Lecture10-11

Data Science → LINEAR ALGEBRA → Theoretical

① Vector
② Matrices
③ System of Equation

Vectors → 2D

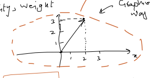
→ magnitude + direction → Velocity, weight

→ $\begin{bmatrix} x \\ y \end{bmatrix}$; (x, y)

→ $\begin{bmatrix} x \\ y \end{bmatrix}$ magnitude ≥ 0, " ≤ 0

Cartesian Coordinates

Graphical way



$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2+5 \\ 3+4 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \end{bmatrix}$; $\begin{bmatrix} 2-5 \\ 3-4 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 2 \times 5 \\ 3 \times 4 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \end{bmatrix}$

Scaling

$3 \times \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$

$\begin{bmatrix} 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 6 \end{bmatrix} = (3 \times 5) + (4 \times 6)$

$\Rightarrow 15 + 24$

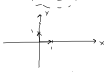
$\Rightarrow 39$ ✓

scalar

unit Vector

$\hat{i} = \hat{X} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \hat{Y} = \hat{j}$

hat



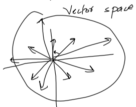
* Any vector can be created by unit Vectors.

$A = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

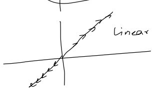
$= 3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 4 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

Vector space



Linear



Cross product

$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} (5 \times 6) - (6 \times 6) \\ (6 \times 5) - (4 \times 7) \\ (4 \times 6) - (5 \times 5) \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$\begin{bmatrix} 5 \\ 6 \end{bmatrix} \times \begin{bmatrix} 6 \\ 7 \end{bmatrix}$

$\begin{bmatrix} 4 \\ 6 \end{bmatrix} \times \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$\begin{bmatrix} 4 \\ 5 \end{bmatrix} \times \begin{bmatrix} 5 \\ 6 \end{bmatrix}$

Matrices → image dataset → conv2D

A + B

element of matrix

transformed vector

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

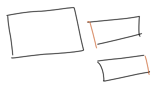
$= \begin{bmatrix} 1+3 & 0+4 \\ 0+5 & 1+6 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 5 & 7 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

$= \begin{bmatrix} 1-3 & 0-4 \\ 0-5 & 1-6 \end{bmatrix} = \begin{bmatrix} -2 & -4 \\ -5 & -5 \end{bmatrix}$

cross product | Dot product

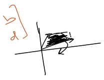
① Addition
② Subtraction
③ dot product → scalar quantity
④ cross product → vectors/Matrices
⑤ determinant
⑥ Inverse



$\Rightarrow -0.5$

Determinant

$A = \begin{bmatrix} 5 & 7 \\ 6 & 10 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$



$|A| \Rightarrow (a \times d) - (c \times b) = \square$

$\Rightarrow (5 \times 10) - (7 \times 6) = 50 - 42 \Rightarrow 8 \rightarrow$

Inverse

A^{-1}, $AA^{-1} \Rightarrow I$

2x2 3x3

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix} = (4 \times 6) - (7 \times 2) = 24 - 14 \Rightarrow 10$

step ① → Determinant

step ② → swap the element in diagonal & off diagonal

step ③ → swap the signs off diagonals

step ④ → Divide the result of ③ by det.

$\begin{bmatrix} 6 & -2 \\ -7 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{10} \begin{bmatrix} 6 & -2 \\ -7 & 4 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.2 \\ -0.7 & 0.4 \end{bmatrix}$

System of Equation

equation → $2x+5y=3$ → multiple

→ Graphical solution
→ Substitution
→ Elimination

$\left.\begin{array}{l} 2x+5y=3 \\ 3x+8y=4 \end{array}\right\}$ → solve → X ? Y ? $X=a$, $Y=b$

$2(a)+5(b) = 3$
$3(a)+8(b) = 4$ → Solution of my System of Eqs

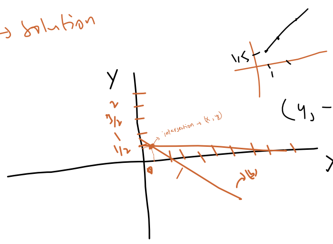
① Graphical → solution

(a) → $2x+5y=3$

(b) → $3x+8y=4$

a) $y = \frac{-2x}{5} + \frac{3}{5}$

b) $y = \frac{-3x}{8} + \frac{4}{8}$



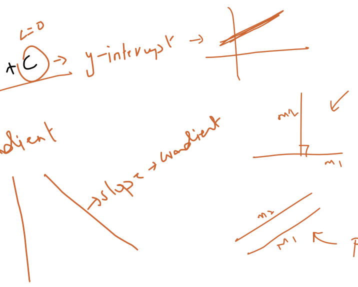
$(4, -1)$

$y = mx + c$ → y-intercept →

$c=0$

Gradient

slope → Gradient



perpendicular

$m_1 \times m_2 = -1$

$m_1 = \frac{-1}{m_2}$

$m_1 = m_2$

parallel

(a)

X	1	3	5	7	9
Y	1/5				

(b)

X	1	3	5	7	9
X	1/8				

② Substitution

$y = \frac{-2x}{5} + \frac{3}{5}$

$y = \frac{-3x}{8} + \frac{4}{8}$

$\frac{-3x}{8} + \frac{4}{8} = \frac{-2x}{5} + \frac{3}{5}$

$\frac{-3x}{8} + \frac{2x}{5} = \frac{3}{5} - \frac{4}{8}$

$3x - 20$

Y, X → Subject

$2x+5y = 3$

$2x = 3 - 5x$

$\boxed{x = \frac{3-5y}{2}}$

$\boxed{x = 4}$

③ Elimination

$2x+5y=3$

$3x+8y=4$ +

$-3(2x+5y=3)$

$2(3x+8y=4)$

$-\cancel{6x} - 15y = -9$

$\cancel{6x} + 16y = 8$

$y = -1$

$2x + 5(-1) = 3$

$2x - 5 = 3$

$2x = 3+5$

$$\frac{-15x+16x}{\cancel{40}} = \frac{\ldots}{\cancel{40}}$$

$$\boxed{y=-1}$$

$$y = -\frac{2}{5}(4) + \frac{3}{5}$$

$$y = \frac{-8+3}{5}$$

$$y = -\frac{\cancel{5}}{\cancel{5}}$$

$$y=-1$$

$(4, -1)$

$$x = \frac{8}{2}$$

$$\boxed{x=4}$$