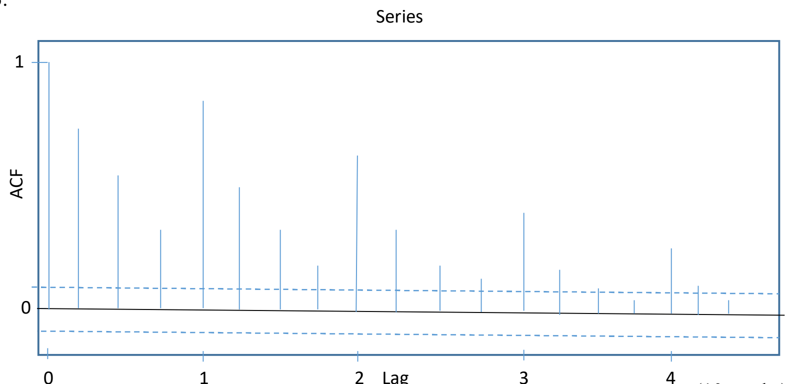
Department of Statistics
STATS 326: Applied Time Series
First Semester, 2020
Test 1 – Answer Guide
Total Marks = 100

1. The plot of the acf is used to assist in determining whether the assumptions of the model are satisfied. The errors must be independently and identically distributed from a normal distribution with 0 mean constant variance: $\varepsilon_t \sim iidN(0,\sigma^2)$. The plot of the acf of the residuals is used to assess the independence aspect of the assumptions. While there will nearly always be an autocorrelation structure in the raw data, the model's components (trend and seasonality) will remove some, or on rare occasions all of that autocorrelation pattern. The autocorrelation that remains appears in the Residual Series.
(10 marks)

2. A lagged response variable will take care of all of the autocorrelation in the initial model when the plot of the acf shows exponential decay and the plot of the pacf shows only 1 significant lag at $k = 1$.
(5 marks)

3.



(10 marks)

4. $\pm \frac{1.96}{\sqrt{625}} = \pm 0.078$
(5 marks)

5.
$$(1-\text{B})(1-\text{B}^4)^2 y_t = \varepsilon_t$$
$$\Rightarrow (1-\text{B})(1-\text{B}^4)(1-\text{B}^4) y_t = \varepsilon_t$$
$$\Rightarrow (1-\text{B})(1-2\text{B}^4+\text{B}^8) y_t = \varepsilon_t$$
$$\Rightarrow (1-2\text{B}^4+\text{B}^8-\text{B}+2\text{B}^5-\text{B}^9) y_t = \varepsilon_t$$
$$\Rightarrow y_t = (\text{B}+2\text{B}^4-2\text{B}^5-\text{B}^8+\text{B}^9) y_t + \varepsilon_t$$
$$\Rightarrow y_t = y_{t-1} + 2y_{t-4} - 2y_{t-5} - y_{t-8} + y_{t-9} + \varepsilon_t \quad where \quad \varepsilon_t \sim iidN(0,\sigma^2)$$
(20 marks)

6. The Monthly Global N2O atmospheric concentration shows an increasing reasonably linear trend that appears to have a change in slope around August 2007 (observation 80). There appears to be a seasonal pattern with the seasonal peak around January and the seasonal trough around August.
(5 marks)

7. July 2019: 331.7998 + 0.0787*1 – 0.1766 = 331.7019 = 331.7 ppb
Aug 2019: 331.7998 + 0.0787*2 – 0.1898 = 331.7674 = 331.8 ppb
Sept 2019: 331.7998 + 0.0787*3 – 0.1403 = 331.8956 = 331.9 ppb
Oct 2019: 331.7998 + 0.0787*4 – 0.0377 = 332.0769 = 332.1 ppb
(10 marks)

8a. The plot shows that the trend is increasing and is reasonably linear until around observation 80 (August 2007) when the slope increases, but remains reasonably linear.
(3 marks)

8b. The plot of the Residual Series shows reasonably constant scatter with a zero mean. The acf of the Residual Series shows 1 significant positive lag (lag 1) which is small enough to ignore (especially given the autocorrelation estimate is 0.9). The 5-number summary of the Residual Series has a median close to 0, similar absolute values for the 1st and 3rd quartiles and similar absolute values for the minimum and maximum indicating the distribution is very symmetric. Normality appears satisfied. The Residual Series appears to be White Noise - $\varepsilon_t \sim iidN(0,\sigma^2)$ as required for the assumptions to be satisfied.
(12 marks)

9. Aug 2019:
32.04 + 0.006196*224 + 0.001899*144 - 0.18823189 + 0.8987*(331.7213 + 0.18337078) = 331.7959 = 331.8 ppb

Oct 20129:
32.04 + 0.006196*226 + 0.001899*146 - 0.04236004 + 0.8987*(331.9275 + 0.13198189) = 332.097 = 332.1 ppb

STL SA RMSEP:
sqrt(1/4 * [(331.9 – 331.7213)^2 + (331.9 – 331.7959)^2 + (331.9 – 331.9275)^2 + (332.1 – 332.097)^2]) = 0.1043261 (= 0.1 ppb)
(15 marks)

10. The Moving Average Seasonally Adjusted model is the best predicting model as it has the lowest RMSEP (0.104 ppb compared to HW at 0.120 ppb).
(5 marks)