ANALYSIS OF APPLIED CONTROL SYSTEM

Control system design report

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1.1 Introduction

The design, development, and implementation of an effective control system to achieve a stable, desired, and optimized output pitch motion of the vanguard missile shall be the subject of technical work. The system shall be fully and systematically defined, either by means of a mathematical Laplace transfer function domain or the state space model domain, to design and configure control systems or feedback controllers for an individual dynamical system. For the given scenario the missile pitch rate dynamic transfer function model as well as actuator dynamic transfer function expression is provided, there is required to design and implement the feedback PID controller for the system. In the figure above, lateral, and horizontal forces of the missile based on the coordinate frame axis corresponding to the refence axis are indicated. The pitch rate, pitch angle, fin angle, center of gravity frame, control force, longitudinal speed U axis shown in above figure. Firstly, the design and implementation of a feedback PID controller for missile pitch angle control with an actuator as ideal form needs to be considered using time domain and frequency domains analysis to fully analyze the established control system. The second objective is to add a dynamic model of the first order transfer function with dead time or time delay dynamics to the actuator block and design and to develop a controller to investigate the system's time domain and frequency performance. The performance of the system shall be analyzed in terms of time domain parameters such as growth, period, settling and percentage overshoot or frequency field parameters which include phase margin, gain margin and error margin.

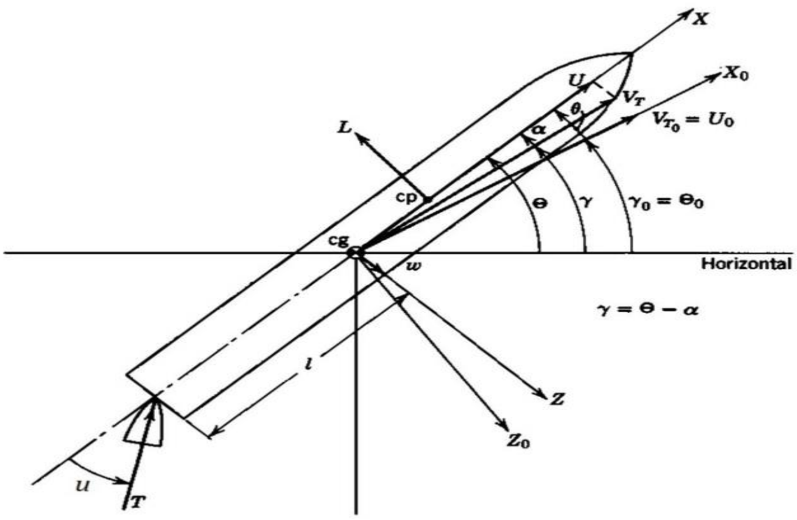


Fig. 1: Basic Co-Ordinate Axes and forces on Vanguard missile

1.2 Design Methodology and Block Diagram

The dynamic system parameters and transfer function model for the missile dynamics for a control actuation system provided as:

$$M(s) = \frac{q(s)}{\delta(s)} = \frac{7.21(s+0.526)s}{(s+1.6)(s-1.48)(s-0.023)} \qquad (1)$$

As regards the input thrust vector angle $\delta(s)$ shown in expression 1, the given transfer function represented the output pitch rate.

1.3 Block Diagram

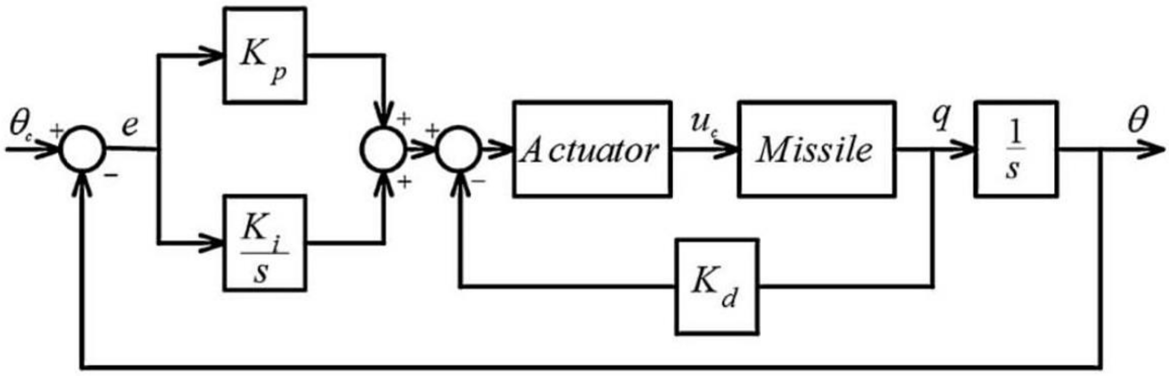


Fig. 2: A PID controller with rate feedback

Above given block diagram has the actuator block connected in forward path of missile that deliver the control input thrust to missile. To ensure a stable response to pitch rate, the missile's output will be fed with derivative rates feedback loop. The measured pitch rate of missile is integrated to obtain the actual pitch angle theta. When the actual pitch angle is compared to the desired reference pitch angle, the actual pitch angle is feedback from the unity gain to the difference block. The generated error is supplied to PI controller where the controlled output for pitch rate inner control loop is obtained.

1.4 Given design criteria for solution.

No	Criteria	Desired Value
1	Settling time $t_s(s)$	$t_s < 3(s)$
2	Rise time $t_r(s)$	$t_r < 1.5(s)$
3	Maximum overshoot $M_p(\%)$	$M_p < 15(\%)$
4	Steady-state error $e_{ss}(\%)$	$e_{ss} < 2(\%)$
5	Maximum undershoot	NA in this system
6	Integral of the Absolute Error $ITAE$	Minimum
7	Integral of the Absolute Control Effort $IACE$	Minimum
8	Gain Margin $GM(db)$	$GM > 6db$
9	Phase Margin $PM(^0)$	$PM > 60^0$
10	Delay Margin $DM(s)$	$DM > 0.8s$
11	Integral of the Absolute Control Effort $IACE$	Minimum
12	Integral of the Absolute Control Effort Rate $IACRE$	Minimum

2.1 (Q-1)

Consider the actuator is ideal and its transfer function is Gac =1.0. Using the obtained knowledge in this module, design and simulate a stable controller (for unit step response in 20s) with the best achievable performance.

2.2 Answer

As the actuator's dynamics are considered ideal, there will be a transfer function in the actuators:

$$G_{ac} = 1 \qquad (2)$$

The inner close loop system transfer function for pitch rate-controlled system will be obtained as:

$$G_{inner\ loop} = \frac{G_{ac}G_{missile}}{1+G_{ac}G_{missile}K_d} \qquad (3)$$

As:

$$G_{ac}G_{missile} = \frac{7.21(s+0.526)s}{(s+1.6)(s-1.48)(s-0.023)} \qquad (4)$$

$$G_{inner\ loop} = \frac{7.21(s+0.526)s}{(s+1.6)(s-1.48)(s-0.023)} \div 1 + \frac{7.21(s+0.526)sKd}{(s+1.6)(s-1.48)(s-0.023)} \qquad (5)$$

$$G_{inner\ loop} = \frac{7.21(s+0.526)s}{(s+1.6)(s-1.48)(s-0.023)} \div \frac{(s+1.6)(s-1.48)(s-0.023)+7.21(s+0.526)sKd}{(s+1.6)(s-1.48)(s-0.023)} \quad (6)$$

$$G_{inner\ loop} = \frac{7.21(s+0.526)s}{(s+1.6)(s-1.48)(s-0.023)+7.21(s+0.526)sKd} \quad (7)$$

Transfer function with PI controller for the outer loop:

$$G_{outer\ loop} = \frac{\theta}{\theta_c} = \frac{G_{inner\ loop} * \frac{1}{s} * \left(\frac{K_p s + K_i}{s}\right)}{1 + G_{inner\ loop} * \frac{1}{s} * \left(\frac{K_p s + K_i}{s}\right)} \quad (8)$$

$$G_{outer\ loop} = \frac{\theta}{\theta_c} = G_{inner\ loop} * \frac{1}{s} * \left(\frac{K_p s + K_i}{s}\right) \div 1 + G_{inner\ loop} * \frac{1}{s} * \left(\frac{K_p s + K_i}{s}\right) \quad (9)$$

Insert values of $G_{inner\ loop}$ in expression 9:

$$G_{outer\ loop} = \frac{\theta}{\theta_c} = \frac{7.21(s+0.526)}{(s+1.6)(s-1.48)(s-0.023)+7.21(s+0.526)sKd} * \left(\frac{K_p s + K_i}{s}\right) \div 1 + \frac{7.21(s+0.526)}{(s+1.6)(s-1.48)(s-0.023)+7.21(s+0.526)sKd} * \left(\frac{K_p s + K_i}{s}\right) \quad (10)$$

$$G_{outer\ loop} = \frac{\theta}{\theta_c} = \frac{7.21(s+0.526)(K_p s + K_i)}{s((s+1.6)(s-1.48)(s-0.023)+7.21(s+0.526)sKd)} \div 1 + \frac{7.21(s+0.526)(K_p s + K_i)}{s((s+1.6)(s-1.48)(s-0.023)+7.21(s+0.526)sKd)} \quad (11$$

$$G_{outer\ loop} = \frac{\theta}{\theta_c} = \frac{7.21(s+0.526)(K_p s + K_i)}{s((s+1.6)(s-1.48)(s-0.023)+7.21(s+0.526)sKd)} \div \frac{((s+1.6)(s-1.48)(s-0.023)+7.21(s+0.526)sKd)+7.21(s+0.526)(K_p s + K_i)}{s((s+1.6)(s-1.48)(s-0.023)+7.21(s+0.526)sKd)} \quad (12)$$

$$G_{outer\ loop} = \frac{\theta}{\theta_c}$$

$$= \frac{7.21(s+0.526)(K_p s + K_i)}{s((s+1.6)(s-1.48)(s-0.023) + 7.21(s+0.526)sKd)}$$

$$\times \frac{s((s+1.6)(s-1.48)(s-0.023) + 7.21(s+0.526)sKd)}{((s+1.6)(s-1.48)(s-0.023) + 7.21(s+0.526)sKd) + 7.21(s+0.526)(K_p s + K_i)} \tag{13}$$

$$G_{outer\ loop} = \frac{7.21(s+0.526)(K_p s + K_i)}{((s+1.6)(s-1.48)(s-0.023) + 7.21(s+0.526)sKd) + 7.21(s+0.526)(K_p s + K_i)} \tag{14}$$

The proportional, integral, and derivative gains Kp, Ki and Kd will be determined to stable the system and to obtain the desired performance of system.

2.3 Frequency Domain Analysis

2.4 MATLAB coding work:

MATLAB_code.m

```
%% Section 1  Ideal Actuator Gac=1
%%% Kp=10, ki=1, Kd=1
clc;close all;  %%% Clear all
s=tf('s');      %%% define system in s domain
Mqu=(7.21*(s+0.526)*s)/((s+1.6)*(s-1.48)*(s-0.023))  %% Missile TF
Gac=1;          %% Actuator Transfer Function
Kp=10;           %% Proportional gain
Ki=1;           %% Integral gain
Kd=1;           %% derivative gain rate feedback
Simulation_Time=20;  %%% simulink simlation Time
sim('Simulink_Model_Solutions',Simulation_Time) %%% RUN
Ginner=feedback(Gac*Mqu,Kd)  %%% inner loop pitch rate control with rate feedback
C=(Kp+(Ki/s));  %% PI controller
Gouter=feedback(C*(Ginner*(1/s)),1); %%% outer loop TF
Margins=allmargin(C*(Ginner*(1/s)))  %%% Frequncy domian Performance
stepinfo(Gouter)  %%% Time domain Performance Parameters
margin(C*(Ginner*(1/s)))  %%% bode plot
```

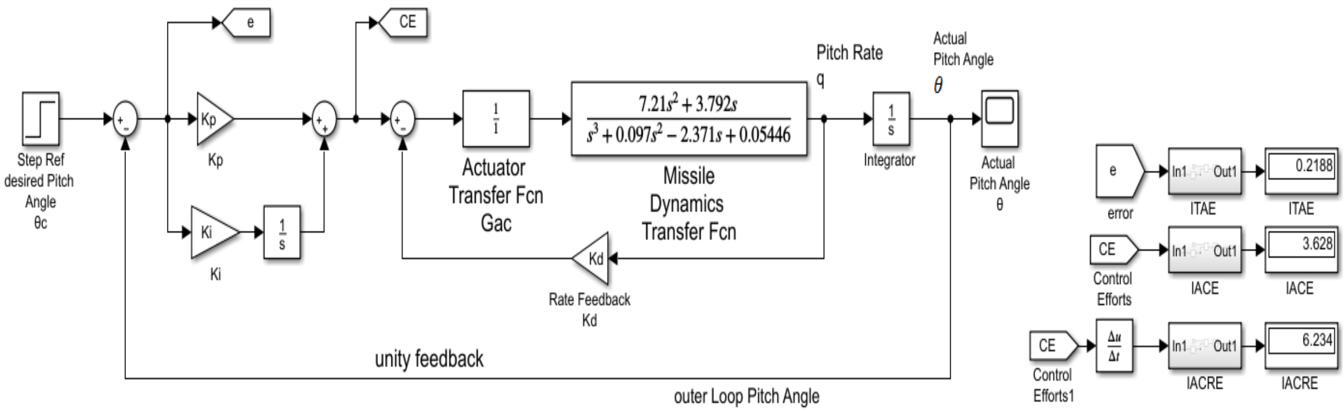
- Output frequency domain parameters with Kp=10, Ki=1, Kd=1 parameters:

Command Window

```
Margins =

  struct with fields:

     GainMargin: Inf
    GMFrequency: Inf
    PhaseMargin: 41.3731
    PMFrequency: 7.1623
    DelayMargin: 0.1008
    DMFrequency: 7.1623
         Stable: 0
```

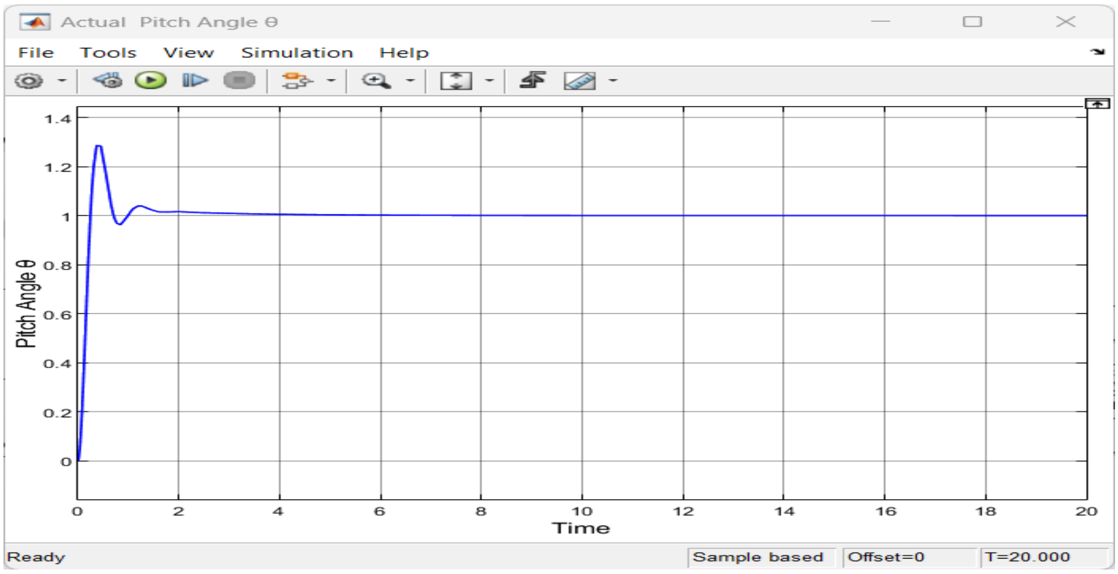
The above figure shows the frequency domain parameters such as the phase margin, the gain margin, the delay margin for the closed loop feedback system with Kp=10, Ki=1, and Kd=1, along the ideal actuator feedback gain.

2.5 Ideal Actuator Gac=1 Simulink block model



Above Simulink Block Model developed with assuming the ideal actuator Gac=1 along rate feedback gain Kd=1, and Proportional Kp=10 as well as integral gain Ki=1. There's a double loop. One is the internal loop, which controls the pitch rate of a missile represented by the dynamics of the Control Actuation System CAS for missiles' Pitch Rate Control Loops and the outer loop ensured to give an accurate response to the output plate angle.

2.6 Step response of the system with Kp=10, Ki=1, Kd=1 controller gains with ideal actuator

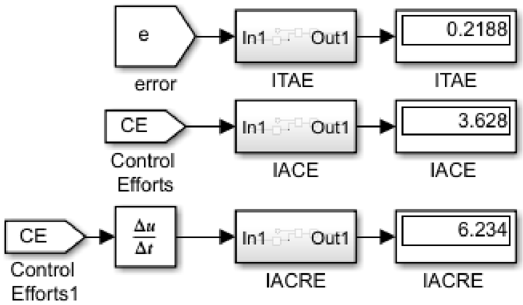


2.7 Describe above graph.

The output response of the system, which is controlled by transient and constant state behaviors, has been shown to be consistent with a closed loop system that assumes ideal dynamic properties for an actuator. The response is solid.

2.8 Time domain performance parameters and ITAE, IACE and IACRE measurements

```
    RiseTime: 0.1691
SettlingTime: 1.4959
 SettlingMin: 0.9420
 SettlingMax: 1.3005
   Overshoot: 30.0481
  Undershoot: 0
        Peak: 1.3005
    PeakTime: 0.4150
```



The following figure shows the transient and constant state performance parameters. Kp=10, Ki=1, and KD=1 gains are measured for rise time, settling time, etc. Moreover, the Kp=10, Ki=1, and Kd=1 gain parameters are used to measure the integral time absolute error, the integral of absolute control effort, and the integral of absolute control effort rate.

2.9 Ideal Actuator Gac=1 Performance parameter table and describe table

It is observed from the close loop system design with ideal actuator dynamics included and simulation-based analysis ensured with selection of different proportional, integral, and derivative gains of PID control architecture. The system has been observed to be stable with a controlled output response. Depending on the gain of the different controllers, the transient and steady state performance parameters and the frequency domain parameters are different. The system's output response speeds up with the shortening of rise times because of increased proportional gain. The percentage overshoot was also reduced when the proportional gain was reduced. The phase margin, ITAE and IACE values also increased when the integrated gain reduced the value of the settling time, but these values have been lowered for IACRE. When derivative gain increased the percentage overshoot decreased or in other words with decreasing derivative gain the percentage overshoot increased. The requirements of the gain margin are not met and there is no satisfaction.

Performance Parameters	Kp=10 Ki=1 Kd=1	Kp=5 Ki=0.1 Kd=1	Kp=5 Ki=0.001 Kd=1.1
Settling Time (s)	1.49 seconds	2.44 seconds	2.3 seconds
Rise Time (s)	0.16 seconds	0.28 seconds	0.3 seconds
Maximum Overshoot %	**30.04%**	**17.31%**	**13.10%**
Steady state error	0	0	0
Maximum undershoot %	NA in this system	NA in this system	NA in this system
ITAE	0.2188	0.44	0.6
IACE	3.6	3.5	3.5
GM (dB)	Infinity	Infinity	Infinity
PM 0	41.4 o	54o	58o
DM (s)	0.1 seconds	0.21 seconds	0.24 seconds
IACRE	6.2	3	2.75

3.1 (Q-2)

Consider the dynamics of the actuator can be explained by the following first order plus dead-time transfer function.

$$G_{ac} = \frac{e^{-\tau_d s}}{\tau s+1}, \tau = 0.1, \tau_d = 0.02 \qquad (2)$$

3.2 Answer

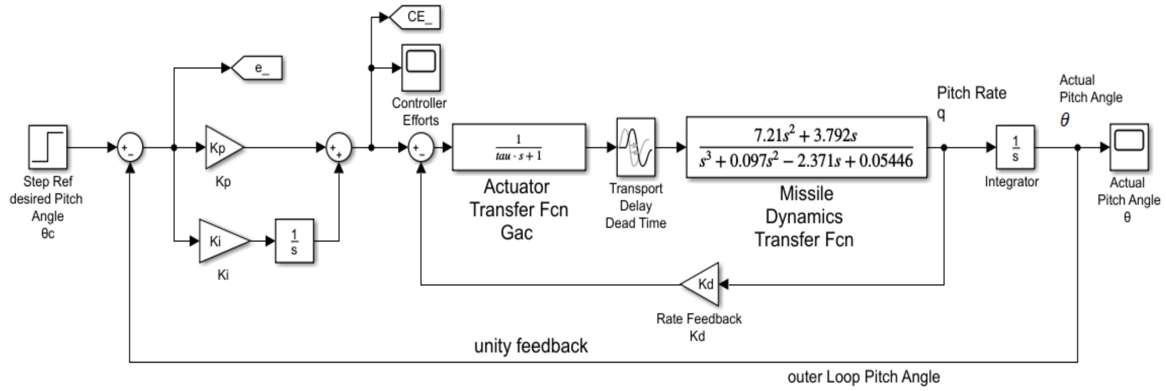
actuator dynamic transfer function expression given.

$$G_{ac} = \frac{e^{-\tau_d s}}{\tau s + 1} \qquad (1$$

Where;

$$\tau_d = 0.02\ seconds \quad \tau = 0.1\ seconds$$

3.3 Actuator dynamics Simulink block model



Above Simulink block model developed with including actuator dynamic transfer function expression. The time delay or dead time value of the actuator is 0.02 seconds. The system was analysed based on gain values from different control systems.

3.4 Dynamic actuator transfer function performance parameter table

Performance Parameters	Kp=1 Ki=1 Kd=1	Kp=1.5 Ki=0.02 Kd=0.8	Kp=1.5 Ki=0.0025 Kd=0.45
Settling Time (s)	10.7 seconds	4.2 seconds	5.8 seconds
Rise Time (s)	0.6 seconds	0.4 seconds	0.34 seconds
Maximum Overshoot %	**51%**	**14.90%**	**71%**
Steady state error	0	1%	1%
Maximum undershoot %	NA in this system	NA in this system	NA in this system
ITAE	5.2	20	20
IACE	7.4	3.5	3.5
GM (dB)	15 dB	11 dB	5.7 dB
PM 0	30.6o	56o	22.4o
DM (s)	0.38 seconds	0.4 seconds	0.1 seconds
IACRE	**7.2**	**2.8**	**3.3**

3.5 Describe above table.

It is noticed in the close loop system design with actuator dynamics transfer function incorporated and simulation-based analysis assured with selection of different proportional, integral, and derivative gains of PID control architecture. With a controlled output response, the system is intact and steady. The transient and steady state performance of the controller, as well as frequency domain parameters, are changing according to changes in its gain. The system output response is accelerated when the proportional gain increases, with a slower rise period. The percentage overshoot was also reduced because of the reduction in the proportionate gain. The settling time value, phase margin and ITAE, as well as the IACE and IACRE values are increased when the integral gain is reduced. The percentage overshoot decreased as the derivative gain increased, whereas the percentage overshoot increased as the derivative gain decreased.

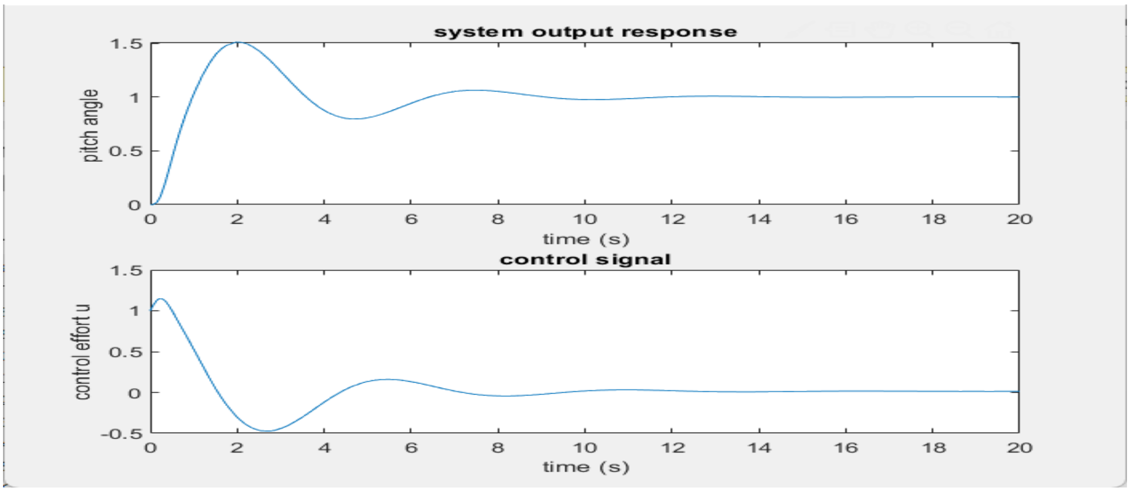
4.1 (Q-3)

Our recommendation is to utilise illustrative figures that plot system output and the control signal with each other, and all frequency domain charts (like Bode, Nyquist and Nichols) and analyse the system based on them. Use tables to list and compare the design criteria for each tuning and prove your controller is the best design. By adjusting controller parameters and using time and frequency domain approaches such as Bode, Nyquist, and Nichols charts analyse the system behaviour based on the provided 12 criteria.

4.2 Answer: - MATLAB coding work

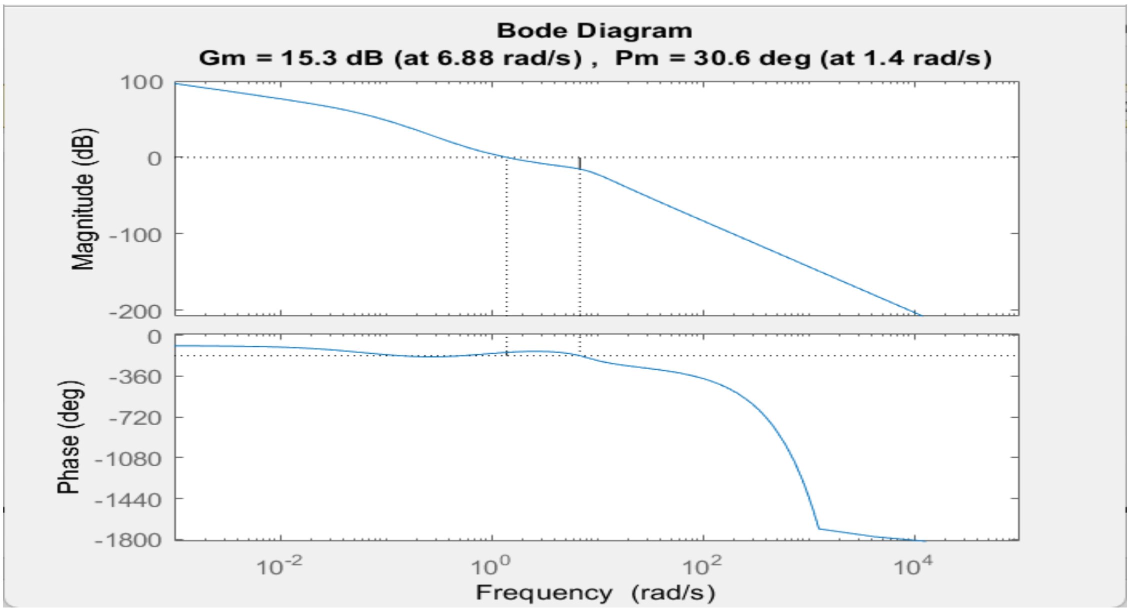
```
stepinfo(Gouter)  %%% Time domain Performance Parameters
figure(1)
subplot(2,1,1)
plot(t,y)
title('system output response')
xlabel('time (s)')
ylabel('pitch angle ')

subplot(2,1,2)
plot(t,u)
title('control signal')
xlabel('time (s)')
ylabel('control effort u')

figure(2)
margin(C*(Ginner*(1/s)))  %%% bode plot
figure(3)
nicholsplot(C*(Ginner*(1/s)))   %%% nichols plot
figure(4)
nyquist(C*(Ginner*(1/s)))  %%% nyquist diagram
```

4.3 Control signal and output response



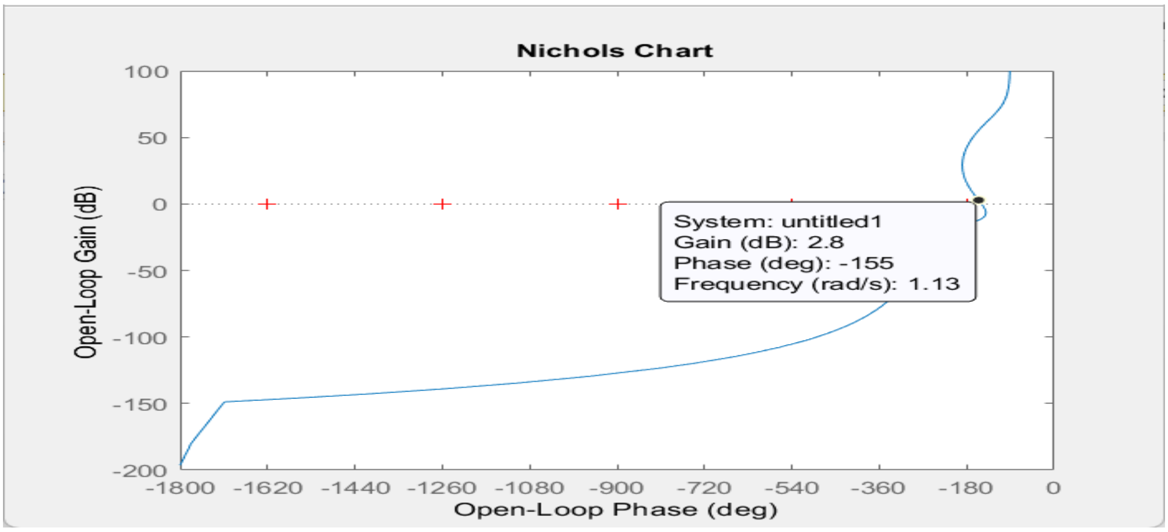
The controlled output desired pitch angle response of the system output and the PID controller output control effort signal was shown in the above response. It noted that, in addition to the design and implementation of an effective PID controller gain value, a desired steady state is achieved for 1 radian pitch angle without any stable state error or transient parameters.

4.4 Frequency domain plot and bode plot:



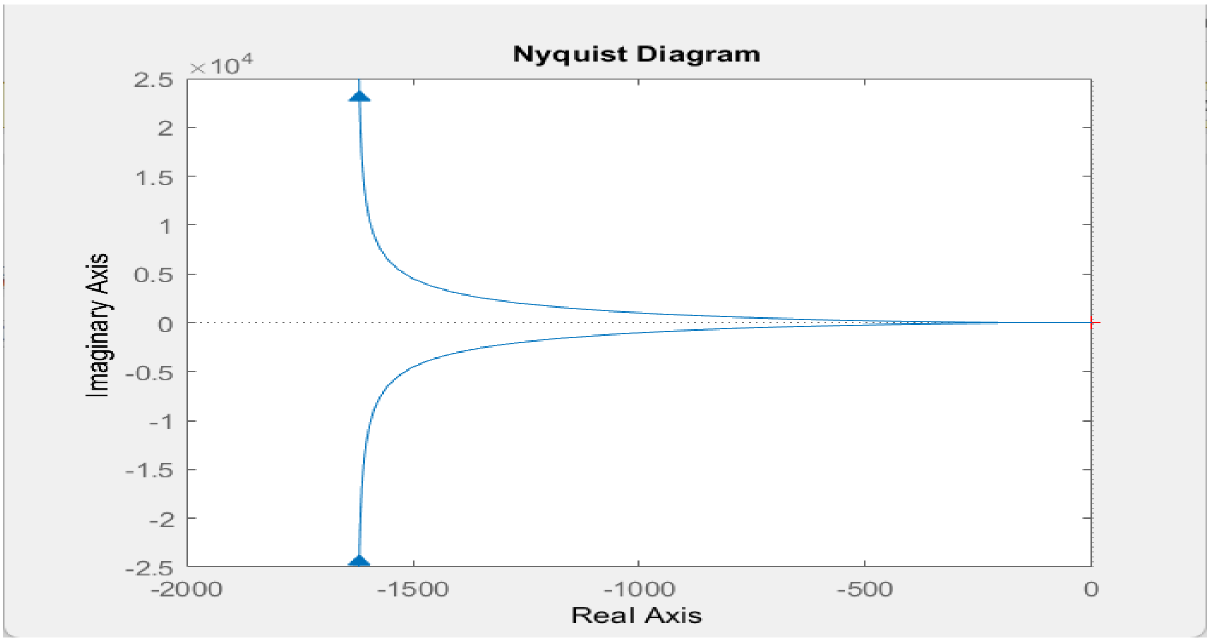
The output response of the frequency domain with the default set of proportional, integral, and derivative gains of 1 is shown in the above obtained response. It is noticed that the phase margin is 30 degree and gain margin is 15.3 dB. To make the system unstable or marginally stable, it means a minimum delay of 30 degrees between controller and comparator.

4.5 Nichol's chart



The above diagram represented the frequency domain plot for open loop phase angle achieved with open loop gain for controller design and implementation of pitch angle-controlled system. When the natural frequency of the system is 1 radium per second, which is controlled by an integral gain, the gain margin is observed to be less than 6dB.

4.6 Nyquist diagram



The Nyquist plot of the developed feedback close loop system is shown in the figure. The lines of the root curve appear to be in the left-hand side, with a clockwise rotation of the critical point available at the location of jw=+/-1 point. If the Nyquist plot is located on the left side of critical point clockwise with rotation in s.plane or jw plane, then the system's root line can be considered to lie within that right hand direction.

4.7 Effective, optimized Controller gains and Performance parameters

Performance Parameters	Kp=1.5 Ki=0.02 Kd=0.8
Settling Time (s)	4.2 seconds
Rise Time (s)	0.4 seconds
Maximum Overshoot %	**14.90%**
Steady state error	1%
Maximum undershoot %	NA in this system
ITAE	20
IACE	3.5
GM (dB)	11 dB
PM 0	56o
DM (s)	0.4 seconds
IACRE	**2.8**

Transient and steady state parameters as well as frequency response parameters obtained by choosing effective and optimized values of Kp, Ki and Kd obtained shown in above table.

5.1 (Q-4)

Through simulations in the time and frequency domain, prove the calculated GM and DM match the time domain simulations (by increasing the gain and delay in simulations).

5.2 Answer

To obtain the desired simulation response, delay and gain margins shall be added to the system.

5.3 Gain margin

When Kp=1, Ki=1, Kd=1 then the obtained gain margin of system in dB is:

$$20\log[K] = 15.3\ dB$$

$$\log[K] = \frac{15.3}{20}$$

$$\log[K] = 0.77$$

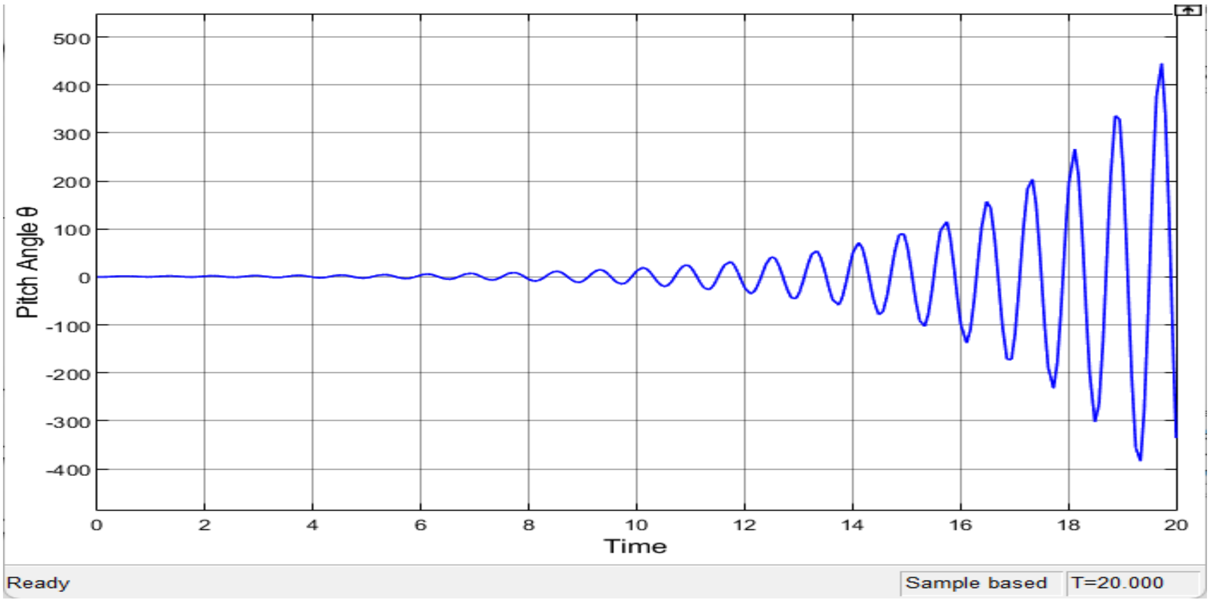
On both sides, take an inverse log.

$$K = 5.88$$

5.4 MATLAB coding work

```
%% Section 4 GM verification
%%% Kp=7, ki=1, Kd=1
clc;close all;  %%% Clear all
s=tf('s');      %%% define system in s domain
Mqu=(7.21*(s+0.526)*s)/((s+1.6)*(s-1.48)*(s-0.023))  %% Missile TF
tau=0.1;        %%% Time constant of actuator
td=0.02;        %%% dead time of actuator
num = 1;
den = [tau 1];
Gac = tf(num,den,'InputDelay',0.02) %% Actuator Transfer Function
%Gac=1;
Kp=7;           %% Proportional gain
Ki=1;           %% Integral gain
Kd=1;           %% derivative gain rate feedback
Simulation_Time=20;  %%% simulink simlation Time
sim('Simulink_Model_Solutions__',Simulation_Time) %%% RUN
Ginner=feedback(Gac*Mqu,Kd)  %%% inner loop pitch rate control with rate feedback
C=(Kp+(Ki/s));  %% PI controller
Gouter=feedback(C*(Ginner*(1/s)),1); %%% outer loop TF
Margins=allmargin(C*(Ginner*(1/s)))  %%% Frequnecy domian Performance
margin(C*(Ginner*(1/s)))  %%% bode plot
```

5.5 Open loop gains greater than gain margin value Kp=7



The system will be slightly stable if the forward gains are equal to the gain margin value, and then it's unstable when the profits exceed the profit margin value.

5.6 Delay margin

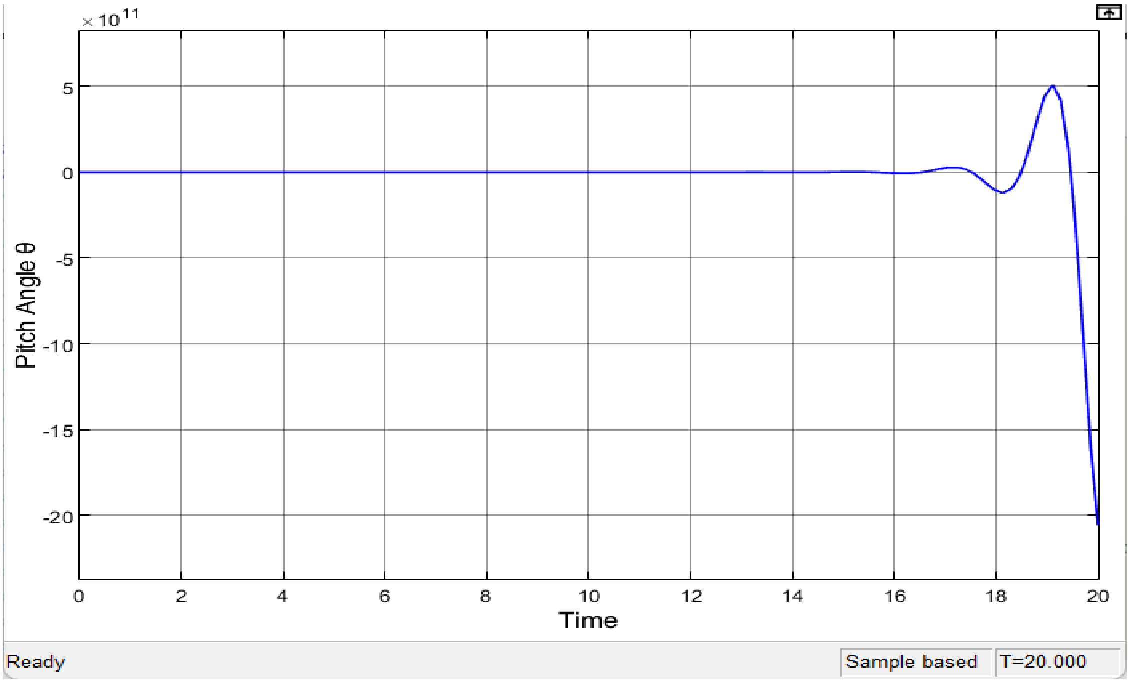
At kp=1, ki=1, and Kd=1 the delay margin value is obtained as:

```
PhaseMargin: 30.6026
PMFrequency: 1.3987
DelayMargin: 0.3819
DMFrequency: 1.3987
```

The output simulation response shall be provided when the delay margin is set at DM=0.38 for the actuator transfer function.

```
%% Section 4 b DM verification
%%% Kp=1, ki=1, Kd=1
clc;close all;  %%% Clear all
s=tf('s');      %%% define system in s domain
Mqu=(7.21*(s+0.526)*s)/((s+1.6)*(s-1.48)*(s-0.023))  %% Missile TF
tau=0.1;        %%% Time constant of actuator
td=0.38;        %%% dead time of actuator
num = 1;
den = [tau 1];
Gac = tf(num,den,'InputDelay',0.02) %% Actuator Transfer Function
%Gac=1;
Kp=1;           %% Proportional gain
Ki=1;           %% Integral gain
Kd=1;           %% derivative gain rate feedback
Simulation_Time=20;  %%% simulink simlation Time
sim('Simulink_Model_Solutions__',Simulation_Time) %%% RUN
Ginner=feedback(Gac*Mqu,Kd)  %%% inner loop pitch rate control with rate feedback
C=(Kp+(Ki/s));  %% PI controller
Gouter=feedback(C*(Ginner*(1/s)),1); %%% outer loop TF
Margins=allmargin(C*(Ginner*(1/s)))  %%% Frequncy domian Performance
margin(C*(Ginner*(1/s)))  %%% bode plot
```

The output simulation response shall be obtained as follows when the delay margin is set to DM=0.38 for an actuator transfer function.



The system will be unstable if the delay margin is greater than DM=0.38, which is verified by the simulation plot.

6.1 Conclusion

The analysis of the design, development, and implementation of a proficient PID controller for the pitch angle control of Missile output response leads to the conclusion that the system exhibits a resilient, regulated, desired, and efficient response, meeting the maximum design requirements. The optimized values of Proportional, Integral, and Derivative gains successfully achieve the transient and steady state performance parameters of the system output, even in the presence of dead time or delay time within the actuator transfer function dynamics. Finally, the determined delay margin and gain margin values are validated through simulation-based analysis using MATLAB Simulink.