

# CS 111. Homework 7

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1. 1.1. We need to change all of the element at the boundaries top, bottom, left and right,to make the row sum = 0, which in total 12 element

1.2. 4k-4

1.3. grid of  $n^2$  points are connected

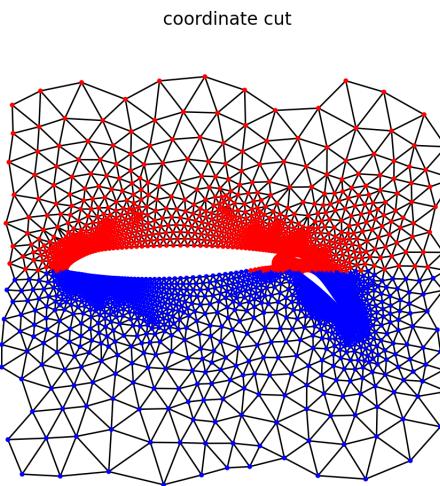
2. 2.1. We can describe a partition by labeling each vertex with either a 1 or a -1

$x(i), x(j)$  belongs to  $\{-1, 1\}$

$(x(i) - x(j))^2 = 4$ , the squared differences of vertex of different labels is 4, so 4 is added to the LQF every time the edge has vertices with different labels(cuts), the quadratic sum is 4 times the cuts.

$$\alpha = 1/4$$

2.2. Coordinate cut



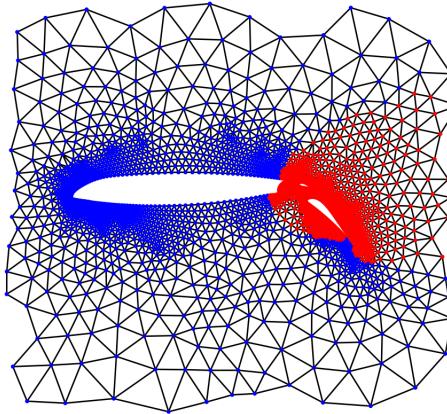
```
y=[]
for v in range(len(xycoords)):
    y.append(xycoords[v][1])
median = np.median(y)
print(median)
colors = []
x = []
for v in G.nodes():
    x_v, y_v = xycoords[v]
    if y_v > median:
        colors.append('r')
        x.append(1)
    else:
        colors.append('b')
        x.append(-1)
x = np.array(x)
x.T@L@x/4
-0.023435
148.0
```

the median = -0.023435, which means the y coordinates cut = -0.023435

the number of cuts = 148

### 2.3. Spectral cut

random partition



```
G, xycoords = cs111.read_mesh('airfoil1')
```

```
L = nx.linalg.laplacian_matrix(G).toarray()
lam, Q = spla.eigh(L)
fiedler_value = lam[1]
fiedler_vector = Q[:,1]
print('Fiedler value:', fiedler_value)
print('Fiedler vector:')
print(fiedler_vector)
fmedian = np.median(fiedler_vector)
print('Fiedler vector median:', fmedian)
```

Fiedler value: 0.0018479302795141638  
 Fiedler vector:  
 $[-0.0154 \ -0.0163 \ -0.0172 \ \dots \ 0.0075 \ 0.0061 \ 0.0078]$   
 Fiedler vector median: 0.007527750478250471

```
colors = []
x = []
for v in G.nodes():
    if Q[v,1] > fmedian:
        colors.append('r')
        x.append(1)
    else:
        colors.append('b')
        x.append(-1)
x = np.array(x)
print(x.T@L@x/4)
```

132.0

the number of cuts = 132