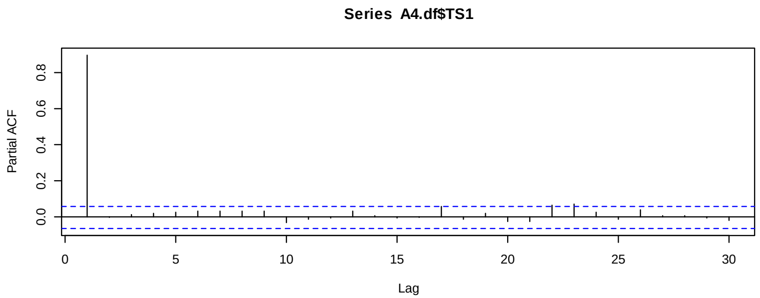
STATS 326
Applied Time Series
ASSIGNMENT FOUR
ANSWER GUIDE

<u>Question One</u>: TS1

```
> plot.ts(A4.df$TS1,main="TS1")
> acf(A4.df$TS1)
> pacf(A4.df$TS1)
```







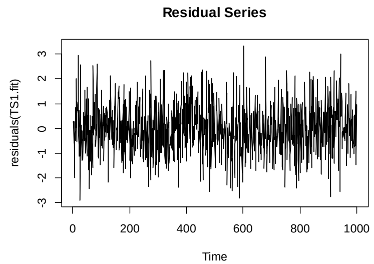
$$y_t = \rho_1 y_{t-1} + \varepsilon_t$$

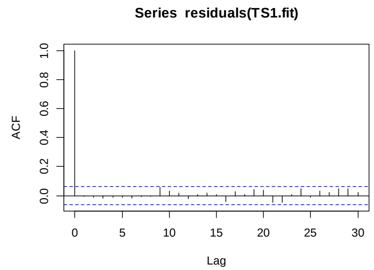
The plot of the series shows clustering indicating positive autocorrelation. The acf shows decay while the pacf shows cut-off at lag 1. This suggests an AR(1) is the most suitable model.

```
> TS1.fit = arima(A4.df$TS1,order=c(1,0,0))
> TS1.fit

Call:
arima(x = A4.df$TS1, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.8960    -0.1390
s.e.  0.0139     0.3006

sigma^2 estimated as 0.9944: log likelihood = -1416.93, aic = 2839.86
```

$$y_t = 0.896 y_{t-1} + \varepsilon_t$$

```
> plot(residuals(TS1.fit),main="Residual Series")
> acf(residuals(TS1.fit))
```





The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows no significant lags.

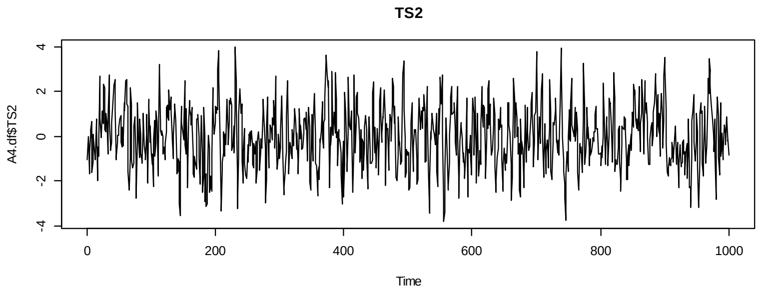
Other models tried:

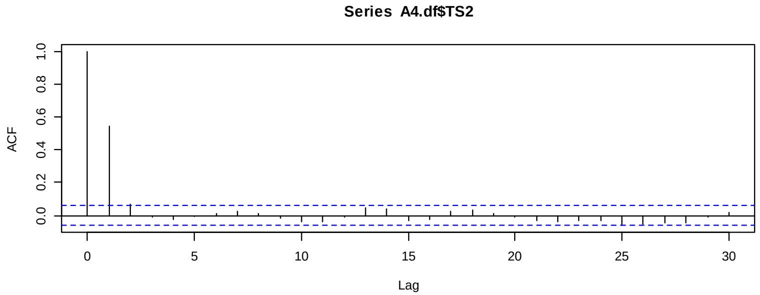
AR(2)	AIC = 2841.86	2^{nd} AR term not significant
ARMA(1,1)	AIC = 2841.86	MA term not significant

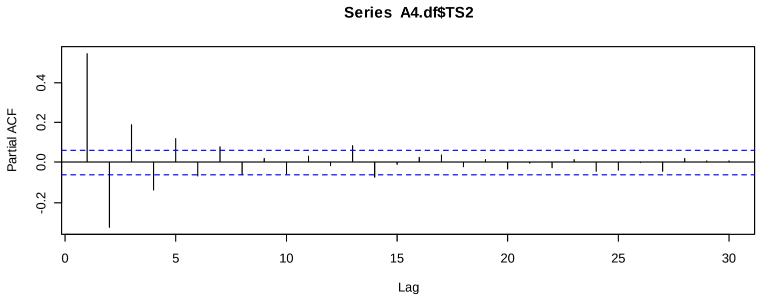
The AR(1) model had the smallest AIC and all terms were significant.

Question Two: TS2

```
> plot.ts(A4.df$TS2,main="TS2")
> acf(A4.df$TS2)
> pacf(A4.df$TS2)
```







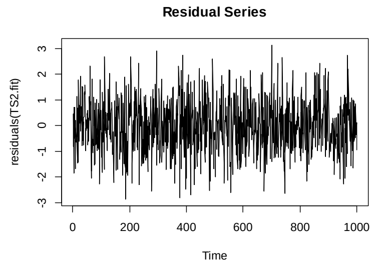
$$y_t = \varepsilon_t + \alpha_1 \varepsilon_{t-1}$$

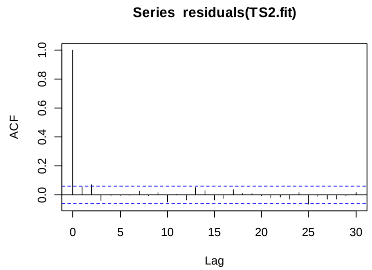
The plot of the series shows little in the way of a pattern. The acf shows cut-off at lag 2 and the pacf shows decay (or persistence). This suggests a MA(1) is the most suitable model.

```
> TS2.fit = arima(A4.df$TS2,order=c(0,0,1))
> TS2.fit

Call:
arima(x = A4.df$TS2, order = c(0, 0, 1))

Coefficients:
         ma1  intercept
      0.8073    -0.0182
s.e.  0.0195     0.0588

sigma^2 estimated as 1.06: log likelihood = -1448.8, aic = 2903.6
```

$$y_t = \varepsilon_t + 0.8073\varepsilon_{t-1}$$

```
> plot.ts(residuals(TS2.fit),main="Residual Series")
> acf(residuals(TS2.fit))
```





The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows no significant lags.

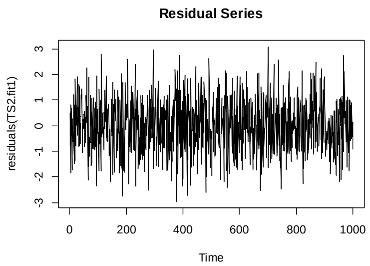
Better model:

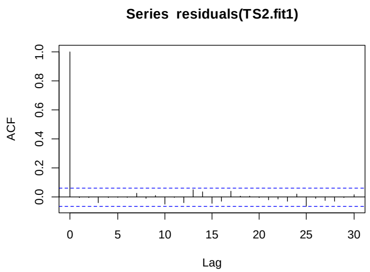
MA(3) AIC = 2900.01

```
> TS2.fit1 = arima(A4.df$TS2,order=c(0,0,3))
> TS2.fit1

Call:
arima(x = A4.df$TS2, order = c(0, 0, 3))

Coefficients:
         ma1     ma2     ma3  intercept
      0.8625  0.1162  0.0569    -0.0186
s.e.  0.0319  0.0433  0.0322     0.0660

sigma^2 estimated as 1.052:  log likelihood = -1445.05,  aic = 2900.1
```

$$y_t = \varepsilon_t + 0.8625\varepsilon_{t-1} + 0.1162\varepsilon_{t-2} + 0.0569\varepsilon_{t-3}$$

```
> plot.ts(residuals(TS2.fit1),main="Residual Series")
> acf(residuals(TS2.fit1))
```

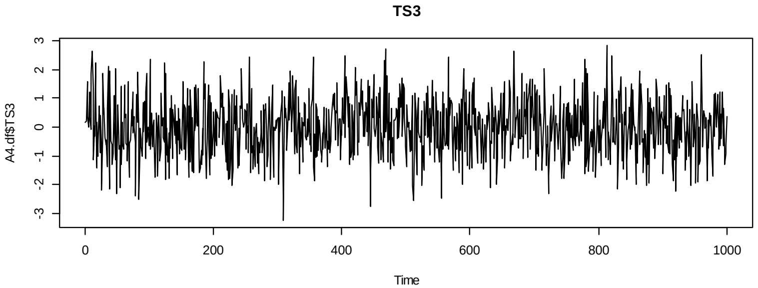


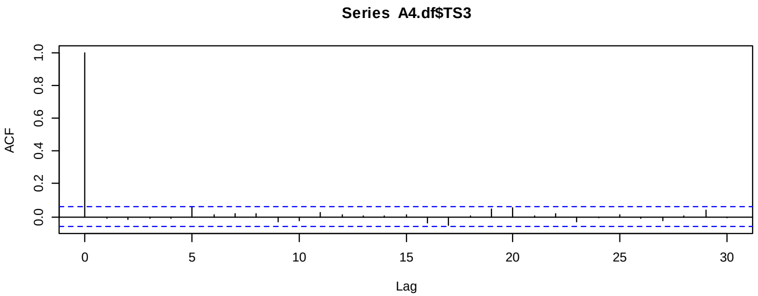


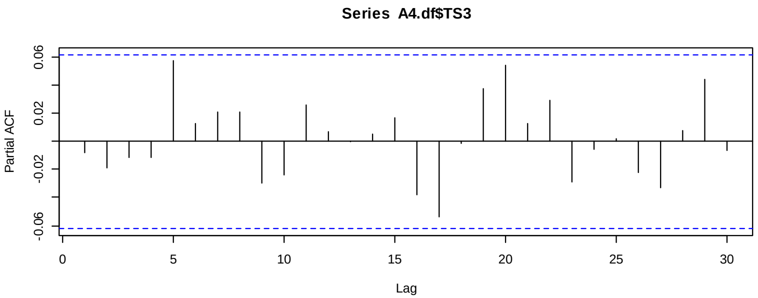
The MA(3) term is just significant at the 10% level.

<u>Question Three</u>: TS3

```
> plot.ts(A4.df$TS3,main="TS3")
> acf(A4.df$TS3)
> pacf(A4.df$TS3)
```







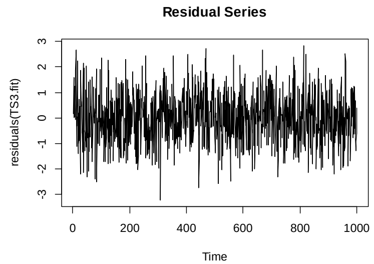
$$y_t = \varepsilon_t$$

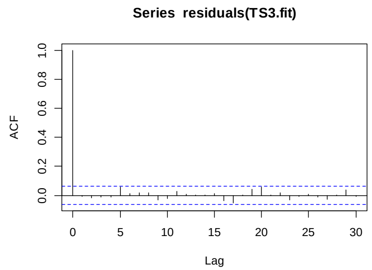
The plot of the series shows little in the way of a pattern. The acf and the pacf have no significant lags. This suggests the series is White Noise.

```
> TS3.fit = arima(A4.df$TS3,order=c(0,0,0))
> TS3.fit

Call:
arima(x = A4.df$TS3, order = c(0, 0, 0))

Coefficients:
      intercept
        -0.0138
s.e.     0.0318

sigma^2 estimated as 1.012:  log likelihood = -1425.11,  aic = 2854.22
```

$$y_t = \varepsilon_t$$

```
> plot.ts(residuals(TS3.fit),main="Residual Series")
> acf(residuals(TS3.fit))
```





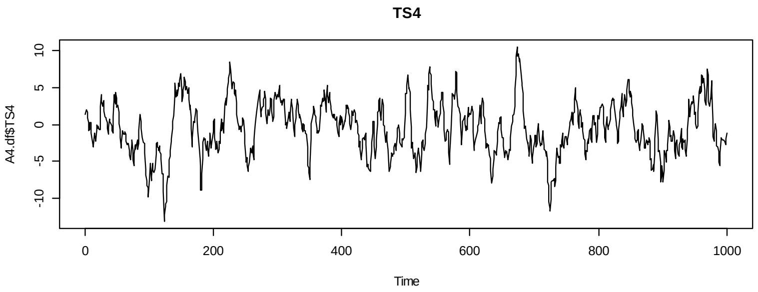
The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows no significant lags.

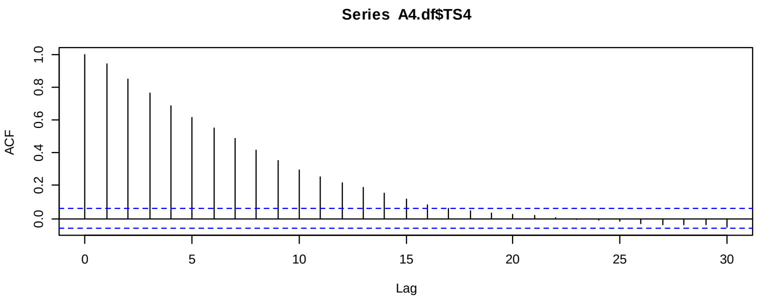
Other models tried:

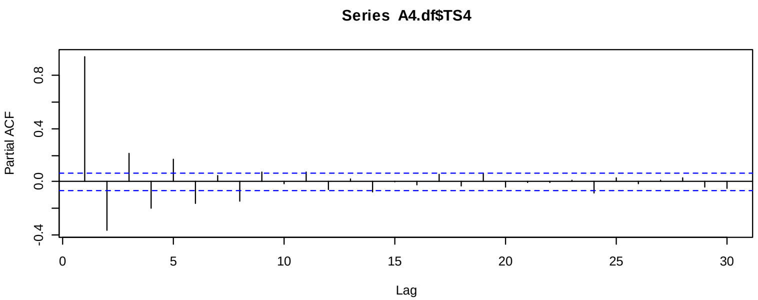
AR(1)	AIC = 2856.16	AR term not significant
MA(1)	AIC = 2856.16	MA term not significant

Question Four: TS4

```
> plot.ts(A4.df$TS4,main="TS4")
> acf(A4.df$TS4)
> pacf(A4.df$TS4)
```







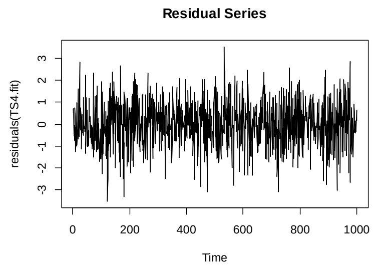
$$y_t = \rho_1 y_{t-1} + \varepsilon_t + \alpha_1 \varepsilon_{t-1}$$

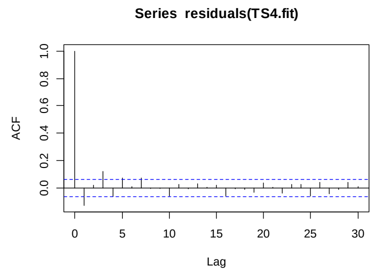
The plot of the series shows clustering indicating positive autocorrelation. The acf shows decay and the pacf also shows decay. This suggests an ARMA(p,q) is the appropriate model, but as we have no indication from the plots of the order we begin with an ARMA(1,1).

```
> TS4.fit = arima(A4.df$TS4,order=c(1,0,1))
> TS4.fit

Call:
arima(x = A4.df$TS4, order = c(1, 0, 1))

Coefficients:
         ar1     ma1  intercept
      0.8680  0.7647    -0.4931
s.e.  0.0162  0.0307     0.4342

sigma^2 estimated as 1.07:  log likelihood = -1454.32,  aic = 2916.65
```

$$y_t = 0.868 y_{t-1} + \varepsilon_t + 0.7647 \varepsilon_{t-1}$$

```
> plot.ts(residuals(TS4.fit),main="Residual Series")
> acf(residuals(TS4.fit))
```





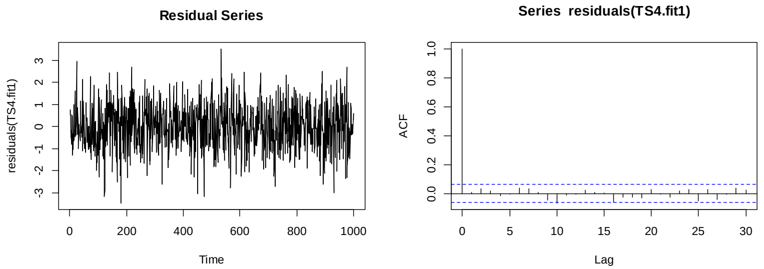
The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows 2 significant lags.

Better model:

```
> TS4.fit1 = arima(A4.df$TS4,order=c(2,0,1))
> TS4.fit1

Call:
arima(x = A4.df$TS4, order = c(2, 0, 1))

Coefficients:
         ar1     ar2     ma1  intercept
      0.6245  0.2607  0.8865    -0.4903
s.e.  0.0375  0.0374  0.0181     0.5205

sigma^2 estimated as 1.025: log likelihood = -1432.98, aic = 2875.978
```

$$y_t = 0.6245y_{t-1} + 0.2607y_{t-2} + \varepsilon_t + 0.8865\varepsilon_{t-1}$$

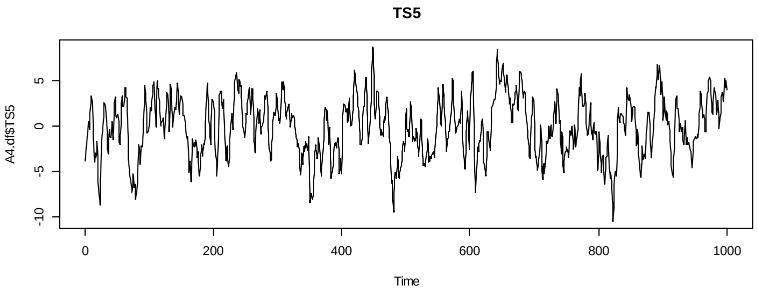
```
> plot.ts(residuals(TS4.fit1),main="Residual Series")
> acf(residuals(TS4.fit1))
```

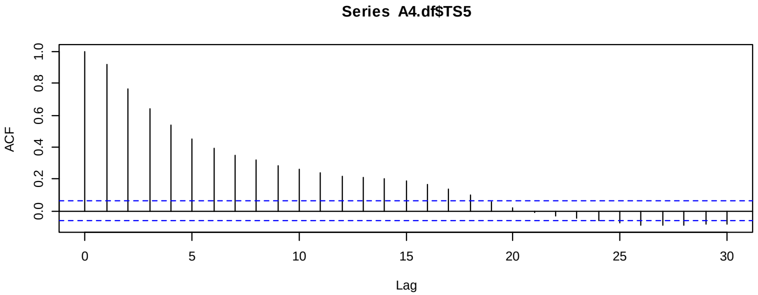


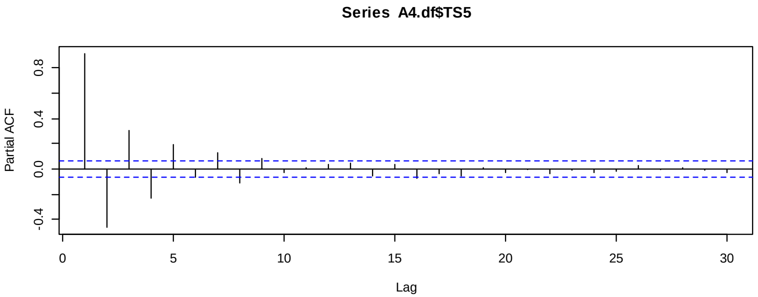
The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows no significant lags.

Question Five: TS5

```
> plot.ts(A4.df$TS5,main="TS5")
> acf(A4.df$TS5)
> pacf(A4.df$TS5)
```







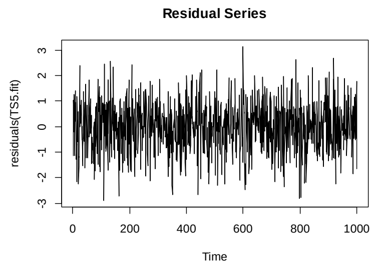
$$y_t = \rho_1 y_{t-1} + \varepsilon_t + \alpha_1 \varepsilon_{t-1}$$

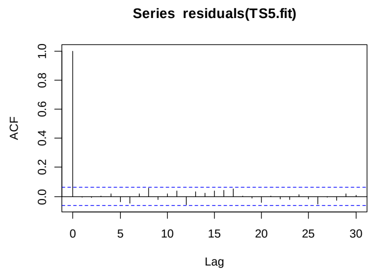
The plot of the series shows clustering indicating positive autocorrelation. The acf shows decay and the pacf also shows decay. This suggests an ARMA(p,q) is the appropriate model, but as we have no indication from the plots of the order we begin with an ARMA(1,1).

```
> TS5.fit = arima(A4.df$TS5,order=c(1,0,1))
> TS5.fit

Call:
arima(x = A4.df$TS5, order = c(1, 0, 1))

Coefficients:
         ar1     ma1  intercept
      0.8379  0.8646    -0.0943
s.e.  0.0175  0.0173     0.3538

sigma^2 estimated as 0.9566: log likelihood = -1398.58, aic = 2805.1
```

$$y_t = 0.8379 y_{t-1} + \varepsilon_t + 0.8646 \varepsilon_{t-1}$$

```
> plot.ts(residuals(TS5.fit),main="Residual Series")
> acf(residuals(TS5.fit))
```





The Residual Series appears to be random scatter about 0. The plot of the autocorrelation function of the Residual Series shows no significant lags.

Other models tried:

ARMA(2,1)	AIC = 2807.16	2nd AR term not significant
ARMA(1,2)	AIC = 2807.16	2nd MA term not significant

The AR(1) model had the smallest AIC and the AR term was significant.