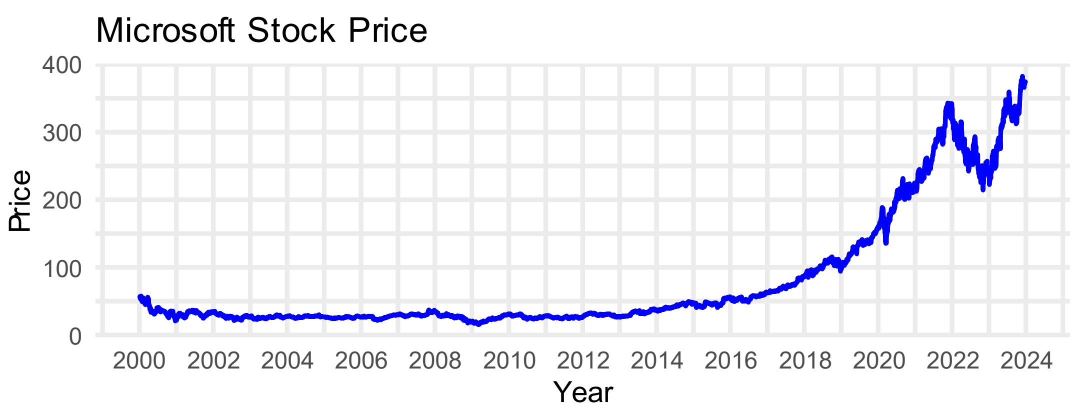
42472 FM321 Project

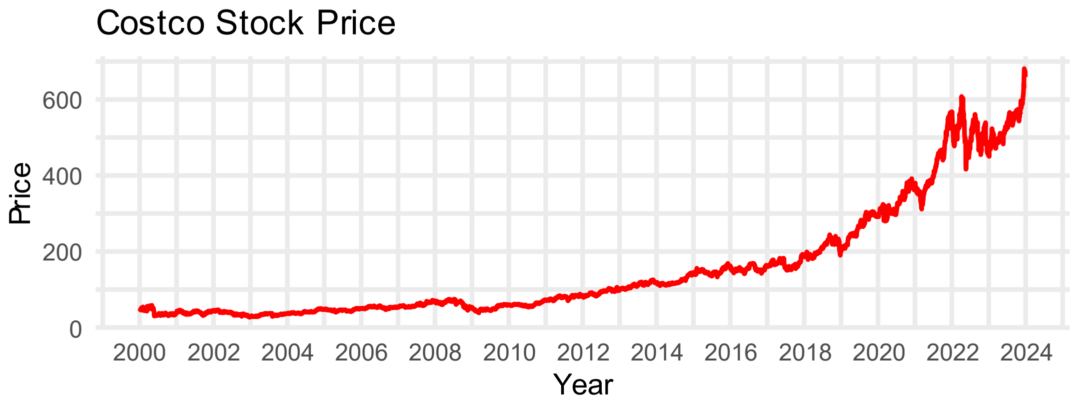
Diversification is essential to portfolio management, aiming to reduce risk by investing across various weakly/negatively correlated assets. Value at risk is not a coherent risk measure as it can violate subadditivity meaning the potential losses on a portfolio may exceed the sum of its individual asset's losses. This violation can lead to misleading conclusions about the benefits of diversification, especially during market stress (periods of non-linear dependence) when asset correlations increase. This project explores how to effectively provide a practical recommendation to project better risk estimates during high volatility/fewer stable periods (reducing risk doesn't necessarily mean reduction in potential losses in monetary terms).

I downloaded stocks data (daily returns, daily prices) from Wharton Research Data Services (WRDS) for two companies traded on the NASDAQ during the period 03/01/2000 to 29/12/2023 to help with my analysis. Microsoft is a technology company that develops and sells software, services, and hardware to individuals and businesses. Costco is a membership only warehouse retail store that offers various products, including groceries, appliances at discounted prices to its members.

Individual Stock Analysis – Sample Period 3/01/2000 to 29/12/2003

Statistic	MSFT	COST
Average Return	0.00	0.00
Std Dev Return	0.02	0.02
Max Return	0.18	0.14
Min Return	-0.17	-0.24
Skewness	-0.15	-0.78
Kurtosis	12.16	17.96
Ljung–Box test statistic	1158.60	484.97
Ljung–Box p-value	0.00	0.00
Jarque–Bera test statistic	21104.31	56906.49
Jarque–Bera p-value	0.00	0.00




Costco Stock Price
Price
Year
0
200
400
600
2000
2002
2004
2006
2008
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2012
2014
2016
2018
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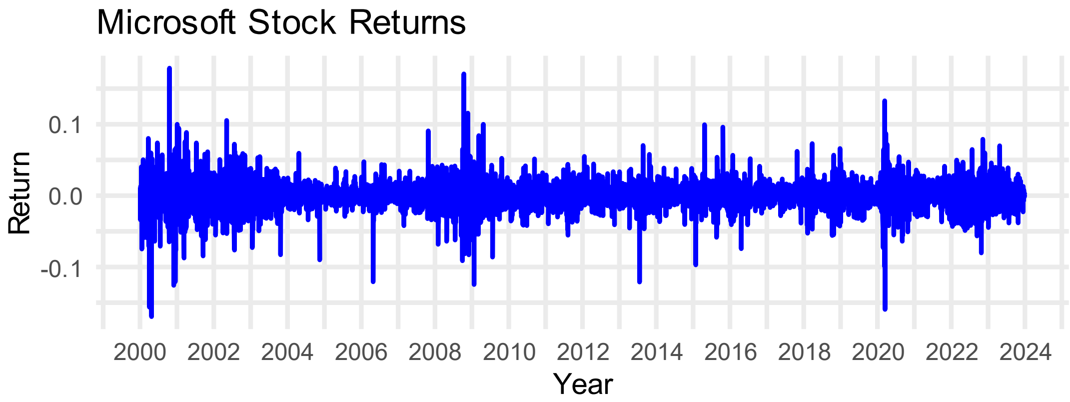
2022
2024


Microsoft Stock Returns
Return
Year
-0.1
0.0
0.1
2000
2002
2004
2006
2008
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2016
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2022
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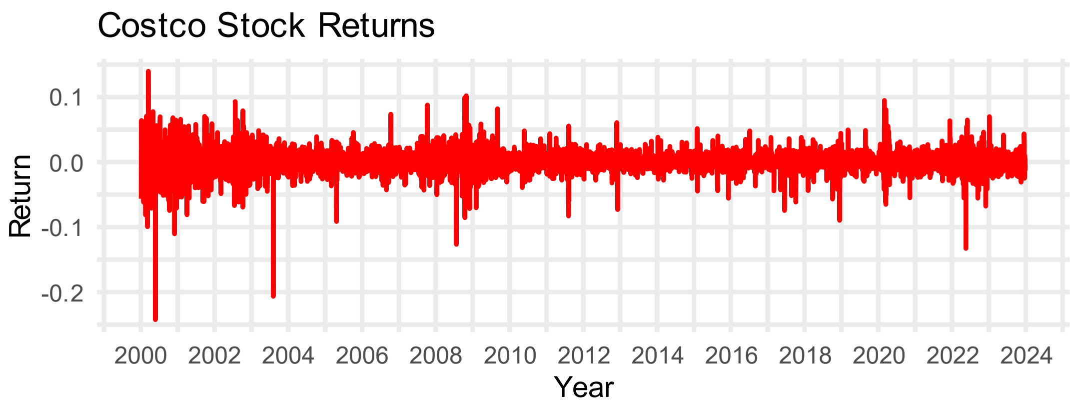
Costco Stock Returns
Return
Year
-0.2
-0.1
0.0
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2000
2002
2004
2006
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2020
2022
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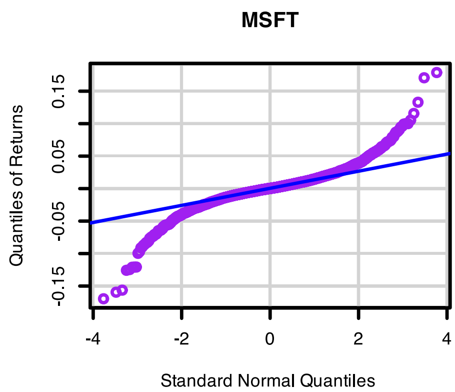
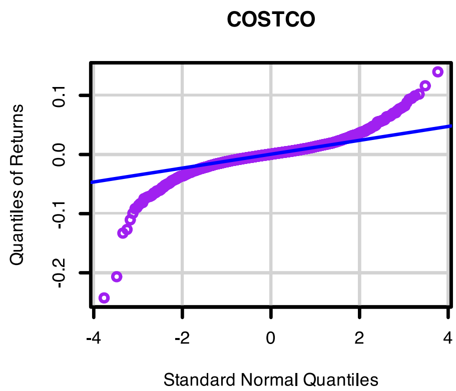
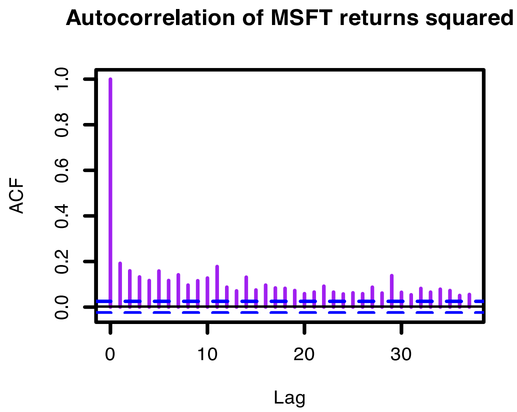
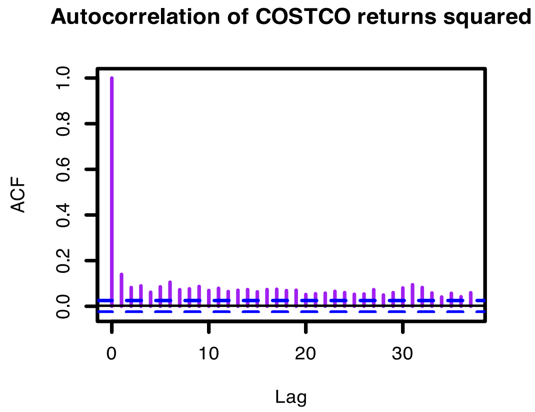

MSFT
Quantiles of Returns
Standard Normal Quantiles
-0.15
-0.05
0.05
0.15
-4
-2
0
2
4


COSTCO
Quantiles of Returns
Standard Normal Quantiles
-0.2
-0.1
0.0
0.1
-4
-2
0
2
4





Microsoft's and Costco's returns have a weak to moderate positive correlation (0.41), likely driven by their shared exposure to consumer spending trends, but their returns differ due to focus on different industries.

Technology Bubble 2000:

Microsoft recorded its lowest return of -17% on 24/04/2000 during the dot com bubble burst, a period following the tech boom when investors realised their predictions about the rapid growth of technology companies were too optimistic. Although being a profitable and well recognised company, Microsoft's stock price fell as investors panicked and revaluated the worth of tech firms. Many investors wrongly linked Microsoft with unprofitable internet startups, assuming all tech companies faced the same risks. This widespread selling, driven by fear and corrections of earlier predictions, caused a sharp decline in the entire sector's value. Costco recorded its lowest return of -24% on 24/05/2000, as the dot com bubble burst created widespread market panic and economic uncertainty. Its stock was affected by a general selling of equities and fears of reduced consumer spending during future period.

Autocorrelation:

Markets are weak-form efficient (average return 0%), thus meaning that prices fully reflect all available information (asset prices adjust instantly as soon as new information arrives). We cannot always predict future prices or make profits by just analysing past prices trends to consistently beat the market.

In the ACF graphs, autocorrelation of returns squared is analysed up to the 37th lag, with the light blue dashed boundary representing the 5% significance level. The 0th lag has an expected ACF of 1. Several lags with autocorrelations exceeding this boundary indicate constant volatility clusters in squared returns for both stocks. For Microsoft squared returns the ACF plot displays a wavy pattern, with strong autocorrelation at early lags that gradually diminishes, followed by periodic increases. This behaviour suggests persistent but decaying volatility with cyclical patterns. We observe a similar pattern for Costco as well but with a lower magnitude. The Ljung-Box test results in test statistics of 1158.60 for Microsoft's squared returns and 484.97 for Costco's squared returns over 10 lags, which follow a chi-squared distribution with 10 degrees of freedom. These statistics are far above the critical value of 18.31, leading us to reject the null hypothesis of no autocorrelation for both stocks' squared returns. This suggests strong, predictable volatility clustering, meaning high-volatility days are often followed by additional high-volatility periods, and vice versa.

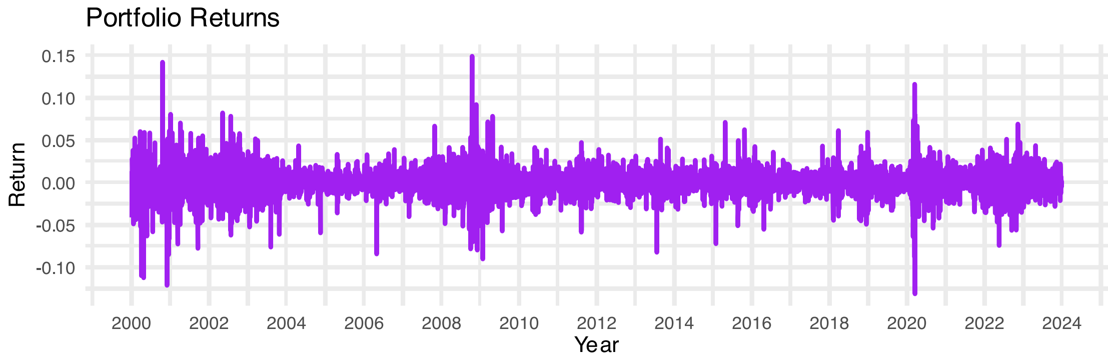
Predictability in returns squared does not violate market efficiency, as it doesn't tell us about the direction of price movements. Even if traders may be able to predict risk, they cannot systematically use that information to predict profits after accounting for transaction cost and prediction outcome.

Normality:

The Q-Q plots compare the quantiles of stock returns with those of a standard normal distribution. The blue line represents where the returns quantiles would fall if they followed exactly a standard normal distribution. Significant deviations of Microsoft's and Costco's returns from this line indicate that their distributions do not follow a standard normal pattern. This means that the returns distributions have heavier/fat tails, implying a higher probability of extreme outcomes compared to a standard normal distribution. Both return distributions are slightly negatively skewed increasing the likelihood of negative returns. Any normal distribution regardless of mean or variance has a kurtosis of 3, but Microsoft and Costco returns Kurtosis > 3. The Jarque-Bera test yields a test statistic based on sample size, skewness, kurtosis of 21104.31 for Microsoft's returns and 56906.49 for Costco's returns. These test statistics follow chi-squared distribution with 2 degrees of freedom. Comparing it to the critical value of 5.99 for a 5% significance test, we find them to be high. As a result, we reject the null hypothesis of returns following a normal distribution. Instead, we found that Microsoft's and Costco's returns are much closely related to a t – distribution with 3 degrees of freedom.

Portfolio Analysis

I formulated a portfolio with 70% in Microsoft and 30% in Costco to achieve an optimal balance between growth and stability. The heavier allocation to Microsoft leverages its strong growth potential and higher risk profile, while the inclusion of Costco provides a stabilising effect with its consistent performance in retail.



Portfolio	Avg Return	Std Ret	Max Return	Min Return
Statistic	0.00	0.016	0.15 (13 Oct 2008)	-0.13 (16 Mar 2020)

13 Oct 2008 - global efforts like the U.S. $2 trillion bank rescue plan to stabilise markets during the Great Financial Crisis.

16 Mar 2020 - panic selling after the US Federal Reserve's emergency rate cut to near-zero failed to ease fears over the escalating COVID-19 pandemic and ongoing lockdowns.

Value at Risk (VaR)

The loss on a trading portfolio such that there is a probability ρ of losses equalling or exceeding VaR in a given trading period and a (1− ρ) probability of losses being lower than the VaR.

$q \rightarrow \textit{Profit and Loss} \qquad q_t = \vartheta_t y_t = \vartheta \frac{p_t - p_{t-1}}{p_{t-1}}$

$\mathbb{P}[q \leq -VaR(\rho)] = \rho \quad (VaR(p) \rightarrow \textit{Quantile on Profit/Loss distribution})$

VaR assumptions:

- Portfolio Value
- Probability (1% or 5%)
- Holding Period (a day)
- Returns follow a conditional distribution (normal or student – t)

VaR calculations:

Non-Parametric Models (No distribution of data assumed, no estimation, and hence no distributional parameters)

- Historical Simulation calculates VaR by identifying the negative of the (WE × p) th smallest return from a rolling window of past WE days of returns, scaling it by the monetary value of the portfolio, and repeating this process for each day in the dataset starting from WE + 1.

Parametric Models (Assume a distribution and estimate its parameters)

Single Asset ⟶ $\boldsymbol{VaR_{t+1} = -\sigma_{t+1}\Phi^{-1}(\rho)\vartheta_t}$

- Univariate Volatility Modelling ⟶ Conditional Volatility σ_t

  $\boldsymbol{EWMA: \sigma_{t+1}^2 = (1-\lambda)y_t^2 + \lambda\sigma_t^2}$

  $\boldsymbol{GARCH: \sigma_{t+1}^2 = \omega + \alpha y_t^2 + \beta\sigma_t^2}$

  $\textit{Returns follow a conditional normal distribution with mean } 0 \textit{ and time varying variance - } y_t \sim N(0, \sigma_t^2)$

  $y_t = \sigma_t \epsilon_t \qquad \epsilon_t \sim N(0,1) \textit{ Shock Term}$

  t-GARCH: $\textit{Returns follow a conditional student t distribution}$

  $\boldsymbol{VaR_{t+1} = -\sigma_{t+1}F_{(v)}^{-1}(\rho)\vartheta_t\sqrt{\frac{(v-2)}{v}}}$

Portfolio: $\boldsymbol{VaR_{t+1} = -\sigma_{t+1(portfolio)}\Phi^{-1}(\rho)\vartheta_t}$

- $\sigma_{t(portfolio)}^2 = w'\Sigma_t w$

  $\Sigma_t = D_t R_t D_t$

  $\Sigma_t \textit{ Conditional Covariance Matrix estimated by DCC method}$

  Estimate each asset conditional volatilities separately (EWMA/GARCH) then run the DCC equation for updating correlations

  $\boldsymbol{D_t}$ *diagonal matrix where each element is the volatility of each asset*

  $\boldsymbol{R_t} \textit{ Time Varying correlation of residuals/shocks matrix}$

  DCC captures both linear and non-linear dependencies between different stocks

VaR estimates 2019 – 2023 (Covid – 19, high inflation/ high cost of living)

- Quantile ρ = 1%, for the quantile of the profit and loss distribution (worst possible losses), standard practice in risk management.
- λ = 0.94 (univariate and multivariate cases) the decay factor for EWMA prioritises recent data while gradually reducing the impact of older observations.
- We assume portfolio value 1 for simplicity and comparisons.
- Sample period of 2250 observations is selected to ensure sufficient data for robust estimation and validation.
- Rolling estimation window of 1000 observations is used to compute VaR based on the most recent data. This approach ensures adaptability to changing market conditions while maintaining enough historical context for stable estimates.
- VaR is estimated over a testing window of 1250 observations, corresponding to the years 2019–2023. This period allows for evaluation under market stress conditions (Covid – 19, Inflation rise/Cost of Living)
- A burn-in period of 30 days is included to allow the models, particularly those with iterative components like GARCH, to stabilise their parameter estimates.

Single Asset VaR:

- rugarch library is employed for univariate volatility modelling**.**
- GARCH and t-GARCH models parameters (ω, α, β, ν) are estimated using the ugarchfit function, which applies maximum likelihood estimation (MLE) to iteratively optimise parameters based on the log-likelihood of observed returns over time.
- Models dynamically adjust to changes in market volatility.

Portfolio VaR:

- rmgarch and rugarch libraries are employed for multivariate volatility modelling, focusing on Dynamic Conditional Correlation (DCC).
- dccfit function estimates conditional covariance matrices by optimising parameters through maximum likelihood estimation.
- DCC – (GARCH/EMWA) models capture time-varying correlations between assets.

VaR Graphical Analysis:

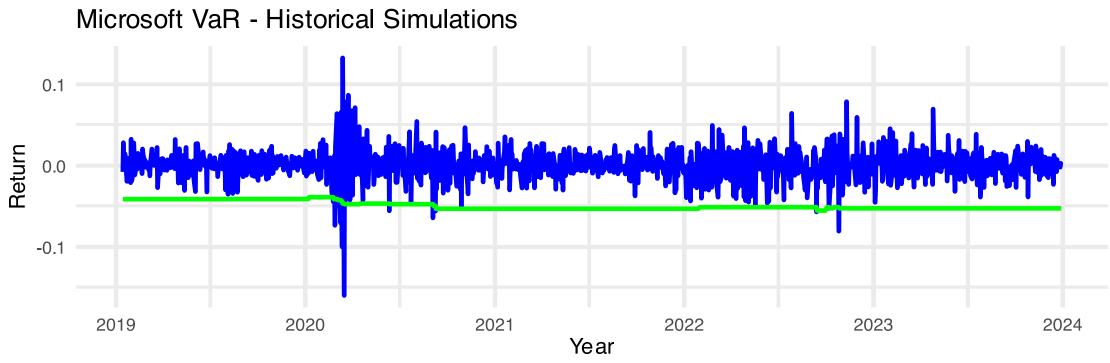
- Visualisations of VaR estimates, VaR violations and violations clustering, how strongly do statistical test back/ agree with the graphical analysis.

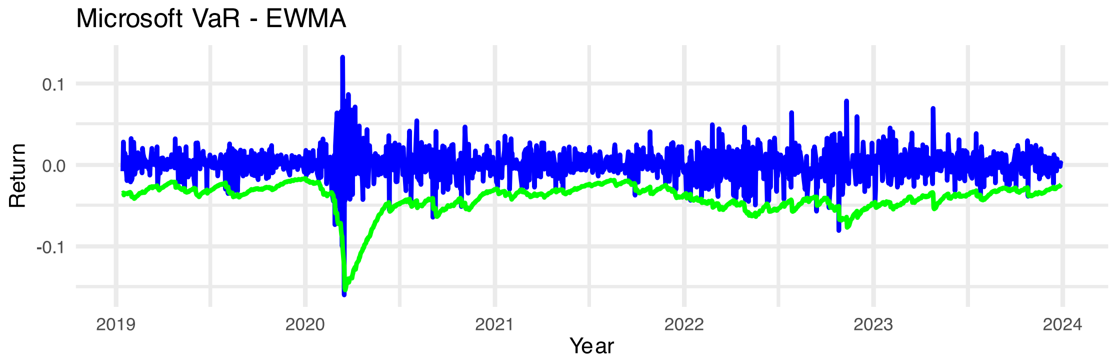
Statistical Test:

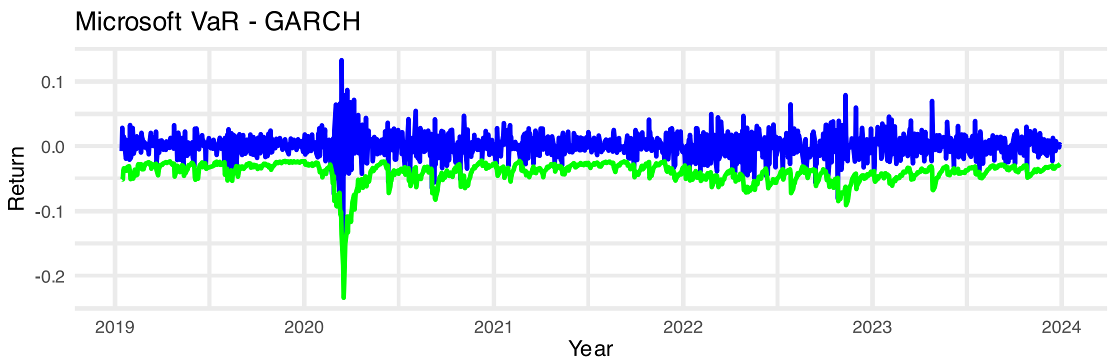
- Bernoulli Coverage Test: Test for VaR violations distribution, $H_0 : \eta_t \sim B(\rho)$

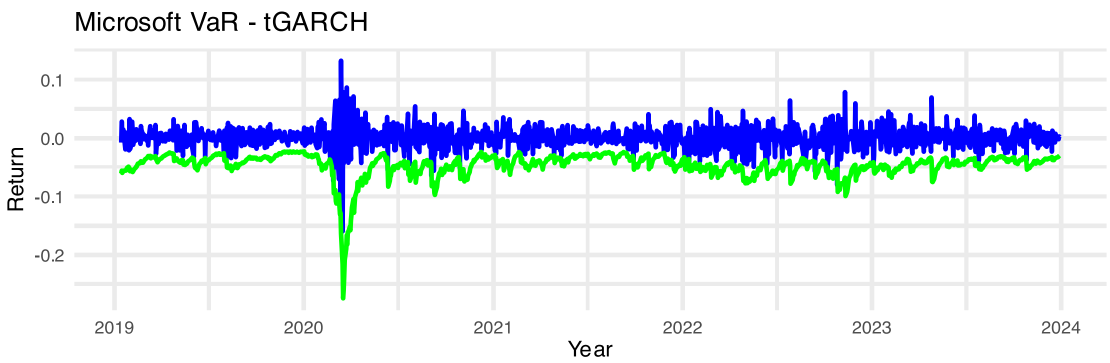
History:

- Swiss Franc appreciation in 2015, where the currency jumped 30% in a day, and the COVID-19 market crash in March 2020, are extremely rare events within markets.
- Relying on past data can underestimate such extreme risks, leaving portfolios exposed to unexpected major shocks.

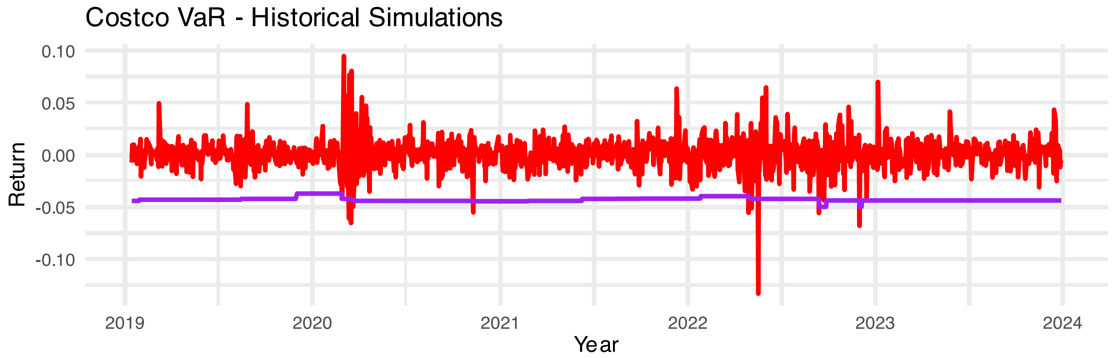


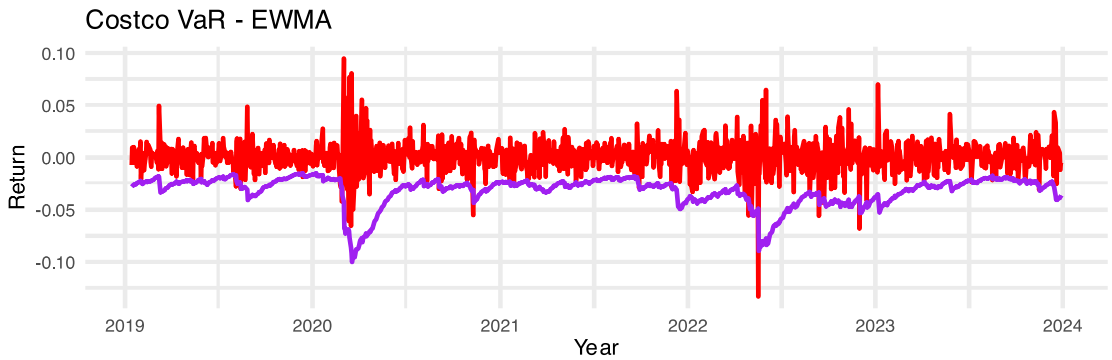


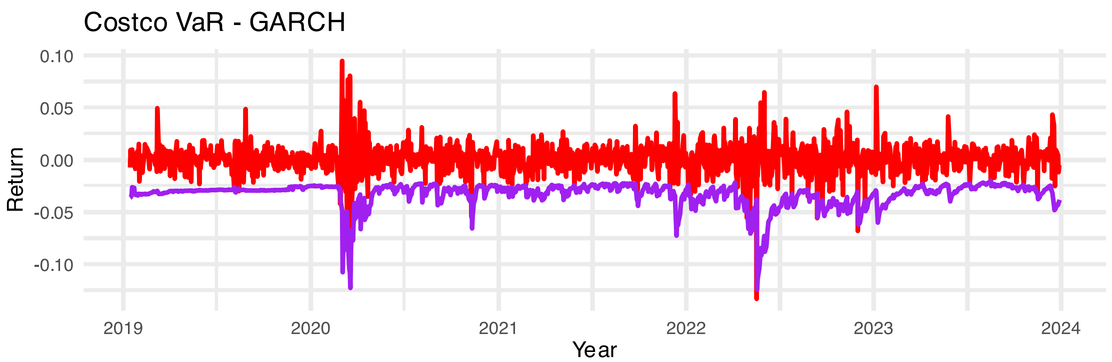


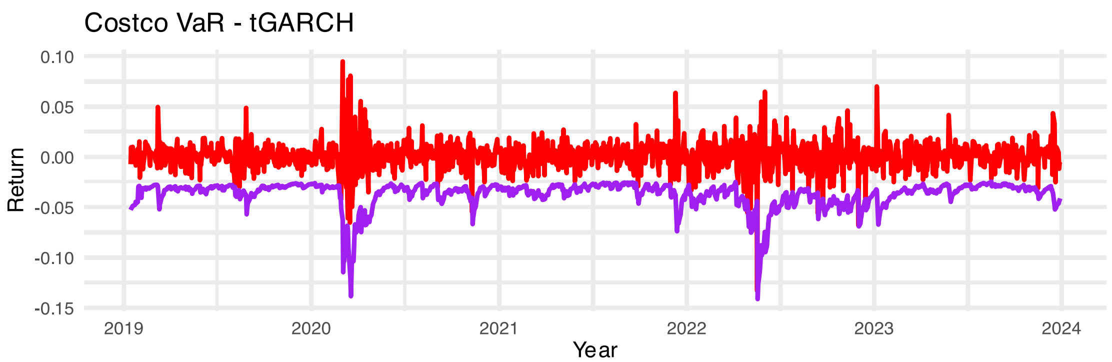


tGARCH and GARCH VaR models show minimal VaR violations, effectively capturing volatility spikes and tail risks during the highly uncertain periods of early 2020 (COVID-19 pandemic) and late 2022. These periods reflect the global pandemic's rapid influence, which disrupted supply chains and tech demand, and late 2022, when rising interest rates and inflation fears led to a valuation reset in the tech sector, particularly affecting Microsoft. Historical simulation model exhibit violation clustering and can significantly underestimate risk during these times due to their static nature, failing to adapt to the sharp volatility shifts caused by declining PC sales, Azure growth deceleration due to competition from AWS and google cloud. The EWMA VaR model performance seems better but still struggled to fully capture very extreme tail events during these crises.





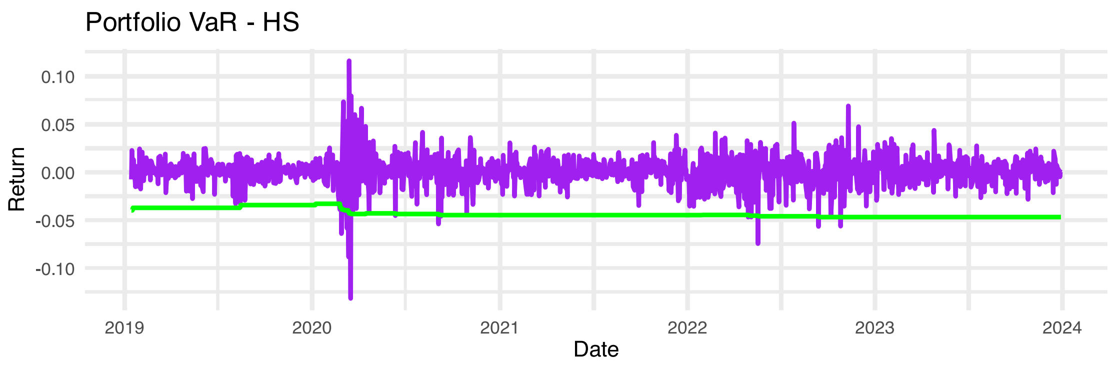


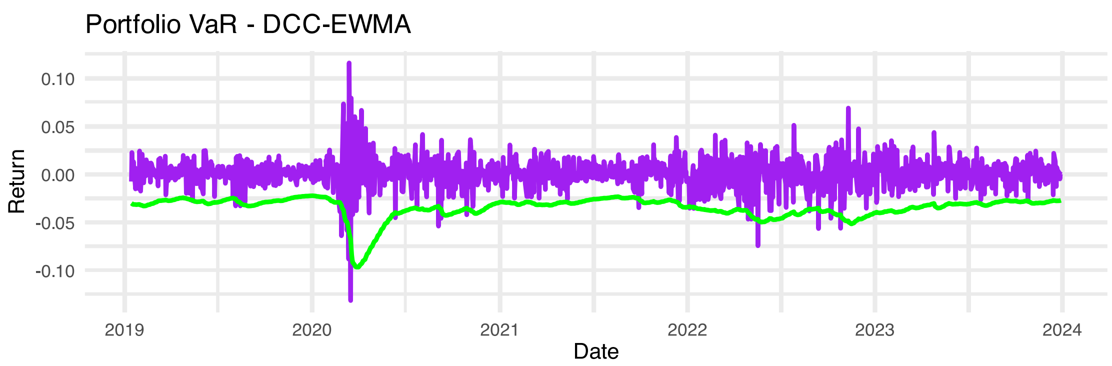


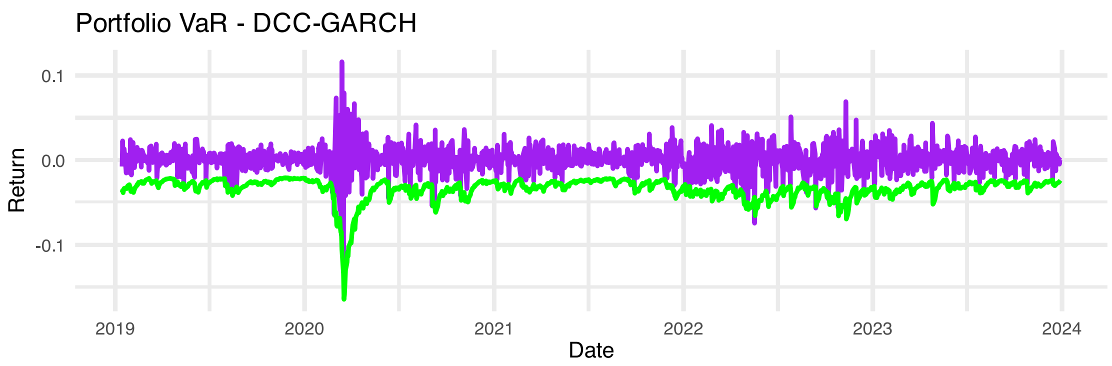
Historical simulation (HS) model showed small VaR violations and clustering a during COVID-19 but massive ones towards mid 2022 as inflation and cost of living intensified, followed by other minor violations at the year's end due to consumer spending concerns. EWMA VaR recorded moderate violations in the first half and late 2022, reflecting limited responsiveness to market shifts. t-GARCH and GARCH VaR adapted to Costco's volatility dynamics during crises, demonstrating decent risk estimation due to minimal violations. This highlights the importance of dynamic models for businesses impacted by macroeconomic volatility.

Comparison:

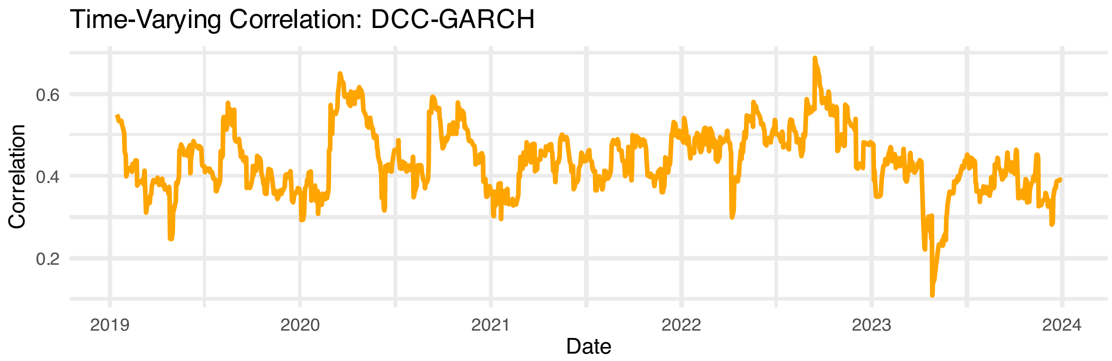
While Microsoft's 2020 violations reflected the tech sector's sensitivity to sudden global disruptions, Costco's 2022 violations were tied to macroeconomic inflationary challenges and shifts in retail spending, highlighting sector-specific drivers during these distinct periods.







DCC-GARCH model demonstrates consistent performance in the portfolio, visualising minimal VaR violations even during periods of extreme market turbulence accounting for non-linear dependence. DCC-EMWA model due to its fixed lambda parameter, has limited ability to fully adapt to rapidly changing volatility patterns. Historical Simulations exhibits some violations clustering due to relying on history and past trends.



2020 (Start of COVID-19): Correlation peaked positively during the onset of the pandemic as both Microsoft and Costco reacted to unprecedented market conditions. Panic-buying boosted Costco's revenue, while Microsoft saw increased demand for remote work solutions like Teams and Azure. However, despite this alignment, overall returns were volatile and often negative during the broader market sell-off triggered by pandemic uncertainty, reflecting synchronised downside risks during the initial shock.

2022 (3rd/4th Quarter): Correlation peaked very strongly positive as global markets faced high volatility driven by inflation, cost of living rises, and slow growth. Both stocks moved in tandem as investors reacted to shared macroeconomic pressures.

Mid-2023 (Lowest Correlation): Correlation dropped significantly as sector-specific factors dominated. Costco's stock stabilised with the normalisation of consumer spending and easing inflation, while Microsoft's returns were increasingly influenced by growth in AI technologies and strong competition.

Bernoulli Coverage Test:

$$Violations\ \eta_t = \begin{cases} 1, & y_t < -VaR_t \\ 0, & otherwise \end{cases} \qquad H_0 : \eta_t \sim B(\rho)$$

$$LR = 2(log\mathcal{L}_U(\rho\ hat) - log\mathcal{L}_R(\rho)) \qquad p\ hat = \frac{\#Violations}{Testing\ Window\ Size\ (W_T(days))}$$

$$LR \sim \chi^2_{(1)}$$

VaR Model	W_T	$\#Violations$	$\#No-Violations$	$p\ hat$	LR
MSFT HS	1250	13	1237	0.0104	0.0199
MSFT EWMA	1250	23	1227	0.0184	7.14
MSFT GARCH	1250	26	1224	0.0208	11.2
MSFT t-GARCH	1250	16	1234	0.0128	0.909
COST HS	1250	10	1240	0.008	0.542
COST EWMA	1250	26	1224	0.0208	11.2
COST GARCH	1250	21	1229	0.0168	4.85
COST t-GARCH	1250	8	1242	0.0064	1.88
Portfolio HS	1250	15	1235	0.012	0.475
Portfolio DCC-EWMA	1250	20	1230	0.016	3.846
Portfolio DCC-GARCH	1250	26	1224	0.0208	11.23

We reject the null hypothesis if the LR is above the critical value 3.84 meaning that the observed number of VaR violations significantly deviates from the expected number of violations based on the model's specified confidence level. This indicates that the VaR model is either underestimating or overestimating the probability of (extreme) losses.

t-GARCH VaR performed well because it incorporates conditional volatility and accounts for the fat tails in conditional return distributions, which are common in financial markets. The t-distribution used in these models better captures extreme movements, leading to more accurate estimation of VaR. This improves the alignment between expected and actual violations, reducing the likelihood ratio and making it less likely to reject the null hypothesis.

Historical simulation performed well because it directly uses historical return data to estimate VaR without relying on parametric assumptions about return distributions. This approach captures real market behaviours, including non-normality and extreme events, as observed in past data. Its simplicity and reliance on actual historical patterns often result in a model that aligns well with observed violations, leading to lower likelihood ratios and reduced model misspecification.

Models like GARCH, EWMA, and DCC rely heavily on assumptions about how volatility evolves over time. These models struggle to quickly adapt to sudden market shocks, leading to underestimation of risk during periods of turbulence. Their reliance on lagged data can make them slow to capture abrupt changes, which is reflected in higher violation rates and rejection of the null hypothesis in the tests.

On average, portfolio models have lower p-hat values compared to individual GARCH and EWMA models, demonstrating how diversification smooths extreme movements. This effect makes portfolios less sensitive to sector-specific shocks, but the models can underestimate/overestimate risks. However, all models are mis specified and only some are useful.

Recommendation:

For robust risk management, a hybrid approach is recommended. Using t-GARCH for high-volatility periods to account for tail risks and time-varying dynamics, supplemented by HS for its simplicity during stable markets. For portfolios, a DCC-t-GARCH model should be employed to capture non-linear dependencies between assets while maintaining accuracy in estimating tail risks. This hybrid method provides flexibility and accuracy across different market conditions, aligning well with both trading and regulatory requirements.

Conclusion:

Diversification remains an essential part of risk management, effectively reducing portfolio volatility through imperfect correlations. Incorporating machine learning techniques, such as neural networks or tree methods using R or python, could further enhance risk estimation by detecting non-linear patterns and improving tail risk predictions. By blending dynamic parametric models, historical approaches, and advanced data-driven methods, portfolio managers can better address risk forecasting.