3 Assignments

3.1 Part A : Linear Programming

The problem you have to solve depends on your student ID. Let $x = \text{StudentID} \mod 4$, i.e. the remainder when you divide your student ID by 4. You have to solve problem Ax.

Deliverables: a written report with your LP formulation of the problem and the answers to the questions, an optimal solution to the given instance, and your source codes.

3.1.1 Assignment A0: Investment Plan I

Fox Enterprises is considering six projects for possible construction over the next four years. The expected returns and cash outlays for the projects are given below. Fox can undertake any of the projects partially or completely, during the whole 4-year period. A partial undertaking of a project will prorate both the return and cash outlays proportionally.

	Cash outlay ($1000)				
Project	Year 1	Year 2	Year 3	Year 4	Return ($1000)
1	10.5	14.4	2.2	2.4	32.40
2	8.3	12.6	9.5	3.1	39.80
3	10.2	14.2	5.6	4.2	37.75
4	7.2	10.5	7.5	5.0	34.80
5	12.3	10.1	8.3	6.3	38.20
6	9.2	7.8	6.9	5.1	27.35
Available funds ($1000)	60	70	35	20	

1. Formulate the problem as a linear problem, and determine the optimal project mix that maximizes the total return. Ignore the time value of money.
2. Use CPLEX to get the information needed to answer the following questions. For each question, explain which piece of information enables you to answer.
   (a) What are the binding constraints?
   (b) What would you gain if the RHS of one of those binding constraints increased of one unit?
   (c) To what extend could the return of project 6 be increased without changing the optimal solution?

3. How would you modify your linear formulation to take into account the following additional features:
   Suppose that if a portion of project 2 is undertaken then at least an equal portion of project 6 must be undertaken. Suppose that any funds left at the end of a year are used in the next year.
   What is the impact of those two new characteristics on your first optimal solution?

Grading scheme A0		
	Report	Java+CPLEX
Q1	/5	/7
Q2	/3	/5
Q3	/3	/2
Total	/11	/14

3.1.2 Assignment A1: Investment Plan II

Investor Doe has \$10 000 to invest in four projects. The following table gives the cash flow for the four investments.

	Cash flow (\$1000) at the start of				
Project	Year 1	Year 2	Year 3	Year 4	Year 5
1	-1.00	0.50	0.30	1.80	1.20
2	-1.00	0.60	0.20	1.50	1.30
3	0.00	-1.00	0.80	1.90	0.80
4	-1.00	0.40	0.60	1.80	0.95

The information in the table can be interpreted as follows. For project 1, \$1.00 invested at the start of year 1 will yield \$0.50 at the start of year 2, \$0.30 at the start of year 3, \$1.80 at the start of year 4, and \$1.20 at the start of year 5. The remaining entries can be interpreted similarly. The entry 0.00 indicates that no transaction is taking place. Each year, Doe has the additional option of investing in a bank account that earns 6.5% annual profit. All funds accumulated at the end of one year can be reinvested in the following year. Doe can invest in projects 1, 2, and 4 only at the start of year 1. He can invest in project 3 only at the start of year 2. However, investing in the bank account is possible at the start of years 1, 2, 3, and 4.

1. Formulate the problem as a linear program to maximize the total earned money at the start of year 5. Implement your mathematical program to get the optimal solution and optimal value.

2. Use CPLEX to get the information needed to answer the following questions. For each question, explain which piece of information enables you to answer.

3.2 Part B: Integer Linear Programming in logistic context

The problem you have to solve depends on your student ID. Let $x = \text{StudentID} \mod 4$, i.e. the remainder when you divide your student ID by 4. You have to solve problem Bx.

Deliverables: a written report with your ILP formulation of the problem an optimal solution to the given instance, and your source codes.

3.2.1 Assignment B0: capacitated facility location I

TrendCoats is a manufacturer of jeans and sells its products on the North America market. Actually, the firm has a plant in Denver with a capacity of 1500 thousands of units, two warehouses: one in Chicago and one in Salt Lake City, each with a capacity of 1000 thousands of units, and serves markets in Seattle, Sacramento, Houston, Toronto, Miami and Detroit. The market demand and the transportation costs are shown below:

Transportation Cost per thousand units ($)	Seattle	Sacramento	Houston	Toronto	Miami	Detroit
Chicago	165	183	124	86	132	67
Salt Lake City	110	75	132	210	153	195
Demand (thousand units)	190	150	90	200	240	130

The transportation cost of one thousand units between the plant in Denver and the warehouse in Chicago is $105 and from the plant in Denver to the warehouse in Salt Lake City is $68. Assume that the jeans are packed in boxes of one thousand units and that they cannot be split.

The demand is growing dramatically. In order to satisfy this demand, the managers of TrendCoats have decided to change their network design, several options are studied:

- A new plant can be opened in Wichita (in addition to the low capacity plant already existing in Denver) **or** the capacity of the plant in Denver can be increased (a high capacity plant would take the place of the low capacity one).
- The capacity of the warehouse in Chicago and/or Salt Lake City can be increased to 2000 thousands of units if needed.

Plant capacities, market demand, fixed costs incurred by the transformations and transportation costs are shown below:

Transportation Cost per thousand units ($)	Chicago	Salt Lake City	Capacity (thousand units)	Fixed Cost (thousand $)
Wichita	87	75	1500	2000
Denver (low capacity)	105	68	1500	0
Denver (high capacity)	105	68	2500	1500

Transportation Cost per thousand units ($)	Seattle	Sacramento	Houston	Toronto	Miami	Detroit	Fixed Cost (thousand $)
Chicago (low cap.)	165	183	124	86	132	67	0
Chicago (high cap.)	165	183	124	86	132	67	250
Salt Lake City (low cap.)	110	75	132	210	153	195	0
Salt Lake City (high cap.)	110	75	132	210	153	195	200
Demand (thousand units)	480	420	220	500	450	320	

1. Write down the mixed integer linear programming formulation that would minimize the total transportation and fixed cost of TrendCoats (taking the best decisions among the propositions above).

2. Solve the formulation with CPLEX. Describe the optimal solution and interpret the value of the variables in the context. Give the optimal value associated to this solution.

Grading scheme B0		
	Report	Java+CPLEX
Q1	/7	
Q2	/3	/15
Total	/10	/15

3.2.2 Assignment B1: capacitated facility location I

SC Consulting, a supply chain consulting firm, must decide on the location of its home offices. Its clients are located primarily in the 16 states listed in Table 1. There are four potential sites for home offices : Los Angeles, Tulsa, Denver and Seattle. The annual fixed cost of locating an office in Los Angeles is $165 428, Tulsa is $131,230, Denver is $140 000 and Seattle is $145 000. The expected number of trips to each state and the travel costs from each potential site are shown in Table 1.

3.3 Part C: Combinatorial Optimization

The problem you have to solve depends on your student ID. Let $x = \text{StudentID} \mod 4$, i.e. the remainder when you divide your student ID by 4. You have to solve problem Cx.

For your assignment, you are provided with five instances that you have to solve. You are supposed to:

1. Develop an integer linear program for the given problem. Then, implement it using CPLEX.
2. For each of the five instances, find the optimal integer solution.
3. For each of the five instances, find the optimal solution to the corresponding linear programming relaxation, i.e. when all integer/binary variables are relaxed to take non–negative real values.
4. For each of the five instances, measure the running times needed to solve both versions.
5. For each of the five instances, determine the GAP of the solution of the relaxation compared to the optimal integer solution. The GAP is defined as the ratio between the difference of the best bound (provided by the relaxation in this case) and the best integer solution value over the best integer solution value. The GAP is expressed in percentages and measures how good a solution is (the closer to 0, the better):

$$\frac{|\text{best\_relaxation\_value} - \text{best\_integer\_value}|}{|\text{best\_integer\_value}|}$$

6. For one of the five instance, describe the optimal solution in the context (= in words).

Deliverables: a written ILP formulation of the problem, the following table (filled with values), a brief description of the results in the table, the description of one optimal solution in the context of the assignment, and your source codes.

Instance	Optimal value (ILP)	Run time ILP (ms)	Optimal value (LP)	Run time LP (ms)	GAP
1					
2					
3					
4					
5					

Important: Set a limit of 3600 seconds to the computational time of CPLEX. If the optimal solution cannot be reached within one hour, then give the current best bound and associated GAP for the instance.

Grading scheme C		
	Report	Java+CPLEX
ILP	/5	/8
Results Table Sol. description	 /6 /1	
Relaxation		/5
Total	/12	/13

3.3.1 Assignment C0 - The Bin Packing Problem

Given a set of items $I = \{1, \ldots, n\}$, each having a size $s_i \in [0, 1]$ for $i \in I$, determine an allocation of items to bins of size 1, that minimizes the number of bins used.

3.3.2 Assignment C1 - The Knapsack problem

Given a set of items $I = \{1, \ldots, n\}$, each having a required space s_i in cm^2, a duration d_i in minutes, and a value $v_i \in \mathbb{R}^+$ for $i \in I$. The plant has an available space of 300 cm^2 and a workforce working 8 hours. Select the items such that the value of the selected items is maximized.

3.3.3 Assignment C2 - Minimizing weighted completion times

Given a set $J = \{1, \ldots, n\}$ of jobs, each having a processing time $p_j \in \mathbb{R}^+$ and a weight $w_j \in \mathbb{R}^+$, determine an allocation of the jobs to a single machine that minimizes the weighted sum of completions times of the jobs, i.e. $\sum_{j \in J} w_j C_j$, where C_j is the time that job j is finished.

Hint: you have to declare as a decision whether a job precedes another one.

3.3.4 Assignment C3 - The Scheduling Parallel Machines Problem

Given a number m of machines, and a set $J = \{1, \ldots, n\}$ of jobs, each having a processing time $p_j \in \mathbb{R}^+$, determine an allocation of the jobs to the machines that minimizes the makespan of the schedule, i.e. the finish time of the last machine.

3.4 Part D: Discrete Network Location Models

For the assignment, you need first to be able to solve the shortest path problem:

> Given an undirected graph $G = (V, A)$, arc weights $w : A \to \mathbb{R}^+$, a source $s \in V$ and a destination $t \in V$, determine the shortest path in G between s and t. Then, use this algorithm to determine the shortest distance between any pair of vertices in V. Present your results as a distance matrix.

Using this as a tool to compute a distance matrix, four location problems are then presented. The problem you have to solve depends on your student ID. Let x = StudentID mod 4, i.e. the remainder when you divide your student ID by 4. You have to solve problem Dx.

For your assignment, you are provided with five instances that you have to solve. You are supposed to:

1. Present your ILP or algorithm to solve the shortest path.
2. For your specific assignment, find the name of this type of problem in the literature. Provide the reference(s) you used.
3. Develop an integer linear program for the given assignment. Then, implement it using CPLEX.
4. For each of the five instances, find the optimal integer solution and provide the running time (without the time to compute the distance matrix).
5. For **one** of the five instances, interpret the solutions in the actual context.

Deliverables: a written report with all the above elements, and your source codes.

Grading scheme D		
	Report	Java+CPLEX
Shortest path	/3	/4
Positioning	/1	
ILP	/5	/8
Results	/3	
Sol. description	/1	
Total	/13	/12

3.4.1 Assignment D0 - Facility location

The customers and the potential facilities of a country have been aggregated in n regions. Some of the regions are connected to each others through a set of m arcs.

A company would like to open p facilities among these regions to supply all the customers in minimizing the total distance to travel.
We assume that all the facilities have the same fixed cost to get opened and that they have an infinite capacity. A customer can be supplied by only one facility.

Write down a linear model for this problem. Solve it with $p = 2, 3, 4$. Include the following table filled in in your report. Comment the results.

	Optimal value	Run time (ms)
Instance 1		
$p = 2$ $p = 3$ $p = 4$		
Instance 2		
$p = 2$ $p = 3$ $p = 4$		
Instance 3		
$p = 2$ $p = 3$ $p = 4$		
Instance 4		
$p = 2$ $p = 3$ $p = 4$		
Instance 5		
$p = 2$ $p = 3$ $p = 4$		

3.4.2 Assignment D1 - Hospital location

Inhabitants have been grouped in n regions. Some of the regions are connected to each others through a set of m arcs. We want to locate new hospitals in some regions such that each person in each region can reach at least one of them in less than a maximum distance M. Determine the minimum number of hospitals to build.

Write down a linear model for this problem. Solve it with $M = 3, 5, 10$. Include the following table filled in in your report. Comment the results.