1. Background and Motivation

Open swimming pools get naturally heated by absorbing solar energy directly from the sun. Majority of the solar radiation striking the swimming pool surface is absorbed by the water and converted into heat [1]. However, due to their vast surface area, heat losses to the surroundings are relatively high, and swimming pools maintain their average temperature relatively close to the average surrounding air temperature. As a result, during cooler months when the temperature outside is too low, heaters are used to heat the pool water to bring it to one's desired temperature, which results in a substantial amount of electricity consumption.

The simplest suggested solution to this high heat loss issue to increase water temperature is an introduction of a transparent plastic cover that will allow solar radiation to pass through [2]. Plastic also meets the requirements of being a vapor barrier. Accordingly, the objective of this project is to design and analyze a transparent swimming pool cover that will address the aforementioned issues and should significantly reduce pool heating costs.

Heat loss from a swimming pool mainly occur through evaporation (ignored in this report), convection, radiation, and some negligible conduction to the ground. In order to retain the water at a higher temperature than that of the surrounding air, the amount of heat to be supplied to the water must be equal to the heat loss at this temperature. The water temperature will increase by keeping the heat input unchanged and reducing this heat loss [1]. With the introduction of a transparent cover, there will also be an introduction of heat loss due to reflection of the interfaces between the sun and the water. A pool cover will also decrease the solar gain contribution to some extent.

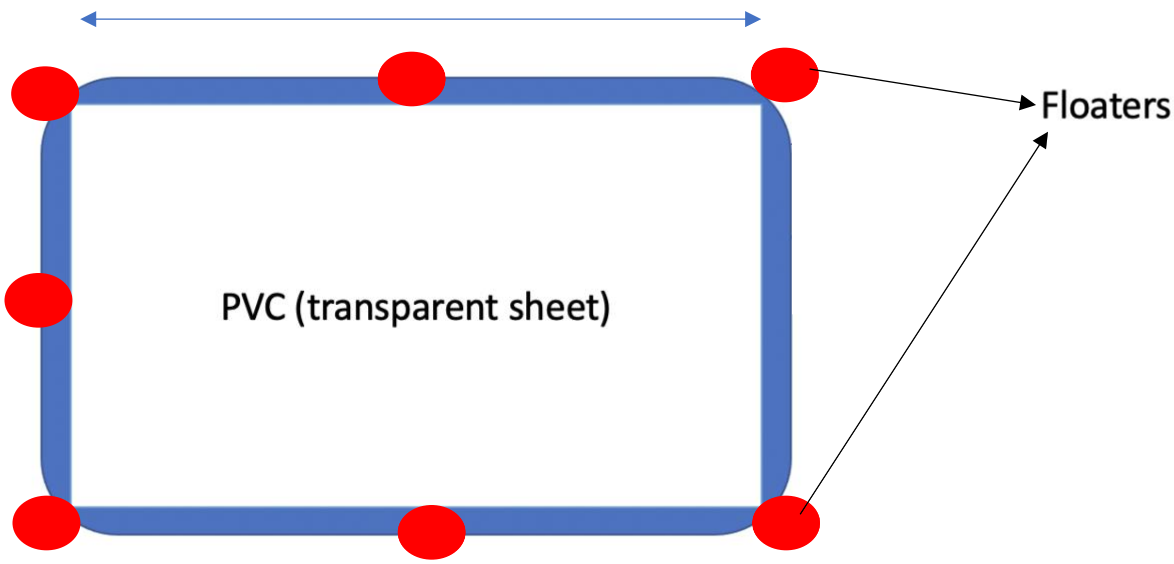
This project aims to analyse ad compare the heat losses from an open swimming pool with and without the incorporation of a transparent cover and see how much money can be saved through utilising a cover. Quite a few assumptions are introduced in the calculations, therefore the final calculated values may not be ideal or extremely accurate, however, it will still provide an insight to the suggested solution.

This project has used the weather dataset for Montreal from the months of March-May, and took advantage of MATLAB to calculate the hourly and total forced and natural convection heat transfer coefficients, corresponding resistances, reflection losses, and the Q values for convection, radiation and combined values for both with and without the presence of a sheet, all of which are presented and discussed in the following pages.

2. Proposed Design

We are assuming that the swimming pool being analysed has the following specifications:

- Depth: 2m
- Surface Area: 5m X 5 m = 25 m^2
- Water Temperature: 27 degrees Celsius = 300 K
- Floaters: Polyform 6" x 15" Red Buoys on the corner and middle edges
- 3-centimeter air gap between PVC sheets



In this project, we are trying to achieve minimal spending on raw materials which would have a greater impact towards our goal. That is why we decided to have square shaped pool, which makes it easier to manufacture the insulating sheet as well as making the calculations easier.

Since the dimension are 5mx5mx2m therefore the total volume is 50 m^3. Relatively this is a smaller swimming pool but having a compact volume enables us to achieve better heat gain with less amount of heat supplied.

The density of the PVC sheet is 1600 kg/m^3. Therefore, it will sink when placed on the surface of the pool (density of water = 1000 kg/m3). So, floaters on the sides of the PVC sheet are attached in order for the sheet to stay on top of the water surface. The sheet is designed such that the first layer is of PVC, then air and lastly PVC again. Hence, it would require 2x20mm thick PVC sheets covering $2x25m^2=50m^2$ and 8 buoys holding the sheets afloat from 4 corners and 4 middles.

We are also assuming the desired water temperature to be 27 degrees Celsius (300 K), since most people prefer slightly warmer temperature water. According to swim university [3], the most common water temperature for residential pools for leisure is 26-28 degrees Celsius, and hence, we have decided to go with 27 degrees Celsius.

3. Critical Analysis

i. Selected material properties and environmental data relevant to the problem.

We will be choosing a clear PVC sheet as our material. The material is transparent and requires minimal investment.

Where Density, ρ = 1600 kg/m^3, Thermal conductivity, k = 0.16 $\frac{\mathrm{Kw}}{\mathrm{mk}}$ [4]

ii. Identification of relevant heat transfer mechanisms

In this project all three modes (taught in Mech 346) of heat transfer are active. Therefore, in the following paragraph we will discuss their roles below.

Conduction:

Conduction is mode of heat transfer in which heat moves through the material also if through the subsequent material if there is any. In our case heat can either move from the PVC sheet to the water body or from the water body to the sheet.

Convection:

In convection heat is transported through the movement of fluids. It flow direction is from hot to cold environment. For our project we will deal with convection in the medium of air. The wind and temperature difference between the sheet as well as the outside atmosphere will cause convection effects. This will either result in heat loss or gain by the water body.

Radiation:

Unlike previous heat transfer methods, radiation does not require a method to transfer heat. It is the transmission of energy through waves. This method would be the main source of heat gain in the water body. Although it should be kept in mind that not all the radiation energy emitted by the sun would absorb because of reflection of the PVC sheet as well as water.

iii. Main assumptions:

- Evaporation losses ignored
- Negligible solar absorption in the transparent material, but not negligible thermal radiation emitted and absorbed by the transparent material.
- The heat transfer through the floaters is negligible
- Radiation energy losses with the addition of sheet are ignored
- Montreal-est dataset used for the 3 months were March-May.

- Kinematic viscosity of air, v, assumed as an average constant of 1.47×10^{-5} m^2/s [5] at 15 degrees Celsius temperature (considered average temperature during March-May) for easier manipulation of our code.
- Prandtl number of air, Pr, assumed as an average constant of 0.7323 [5] at 15 degrees Celsius temperature (considered average temperature during March-May) for easier manipulation of our code.
- Thermal Conductivity of air, k, 0.02476 W/m-K [5] assumed as an average constant of 0.7323 at 15 degrees Celsius temperature (considered average temperature during March-May) for easier manipulation of our code.
- Critical Reynold's number $=5 \times 10^5$

iv. Equations

Without the sheet

Natural Convection Loss:

In order to find the find the convection loss we had to develop an algorithm which would extract the necessary weather data such as wind speed and temperature of our selected area. This information was utilized to find the natural convection factors and forced convection factors. In the passage below we will do a sample calculation in order to illustrate the MATLAB code.

For natural convection,

$$\mathrm{T_{film}} = \frac{\mathrm{T_w} + \mathrm{T_{sky}}}{2}$$

$$\mathrm{Gr} \;=\; \mathrm{g} \times \left(\frac{1}{\mathrm{T_{film}}} \times (\mathrm{T_w} - \mathrm{T_{sky}}) \times \frac{\mathrm{lc}^3}{(\upsilon)^2}\right)$$

$$\mathrm{Ra_{lc}} \;=\; \mathrm{Gr} \;\times Pr_{air}$$

Note: we assume the value of υ to be constant. It is calculated at 15 degrees Celsius.

After we compute the Ra value, a condition was enforced in order to utilize the correct formula for finding the Nusselt number according to the value.

```
if 1E4 < Ralc(i) && Ralc(i) <1E7
        Nulc(i) = 0.54 * Ralc(i)^0.25.
elseif 1E7 < Ralc(i) && Ralc(i) <1E11
        Nulc(i) = 0.15 * Ralc(i)^ (1/3);
End
```

Moving on we then find the H coefficient and the power lost due to natural convection

$$\mathrm{h_{nconv}} \;=\; \mathrm{Nu_{lc}} \times \frac{\mathrm{k_{air}}}{\mathrm{lc}}$$

$$\mathrm{Q_{nconv}} \;=\; \mathrm{h_{nconv}} \times \mathrm{A} \times (\mathrm{T_w} - \mathrm{T_{sky}})$$

Forced Convection Loss:

In this part we begin by calculating the Reynolds number with help of wind speed data.

$$Re = \frac{U_\infty l_c}{\upsilon}$$

Next according to the value of Reynolds number we apply the formula to find the Nusselt number which eventually leads us to get h, just like how we calculated in natural convection.

```
Re(i) = wind(i)*lc/vair;% Re #
     if Re(i) > 5E5 % check for Laminar or Turbulent flow
            Nu_lf(i) =
0.664*Recr^0.5*Prair^(1/3)+0.037*Re(i).^0.8*Prair^(1/3)*(1-
(Recr/Re(i)).^0.8);% Nu # for turbulent flow
     else
        Nu_lf(i) = 0.664*Re(i).^0.5*Prair^(1/3);% Nu# for laminar flow
```

$$\mathrm{h_{fconv}} = \mathrm{Nu_{lf}} \times \frac{\mathrm{k_{air}}}{\mathrm{lc}}$$
$$\mathrm{Q_{fconv}} = \mathrm{h_{fconv}} \times \mathrm{A} \times (\mathrm{T_w} - \mathrm{T_{sky}})$$

Radiation Loss:

$$Q_{rad} = A\varepsilon\sigma \times \frac{4\left(T_w + T_{sky}\right)^3}{2} \times \left(T_w - T_{sky}\right)$$

The above formula was implanted on the array of atmospheric temperature. $\varepsilon = 0.96$ [5]

Total loss without sheet

Lastly all the power loss's were summed up to get an idea of how much energy is lost without the sheet.

With sheet

For this scenario we will create a thermal circuit. The first layer is of PVC then air and lastly PVC again.

$$R_t = \frac{1}{A}\left(\frac{L_s}{K_s} + \frac{L_a}{K_a} + \frac{L_s}{K_s} + \frac{1}{h_c}\right)$$

Where: Tm = 287.25 K, $K_a = 0.026$ [5], $K_s = 160$, $L_a = 0.03m$, $L_s = 0.02m$

Since we already calculated the h in the last part, so we simply constructed a new formula using h values and the values mentioned above.

```
R(i)= (2*(0.02))/160 +0.03/kair +1/(hnconv(i)+hfconv(i)));
```

$$Q_{conv} = UA(T_w - T_a)$$

```
Qsconv(i) = (25*(R(i).^-1)*(Tw-Tempsky(i)))
```

Radiation Loss:

The radiation loss was neglected due to it being complicated for this type of design. However, it is acknowledged that this assumption will introduce a significant error margin.

Reflection Loss with sheet:

Because we are adding transparent PVC sheets for our cover, there is an introduction of reflection losses between the sun and the water for each interface in between. The estimated total fraction of solar energy absorbed by the water (α_{tot}) can be calculated follows:

$$\alpha_{tot} = (1 - R_w) \times (1 - R_1)^2 \times (1 - R_2)^2$$

Where, R_1= R_2= The fraction of solar energy reflected by a transparent PVC sheet of material with refractive index $n_1 = n_2$ [6]and R_w is the reflection from the water surface with refractive index n_w [5]

$$R_w = \left(\frac{n_w - 1}{n_w + 1}\right)^2 = \left(\frac{1.33 - 1}{1.33 + 1}\right)^2 = 0.02$$

$$R_1 = R_2 = \left(\frac{n_1 - 1}{n_1 + 1}\right)^2 \left(\frac{1.54 - 1}{1.54 + 1}\right)^2 = 0.045$$

Hence,

$$\alpha_{tot} = (1 - R_w) \times (1 - R_1)^2 \times (1 - R_2)^2$$
$$= (1 - 0.02) \times (1 - 0.045)^2 \times (1 - 0.045)^2$$
$$= \mathbf{0.815}$$

Using the value of alpha found above we can find how much energy was lost due to reflection.

$$Q_{reflection} = (1 - \alpha_{tot}) \frac{(area)(solar_{irradiecne})}{3600}$$

The above equation was divided by 3600 in order to convert energy to power since the units for solar irradiance are $\frac{KJ}{m^2}$.

Total loss with sheet

Lastly, we added the losses due to convection and reflection to sum the loss in power with having a sheet on top of the pool.

Note: there should be an error in the final heat loss with sheet as radiation is ignored.

With two surfaces the formula for heat loss with radiation is $Q_{rad} = \frac{A\alpha(T_1^4 - T_2^4)}{\frac{2}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$(ignoring air)

Since we have an airgap between our sheets the calculation becomes complicated and therefore, we ignore radiation.

The power reserved in the pool due to the sheet is:

$$Q_{withoutsheet} - Q_{withsheet} = \mathbf{30857.349}\ \boldsymbol{KWH}$$

v. Monthly heating costs savings for three representative months:

According to Hydro-Quebec's Rate D plan, which is a two-tiered pricing plan [7]:

- 6.08¢/kWh for energy consumed up to 40 kWh per day times the number of days in the consumption period (1^{st} tier)

Once we have found the total power saved, we then multiply it by the rate mentioned above. It should be noted that we assumed that the power saving does not go above 40 KWH per day in order to use a constant rate.

Therefore, savings for 3 months:

$$30857.349\ KWH * 0.0608\$ = \mathbf{1876.13}\ \$$$

However, again, it is to be noted that the real amount of money being saved would definitely be lower than the above because we are not accounting for the losses in radiation. There should be an approximate error of 15-20%.

4. Discussion and Conclusion

In our results, we have analysed the losses considering the sheet be placed on the pool at all times through out the day, however this will not be the case in an ideal scenario, since it will be taken on and off during swim time. That will considerably increase our Q losses and therefore, reduce our cost saving. Furthermore, the introduction of the assumption of no radiation loss between our sheets has introduced quite a significant error, assumed to be at least 15-20%. If these had been taken into consideration, the cost saving over the 3 months would have been a much lower number than the one calculated above. However, having taken everything into consideration, the results still show quite a good amount of Power and money saving with the introduction of a very simple PVC cover.

In terms of installation cost, the only cost to be considered is that of our 2x20mm thick PVC sheets covering 2x25m^2=50m^2 and the 8 buoys. This would require about 19 sheets of 3'x6' PVC sheets of 2-3mm thickness from a supplier on Indiamart [8]. That would result in CAD $40 for sheets+$187 for buoys [9]+shipping costs, which is still significantly lower than the cost savings.

Because we are considering a simple PVC sheet with floaties, there is no significant maintenance cost. This is not an automatic opening/closing mechanism, and therefore, there are no pulleys or mechanisms that will require lubrication or tightening. There are also no cover tracks that would require cleaning since the cover is being supported by floaties. The only issue to be considered is a substantial amount of snowfall. If the snow on top of the cover, near the middle section gets too heavy, then the floaties will not be able to provide their support and the cover will sink in the middle, rendering the cover to be very inefficient during months of heavy snowfall.

However, all things considered, the transparent PVC sheets are a simple and relatively cheap method of reducing heat losses from pools.

5. Appendices

References:

[1] J. T. Czarnecki, “A Method of Heating Swimming Pools by Solar Energy”, Commonwealth Scientific and Industrial Research Organization, 7 (1) (1963)

[2] Brooks, F. A., "Solar Energy Research." Edited by F. Daniels and J. A. Duffle, Madison, (76) (1955)

[3] Giovanisci, M., “What is the perfect pool Temperature?”, Swim University (2019). Available at: https://www.swimuniversity.com/pool-temperature/

[4] Lienhard IV, J. H., Lienhard IV, J. H., “A Heat Transfer Textbook”, 5th Ed., Phlogiston Press (2020).

[5] Cengel, Y. A., Ghajar, A. J., “Heat and mass transfer: fundamentals and applications” 5th Ed., McGraw-Hill Higher Education (2015).

[6] Shackelford, J. F., Introduction to Materials Science for Engineers, 5th Ed., McGraw-Hill (2000)

[7] 2019 Electricity Rates, *HydroQuebec.com*, Section 2 (12) (2019). Available at: https://www.hydroquebec.com/data/documents-donnees/pdf/electricity-rates.pdf?#page=16

[8] PVC Sheet Supplier quote: https://www.indiamart.com/proddetail/transparent-rigid-pvc-sheet-22416477897.html

[9] Polyform S-1 RED S Series Buoy - 6" x 15", Red: https://www.amazon.com/Polyform-Boat-Fender-Small-1-Red/dp/B000KORR5C/?&_encoding=UTF8&tag=polyformuscom-20&linkCode=ur2&linkId=1a4cba48e59ac50f87338b016fd9c5cc&camp=1789&creative=9325

MATLAB Code:

```
clc;
clear all;

X= readtable('Montreal-est.xls'); %data to be extracted from excel
solar = transpose(((X{:,13}))); %irradiance in KJ/m^2
Tempsky = transpose(((X{:,12})+273.15)); %sky temp in K
wind = transpose(((X{:,9})*0.1)); %windspeed in m/s

%constants
A = 25; %area of pool
P =20; %perimeter of pool
n=2208; %number of datasets
g=9.81; %m/s^2
Tw=27+273.15; %K, water tempertaure
vair=1.47*1E-5; %m^2/s, assume number for vair at 15 degrees celcius
Prair=0.7323; % no unit, Avg Pr assumed at 15 degrees celcius at 1 atm
lc = (A/P); %m, critical length of pool
kair=0.02476; %Wm/K, at 15 degrees celcius Air
Recr=5E5; %critical Reynolds number
alpha = 0.815; %fraction of solar energy absorbed by water

%For combined HT
```

```
Rcombinedconv=zeros(1,n); %Stores combined Forced and Natural
Convection R values
Qcombinedconv=zeros(1,n); %Stores combined Forced and Natural
Convection Q values
Qcombined=zeros(1,n); %Stores combined Qradiation and Qconvection
R= zeros(1,n); %Stores Resistance values
Qsconv= zeros(1,n); %stores Qconvection with sheet

%For natural convection loop arrays
hnconv=zeros(1,n); %stores natural convection hconv values
Gr=zeros(1,n); %stores Gr values
Ralc=zeros(1,n); %stores Ralc
Tfilm=zeros(1,n); %stores Tfilm
Nulc=zeros(1,n); %stores Nusselt number
Rnconv=zeros(1,n); %stores natural convection R values
Qnconv=zeros(1,n);  %stores natural convection values
Qref=zeros(1,n); %stores Qreflection loss
Qtotwsheet= zeros(1,n); %stores total Q loss with sheet included

%Forced convection loop arrays
Re=zeros(1,n); %stores Reynolds number
hfconv=zeros(1,n); %stores h for forced convection
Rfconv=zeros(1,n);%stores R, resistance for forced convection
Nu_lf=zeros(1,n); %Nusselt number for forced convection
Qfconv=zeros(1,n); %stores Q for forced convection

%Radiation loss array+variables
Qrad=zeros(1,n); %stores Qradiation
sigma=5.67E-8; %Boltzman constant
e=0.9; %emissivity


%%Natural Convection at top surface of the cover

for i=1:n
    Tfilm(i)=(Tw+Tempsky(i))/2; %Tfilm calculation
    Gr(i) = g.*(1./(Tfilm(i))).*(Tw-Tempsky(i))*lc.^3./(vair).^2; %Gr
formula
    Ralc(i) = Gr(i) * Prair;
    if 1E4 < Ralc(i) && Ralc(i) <1E7
        Nulc(i) = 0.54 * Ralc(i)^0.25;
    elseif 1E7 < Ralc(i) && Ralc(i) <1E11
        Nulc(i) = 0.15 * Ralc(i)^(1/3);
    end
    hnconv(i) = Nulc(i)*kair/lc;
    Rnconv(i) = 1/hnconv(i)/A;
    Qnconv(i) = hnconv(i)*A*(Tw-Tempsky(i));
end

%%%Forced Convection at top surface of the cover
for i=1:n
```

```
    if wind(i)==0 %checks is windspeed is zero
        hfconv(i)=0;
        Rfconv(i)=0;
        Rcombinedconv(i)=Rnconv(i);
    else
        Re(i) = wind(i)*lc/vair;% Re #
        if Re(i) > 5E5 % check for Laminar or Turbulent flow
            Nu_lf(i) =
0.664*Recr^0.5*Prair^(1/3)+0.037*Re(i).^0.8*Prair^(1/3)*(1-
(Recr/Re(i)).^0.8);% Nu # for turbulent flow
        else
           Nu_lf(i) = 0.664*Re(i).^0.5*Prair^(1/3);% Nu # for laminar
flow
           hfconv(i) = Nu_lf(i)*kair/lc;
           Rfconv(i) = 1/(hfconv(i)*A); %forced Conv R
           Rcombinedconv(i) = 1/((1/Rnconv(i))+(1/Rfconv(i)));
%Combine Forced and Natural Convection at cover top
        end
    end
 Qfconv(i)=  hfconv(i)*A*(Tw-Tempsky(i));
 Qcombinedconv(i)=Qfconv(i)+Qnconv(i);
end


%%%Radiation Loss
for i=1:n
    Qrad(i)=A*sigma*e*4/2*(Tw+Tempsky(i)).^3*(Tw-Tempsky(i));
    Qcombined(i)=Qrad(i)+Qcombinedconv(i);
end
%%%Q convecction with sheet
for i = 1:n
   R(i)= (0.04/160 +0.03/kair +1/(hnconv(i)+hfconv(i)));
   Qsconv(i) = (25*(R(i).^-1)*(Tw-Tempsky(i)));
end
%%% reflection loss with sheet
for i = 1:n
    Qref(i) = (0.185*25*solar(i))/3.600;
    Qtotwsheet(i)= Qref(i)+Qsconv(i);
end
d =nansum(Qtotwsheet(1,:)) %total Q loss with sheet
t = nansum(Qcombined(1,:)) %total Q loss without sheet
Power_Saving = (t - d)/1000 %total Q saved in KW
money_saved = ((0.0608)*Power_Saving) %total $$$ saved
```