

Exercise 1. (Multi-item auctions)

Suppose there are 3 bidders, and we have 6 items for sale. The bidders have utility functions that feature decreasing marginal utility additional items. In the table below the marginal utilities for each additional item are given for each of the bidders.

# items	Bidder 1	Bidder 2	Bidder 3
1	100	110	90
2	100	100	80
3	45	85	60
4	30	35	40
5	0	20	30
6	0	0	10

- (a) Execute the VCG mechanism. Compute the allocation plus the payments.
- (b) Execute the Ausubel auction. Show that both allocation and payments coincide with the VCG outcome.
- (c) What is the outcome under a uniform price auction when bidders report truthfully? Compare the net utilities with the VCG outcome.
- (d) Show that under a uniform price rule bidder 2 can profit from demand reduction by misreporting his utility function.

Exercise 2. (CE in economy with production)

Consider the following two-person economy consisting of agents A and B . Agent A owns 12 oranges. His utility of consuming x_A oranges and drinking y_A liters of orange juice is given by $u_A(x_A, y_A) = x_A y_A^2$. Agent B does not own any oranges, but he owns an orange squasher that can make $y = \sqrt{x}$ liters of orange juice out of x oranges. The utility of agent B of consuming x_B oranges and drinking y_B liters of orange juice is given by $u_B(x_B, y_B) = x_B y_B$. The price of an orange is p and the price of one liter of orange juice is q .

- (a) Compute the supply and demand functions for oranges and orange juice for both agents, and for the orange squasher.
- (b) Show that prices p and q form a competitive equilibrium exactly when $64p^2 = 3q^2$.
- (c) Show that in equilibrium $x_A = 4$, $y_A = \sqrt{3}$, $x_B = \frac{8}{3}$ and $y_B = \frac{1}{\sqrt{3}}$. Show that the resulting utilities are $u_A = 12$ and $u_B = \frac{8}{3\sqrt{3}}$.
- (d) Your economic gut feeling dictates that when the price of a good goes up, the demand for this good goes down. Nevertheless, the demand $y_B = \frac{q}{8p}$ for orange juice of agent B is increasing in the price q of orange juice and the demand $x_A = 4$ for oranges of agent A does not depend at all on the price p for oranges. Explain why this makes perfect sense in this particular model.

Exercise 3. (CE in exchange economy with quasi linear utility)

Consider an economy with two consumers, 1 and 2, and two goods, ‘money’ and ‘manna’. The utility functions of the two consumers are

$$u_1(x_1, y_1) = x_1 + \ln(y_1 + 1) \quad \text{and} \quad u_2(x_2, y_2) = x_2 + 2 \ln(y_2 + 1),$$

where x_i denotes the amount of money and $y_i \geq 0$ the amount of manna ($i = 1, 2$). Consumer 1 is endowed with 1 unit of manna and consumer 2 with 2 units of manna. Additionally, each consumer has 2 units of money.

- (a) Compute the allocation of manna where the sum of the utilities of the two consumers is maximal.
- (b) Describe all efficient allocations of money and manna between the two consumers. Motivate your answer.
- (c) What is the allocation of manna in a competitive equilibrium? (Do not make any new calculations.)
- (d) Let the price of money be 1 and the price of manna $p \leq 1$. Compute the competitive equilibrium price of manna, and the associated allocation of manna and money.