ROB310 - Problem Set 10

Problem 10.1

r.v.s : x, $x \in \mathbb{R}$, $x \sim \mathcal{N}(\mu_x, \sigma_x^2)$
y, $y \in \mathbb{R}$, $y \sim \mathcal{N}(\mu_y, \sigma_y^2)$ $\Big\}$ => x & y = ind.
$z = x + y$, $z \sim \mathcal{N}(\mu_z, \sigma_z^2)$

$$\Rightarrow \mu_z = E[z] = E[x+y] = E[x] + E[y] = \mu_x + \mu_y$$

$$\Rightarrow \sigma_z^2 = E[(z-\mu_z)^2] = E[((x-\mu_x)+(y-\mu_y))^2]$$
$$= E[(x-\mu_x)^2] + 2E[(x-\mu_x)(y-\mu_y)] + E[(y-\mu_y)^2]$$
$$= \sigma_x^2 + \underbrace{2(E[x]-\mu_x)(E[y]-\mu_y)}_{\text{b/c } x \,\&\, y \text{ are ind.}} + \sigma_y^2$$

Problem 10.2

-> continuous r.v.s : $\underline{x} \in \mathbb{R}^n$ and $\underline{y} \in \mathbb{R}^n$ w/ joint Gaussian Distribution.

$$\begin{bmatrix} \underline{x} \\ \underline{y} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \underline{\mu}_x \\ \underline{\mu}_y \end{bmatrix}, \begin{bmatrix} \underline{C}_x & \underline{C}_{xy} \\ \underline{C}_{xy}^T & \underline{C}_y \end{bmatrix} \right)$$

-> let $\underline{c} := \begin{bmatrix} \underline{x} \\ \underline{y} \end{bmatrix}$

$\Rightarrow \underline{z} = \underline{x} + \underline{y} = \underline{M} \begin{bmatrix} \underline{x} \\ \underline{y} \end{bmatrix} = \underline{M}\underline{c}$. , $\underline{M} = [\underline{I} \ \underline{I}]$, $\underline{I} \in \mathbb{R}^{n \times n}$ = identity matrix.

$$\Rightarrow \underline{\mu}_z = E[\underline{M}\underline{c}] = \underline{M}E[\underline{c}] = \underline{M}\underline{\mu}_c = \underline{M}\begin{bmatrix} \underline{\mu}_x \\ \underline{\mu}_y \end{bmatrix} = \underline{\mu}_x + \underline{\mu}_y$$

$$\Rightarrow \underline{C}_z = E[(\underline{z}-\underline{\mu}_z)(\underline{z}-\underline{\mu}_z)^T] = E[(\underline{M}\underline{c} - \underline{M}\underline{\mu}_c)(\underline{M}\underline{c} - \underline{M}\underline{\mu}_c)^T]$$
$$= E[\underline{M}(\underline{c}-\underline{\mu}_c)(\underline{c}-\underline{\mu}_c)^T\underline{M}^T] = \underline{M}\,E[(\underline{c}-\underline{\mu}_c)(\underline{c}-\underline{\mu}_c)^T]\underline{M}^T$$
$$= \underline{M}\underline{C}_c\underline{M}^T = \underline{M}\begin{bmatrix} \underline{C}_x & \underline{C}_{xy} \\ \underline{C}_{xy}^T & \underline{C}_y \end{bmatrix}\underline{M}^T = \underline{C}_x + \underline{C}_{xy} + \underline{C}_{xy}^T + \underline{C}_y$$