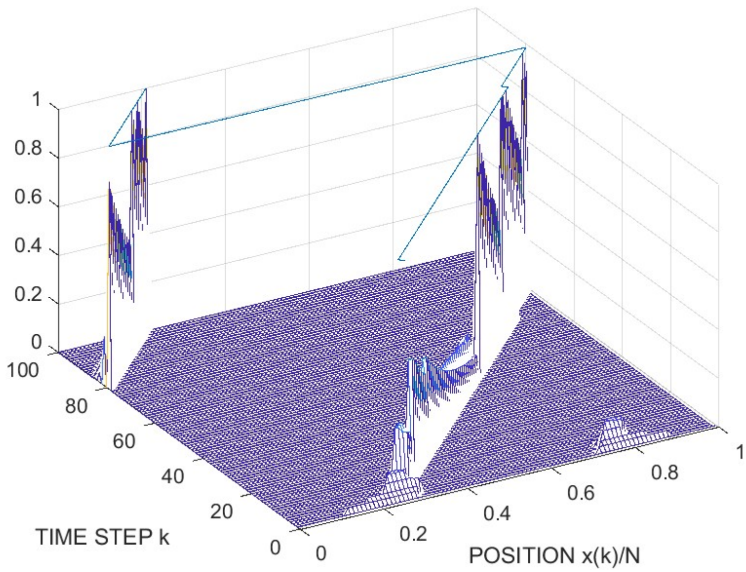
ROB310 Problem Set 9

Problem 9.1

Part (a)

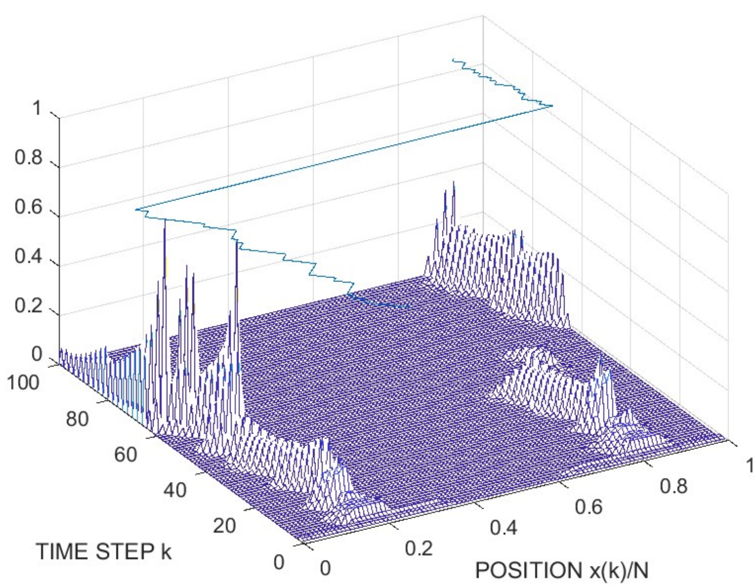
i) $L = 2, p = 0.95$



-> Since p is very high, the robot moves counterclockwise, and very rarely clockwise.

-> Initially, the robot position is unknown, so being at the top or the bottom of the circle is equally likely. However, the PDF quickly converges to the actual position of the robot because the model of the robot motion is with p = 0.95, which is the correct value.

ii) $L = 0.4, p = 0.35$



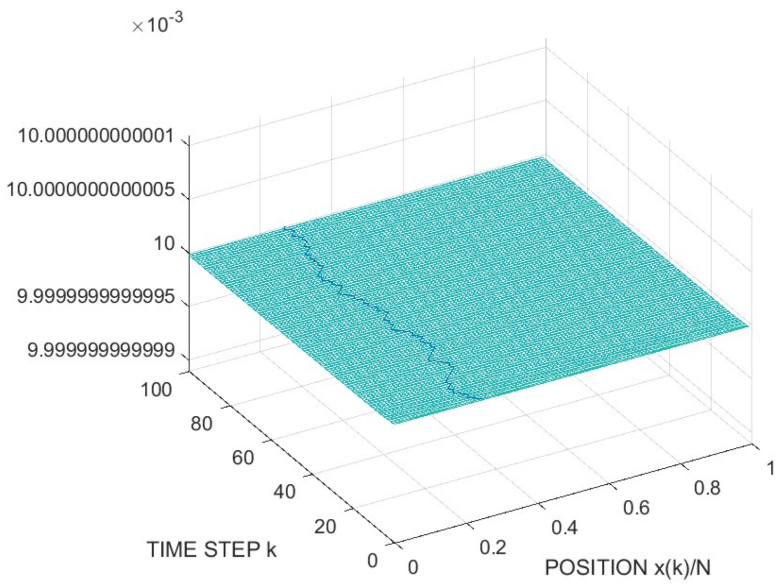
-> The robot is mainly moving in clockwise since p is smaller than 0.5. However, it is kind of moving randomly comparing relatively to part (i) since there is still 35% likelihood that it will move in a counterclockwise direction.

-> Initially, the robot could have been at the top or bottom of the circle similar to part (i), but it does not converge as fast, and there is still uncertainty with the robot's position.

-> The measurement sensor is within the circle

-> The largest peaks of the PDF are consistent with the robot's position.

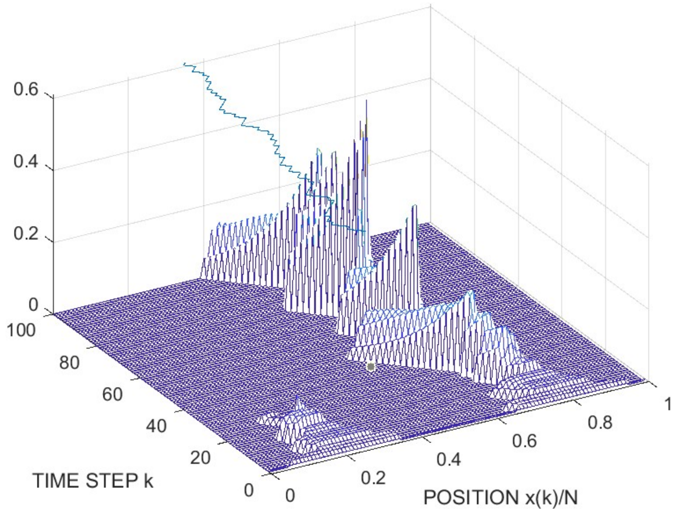
iii) $L = 0, p = 0.5$



-> The measurement sensor is in the centre of the circle, so the distance between the robot and the sensor is always the same. Therefore, the measurement would be constant, except for the noise. As a result, with the initial condition, the PDF is uniform, and all positions of the robot are equally likely.

Part (b)

i) $\hat{p} = 0.1, \hat{e} = 0.8$



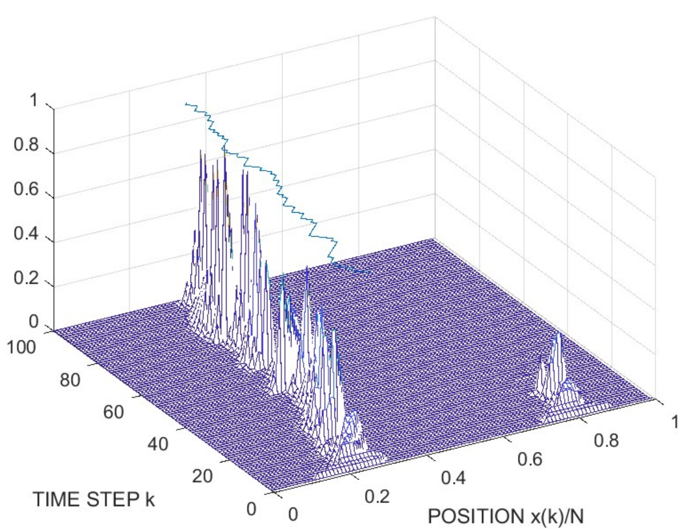
-> Since p = 0.55, which is around 0.5, the robot stays relatively in the same position. However, it will slowly move counterclockwise since p is still slightly bigger than 0.5.

-> The PDF is initially a bimodal distribution because the initial conditions are unknown

-> The PDF converges to the wrong peak around $\frac{3N}{4}$ since our $\hat{p}$ is smaller than 0.5 while p is larger than 0.5.

-> The PDF has wide peaks since our $\hat{e}$ is quite high.

ii) $\hat{p} = 0.85, \hat{e} = 0.5$



-> Since p and $\hat{p}$ are now both higher than 0.5, the PDF converges to the correct robot position. However, since the $\hat{p}$ is larger than p, the PDF peak around $\frac{3N}{4}$ quickly vanishes.

-> The PDF peaks are narrower since e and our $\hat{e}$ are the same.

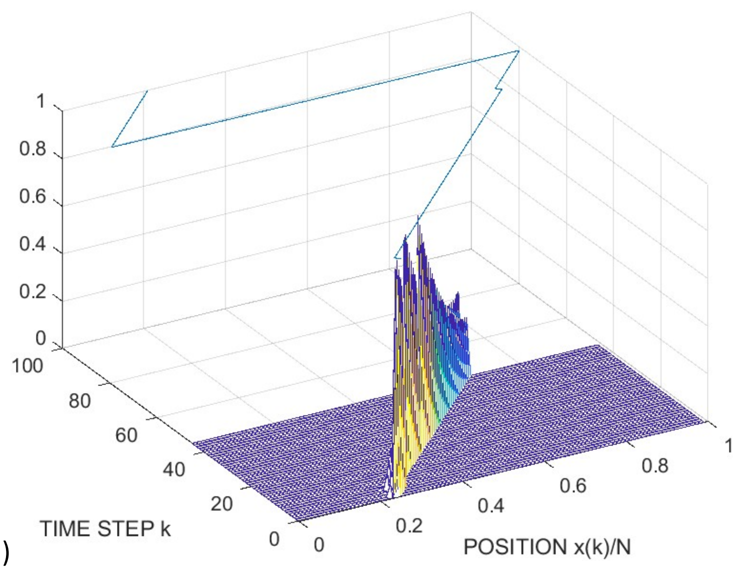
Part (c)

<u>A) The initial position of the robot is perfectly known and is $\frac{N}{4}$</u>

-> Since the initial position of the robot is perfectly known to be the upper half of the centre, the PDF starts with a unimodal distribution with a large peak at $x = \frac{N}{4}$ and no initial peak at $x = \frac{3N}{4}$

For Part (a):

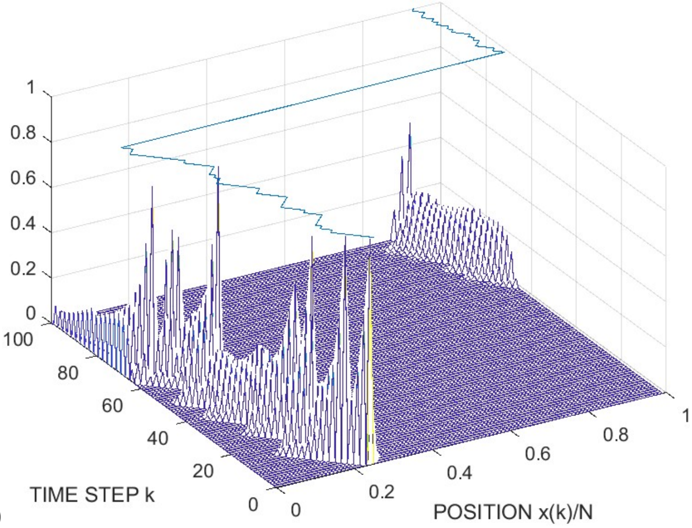
i)



-> Initially, the PDF is quite consistent with the robot's position.

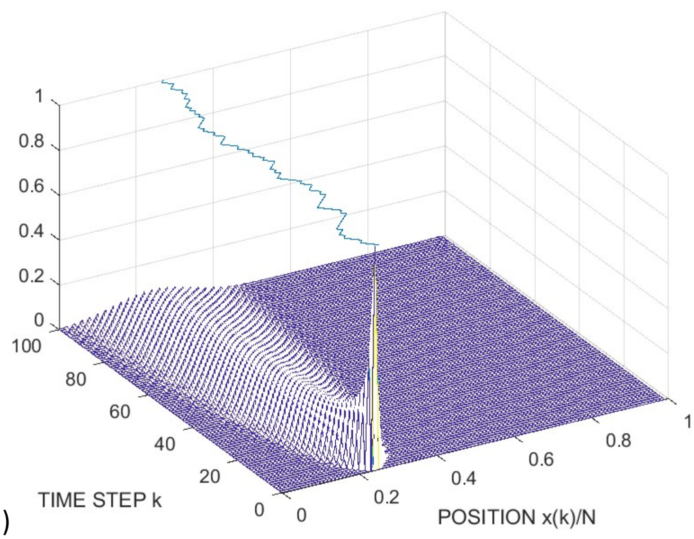
-> At some point, the program crashes. This is because there is no position that satisfies the conditional likelihood with the measurement and our prediction model.

-> The issue may be improved by increasing the value for measurement noise in the estimation model to be slightly higher than the true system.



ii)

-> The estimation scheme works fairly well. The PDF is consistent with the robot's position.



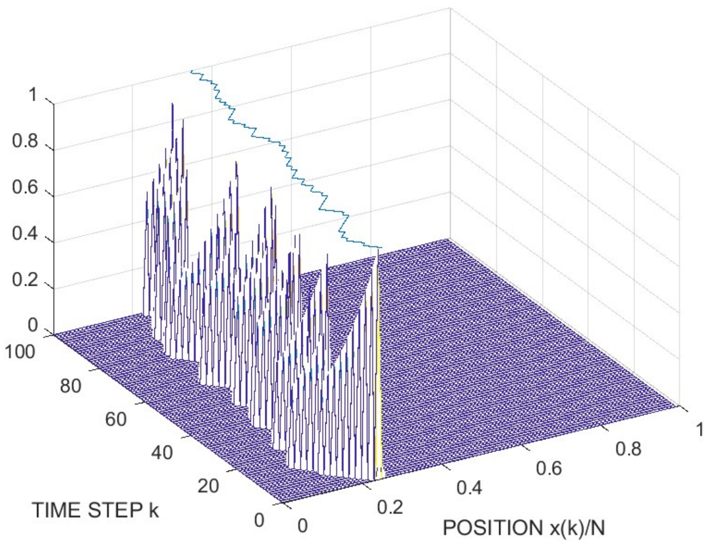
iii)

-> The estimation initially is fairly good.

-> However, since the measurements do not give us any information about the robot's position, we can only estimate it based on our prediction model.

-> Eventually, the PDF will return to a uniform distribution, and we will not know where the robot actually is.

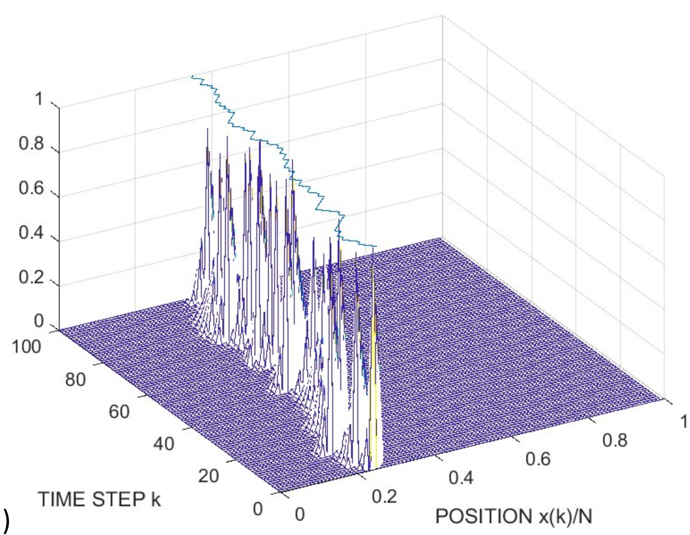
For Part (b):



i)

-> The estimate scheme works fairly well, unlike the situation above without the initial condition.

-> Since the initial condition is given, assuming opposite direction for p and $\hat{p}$ is not an issue.



ii)

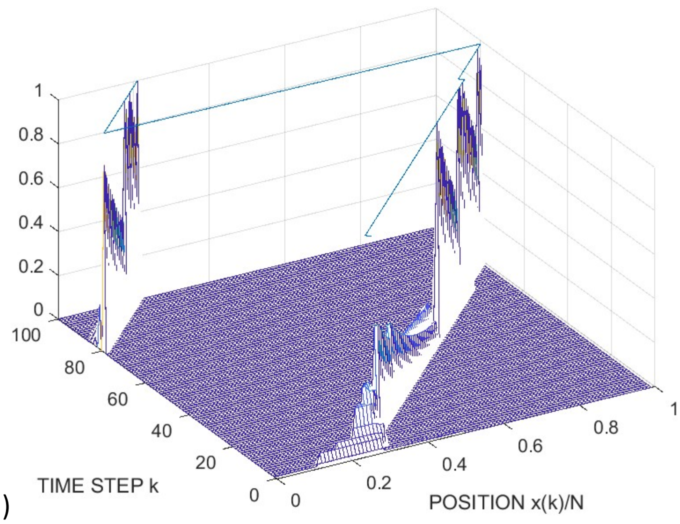
-> The estimation scheme works fairly well, similar to the above situation. However, it starts with a unimodal distribution instead of a bimodal one.

<u>B) We only know that the robot is somewhere on the upper half of the circle</u>

For Part (a):

-> Since we know the robot is somewhere on the upper half of the circle, there is a unimodal distribution initially.

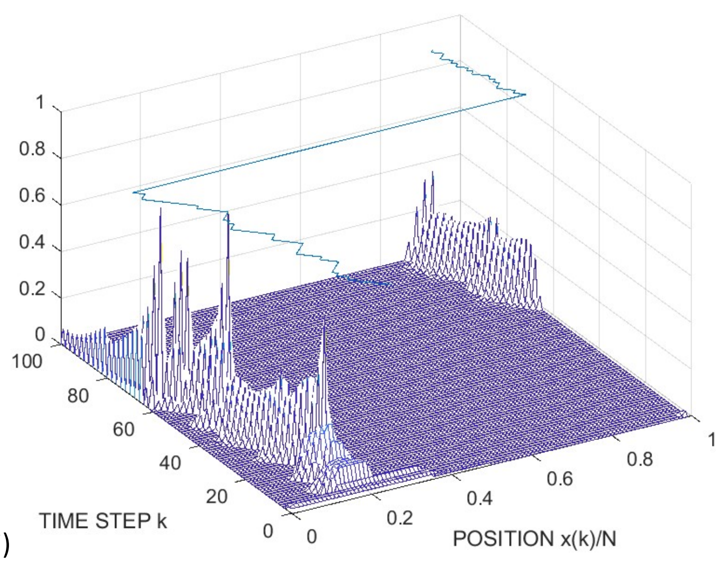
-> No initial peak at $x = \frac{3N}{4}$



i)

-> The estimation scheme works fairly well.

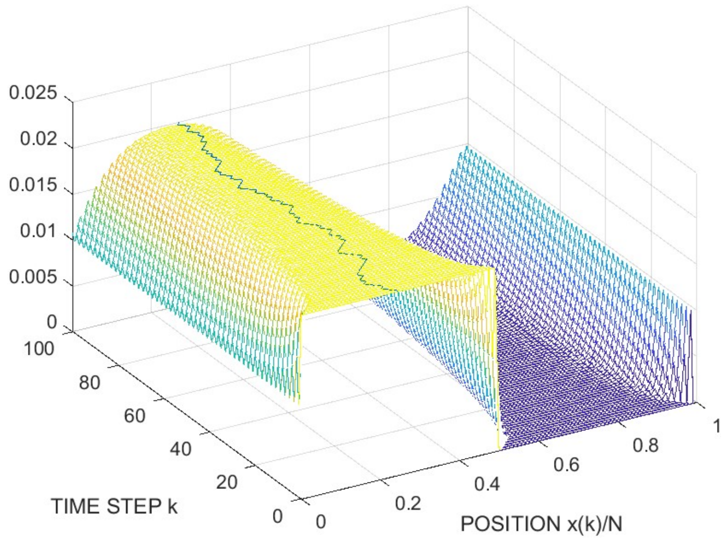
-> However, there is no initial PDF peak near the bottom half of the circle, unlike the situation of it in part (a)



ii)

-> The estimation scheme works fairly well.

-> Similar to part (ii) right above this question, there is also no initial PDF peak near the bottom half of the circle, unlike the situation of it in part (a).

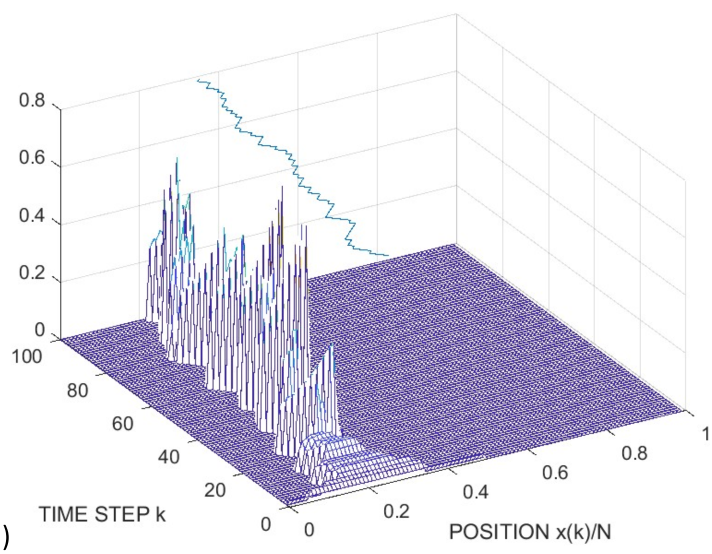


iii)

-> We do not get any measurement information similar to the situations above.

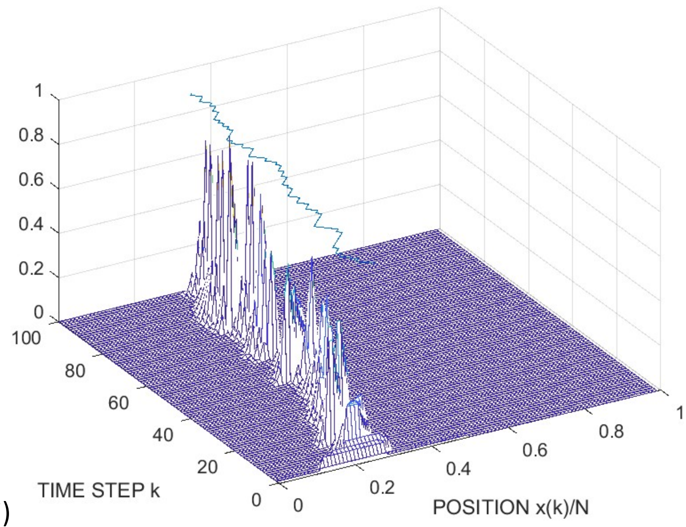
-> The distribution is based on the initial condition initially, and will return to a uniform distribution as seen before.

For Part (b):



i)

-> Similar to the situation in part (a) above, since there is an initial condition, assuming p and $\hat{p}$ to be in the opposite direction does not cause an issue.



ii)

-> The estimation scheme works fairly well.

-> Except that there is a unimodal distribution initially, it is pretty similar to part (a) of it. The unimodal distribution is due to the given initial condition.

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Problem 9.2

$\hat{x}^{ML} := \arg\max_x f(y|x)$ when $y_i = x + w_i$, $i = 1, 2,$

$\rightarrow$ w_i are ind. w/ $w_i \sim N(0, \sigma_i^2)$ & $z_i, w_i, x \in \mathbb{R}$

$\rightarrow f(w_i) \propto \exp\left(\frac{-w_i^2}{2\sigma_i^2}\right)$

$\rightarrow f(y_1, y_2 | x) \Rightarrow$ b/c conditional ind.

$\Rightarrow f(y_1|x)\, f(y_2|x) \propto \exp\left(\frac{-1}{2}\left(\frac{(y_1-x)^2}{\sigma_1^2} + \frac{(y_2-x)^2}{\sigma_2^2}\right)\right)$

$\rightarrow \left.\frac{\partial f(y_1, y_2|x)}{\partial x}\right|_{x=\hat{x}^{ML}} = 0 = \frac{(y_1 - \hat{x}^{ML})}{\sigma_1^2} + \frac{(y_2 - \hat{x}^{ML})}{\sigma_2^2}$

$\Rightarrow \boxed{\hat{x}^{ML} = \frac{y_1\sigma_2^2 + y_2\sigma_1^2}{\sigma_1^2 + \sigma_2^2}}$

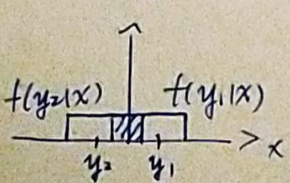
Problem 9.3

$f(w_1, w_2) = \begin{cases} 1/4 & \text{for } -1 \le w_1 \le 1 \text{ and } -1 \le w_2 \le 1 \\ 0 & \text{otherwise} \end{cases}$

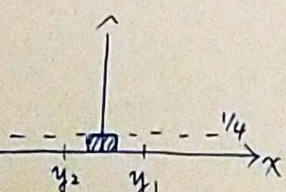
$\Rightarrow f(w_i) = \begin{cases} 1/2 & \text{for } w_i \in [-1, 1] \\ 0 & \text{otherwise} \end{cases}$

$\Rightarrow f(y_i|x) \begin{cases} 1/2 & \text{for } \overbrace{-1 \le y_i - x \le 1}^{\rightarrow\ y_i - 1 \le x \le y_i + 1} \\ 0 & \text{otherwise} \end{cases}$

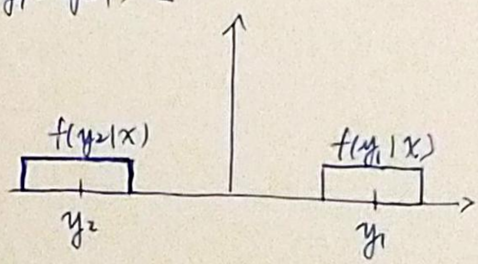
① $\rightarrow$ for $|y_1 - y_2| \le 2$:



$\Rightarrow$



② for $|y_1 - y_2| > 2$:



$\Rightarrow f(y_1, y_2 | x) = 0$

$$\Rightarrow f(y_1, y_2 | x) \begin{cases} \frac{1}{4} & \text{for } x \in [y_1 - 1, y_1 + 1] \cap [y_2 - 1, y_2 + 1] \\ 0 & \text{otherwise.} \end{cases}$$