

CS 111. Homework 7

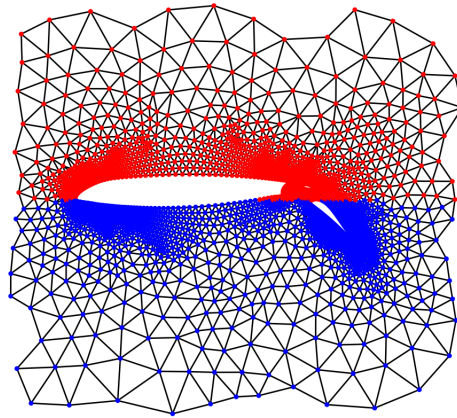
Xiao Zhou

1. 1.1. We need to change all of the element at the boundaries top, bottom, left and right, to make the row sum = 0, which in total 12 element
- 1.2. $4k-4$
- 1.3. grid of n^2 points are connected
2. 2.1. We can describe a partition by labeling each vertex with either a 1 or a -1
 $x(i), x(j)$ belongs to $\{-1, 1\}$
 $(x(i) - x(j))^2 = 4$, the squared differences of vertex of different labels is 4, so 4 is added to the LQF every time the edge has vertices with different labels (cuts), the quadratic sum is 4 times the cuts.

$$\alpha = 1/4$$

2.2. Coordinate cut

coordinate cut



```
y=[]
for v in range(len(xycoords)):
    y.append(xycoords[v][1])
median = np.median(y)
print(median)
colors = []
x = []
for v in G.nodes():
    x_v, y_v = xycoords[v]
    if y_v > median:
        colors.append('r')
        x.append(1)
    else:
        colors.append('b')
        x.append(-1)
x = np.array(x)
x.T@L@x/4
```

-0.023435

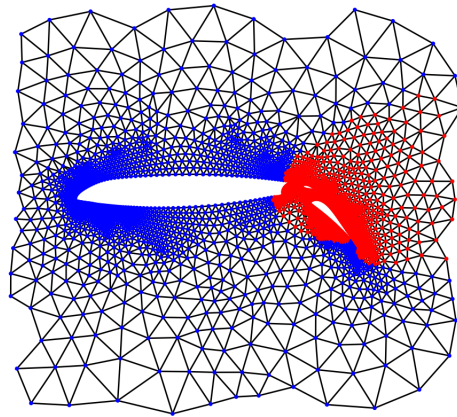
148.0

the median = -0.023435, which means the y coordinates cut = -0.023435

the number of cuts = 148

2.3. Spectral cut

random partition



```
G, xycoords = cs111.read_mesh('airfoill')
```

```
L = nx.linalg.laplacian_matrix(G).toarray()
lam, Q = spla.eigh(L)
fiedler_value = lam[1]
fiedler_vector = Q[:,1]
print('Fiedler value:', fiedler_value)
print('Fiedler vector:')
print(fiedler_vector)
fmedian = np.median(fiedler_vector)
print('Fiedler vector median:', fmedian)
```

```
Fiedler value: 0.0018479302795141638
Fiedler vector:
[-0.0154 -0.0163 -0.0172 ... 0.0075 0.0061 0.0078]
Fiedler vector median: 0.007527750478250471
```

```
colors = []
x = []
for v in G.nodes():
    if Q[v,1] > fmedian:
        colors.append('r')
        x.append(1)
    else:
        colors.append('b')
        x.append(-1)
x = np.array(x)
print(x.T@L@x/4)
```

```
132.0
```

the number of cuts = 132