EXAM 2 (MODULE 3): MONTE CARLO SCHEME TO PRICE EXOTIC OPTIONS

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Outline of the finance problem and numerical procedure used

Overview of Exotic Options

European options are simpler compared to options such as Asian and look-back options since the payout depends on certain characteristics of the stock price at some time during the maturity of the option. The Asian option is different from other options because its payoff depends not on the price of the underlying asset at expiration but on the average price of the asset during a specific time interval (Glória *et al*. 2024). In this way, the averaging stabilises and effectively offers Asian options for markets with fluctuating assets, which is common in Asia.

The Lookback option in contrast offers the holder the advantage of looking back by enabling the payoff to be determined based on the minimum or maximum value of the underlying asset during the entire period of the existence of the option. This feature allows the holder to 'turn back time' and set the strike or exercise price, which makes it very useful in situations with high volatility.

Monte Carlo Simulation for Option Pricing

Monte Carlo methods generate a large number of possible future prices for the underlying asset. This implements the process of calculating average payoffs across those simulations. They are both calculated using the risk-neutral measure under which all investors are assumed to be risk neutral and therefore all cash flows are discounted at the risk-free rate (Gatta *et al*. 2023).

$$V(S,t) = e{-}r^{(T-t)}E_Q[Payoff(S_T)]$$

The payoff for exotic options such as Asian and lookback options depend not only on the final price but on the behaviour of the asset over the entire life of the option. This path dependency can be modelled using Monte Carlo simulations because it helps generate price trajectories over time (Lu, 2024). The expected value of the discounted payoff for each trajectory is then computed, and when these are averaged over a large number of simulations, yield the price of the option.

Euler-Maruyama Scheme

The Euler-Maruyama scheme is a numerical technique to approximate the solutions of the SDE (Stochastic Differential Equation) that models the existent dynamics of stock prices in the financial markets.

$$dS_t - \mu S_t dt + \sigma S_t dWt$$

This is related to analysing options, stock prices are often described by a Geometric Brownian Motion (GBM) an SDE possessing both a deterministic covariate and a stochastic variance (Azzone and Baviera, 2023). This is due to the infeasibility of exact analytical solutions for

SDEs, numerical simulations such as Euler-Maruyama serve as a good means of estimating paths of stock prices.

Problem Setup

The problem focuses on determining the price of the exotic options namely Asian and lookback options using a Monte Carlo simulation applying the Euler-Maruyama scheme for stochastic simulation of the stock prices (Liu, 2024). The initial conditions for the problem are as follows: Currently, the stock price ***"S_0"*** is at $100, the ***"exercise price (E)"*** is also at $100, time remaining until expiry ***"(T-t)"*** is 1 year, the volatility σ is 20% and the "***risk-free rate of interest (r)"*** is at 5%. Some are conventional with many other financial pricing issues while others represent actual market conditions (Yu, 2024).

The changing of the parameters means that the price option is also affected. This is related to the process of raising the degree of volatility that usually raises the price of the option because the asset backing the option is likely to make a big move, improving the option's value. An increase in the interest rate reduces the present value of the future payoff, and this normally results in a lower option price (Wang and Wang, 2024). The time to expiry has an impact on the time available through which the price of the asset has to move in favour of the option holder and a longer time implies a better chance of a favourable payoff.

Results - appropriate tables and comparisons

Option Type	**Price**
Asian Option Price	5.80
Lookback Option Price	18.41

Table 1: Payoff Calculation

(Source: Derived from Jupyter Notebook)

The table that is related to increasing volatility, the price of both the options goes up but the Lookback option price is much more sensitive to increasing volatility than is the case with the standard option. This interprets that the Lookback option gains more from fluctuations in stock prices. The Asian option gains less from increased volatility.

Figure 1: Simulated Stock Price Paths

(Source: Derived from Jupyter Notebook)

This graph shows that the density of the lines close to t=0 is less as compared to the later stage 't=250' because as time progresses, the influence of random fluctuation accumulates thereby increasing the long-term unpredictable nature of a straight line. These simulated stock price paths are useful in calculating the expected payoff of Asian and Lookback options. Monte Carlo can estimate the value of the option since it is possible to simulate many future stocks price movements and calculate the option payoff for each.

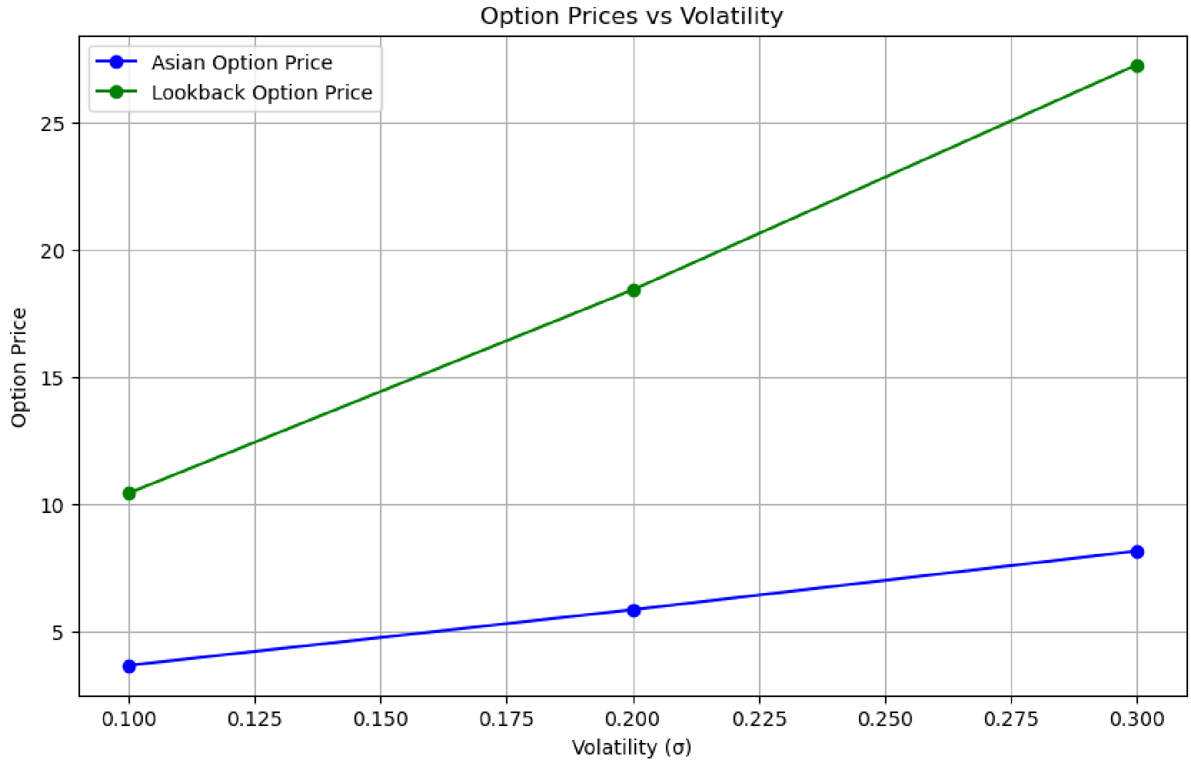

Figure 2: Option Prices vs Volatility

(Source: Derived from Jupyter Notebook)

The graph shows the option prices (of both Asian and Lookback options) to volatilities. The price of both the Asian and Lookback option proves to be directly proportional to the volatility that is defined by (σ). This interprets the future stock price as higher with higher volatility and this increases the value of options. The green line corresponding to Lookback options scales higher, which means that these options are more responsive to changes in volatility. As depicted in the blue line, Asian options require lower levels of volatility compared to standard options.

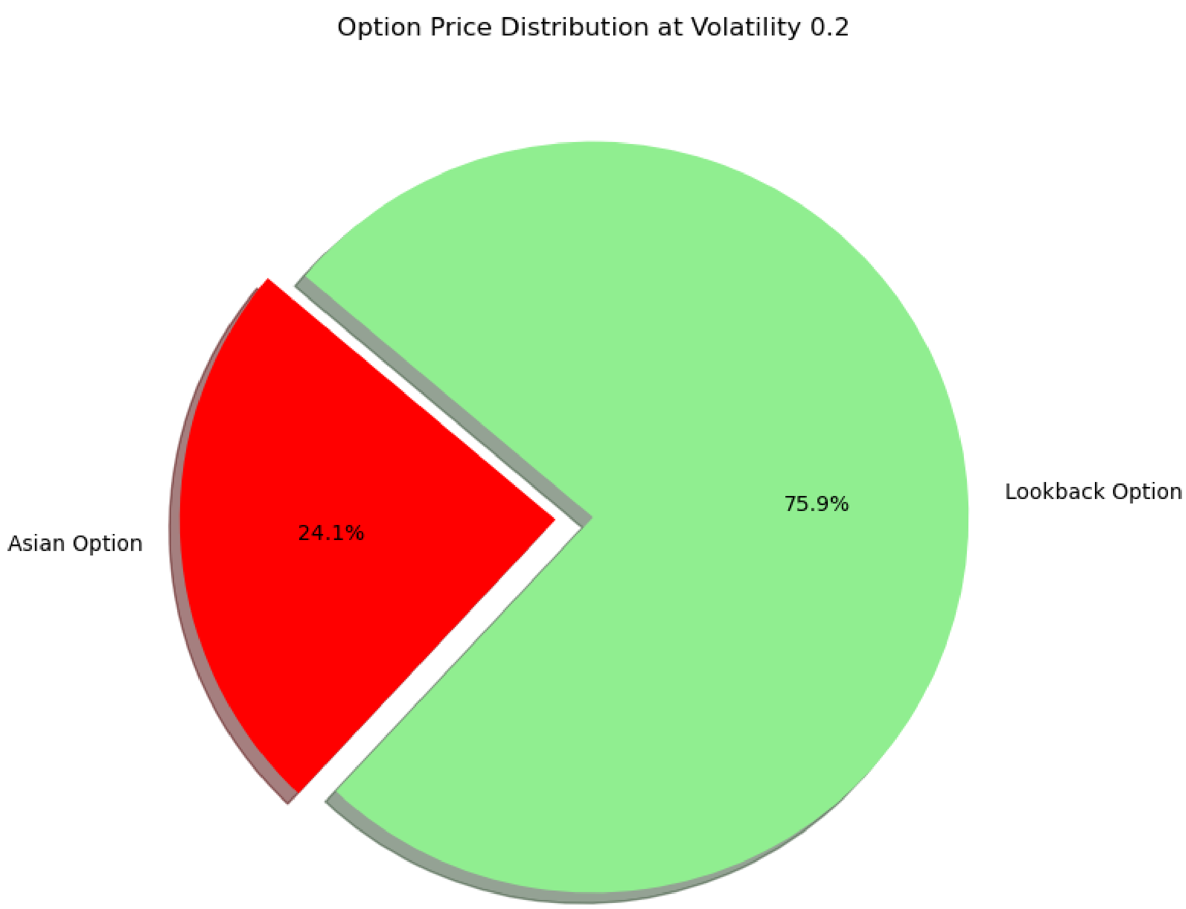

Figure 3: Option Price Distribution vs Volatility at 0.2

(Source: Derived from Jupyter Notebook)

The figure interprets that Lookback options are more costly as compared to Asian options at similar volatility levels. This is because Lookback options are more sensitive to the stock price as compared to the standard European call or put option. The Lookback option allows the holder to "look back" and choose the most beneficial price and hence, the value of this option rises in volatility.

Observations and Challenges

Observations

Lookback options are affected by Volatility in the same manner as Asian options but are more sensitive to it since Volatility leads to high stock prices or low stock prices, thus resulting in higher expected payoffs. Asian options on the other hand use the average of stock prices over a period of time thus minimising the short-term volatility (Tong, 2024). Interest rates also affect the options price; higher rates normally tend to raise the call options' prices since the cost of obtaining the shares is high if one is delayed.

Challenges

Monte Carlo simulations present issues in the option pricing models. This is a problem especially when the options are complex such as Asian or Lookback, which may require several simulations across different time steps, thus slowly converging. Sources of variation in simulation are usually a determinant of precision; methods that lessen variance, such as antithetic or control variates, and improve efficiency.

Conclusion

Monte Carlo simulations for Asian and Lookback options applied by the Euler-Maruyama scheme show the important peculiarities of pricing differences. Due to their nature of being more volatile due to the dependence of lookback options on extreme values of the underlying stock price Asian options are less sensitive to price changes than lookback options. The analysis illustrates how the Euler-Maruyama method can be applied and also illustrates the need to conduct as many experiments and tune parameters to provide optimal and efficient exotic options in pricing.

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