Competitive procurement with corruption

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We study competitive procurement administered by a corrupt agent who is willing to manipulate his evaluation of contract proposals in exchange for bribes. With complete information and no corruption, the efficient firm will win the contract for sure. If the agent is corrupt and has large manipulation power, however, bribery makes it costly for the efficient firm to secure a sure win, so in equilibrium the efficient firm loses the contract with positive probability. The optimal scoring rule for the buyer deemphasizes quality relative to price and does not fully handicap, and may even favor, the efficient firm.

> "*Corruption wins not more than honesty.*"
> —Shakespeare, *King Henry VIII, Act 3, Scene 2, Line 444*

1. Introduction

■   Bribe taking in competitive procurement, whether public or private, is widespread. During the first half of the 1970s, more than 450 U.S. companies, 117 of which were listed in the Fortune 500, had made over $400 million in questionable payments to foreign concerns,[1] and the U.S. firms allegedly lost nearly 100 foreign contracts worth $45 billion to foreign competitors through graft in 1994-1995.[2] Among the recent publicized cases are the 1988 U.S. investigation into defense procurement fraud, dubbed "Operation Ill Wind," which resulted in the conviction of 46 individuals and 6 defense corporations, with fines and penalties totaling $190 million, and the 1993 case in Korea that alleged bribe taking by two former defense ministers and former chiefs

[1] This figure came from the voluntary disclosure program of the U.S. Securities and Exchange Commission. See Rossbacher and Young (1997).

[2] This claim is based on a Department of Commerce calculation. See "An End to Corruption," *Washington Post*, April 16, 1996, p. A15.

of air force and naval operations.[3] Increased anticorruption movements across the world and a toughened stance by international organizations reflect both the seriousness and prevalence of the problem in international procurement.[4]

Corruption would never be an issue if the buyer could procure directly without leaving any discretion to a third party. Delegation is often inevitable, however, since evaluating proposals requires special expertise that the buyer may not possess. Often, the procured goods and services involve new technologies and/or nonstandard designs, which are difficult to objectively measure or evaluate. For instance, consider the procurement of a high-speed train system. Apart from the price, "quality" features such as reliability, after-sale technical support, safety, and environmental impact would be important for the buyer. Although the buyer can instruct the procurement officials to apply a specific weighting rule, evaluation of these quality features would require special expertise.

This need for relying on a third-party assessment of contract proposals creates a potential for bribery and corruption. For instance, a procurement officer in charge of assessing proposals can manipulate her evaluation to "steer" the contract to a bribing company. To some extent, such manipulation can be accomplished without even creating suspicion of impropriety, since evaluating new, untested technologies can be subjective.[5]

We present a model with these properties. A buyer procures a good and cares about both quality and the payment he makes to the winning firm. Firms submit multidimensional contract offers specifying quality and price. Simultaneously, they also offer bribes to the agent in charge of evaluating quality bids. The agent then manipulates the quality assessment to favor the high briber. We assume that there is a limit to the extent to which the agent can misrepresent the quality, which we treat as an exogenous parameter.[6] The firm that achieves the high score, *after manipulation*, is then awarded the contract on the proposed terms and pays the bribe to the agent.

We investigate several issues. First, we analyze how corruption affects the allocation of contracts. While it is widely believed that corruption hinders efficient allocation of contracts, the economic logic behind such a belief is far from clear. In essence, corruption entails just another form of competition, and there is no *a priori* reason why competition in bribery should have a more or less adverse impact on allocative efficiency than competition in contract bids. Our model provides a natural framework for investigating whether these two forms of competition, legal and illegal, differ fundamentally and how they interact with each other.[7]

Suppose that two firms, one more efficient than the other, compete with complete information. If the agent is not corrupt, then the more efficient firm will offer a quality-price pair that the less efficient firm cannot profitably match, so the former will win the contract for sure. At first glance, a similar result may appear to hold even when the agent is corrupt: i.e., the efficient firm will offer

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[3] See Fite (1988) and "Navy Ex-Official Gets 4-Year Sentence; Paisley Also Fined $50,000 in Ill Wind Procurement Fraud Probe," *Washington Post*, October 19, 1991, for the former, and see "South Koreans Arrest 4 Former Officials on Bribery Charges," *The New York Times*, July 18, 1993, for the latter.

[4] U.S. corporations are bound by the Foreign Corrupt Practices Act of 1977, which prohibits transnational bribery and the associated tax deductibility. The United Nations, IMF, and World Bank recently declared it their duty to press for anticorruption reforms in countries seeking to borrow money (*Washington Post*, October 4, 1996). The Organization for Economic Cooperation and Development (OECD) recommended that its members stop allowing tax write-offs on bribes, which used to be common practice in many countries.

[5] Steering of contracts to specific firms is sometimes accomplished by adding a few lines to the contract requiring a certain type of equipment that only one company manufactures (Fite, 1988). Other methods of manipulation discovered in Operation Ill Wind ranged from restructuring procurement procedures and leaking crucial information about engineering requirements, to revealing the bid information of the competing firms. See Burguet and Perry (2000) for a model that explicitly considers the last possibility.

[6] For instance, in the high-speed train procurement, a procurement officer cannot judge a conventional train system to be better than the newest models of the French TGV or German ICE, but he can argue that one of the latter is superior or better suited to the particular country procuring the system.

[7] In particular, the agent in our model does not *a priori* prefer one firm over the other (which would clearly hinder allocative efficiency); rather, she assigns her favor solely based on bribery competition. Furthermore, a first-price auction is used to model bribery competition just like bidding competition in the contract terms. That the agent accepts a bribe only from the winning firm appears consistent with the folklore description of how corruption occurs. See also our concluding remarks.

a bribe-contract pair that the less efficient firm cannot profitably match. This is indeed true if the agent has little manipulation power, so the efficient firm wins. But if the agent has substantial manipulation power, this strategy becomes too costly for the efficient firm. The reason is due to the fundamentally different effects that bribery and the contract bid have on competition. In our model, the agent is willing to manipulate significantly for a high briber. Hence, to secure a win, an outbribed efficient firm must undercut its competitor by a discrete amount (whereas even a slight undercutting was sufficient for winning the contract without corruption). The efficient firm can overcome this problem by outbribing the inefficient firm, but, as will be seen later, the inefficient firm can randomize over bribe/contract offers with different bribes in such a way that makes it very costly (if not impossible) for the efficient firm to win against all these offers. In short, the corruption of the agent makes the inefficient firm a much more versatile competitor, against whom it is very difficult to extract a sure win. Consequently, in equilibrium the efficient firm concedes winning with some probability. This inefficiency prediction is shown to be robust to changes in the extensive form, to the number of competing firms, and to changes in the bribery technology.

Second, we study distributional implications of corruption. If the agent has little manipulation power, corruption does not disrupt allocational efficiency but simply makes the efficient firm compete aggressively. Thus, surprisingly, corruption benefits the buyer. The effects of corruption are quite different if the agent has substantial manipulation power. In that case, corruption results in an inefficient allocation. Moreover, bribery softens price competition, which inflates the cost of procurement. According to some estimates, the cost increase due to corruption is likely to be in the range of 10–45% of contract value in Europe (see Eigen, 1997).[8] Hence, the buyer is worse off from corruption, possibly by more than the bribes paid to the agent. The inefficient firm benefits unambiguously from corruption, since it now has a chance to win. Interestingly, the efficient firm can also benefit from corruption, due to softened price competition, if the agent's manipulation power is large. In this case, corruption can be seen as a device for facilitating collusion among competing firms.

Finally, we examine the question of how the buyer should design his selection rule to limit the adverse effect of corruption. Specifically, we address two issues: (1) Should the buyer handicap the efficient firm? If so, how much? (2) How much relative weight should the buyer place on the quality and the price offer? Absent corruption, a straightforward argument shows that the buyer, given complete information, should handicap the efficient firm to the point of virtually eliminating its competitive advantage, but he should not distort his scoring rule on the quality-price tradeoff. With this optimal handicapping, the efficient firm would still win for sure but would surrender to the buyer the entire social surplus associated with the project. We show that a qualitatively different selection policy is desirable for the buyer when facing a corrupt agent. First, unlike the standard prescription, the optimal policy involves less discrimination against the efficient firm and may even involve "reverse discrimination" against the inefficient firm. Handicapping the efficient firm exacerbates the inefficient allocation, due to bribery, without intensifying the price competition. Second, the optimal policy deemphasizes quality relative to price to limit the rents accruing to the efficient firm.

Most of the work on procurement and auctions ignores the corruption issue, with a few exceptions. Rose-Ackerman (1975) argues that corruption can lead to inefficient contract allocation and inflated costs of procurement.[9] The allocative inefficiency there is due, however, to firms facing different costs of bribing (e.g., an inefficient firm may be less scrupulous than a more efficient firm) and the vague preferences of the government. We assume away the differences in the firms' costs of bribery and vague preferences in order to focus on the fundamental differences in the roles played by bribery competition and contract competition. Laffont and Tirole (1991)

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[8] There are some telltale anecdotes indicating the extent of cost increases resulting from corruption. One of the Italian *mani pulite* ("clean hands") judges reported that the construction of subways cost $227 million per kilometer in 1991; after anticorruption actions, the cost fell to $97 million. Likewise, the costs of city rail links fell from $54 million per kilometer in 1991 to $26 million in the mid-1990s. Germany's auditor general estimated that the government suffered costs resulting from corruption in the construction field of about DM 5 billion a year (see Eigen, 1997).

[9] See also an extension of this model in Rose-Ackerman (1978).

and Celentani and Ganuza (2002) study the issue of favoritism in procurement. The assignment of favor (i.e., who colludes with whom) is exogenous in these articles, whereas the current article endogenizes the assignment of the agent's favor through bribery competition. Finally, Burguet and Perry (2000) and Compte, Lambert and Verdier (2000) consider procurement corruption wherein the agent favors a firm by offering it an opportunity to revise its bid to beat its opponent. Bribery competition does not arise at the same time as contract bidding[10] and the quality manipulation issue does not arise there.

The rest of the article is organized as follows. Section 2 presents a procurement model. Section 3 analyzes the outcome of competitive procurement in the absence of corruption. We show that efficiency is guaranteed in this case. Section 4 introduces corruption to show how efficiency and the payoffs of the parties are affected in equilibrium. Section 5 shows how the buyer can mitigate the adverse effects of corruption through the scoring-rule design. Section 6 discusses the robustness of our findings.

2. Model

■ Two firms, firm 1 and firm 2, compete to supply a good to a buyer. All parties are risk neutral. If the buyer procures a good with quality q at the price of p, then the buyer's utility is $V(q, p) = q - p$ and the supplying firm, say i, receives a profit of $p - c_i(q)$, where $c_i(q)$ denotes firm i's cost of producing "quality" q. We assume that $c_i(q)$, $i = 1, 2$, is common knowledge for the firms involved, that $c_i(\cdot)$ is continuous and strictly convex with $c_i(0) = 0$, $\lim_{q \downarrow 0} c'(q) = 0$ and $\lim_{q \uparrow \infty} c'(q) = \infty$, and $c_2(q) - c_1(q) \geq 0$ for all $q \geq 0$. Given the last assumption, firm 1 is (at least weakly) more efficient than firm 2.

The socially efficient quality level conditional on firm $i = 1, 2$ being selected is then uniquely defined as

$$q_i^* \equiv \arg\max_q q - c_i(q).$$

Firms submit bids that specify both quality and price. Let a pair, (q_i, p_i), denote firm i's bid. The buyer then uses a scoring rule, $S_i(q_i, p_i) : \Re_+^2 \to \Re$ to evaluate the bid (q_i, p_i). For the time being, we assume that this scoring rule corresponds to the buyer's true utility function:

$$S_i(q_i, p_i) = q_i - p_i, \qquad i = 1, 2.$$

We relax this assumption in Section 5. As mentioned earlier, the buyer cannot directly measure q_i, so he employs a third party, called the "agent," to evaluate the quality component of a bid. The agent can manipulate the assessment of quality by pronouncing a good with quality q as being of quality up to $q + m$. We refer to the parameter m as the agent's *manipulation power.*[11] We assume that $q_2^* - c_2(q_2^*) \geq 2m$, which is a sufficient condition for the buyer to desire the good despite corruption, as will be shown below.

The firms offer bribes to the agent at the same time as they submit contract bids. The agent can accept a bribe only from the firm that is successful in winning the contract (see footnote 7 for a rationale for such an assumption). It is straightforward to characterize the agent's optimal manipulation decision in this procurement game. The agent compares all bribes offered. If the highest briber can win the auction with manipulation, the agent will favor that bidder and the contract will be awarded to that firm. If the gap between the two bids is so big that the high briber cannot win even with manipulation, then the agent simply takes the low bribe.[12] In sum, it is in the best interest of the agent to exert her full manipulation power to favor the high briber if that leads the latter to win. Given this corruption behavior, the firm that achieves the highest score, *postmanipulation,* is declared the winner; then it produces the good with the bid-specified quality

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[10] See the remark in Section 6 on the implications of the timing of bribe competition.

[11] Discrete manipulation behavior is not crucial for our results. See Section 6.

[12] Whether the agent manipulates on behalf of the low briber in this case has no effect on our results and thus remains unspecified.

and receives the bid-specifed price,[13] and it pays the bribe offered to the agent. The losing firm pays no bribe and receives no payment from the buyer. Finally, any ties between the two firms are broken in favor of firm 1. This tie-breaking rule is assumed primarily for simplicity and to give efficiency the best chance. It involves no restriction because, given an equilibrium with our tie-breaking rule, the rule itself can be sustained as an equilibrium behavior of a meta game in which the procurer and the agent are active players.[14] Throughout, we will focus on equilibria in undominated strategies—the ones that are not weakly dominated by an alternative strategy.[15] Without this refinement, there may exist other uninteresting equilibria in which a losing firm makes an offer that would yield a strict loss to the firm if accepted.

To illustrate the selection behavior precisely, suppose that firms simultaneously submit bid and bribe offers (q_i, p_i, b_i), $i = 1, 2$. The agent compares b_1 and b_2. If $b_1 \geq b_2$, then the agent favors firm 1 by exaggerating its quality by m, so long as firm 1 wins with the manipulation: i.e.,

$$q_1 + m - p_1 \geq q_2 - p_2.$$

In this event, firm 1 produces a good of quality q_1, receives p_1 from the buyer, and pays a bribe b_1 to the agent. Firm 1's payoff would then be $p_1 - c_1(q_1) - b_1$ and firm 2's payoff would be zero. If $q_1 + m - p_1 < q_2 - p_2$, then firm 1 cannot win the contract even if the agent manipulates in its favor. Then firm 2 gets the contract and pays bribe b_2. Its payoff would then be $p_2 - c_2(q_2) - b_2$ and firm 1's would be zero. An analogous outcome applies if $b_1 < b_2$. It is important, for later purposes, to notice that the resulting selection rule exhibits nontransitivity with respect to the offered bribe/contract pairs. Consider the three (bribe, price) pairs $(0, 1)$, $((2/3)m, 1 + (2/3)m)$, and $((4/3)m, 1 + (4/3)m)$, all with the same quality offer. The second pair wins against the first pair (since the former has a higher bribe but a price within m of the latter's price), and likewise the third pair wins against the second pair. Yet the third pair loses to the first pair, since the former asks for a price that exceeds the latter's price by more than m. This nontransitivity will be seen to play a crucial role in determining the nature of competition between the two firms when the agent is corrupt.

Before proceeding, we establish the following lemma, which will simplify our analysis throughout.

Lemma 1. For any participating firm i, it is weakly dominant to propose q_i^* (as defined above) regardless of whether the agent is corrupt.

Proof. Suppose that firm i proposes an arbitrary triplet (q, p, b). We show that if $q \neq q_i^*$, then this offer is weakly dominated by an alternative offer, (q_i^*, p', b), where p' satisfies $q_i^* - p' = q - p$. Clearly, both offers win with the same probability (regardless of whether the agent is corrupt). They offer the same bribe and obtain the same score both with and without manipulation. Yet, conditional on winning, the new offer yields (weakly) higher profit because

$$p' - c_i(q_i^*) - b = q_i^* - q + p - c_i(q_i^*) - b$$
$$= q_i^* - c_i(q_i^*) - [q - c_i(q)] + p - c_i(q) - b$$
$$> p - c_i(q) - b,$$

where the inequality holds because $q_i^* = \arg\max_q q - c_i(q)$ and $q \neq q_i^*$.   Q.E.D.

This result, reminiscent of a similar result obtained in Che (1993), implies that we can focus

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on q_i^* without loss of generality. That is, with a slight abuse of notation, we can treat q_i^* as an exogenous parameter of firm i and focus on $c_i^* \equiv c_i(q_i^*)$.

It will be useful to define firm 1's bidding advantage over firm 2:

$$\Delta \equiv q_1^* - c_1^* - [q_2^* - c_2^*].$$

Notice that this corresponds to firm 1's technological advantage, given our scoring rule.

3. Benchmark: procurement auctions without corruption

■ We first consider as a benchmark the case of an honest agent who does not manipulate quality. Clearly, firms will have no incentive to offer any bribe in this case.[16] Here, we assume that there is a very low "reserve score" such that the buyer refuses to award the contract (and cancels the project) if the high score falls below that reserve level.[17] The next lemma characterizes the equilibrium bidding strategy for the case of an honest agent.

Lemma 2. Assume that the agent does not manipulate the quality assessment of bids. Then it is an equilibrium for firm i to bid q_i^* and p_i^*, where

$$p_1^* = c_1^* + \Delta$$

and

$$p_2^* = c_2^*.$$

In any equilibrium in undominated strategies, firm 1 offers the same bid and wins with probability one, and the firms receive payoffs of $\pi_1 = \Delta$ and $\pi_2 = 0$.[18]

Proof. See the Appendix.

If the agent is honest, then firms compete in the Bertrand fashion, except for adjusting the price bids to account for the quality differential. Firm 1 wins the contract for a price low enough to make the contract unprofitable for firm 2. The resulting outcome is efficient, since the efficient firm is chosen and delivers the first-best efficient level of quality. Firm 1 enjoys positive rents of Δ whenever it commands a bidding advantage over firm 2.

4. Procurement auction with corruption

■ We now introduce corruption on the part of the agent. As explained before, the agent manipulates the high briber's quality by m, and the firm achieving the highest *post*-manipulation score wins the contract. We analyze the equilibrium bidding and bribing behavior of the firms.

With a corrupt agent, an outbribed efficient firm must undercut its rival by m in order to secure a win, whereas even a slight undercutting was sufficient to secure a win in the absence of corruption. This difference in itself does not imply that bribery disrupts an efficient allocation, however. Since both firms are symmetric in their bribing capabilities, at first glance it may appear that the efficient firm should be able, and willing, to outbribe its rival, similar in spirit to the

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[16] This situation is equivalent to the case in which $m = 0$. In this case, any positive bribe is strictly dominated by a zero bribe unless the firm wins with zero probability.

[17] The sole purpose of this assumption is to rule out an (uninteresting) equilibrium in which firms bid infinite prices in the absence of corruption, which is well known in the Bertrand competition literature (see Baye and Morgan, 1999). The buyer can credibly enforce this minimum standard by imposing some maximum budget (e.g., authorized by Congress) that the procurer can spend for the project or simply by reserving the right not to award the contract to any firm, which the buyer will exercise whenever the contract offers would yield a negative surplus to the buyer.

[18] There are (a continuum of) payoff-equivalent equilibria (in undominated strategies) of the following form: firm 1 bids $p_1^* = c_1^* + \Delta$ and firm 2 randomizes between c_2^* and $c_2^* + \varepsilon$ with probabilities $\pi > 0$ and $1 - \pi$, respectively, for some ε. If ε is sufficiently small, then firm 1 does not raise its price bid above $c_1^* + \Delta$.

FIGURE 1
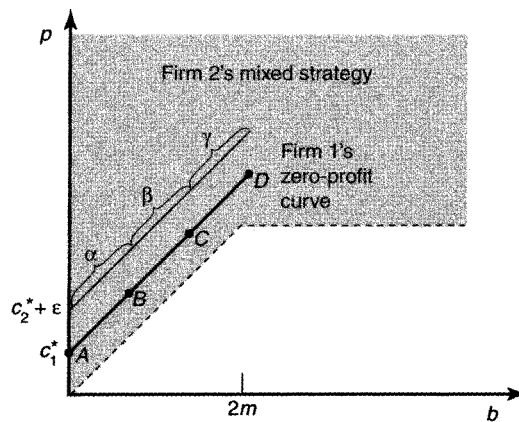
last section. We argue that this intuition does not work because of the lack of transitivity in the selection of the winning firm, observed earlier.

To see this, assume $q_1^* = q_2^* =: q^* > c_2^* + 2m$ and suppose that firm 2 randomizes its bribe b_2 uniformly over $[0, 2m]$ and chooses price $p_2 = c_2^* + b_2 + \varepsilon$ for some small $\varepsilon > 0$. The support of the randomized bribe-price pairs is depicted in Figure 1, as the top line segment.

Efficiency requires that each offer firm 1 may make in equilibrium dominate all these offers. This condition fails if $\Delta = c_2^* - c_1^* < m$. To see this, one can check that each bribe-price pair in the shaded area (above the dotted line) is defeated by *some* pair in firm 2's bid support. If $\Delta < m$, then firm 1's zero profit iso-profit curve lies within the shaded area (for a sufficiently small $\varepsilon > 0$), which means that any nonnegative profit offer firm 1 can make will lose to a set of pairs that firm 2 randomizes over. For instance, if firm 1 offers A in Figure 1, it will lose to the set, α, of firm 2's offers, since these offers have higher bribes and prices within m of the price charged in A. Firm 1 can defeat α by offering B instead, but it will then lose to β. Likewise, the offer C defeats β but loses to γ. Finally, D defeats γ but will lose to α again (this time, due to charging too high a price). This problem arises precisely because of the nontransitivity mentioned earlier. With the randomization strategy, firm 2 exploits the nontransitivity to become a very versatile competitor, against whom firm 1 finds it impossible to profitably win.

If $\Delta > m$, then firm 1 can guarantee a sure win by offering $p_1 = c_1^* + \Delta - m$ (and $b_1 = 0$), which would yield rents of $\Delta - m > 0$. (In this case, the zero profit line of firm 1 will lie below the shaded region.) This strategy is unlikely to be optimal, however, if $\Delta - m$ is not too large. In that case, firm 1 would rather find it profitable to outbribe firm 2 with positive probability, thus taking advantage of the softened competition even at the expense of a sure win. In fact, inefficiency arises in equilibrium whenever $m > (1/3)\Delta$, as shown in the following proposition.

Proposition 1. If $m > (1/3)\Delta$, then it is an equilibrium for firm $i = 1, 2$ to choose a bribe b_i uniformly over $[0, 2m]$ and pick a price bid p_i such that

$$\begin{cases} p_1 = b_1 + m + (1/3)\Delta + c_1^* \\ p_2 = b_2 + m - (1/3)\Delta + c_2^*. \end{cases}$$

Firm 1 wins with probability

$$\frac{1}{2} + \frac{\Delta}{6m} < 1.$$

If $m \leq (1/3)\Delta$, then it is an equilibrium for firm 1 to choose a bribe b_1 over $[0, 2m]$ with any probability distribution and for firm 2 to pick a bribe b_2 uniformly over the same support and for

these firms to choose price bids

$$\begin{cases} p_1 = b_1 - m + \Delta + c_1^* \\ p_2 = b_2 + c_2^*. \end{cases}$$

Firm 1 wins with probability one.

Proof. See the Appendix.

Proposition 1 predicts an efficient allocation, if $m \leq (1/3)\Delta$, just as in the case of an honest agent. But its prediction is qualitatively different if $m > (1/3)\Delta$. In the latter case, the contract is awarded to the inefficient firm with positive probability. In particular, as m gets large relative to Δ, the contract is almost randomly allocated.

The equilibrium stated in the proposition has some other interesting properties. First, each firm picks its price bid deterministically, given a (randomly chosen) bribe level. In particular, in the more interesting case in which the agent has relatively substantial manipulative power ($m > (1/3)\Delta$), firms 1 and 2 randomize uniformly over bribe-price pairs in the bottom and top linear segments, respectively, which are graphed in Figure 2.

Second, different pairs on each segment yield not only the same expected payoff for each firm (which is a necessary condition for a mixed-strategy equilibrium), but they also give the same probability of winning (hence the same payoff conditional on winning). Firms randomize over offers that win and lose against different sets of offers their opponents make but nonetheless win with the same probability across the randomized offers.[19]

Next we turn to the issue of how corruption affects the welfare of different parties. An equilibrium selection issue arises in the case of $m \leq (1/3)\Delta$, since there is a continuum of equilibria. Among them, we select the one in which firm 1 makes no bribe. To see its plausibility, observe that firm 1 is indifferent across all these equilibria: one equilibrium involves a higher price bid, which offsets an equally larger bribe bid, than another. Thus, bribery simply transfers rents from the buyer to the agent, with no other consequence (and irrespective of firm 2's strategy). Given this indifference, even a slight chance of getting caught for bribing will make firm 1 strictly prefer the suggested equilibrium. Given this equilibrium selection, we obtain the following corollary.

Corollary 1. If $m \leq (1/3)\Delta$, then corruption benefits the buyer and harms firm 1. Other parties' welfare remains unaffected. If $m > (1/3)\Delta$, corruption harms the buyer but benefits firm 2 and the agent. Firm 1 is worse off from corruption if $m < (2 + \sqrt{3}/3)\Delta$ but is better off if $m > (2 + \sqrt{3}/3)\Delta$.

FIGURE 2

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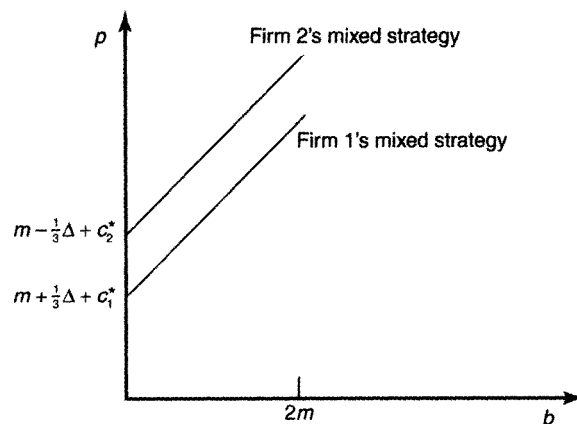
[19] In this respect, our mixed-strategy equilibrium differs from the ones found in Varian (1980) and Bagwell and Ramey (1994). In the equilibrium they focus on, firms randomize over strategies that involve different probabilities of winning (informed) customers, offset by the countervailing, different costs of gaining them.

If $m \leq (1/3)\Delta$, then firm 1 bids more aggressively with corruption in order to fight off the bribery of firm 2. Hence, surprisingly, corruption benefits the buyer in this case. Things are much different, however, if $m > (1/3)\Delta$. Then the buyer is strictly worse off for two reasons: (1) the contract is inefficiently allocated and (2) bribery softens price competition, which raises the price-cost margin.

Firm 2 clearly benefits from corruption, since it wins with positive probability and makes a positive profit. Firm 1's situation is more complex. If $m \leq (2 + \sqrt{3}/3)\Delta$, then its competitive advantage is reduced by firm 2's ability to bribe, so it is hurt by the corruption. If $m \geq (2 + \sqrt{3}/3)\Delta$ ($> \Delta$), however, corruption softens price competition sufficiently so that even firm 1 benefits from corruption. Consequently, the efficient firm would object to a corrupt agent with a moderate manipulative ability but would welcome one with a very strong manipulation capability. Interestingly, the buyer is least likely interested in eliminating corruption when a firm voices complaints against it.

The above proposition offers one equilibrium, which need not be unique. Nevertheless, the above equilibrium is representative of any possible equilibrium in one important respect: In any equilibrium, the allocation of the contract is inefficient whenever $m > (1/3)\Delta$.

Proposition 2. If $m > (1/3)\Delta$, then in any equilibrium firm 2 wins the contract with positive probability.

Proof. See the Appendix.

It may be puzzling at first that an inefficient allocation arises even when $m \in (\Delta/3, \Delta]$. In this case, firm 1 can make a profitable bid that can drive out firm 2. These results imply that while firm 1 can drive out firm 2, it finds this too costly and prefers rather to earn higher rents by conceding the win with some probability. As noted before, this need not imply that firm 1 will be worse off from corruption. If m is sufficiently large relative to Δ, corruption can lead to both firms earning more rents. In this case, corruption can be seen as facilitating a (self-enforcing) collusion among competing firms.

5. Scoring-rule design

■ The results in the above sections naturally raise the question of whether the buyer can limit the negative effect of corruption through the design of his scoring rule. Specifically, we investigate two issues. First, we investigate whether handicapping the efficient firm is desirable for the buyer. Second, we examine how the buyer should choose the relative weights on the quality and price bids. As will be seen below, the optimal scoring rule for the buyer is much different on these two accounts when the agent is corrupt than when the agent is honest.

We consider a class of linear scoring rules. Specifically, our scoring rule assigns to firms 1 and 2 the scores

$$S_1(q_1, p_1) = sq_1 - p_1 - \rho \quad \text{and} \quad S_2(q_2, p_2) = sq_2 - p_2,$$

respectively, if firm i's price bid is p_i and its post-manipulation quality offer is q_i. Note that s represents the relative weight given to quality. Hence, the degree of manipulation, sm, increases with the weight s, which will play an important role in the optimal scoring-rule design. Finally, ρ parameterizes the extent to which firm 1 is "handicapped." (Since ρ can be positive or negative, the scoring rule allows either firm to be handicapped.)

This type of scoring rule is common in government procurement. While the assumed family of scoring rules is not completely general,[20] it includes all plausible, relatively simple, rules and permits us to investigate the two design issues mentioned above. If $s = 1$ and $\rho = 0$, then we

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[20] For instance, it rules out nonlinear rules such as a forcing contract. A forcing contract is a highly nonlinear contract that assigns an infinite penalty (or an infinitely low score) for any quality level other than the one required by the buyer.

are back to the case considered in previous sections. If $s \neq 1$, then the buyer overemphasizes or deemphasizes the relative value of the quality. If $\rho \neq 0$, then the buyer discriminates against one of the firms. As before, the scoring rule is publicly announced before the parties submit bids/bribes and is used to evaluate the bids, subject to possible manipulation by the agent.

Given any such scoring rule, it is straightforward to extend Lemma 1 to show that it is weakly dominant for each firm to select $q_i(s) \equiv \arg \max_{q \geq 0} \{sq - c_i(q)\}$ regardless of whether the agent is corrupt. For this section, we additionally assume that $c_i(q)$ satisfies the single-crossing property in $(q; i)$ (see Milgrom and Shannon, 1994), which, roughly speaking, means that firm 2 has a higher marginal cost than firm 1 in the ordinal sense. Together with strict convexity of $c_i(\cdot)$, this property implies that for any $s \geq 0$, $q_1(s) \geq q_2(s)$.

We can also generalize the definition of firm 1's bidding advantage over firm 2 as

$$\Delta(s, \rho) \equiv sq_1(s) - c_1(q_1(s)) - [sq_2(s) - c_2(q_2(s))] - \rho.$$

We say that firm 1 is *completely* handicapped if $\Delta(s, \rho) = 0$ (i.e., when $\rho = sq_1(s) - c_1(q_1(s)) - [sq_2(s) - c_2(q_2(s))]$). In this case the degree of handicapping is sufficient to eliminate firm 1's bidding advantage over firm 2. We also say that there is *reverse* handicapping if firm 2 is handicapped, i.e., $\rho < 0$.

With this new notation, the main behavioral results in Sections 3 and 4 can be easily generalized. In particular, the following lemma, the proof of which can be obtained along the lines of Lemma 2, shows how firms behave in the absence of corruption.

Lemma 3. Given (s, ρ) and no corruption, it is an equilibrium for firm $i = 1, 2$ to bid $q_i = q_i(s)$ and p_i, where

$$p_1 = c_1(q_1) + \max\{\Delta(s, \rho), 0\}$$

and

$$p_2 = c_2(q_2) + \max\{-\Delta(s, \rho), 0\}.$$

In any equilibrium in undominated strategies, the firms make profits $\pi_1 = \max\{\Delta(s, \rho), 0\}$ and $\pi_2 = \max\{-\Delta(s, \rho), 0\}$. That is, $\pi_1 + \pi_2 = |\Delta(s, \rho)|$.

Absent corruption, there is no need to manipulate the relative weights of quality and price. Indeed, for any $s > 0$, the firms' profits can be reduced to zero by setting ρ such that $\Delta(s, \rho) = 0$. Firm 1 chooses $q_1(s)$ and wins the contract, and the buyer expropriates the entire social surplus $q_1(s) - c_1(q_1(s))$. When there is no corruption, it is thus optimal for the buyer to set $s = 1$ and induce the first-best quality level $q_1^* = q_1(1)$.[21]

Proposition 3. If the procurement agent is not corrupt, then it is optimal to set $s = 1$ and completely handicap firm 1 (i.e., to set ρ such that $\Delta(1, \rho) = 0$).

With corruption, there is again no pure-strategy equilibrium, and the equilibrium in Proposition 1 is generalized, which we report without proof.

Proposition 4. If the procurement agent is corrupt, then the equilibrium is characterized as follows.

(i) If $sm > (1/3)\Delta(s, \rho) > -sm$, then it is an equilibrium for firm $i = 1, 2$ to choose a bribe b_i uniformly over $[0, 2sm]$ and pick a price bid p_i such that

$$\begin{cases} p_1 = b_1 + sm + (1/3)\Delta(s, \rho) + c_1(q_1(s)) \\ p_2 = b_2 + sm - (1/3)\Delta(s, \rho) + c_2(q_2(s)). \end{cases}$$

(ii) If $sm \leq (1/3)\Delta(s, \rho)$, then it is an equilibrium for firm 1 to choose a bribe b_1 from $[0, 2sm]$ with any probability distribution and for firm 2 to pick a bribe b_2 uniformly over the

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[21] If the buyer cannot use handicapping, he will set $s < 1$ in order to reduce the rents accruing to the winning firm.

same support and for these firms to choose price bids

$$
\begin{cases}
p_1 = b_1 - sm + \Delta(s, \rho) + c_1(q_1(s)) \\
p_2 = b_2 + c_2(q_2(s)).
\end{cases}
$$

(iii) If $-sm \geq (1/3)\Delta(s, \rho)$, then it is an equilibrium for firm 2 to choose a bribe b_1 from $[0, 2sm]$ with any probability distribution and for firm 1 to pick a bribe b_2 uniformly over the same support and for these firms to choose price bids

$$
\begin{cases}
p_1 = b_1 + c_1(q_1(s)) \\
p_2 = b_2 - sm - \Delta(s, \rho) + c_2(q_2(s)).
\end{cases}
$$

Firm 1 wins with probability

$$
\max \left\{ \min \left\{ \frac{1}{2} + \frac{\Delta(s, \rho)}{6sm}, 1 \right\}, 0 \right\}.
$$

We now study how the buyer should design his scoring rule, given firms' equilibrium reaction described in Proposition 4. As before, we select the equilibrium in which the stronger firm makes no bribe, in case of multiple equilibria. The following proposition describes the buyer's optimal design of the scoring rule.

Proposition 5. If the procurement agent is corrupt, then it is optimal for the buyer to set $s^* \in (0, 1)$ and to set ρ^* such that $\Delta(s^*, \rho^*) = 3s^*m > 0$. The optimal policy involves reverse handicapping, if firm 2 is sufficiently efficient (but still less efficient than firm 1). In the resulting equilibrium, firm 1 always wins but produces a suboptimally low quality and earns rents of $2s^*m > 0$.

Proof. See the Appendix.

These results show two qualitative differences in the buyer's optimal response relative to the no-corruption case. First, the optimal rule discourages quality competition by setting $s < 1$. Discouraging quality competition has two conflicting effects. On the one hand, it distorts quality choices, so it could lower the surplus that the buyer can extract. On the other hand, lowering s reduces the agent's manipulation capacity, which in turn has the effect of intensifying price competition. At $s = 1$, the former effect is small while the latter is not, so it pays the buyer to induce such distortion.[22] Second, complete handicapping against the efficient firm is never optimal for the buyer. As was clear from the previous section, corruption is most harmful for the buyer when firm 1's bidding advantage is small, whereas it can actually benefit the buyer if the bidding advantage is large. In particular, eliminating the efficient firm's bidding advantage is never in the best interest of the buyer. The same point suggests that if the bidding advantage of firm 1 was not large enough to start with, the buyer may wish to adopt reverse handicapping, or handicapping the weaker firm. This result stands in stark contrast to the standard prescription,[23] but it is rather consistent with nondiscriminatory policies often adopted in public procurement (albeit with a different rationale).[24] In the resulting equilibrium, the allocational inefficiency disappears but the buyer procures a suboptimally low quality and fails to extract rents from the winning firm. In

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[22] While this result may resemble a similar result in Che (1993), the reason for deemphasizing quality is quite different. In the case of Che (1993), information asymmetry necessitates deemphasizing quality, since high quality aggravates the informational problem. With complete information, no such reason exists, as shown by Proposition 3.

[23] See Branco (1994), McAfee and McMillan (1989), and Myerson (1981)

[24] For instance, the European Commission's Green Paper of November 9, 1996, "Public Procurement in the European Union: Exploring the Way Forward," says that the risk of fraud and corruption can be limited if award procedures are fair, transparent, and nondiscriminatory (see Kuhl, 1997). Such a nondiscriminatory policy is advocated because it allegedly reduces the potential for the agent to be corrupt. Our rationale for a nondiscrimination policy comes from its effect on the strategic behavior of the firms.

sum, under the optimal scoring policy, corruption benefits the efficient firm but harms the buyer (while the other parties remain indifferent).

6. Concluding remarks

■ This article has shown how bribery affects the nature of procurement competition and the welfare of the involved parties and how the scoring rule should be designed to mitigate the harmful effect of corruption. The most novel insight we have contributed is that bribery competition and contract bidding, even when modelled the same way, play fundamentally different roles, and that when bribery and contract bidding occur together, the former undermines the effectiveness of the latter in selecting the most efficient contractor. This provides some foundation for the widely held wisdom that corruption hinders the efficient allocation of resources.[25] Our analysis also suggests that the inefficiency cost of bribery is in the same order of magnitude as the agent's manipulation capacity. This inefficiency cost will tend to understate the true costs of corruption, however, which will also include indirect costs, such as distorting/reducing the *ex ante* incentives for innovation to become the efficient supplier and misallocation of the resources across the sectors, which can magnify small distortions into big ones (see Akerlof and Yellen, 1985). The finding suggests, however, that corruption can be controlled by curbing the agent's ability to manipulate, and we have seen how the scoring rule can be designed to mitigate the distortionary effect of corruption.

Throughout, tractability and expositional efficiency have limited our model to be fairly stylized. As we illustrate below, the main insights of the article are generally robust.

□ **The number of firms.** Thus far, we have assumed that only two firms compete in the auction. Our main inefficiency prediction holds even when more than two firms compete. Suppose that there are $n > 2$ firms with $c_k^* - c_2^* < m$ for $k = 1, \ldots, n$, ordered $c_1^* < c_2^* < c_3^* \leq c_4^* \leq \cdots \leq c_n^*$. (There could be firms with higher cost $c_{n+i}^* > c_2^* + m$, which has no effect on the result.) Assume, for simplicity, that all firms have the same optimal quality level. Assume additionally that

$$\sum_{i=1}^{n} c_i^*/n - c_2^* > \left(\frac{n-3}{n}\right) m. \tag{1}$$

When there are three firms, for instance, this condition requires firm 2's cost to be less than the average cost of the three firms. Given (1), it can be shown that firm 2 can secure a strictly positive payoff in any equilibrium, which will imply an inefficient allocation. Suppose that firm 2 chooses its bribe b_2 uniformly from $[0, 2m]$ and its price bid, $p_2 = c_2 + b_2 + \varepsilon$, for some small $\varepsilon > 0$. (Recall that this forms a 45° line, much like in Figure 1.) Now fix any firm $k = 1, 3, \ldots, n$ and an arbitrary bid (b_k, p_k).[26] Any such bid by firm k dominates (or wins against) at most $(m - c_k + c_2 + \varepsilon)/2m$ fraction of firm 2's bid support.[27] Condition (1) then implies that for sufficiently small $\varepsilon > 0$, $n - 1$ rival firms cannot cover the entire bid support of firm 2, even if they coordinate their bids to dominate different segments of the support. This means that firm 2 wins with positive probability in any equilibrium, suggesting that allocational inefficiency is robust generically to the number of firms.[28] Inequality (1) is just a sufficient condition for inefficiency, but its intuition is in line

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[25] There is a difference between corruption (i.e., bribery targeted at the procurer) and collusion (bribery, or side payments, made across colluding firms). In the latter setting, side payments across firms in fact tend to facilitate an efficient allocation. See McAfee and McMillan (1992), Athey and Bagwell (2001), and Athey, Bagwell and Sanchirico (2001).

[26] This need not be a pure strategy. All that is needed is for (b_k, p_k) to be in firm k's support.

[27] Specifically, (b_k, p_k) must satisfy $p_k \geq c_k + b_k$ and win against firm 2's bid only if $b_k \geq b_2$ and $m - p_k \geq -p_2$ or $b_k < b_2$ and $-p_k \geq -m - p_2$. Combining the inequalities, the fraction of firm 2's bids that such a bid can win against is given by $\Pr\{b_2 \in [0, 2m] \mid 0 \leq b_k - b_2 \leq \min\{b_k, m - c_k + c_2\}$ or $b_k - 2m \leq b_k - b_2 \leq -m - c_k + c_2\}$, which equals $(m - c_k + c_2 + \varepsilon/2m)$ if $b_k \in [0, 2m]$ and is less than that amount if $b_k \notin [0, 2m]$.

[28] Efficiency may be achieved in a nongeneric case. Suppose there are four firms with $c_i^* = c^*$, for $i = 2, 3, 4$. Then it is an equilibrium for firm 1 to offer $p_1 = c^*$ and $b_1 = 0$, for firm 2 to offer q^*, $p_2 = c^*$, and $b_2 = 0$, and for the other two firms to offer q^*, $p_j = c^* + m$, and $b_j = m$, $j = 3, 4$. Any deviation to a bribe between zero and m is unprofitable due to

with the similar condition for $n = 2$. Indeed, rewrite (1) as

$$\left(c_2^* - c_1^*\right) - m < \sum_{i=3}^{n} \left[m - \left(c_i^* - c_2^*\right)\right].$$

This condition says, roughly, that the allocation is more likely to be inefficient the smaller the bidding advantage firm 1 enjoys over firm 2, relative to the the the agent's manipulation ability. The right-hand side simply means that firm 2 should have sufficient bidding advantages over other firms, to preclude them from possibly colluding to prevent it from winning.

□   **Alternative extensive forms.** In our model, the bribery competition occurs simultaneously with the contract bidding. This model portrays a plausibles scenario of corruption and also serves the purpose of isolating the fundamental differences between bribery and legal contract bidding, which can be accomplished most effectively when one treats the two forms of competition in the same way. Further, our model is more general than it may at first appear. For instance, our model is equivalent to scenarios in which bribery occurs before or after contract bidding, as long as the information about the first round of competition is unobservable to the other firms when they play the second round of competition. Moreover, the main inefficiency prediction survives even when the firms observe the early round of competition in two sequential variations of our extensive form.

Contract-bribe game. Suppose first that the firms first compete in contract bids and then compete in bribes (after observing the contract bids), followed by evaluation of the contract by the agent. Again, assume for simplicity that $n = 2$ and $q_1^* = q_2^* = q^*$ for some $q^* > c_2^* + 2m$, which implies that $\Delta = c_2^* - c_1^*$. We can show that there cannot be any (pure-strategy) equilibrium with an efficient allocation, provided that $m > \Delta$.[29] Suppose, to the contrary, that an efficient (pure-strategy) equilibrium exists. In that equilibrium, firm 2 must bid away its rents, implying that $p_2 = b_2 + c_2^*$ for some $b_2 \geq 0$, and firm 1 must play its best response: $p_1 = p_2 + m$ and $b_1 = b_2$. Given our tie-breaking rule, firm 1 would then win the contract. We first observe that $p_2 = c_2^*$ (and hence $b_2 = 0$), or else it would pay firm 2 to deviate by lowering its price bid to $p_2' \in (c_2^*, p_2)$ and its bribe to $b_2' = 0$, which will win against $p_1 = p_2 + m > p_2' + m$ (no matter how much firm 1 would bribe). Hence, we conclude that $p_2 = c_2^*$ and $p_1 = c_2^* + m$ in the equilibrium. Now consider a deviation by firm 2 to $p_2'' = c_2^* + 2m - \varepsilon$ in the first round. For a sufficiently small $\varepsilon > 0$, firm 2 will win the contract if it outbribes firm 1 in the second round (since the price difference is less than m). Observe that firm 1 can at most bribe $p_1 - c_1^* = c_2^* + m - c_1^* = \Delta + m$ in the second round. Meanwhile, the higher price allows firm 2 to bribe up to $p_2'' - c_2^* = 2m - \varepsilon$. Since $2m - \varepsilon > \Delta + m$ for small $\varepsilon > 0$, the deviation wins the contract and generates a strictly positive profit for firm 2. We thus conclude that inefficiency must arise in equilibrium.

Bribe-contract game. Suppose next that the firms first compete in bribes and then compete in price bids (after observing the bribes), followed by evaluation/manipulation of the quality by the agent.[30] Again, assume for simplicity that $n = 2$ and $q_1^* = q_2^* = q^*$ for some $q^* > c_2^* + \Delta$. It can be seen that any equilibrium is inefficient if $m > \Delta$. As before, suppose otherwise and imagine an efficient (pure-strategy) equilibrium. In such an equilibrium, firm 2 will bid away its rents, implying that $p_2 = b_2 + c_2^*$, and firm 1 will choose its best response: $b_1 = b_2$ and $p_1 = m + p_2$, which will win the contract for firm 1 (given our tie-breaking rule). Suppose that the procurer does not

the presence of a firm offering a bribe m. Likewise, any deviation to a bribe greater than m is defeated by a firm offering a zero bribe. The equilibrium works because we have a pair of firms offering each level of bribe. This equilibrium is highly nongeneric, though, resting crucially on there being multiple firms with exactly the same technology.

[29] The argument can easily generalize to mixed-strategy equilibria.

[30] One can imagine yet another variation in which the agent's evaluation/manipulation precedes the competition in the price bids. While such a timing assumption yields an efficient outcome (see footnote 30 of Burguet and Che (2001)), this variation seems highly unrealistic. It is unheard of, for instance, that a procurer evaluates one component of a bid (and publicly announces its evaluation outcome) before receiving the rest of the bid.

accept any contract that would generate negative (postmanipulation) surplus. Then, $p_2 \leq q^* + m$, so $b_2 \leq p_2 - c_2^* \leq q^* + m - c_2^* =: \bar{b}$. In equilibrium, we must have $b_1 = b_2 = \bar{b}$, or else firm 2 can deviate by choosing a bribe $b_2' = b_1 + \varepsilon < \bar{b}$ for some small $\varepsilon > 0$, in which case it gets the favor of m and can profitably defeat firm 1 by offering $p_2' = b_1 + c_2^* + 2\varepsilon$ in the price competition, for sufficiently small $\varepsilon > 0$.[31] However, $b_1 = b_2 = \bar{b}$ cannot be an equilibrium either. Suppose that firm 2 deviates by offering $b_2'' = 0$. The lowest price firm 1 can offer in the second-round competition is $\underline{p}_1 = b_1 + c_1^* = \bar{b} + c_1^*$. Firm 2 can profitably outcompete that bid in the second round by offering $p_2'' = \underline{p}_1 - m - \varepsilon$.[32]

The intuition for inefficiency is the same as in our simultaneous version: The possibility of bribery makes the less efficient firm so versatile that a more efficient firm finds it very costly to secure a sure win.

□ **Bribe/manipulation technology.** We have also assumed that only the winning firm pays a bribe, which rules out sunk investments related to lobbying activities or, equivalently, gift giving. If bribes were modelled as sunk investments, as in all pay auctions or other standard rent-seeking models, we would obtain allocational inefficiency simply due to the sunk nature of the bribery (see Tullock (1980), Hillman and Riley (1989), Baye, Kovenock, and de Vries (1996), and Che and Gale (2000) for standard models of rent-seeking games). Hence, our inefficiency prediction would remain valid, if not reinforced, if the sunk-investments nature were included.

Another important feature of our model was the discreteness of the manipulation decision: the agent manipulates by m for a firm no matter how slightly that firm outbribes its opponent. Our inefficiency prediction does not hinge on this feature, either. Our result holds even when the agent's manipulation varies smoothly with a firm's bribe offer. Suppose that the agent manipulates in favor of firm 1 by $m = \mu(b_1 - b_2)$, where $\mu(\cdot)$ is a smooth function satisfying $\mu(0) = 0$ and $\mu(x) = -\mu(-x)$ (symmetry).[33] Such manipulation behavior arises naturally as an optimal decision for an agent if the agent faces a penalty if caught for bribe taking and the probability of getting caught increases with the amount of his manipulation. The nontransitivity feature is still present with this model. Not surprisingly, our inefficiency result holds as long as there exists $x^* > 0$ such that $\mu(x^*) > x^* + \Delta$; i.e., when there exists a bribe that enables firm 2 to overcome the bidding advantage of firm 1.

Appendix

■ Proofs of Lemma 2 and Propositions 1, 2, and 5 follow.

Proof of Lemma 2. We show that the proposed price bids form an equilibrium. Observe that $S_1(q_1^*, p_1^*) = S_2(q_2^*, p_2^*)$. Hence, given firm 2's behavior, firm 1 wins the contract if and only if its price bid is less than or equal to p_1^*, so bidding p_1^* is its best response. (Recall that firm i offers q_i^*, due to Lemma 1.) Given firm 1's behavior, firm 2 wins the contract if and only if it bids strictly less than p_2^*. Bidding strictly greater than p_2^* loses the auction, whereas bidding strictly less wins with a strictly negative payoff. Hence, the bids form an equilibrium.

We now prove the second statement. Fix any equilibrium and let $\underline{s}_i$ and $\bar{s}_i$ denote, respectively, the infimum and the supremum of the score firm $i(= 1, 2)$ offers in that equilibrium. First, we must have $\bar{s}_1 = \bar{s}_2$, or else $\bar{s}_i > \bar{s}_j$, which implies that firm i can gain strictly by lowering its score in $(\bar{s}_j, \bar{s}_i]$. Second, any score exceeding $s_2^* \equiv q_2^* - c_2(q_2^*)$ can only generate a negative payoff and is thus weakly dominated for firm 2. Hence, $\bar{s}_2 \leq s_2^*$. Third, since firm 1 can offer $\bar{s}_2$ and earn $\Delta + s_2^* - \bar{s}_2 \geq 0$, firm 1 must earn at least $\Delta + s_2^* - \bar{s}_2$ in equilibrium. This fact implies that firm 2 must receive

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[31] This deviation price bid can surely win against firm 1, because the lowest price bid firm 1 can make is $\underline{p}_1 = b_1 + c_1^*$ and since firm 2 gets the favor of m by winning the bribe competition, and because $p_2' < b_1 + c_1^* - m + 2\varepsilon = \underline{p}_1 - m - 2\varepsilon$, where the inequality follows from $m > \Delta = c_2^* - c_1^*$. It also yields a strictly positive payoff, since $p_2 - b_2' - c_2^* = \varepsilon > 0$.

[32] To see this, observe first that with that price bid, firm 2 will win against firm 1 even with the favor given to the latter. Observe next that $p_2'' = \underline{p}_1 - m - \varepsilon = q^* + m - c_2^* - c_1^* - m - \varepsilon = q^* - \Delta - \varepsilon > 0$. Hence, the deviation yields a strictly positive payoff to firm 2.

[33] Alternatively, one can imagine bribery and manipulation that are both discrete. Our inefficiency result holds in such an environment as well. Inefficiencies hold, for instance, if the bribery decision is binary, say $b \in \{0, m/2\}$. We thank a referee for pointing out this fact.

zero payoff. Or else, $\bar{s}_2 < s_2^*$, so both firms must receive strictly positive payoffs. This last fact yields a contradiction, since if $\underline{s}_i < \underline{s}_j$, then firm i receives zero payoff, and if $\underline{s}_1 = \underline{s}_2$, then firm 2 must put mass at $\underline{s}_1$ (or else firm 1 would receive zero payoff), which means that firm 2 receives zero payoff.

That firm 2 receives zero payoff implies that $\bar{s}_2 \leq \underline{s}_1$, since otherwise firm 2 can offer $s \in (\underline{s}_1, \bar{s}_2)$ and receive a strictly positive payoff. Since $\bar{s}_1 = \bar{s}_2 \leq \underline{s}_1$, firm 1 puts the entire mass at $\underline{s}_1 = \bar{s}_1$. If $\bar{s} \equiv \underline{s}_1 = \bar{s}_1 = \bar{s}_2 < s_2^*$, then it pays firm 2 to slightly outbid $\bar{s}$ and make a positive payoff. Hence, we conclude that $\underline{s}_1 = \bar{s}_1 = \bar{s}_2 = s_2^*$, from which the statement follows. $Q.E.D.$

Proof of Proposition 1. We prove that the suggested strategy profile forms an equilibrium in each case. Denote the equilibrium quality differential as $\delta \equiv q_1^* - q_2^*$.

(i) $m > (1/3)\Delta$. Suppose that firm 2 follows its equilibrium strategy. Consider any arbitrary bribe-price pair (b, p) for firm 1. We can safely limit ourselves to $b \in [0, 2m]$, since any (b', p') such that $b' > 2m$ is strictly dominated by $(2m, \min\{p', 3m + (1/3)\Delta\})$. For any $b \in [0, 2m]$ consider any arbitrary p. Firm 1 will win if either

$$b \geq b_2 \equiv p_2 - m + \frac{1}{3}\Delta - c_2^* \quad \text{and} \quad p - \delta - m \leq p_2 \tag{A1}$$

or

$$b < b_2 \equiv p_2 - m + \frac{1}{3}\Delta - c_2^* \quad \text{and} \quad p - \delta + m \leq p_2. \tag{A2}$$

(We ignore a possible tie in bribery, since it occurs with zero probability.)

Suppose first that $p - \delta - m > m - (1/3)\Delta + c_2^*$. In this case, $p - \delta + m > 3m - (1/3)\Delta + c_2^*$, so condition (A2) can never hold. Hence, firm 1 will win if and only if (A1) holds, or

$$p - \delta - m < p_2 < b + m - \frac{1}{3}\Delta + c_2^*.$$

The probability of this event is

$$h_1(b, p) \equiv \max\left\{\min\left\{\frac{1}{2m}\left[2m + \frac{2}{3}\Delta + c_1^* - (p - b)\right], 1\right\}, 0\right\}.$$

Suppose now that $p - \delta - m \leq m - (1/3)\Delta + c_2^*$. In this case, it must be that $p - \delta + m \geq b + m - (1/3)\Delta + c_2^*$, or else firm 1 wins with probability one, in which case it pays strictly for the firm to raise p. It follows that firm 1 will win only when condition (A2) holds. In other words, firm 1 will win except when

$$p - \delta + m > p_2 > b + m - (1/3)\Delta + c_2^*.$$

It follows that the probability that firm 1 wins equals $h_1(b, p)$, again. Now consider firm 1's profit associated with (b, p):

$$\pi_1(b, p) \equiv h_1(b, p)(p - b - c_1^*).$$

Note that since $h_1(b, p) \in [0, 1]$, then

$$\pi_1(b, p) = \frac{1}{2m}\left[2m + \frac{2}{3}\Delta + c_1^* - (p - b)\right](p - b - c_1^*),$$

which is maximized at $p_1^*(b) = b + m + (1/3)\Delta + c_1^*$. When $m > (1/3)\Delta$,

$$h_1(b, p_1^*(b)) = \frac{1}{2} + \frac{\Delta}{6m}$$

is strictly between zero and one. This proves that firm 1 chooses $p_1^*(b)$ if it chooses b as the bribe. Now notice that $\pi_1(b, p_1^*(b))$ is constant and strictly positive for $b \in [0, 2m]$. The last fact proves that the described mixed strategy is a best response by firm 1. A symmetric argument works for firm 2 and is thus omitted.

(ii) $m \leq (1/3)\Delta$. We proceed in the same way, assuming that firm 2 behaves according to the described mixed strategy. Suppose that firm 2 picks (b, p), $b \in [0, 2m]$. Firm 1 wins if either

$$b \geq b_2 = p_2 - c_2^* \quad \text{and} \quad p_2 \geq p - \delta - m \tag{A3}$$

or

$$b < b_2 = p_2 - c_2^* \quad \text{and} \quad p_2 \geq p - \delta + m. \tag{A4}$$

Again proceeding as before, the probability of firm 1 winning is given by

$$k_1(b, p) \equiv \max \left\{ \min \left\{ \frac{1}{2m} \left[m + \Delta + c_1^* - (p - b) \right], 1 \right\}, 0 \right\}.$$

Notice that $k_1(b, p) = 1$ for $p \leq b + \Delta + c_1^* - m$ and $k_1(b, p)$ is decreasing in p for $p > b + \Delta + c_1^* - m$. Consequently, firm 1's profit $\pi_1(b, p) \equiv k_1(b, p)(p - b)$, $b \in [0, 2m]$, is strictly increasing in p for $p \leq b + \Delta + c_1^* - m$ and decreasing in p for $p > b + \Delta + c_1^* - m$, and is thus maximized at $p_1^*(b) = b + \Delta + c_1^* - m$. This shows that firm 1 will choose $p_1^*(b)$ if it chooses b. It also follows that $\pi_1(b, p^*(b)) = \Delta - m$ is constant for any $b \in [0, 2m]$. This proves that any mixed strategy by firm 1 over $[0, 2m]$ is a best response.

Given $(b, p_1^*(b))$, for any $b \in [0, 2m]$, firm 2 can never make a strictly positive payoff with any pair (b, p). Hence, facing any mixed strategy over $(b, p_1^*(b))$ for $b \in [0, 2m]$, it is firm 2's best response to randomize according to the stated strategy. $Q.E.D.$

Proof of Proposition 2. Fix any $m > (1/3)\Delta$ and a (possibly mixed-strategy) equilibrium. Suppose that firm 1 wins with probability one in that equilibrium. This implies that any strategy with $p_2 > b_2 + c_2^*$ firm 2 may employ can never win. That is, firm 1 must be playing strategies that defeat all such strategies of firm 2. In particular, firm 1 must be using a (possibly mixed) strategy satisfying (with probability one)

$$p_1 \leq (q_1^* - q_2^*) + c_2^* - m + b_1 = \Delta + c_1^* - m + b_1.$$

To see this, suppose to the contrary that $p_1 = (q_1^* - q_2^*) + c_2^* - m + b_1 + \varepsilon$, for some $\varepsilon > 0$. Then, firm 2 can bid $p_2 = p_1 - (q_1^* - q_2^*) + m - (\varepsilon/3)$ and offer a bribe $b_2 = b_1 + (\varepsilon/3)$. With this new strategy, firm 2 will outbribe firm 1, so its postmanipulation score will become

$$q_2^* + m - p_2 = q_2^* + m - \left[p_1 - (q_1^* - q_2^*) + m - \frac{\varepsilon}{3} \right] = q_1^* - p_1 + \frac{\varepsilon}{3},$$

which exceeds firm 1's (postmanipulation) score. Hence, firm 2 will beat firm 1. Further, such a strategy yields a positive payoff to firm 2, since

$$p_2 - c_2^* - b_2 = \frac{\varepsilon}{3} > 0,$$

so we obtain a contradiction.

Now, given $p_1 \leq \Delta + c_1^* - m + b_1$, firm 1's payoff cannot be higher than $\Delta - m$. Now consider the following (mixed) strategy for firm 1: choose bribe, b_1, uniformly over $[0, 2m]$, and, given b_1, choose $p_1 = b_1 + c_1^* + (m + \Delta)/2$. Notice that against any (pure) strategy of firm 2 with $p_2 \geq b_2 + c_2^*$, this mixed strategy of firm 1 will win with a probability of at least $(m + \Delta)/2$. Hence, firm 1 earns an expected payoff of at least $[(m + \Delta)/2]^2$ no matter what (undominated) strategy firm 2 plays. That means that in equilibrium, firm 1 cannot make profits below this amount. But for $m > (1/3)\Delta$, $[(m + \Delta)/2]^2 > \Delta - m$, which contradicts the fact that firm 1 is winning in equilibrium with probability one. $Q.E.D.$

Proof of Proposition 5. For any s, the buyer can choose any Δ by choosing ρ. We can therefore focus on (s, Δ) as two independent policy instruments for the buyer. Following Proposition 4, write the buyer's payoff as a function of (s, Δ):

$$U(s, \Delta) = \begin{cases} \left(\frac{1}{2} + \frac{\Delta}{6sm} \right) [q_1(s) - c_1(q_1(s)) - 2sm - (1/3)\Delta)] & \\ \quad + \left(\frac{1}{2} - \frac{\Delta}{6sm} \right) [q_2(s) - c_2(q_2(s)) - 2sm + (1/3)\Delta] & \text{if } sm > (1/3)\Delta > -sm \\ q_1(s) - c_1(q_1(s)) - \Delta + sm & \text{if } sm \leq (1/3)\Delta \\ q_2(s) - c_2(q_2(s)) + \Delta + sm & \text{if } (1/3)\Delta \leq -sm. \end{cases}$$

The results are established in several steps.

Claim 1. The optimal scoring rule has $s^* \in (0, 1]$.

Proof. Note first that $s^* > 0$. Otherwise, both firms will propose zero quality, so the buyer will never receive a positive payoff. If the buyer instead chooses $s = 1$ and $\Delta = 0$, then he will receive $(1/2)[q_1^* - c_1(q_1^*) - 2m] + (1/2)[q_2^* - c_2(q_2^*) - 2m] > 0$.

Next we show that $s^* \leq 1$. Suppose, to the contrary, that the optimal policy, (s^*, Δ^*), has $s^* > 1$. There are three cases to consider.

(i) $s^*m > (1/3)\Delta^* > -s^*m$. In this case, consider an alternative policy, $(\hat{s}, \hat{\Delta})$, such that $\hat{s} = 1$ and $\hat{\Delta} = \Delta^*/s^*$. Clearly, $\hat{s}m > (1/3)\hat{\Delta} > -\hat{s}m$, so we have

$$
\begin{aligned}
U(\hat{s}, \hat{\Delta}) &= \left(\frac{1}{2} + \frac{\hat{\Delta}}{6\hat{s}m}\right)\left[q_1(\hat{s}) - c_1(q_1(\hat{s})) - 2\hat{s}m - \frac{1}{3}\hat{\Delta}\right] + \left(\frac{1}{2} - \frac{\hat{\Delta}}{6\hat{s}m}\right)\left[q_2(\hat{s}) - c_2(q_2(\hat{s})) - 2\hat{s}m + \frac{1}{3}\hat{\Delta}\right] \\
&= \left(\frac{1}{2} + \frac{\Delta^*}{6s^*m}\right)[q_1(1) - c_1(q_1(1)) - 2m] + \left(\frac{1}{2} - \frac{\Delta^*}{6s^*m}\right)[q_2(1) - c_2(q_2(1)) - 2m] - \frac{\Delta^{*2}}{9s^{*2}m} \\
&> \left(\frac{1}{2} + \frac{\Delta^*}{6s^*m}\right)[q_1(s^*) - c_1(q_1(s^*)) - 2s^*m] + \left(\frac{1}{2} - \frac{\Delta^*}{6s^*m}\right)[q_2(s^*) - c_2(q_2(s^*)) - 2s^*m] - \frac{\Delta^{*2}}{9s^*m} \\
&= U(s^*, \Delta^*),
\end{aligned}
$$

where the strict inequality follows from $s^* > 1$ and the last equality is obtained from the first equality applied in the reverse order (for (s^*, Δ^*)). Hence, we obtain a contradiction.

(ii) $(1/3)\Delta^* \geq s^*m$. In this case, consider an alternative policy, $(\tilde{s}, \tilde{\Delta})$, such that $\tilde{s} = 1$ and $\tilde{\Delta} = \Delta^* + m - s^*m$. Again, $\frac{1}{3}\tilde{\Delta} \geq \tilde{s}m$. Hence,

$$
\begin{aligned}
U(\tilde{s}, \tilde{\Delta}) &= q_1(1) - c_1(q_1(1)) - \tilde{\Delta} + m \\
&= q_1^* - c_1(q_1^*) - \Delta^* + s^*m \\
&> q_1(s^*) - c_1(q_1(s^*)) - \Delta^* + s^*m \\
&= U(s^*, \Delta^*),
\end{aligned}
$$

where the inequality follows because $s^* > 1$. Hence, we obtain a contradiction.

(iii) $(1/3)\Delta^* \leq -s^*m$. The same argument as in (ii) produces a contradiction.   Q.E.D.

Claim 2. For any $s \in (0, 1]$, the optimal scoring rule has $\Delta^*(s) = 3sm$.

Proof. Differentiating $U(s, \Delta)$ with respect to Δ, we get

$$
\frac{\partial U(s, \Delta)}{\partial \Delta} = \begin{cases} \frac{1}{6sm}\left[q_1(s) - c_1(q_1(s)) - q_2(s) + c_2(q_2(s)) - \frac{4}{3}\Delta\right] & \text{if } sm > (1/3)\Delta > sm \\ -1 & \text{if } sm \leq (1/3)\Delta \\ 1 & \text{if } -sm \geq (1/3)\Delta. \end{cases}
$$

We first characterize the local maxima of the three regions. Clearly, the local maxima in the second and third regions are attained at $\Delta_2(s) = 3sm$ and $\Delta_3(s) = -3sm$, respectively. The local maximum in the first region, if it exists, is attained at $\Delta_1(s) = (3/4)[q_1(s) - c_1(q_1(s)) - q_2(s) + c_2(q_2(s))]$.

We now compare the three local maxima. To this end, first note that $U(s, \cdot)$ jumps down at $\Delta_3(s)$ and jumps up at $\Delta_2(s)$ and $U(s, \cdot)$ is uppersemicontinuous. Hence, a global maximum exists. Further, if the local maximum in the first region does not exist, it means that a local maximum in a different region is a global maximum. Next, observe that for $s \in (0, 1]$,

$$
q_1(s) - c_1(q_1(s)) \geq q_2(s) - c_1(q_2(s)) \geq q_2(s) - c_2(q_2(s)), \tag{A5}
$$

where the first inequality holds because, for $s \leq 1$, $q_2(s) \leq q_1(s) \leq q_1^*$ and since $c_1(\cdot)$ is convex, and the second inequality follows because $c_1(\cdot) \leq c_2(\cdot)$. Now (A5) implies that the local maximum in the second region dominates that in the third region:

$$
U(s, \Delta_2) = q_1(s) - c_1(q_1(s)) - 2sm \geq q_2(s) - c_2(q_2(s)) - 2sm = U(s, \Delta_3).
$$

(The inequality is strict if $c_2(q) > c_1(q) > 0$.) Likewise, the local maximum in the second region dominates that in the first region:

$$
\begin{aligned}
U(s, \Delta_1) &= \left(\frac{1}{2} + \frac{\Delta_1}{6sm}\right)\left[\frac{3}{4}(q_1(s) - c_1(q_1(s))) + \frac{1}{4}(q_2(s) - c_2(q_2(s))) - 2sm\right] \\
&\quad + \left(\frac{1}{2} - \frac{\Delta_1}{6sm}\right)\left[\frac{3}{4}(q_2(s) - c_2(q_2(s))) + \frac{1}{4}(q_1(s) - c_1(q_1(s))) - 2sm\right] \\
&\leq \frac{3}{4}(q_1(s) - c_1(q_1(s))) + \frac{1}{4}(q_2(s) - c_2(q_2(s))) - 2sm \\
&\leq q_1(s) - c_1(q_1(s)) - 2sm \\
&= U(s, \Delta_2),
\end{aligned}
$$

where both inequalities follow from (A5). The inequalities are strict if $c_2(q) > c_1(q)$ for all $q > 0$. Hence, an optimal policy has $\Delta^*(s) = \Delta_2(s) = 3sm$, for $s \in (0, 1]$.   Q.E.D.

Claim 3. It is optimal to set $s^* \in (0, 1)$ and $\Delta^* = 3s^*m$.

Proof. Given Claim 2, $s^* \in \arg\max_{s\in[0,1]} U(s, \Delta_2(s)) = q_1(s) - c_1(q_1(s)) - 2sm$. In light of Claim 1, it suffices to show that $s^* \neq 1$, which holds because

$$\frac{dU(s, \Delta^*(s))}{ds}\Bigg|_{s=1} = q'(1)[1 - c_1'(q(1))] - 2m = -2m < 0,$$

where the second equality holds because $q(1) = q_1^*$. Hence, it is optimal to set $s^* \in (0, 1)$ and, given Claim 2, to set $\Delta^* = 3s^*m$.   Q.E.D.

Claim 4 ("reverse handicapping"). Let $c_2(q; \alpha)$ be firm 2's cost function, for some $\alpha \geq 0$, and assume that $c_2(q; \alpha)$ is continuous in $\alpha \geq 0$ and convex in q, that $c_2(\cdot; \alpha) > c_1(\cdot)$ for all $\alpha > 0$, and that $c_2(\cdot; 0) = c_1(\cdot)$. Then $\rho^* < 0$, for sufficiently small $\alpha > 0$.

Proof. Claim 3 implies that

$$\rho^* = s^*q_1(s^*) - c_1(q_1(s^*)) - [s^*q_2(s^*) - c_2(q_2(s^*); \alpha)] - 3s^*m.$$

Then, by Berge's theorem of maxima, $s^*q_2(s^*) - c_2(q_2(s^*); \alpha)$ converges to $s^*q_1(s^*) - c_1(q_1(s^*))$ as $\alpha \downarrow 0$. Since s^* is strictly positive and does not vary with α, $\rho^* < 0$ for sufficiently small $\alpha > 0$.   Q.E.D.

The last statement follows directly from Proposition 4.   Q.E.D.

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