ROB521 – Assignment 1 Report

In this assignment, we implement the Probabilistic roadmap (PRM) algorithm to construct a graph connecting start to finish nodes for a maze. The initial maze is in a shape of a 7 x 5 units' rectangle. Subsequently, we find the shortest path from the graph using the A* algorithm. Finally, we modify the sampling method from random to uniform to reduce the runtime. Using the optimized method, we could find the shortest path for mazes larger than 40 x 40 units in under 20 seconds.

Question 1: Implement the PRM algorithm to construct a graph connecting start to finish nodes

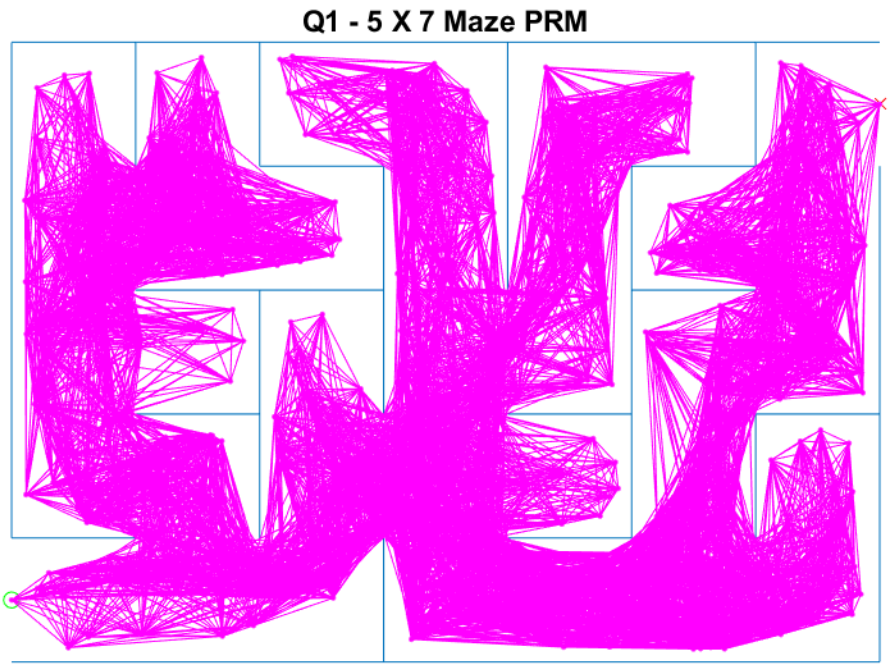


Figure 1: PRM Alogorithm Constructed Graph

Observation: Since we are using 500 samples in a relatively small maze of 7 x 5 units, the graph we construct using the PRM algorithm has a fairly similar shape to the actual maze walls. However, the payoff for having a large number of nodes in close proximity of each other will increase the runtime to construct edges since we are using the nearest neighbour connection strategy that results in lots of edges. Collision checking is

very time-consuming, and each edge would need to be checked against the maze walls and removed if it collides with them. As a result, the runtime is long, and the same method would be hard to scale to bigger mazes.

Question 2: Find the shortest path over the graph by implementing the Dijkstra's or A* algorithm

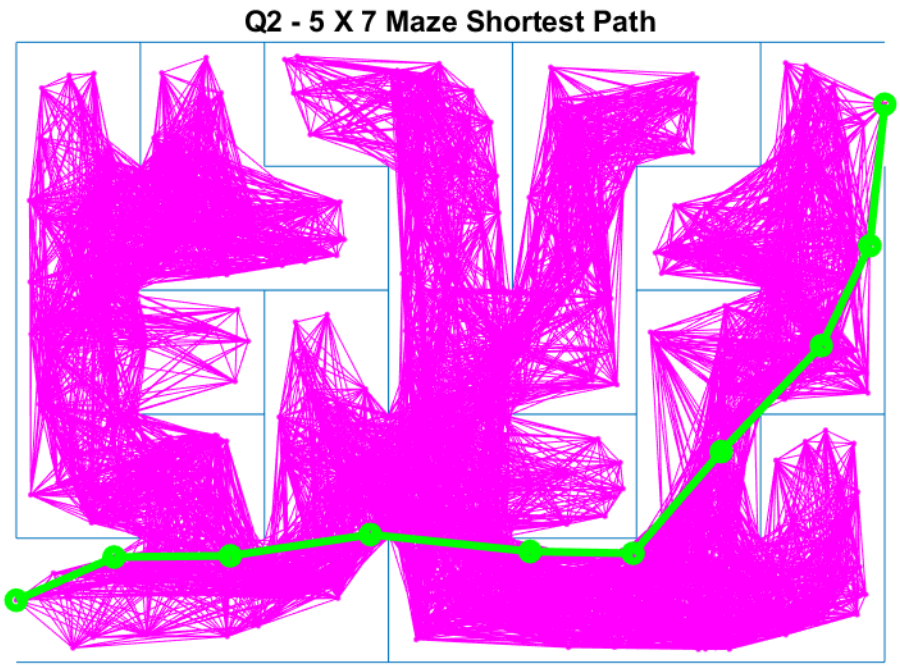


Figure 2: Shortest Path for 7 x 5 Maze Using A-Star Algorithm

Observation: As mentioned in the observation of question 1, the abundance of nodes in close proximity allows us to have a fairly accurate representation of the maze. This supports the algorithm that generates the shortest path to cut corners and generate a path that is close to the maze walls which would reduce the overall length of the shortest path found. The shortest path only takes around 0.2 seconds to find after the PRM graph is constructed, while it takes approximately 8 seconds to construct the graph, confirming that most of the time is used to generate the graph.

Question 3: Identify sampling, connection or collision checking strategies that can reduce runtime for mazes

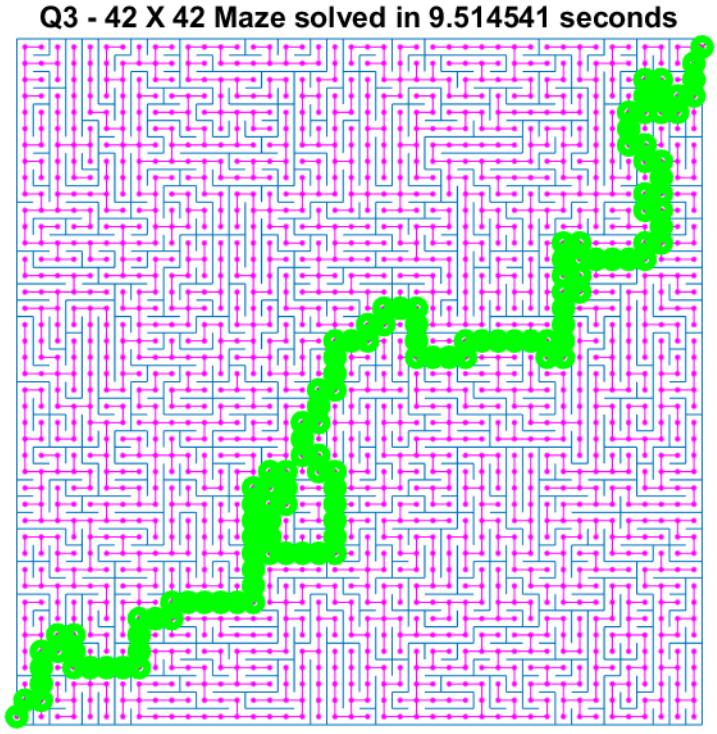


Figure 3: Optimized Algorithm Used for 42 x 42 Maze

Observation: By simply modifying the sampling method, we are able to greatly decrease the runtime, allowing the generation of the shortest path for a maze larger than 40 x 40 units in under 20 seconds. Since the maze and the maze walls are uniformly shaped, we take advantage of this setup and use uniform sampling instead of random sampling. Uniformly spaced-out nodes with our original nearest neighbour connections generation of a neighbour distance of 2 units increases the path length. Nonetheless, it greatly reduces the runtime. The uniform sampling method takes around 9 seconds for a 42 x 42 units maze, so the trade-off is reasonable.

---------- Matlab code is attached below ----------

****Note: The written helper functions start from page 10 of the pdf.**

```
% ======
% ROB521_assignment1.m
% ======
%
% This assignment will introduce you to the idea of motion planning for
% holonomic robots that can move in any direction and change direction of
% motion instantaneously.  Although unrealistic, it can work quite well for
% complex large scale planning.  You will generate mazes to plan through
% and employ the PRM algorithm presented in lecture as well as any
% variations you can invent in the later sections.
%
% There are three questions to complete (5 marks each):
%
%    Question 1: implement the PRM algorithm to construct a graph
%    connecting start to finish nodes.
%    Question 2: find the shortest path over the graph by implementing the
%    Dijkstra's or A* algorithm.
%    Question 3: identify sampling, connection or collision checking
%    strategies that can reduce runtime for mazes.
%
% Fill in the required sections of this script with your code, run it to
% generate the requested plots, then paste the plots into a short report
% that includes a few comments about what you've observed.  Append your
% version of this script to the report.  Hand in the report as a PDF file.
%
% requires: basic Matlab,
%
% S L Waslander, January 2022
%
clear; close all; clc;

% set random seed for repeatability if desired
%rng(1);

% ==========================
% Maze Generation
% ==========================
%
% The maze function returns a map object with all of the edges in the maze.
% Each row of the map structure draws a single line of the maze.  The
% function returns the lines with coordinates [x1 y1 x2 y2].
% Bottom left corner of maze is [0.5 0.5],
% Top right corner is [col+0.5 row+0.5]
%

row = 5; % Maze rows
col = 7; % Maze columns
map = maze(row,col); % Creates the maze
start = [0.5, 1.0]; % Start at the bottom left
finish = [col+0.5, row]; % Finish at the top right

h = figure(1);clf; hold on;
```

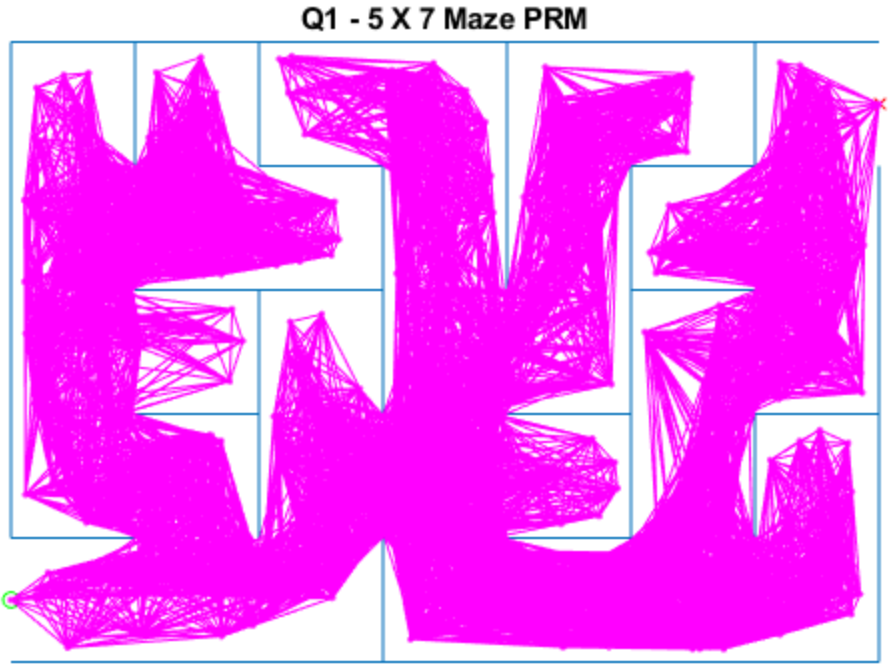
```
plot(start(1), start(2),'go');
plot(finish(1), finish(2),'rx');
show_maze(map,row,col,h); % Draws the maze
drawnow;

% =======================================================
% Question 1: construct a PRM connecting start and finish
% =======================================================
%
% Using 500 samples, construct a PRM graph whose milestones stay at least
% 0.1 units away from all walls, using the MinDist2Edges function provided for
% collision detection.  Use a nearest neighbour connection strategy and the
% CheckCollision function provided for collision checking, and find an
% appropriate number of connections to ensure a connection from  start to
% finish with high probability.


% variables to store PRM components
nS = 500;  % number of samples to try for milestone creation
milestones = [start; finish];  % each row is a point [x y] in feasible space
edges = [];  % each row is should be an edge of the form [x1 y1 x2 y2]

disp("Time to create PRM graph")
tic;
% ------insert your PRM generation code here-------


% random sampling 500 milestones
for i = 1:nS
    % for x coordinate
    ran = rand;
    pt_x = (col - 0.2) * ran + 0.6;
    % for y coordinate
    ran = rand;
    pt_y = (row - 0.2) * ran + 0.6;
    pt = [pt_x pt_y];
    % ensure each milestone is at least 0.1 units away from maze walls
    if (MinDist2Edges(pt, map) > 0.1)
        milestones = [milestones;pt];
    end

end

% ensure each milestone is unique
milestones = unique(milestones, 'rows', 'stable');

% nearest neighbour connection
% compute nearest neighbour distance
% (any milestone within this distance is a neighbour)
neigh_dist = row / 4;
if (col > row)
    neigh_dist = col / 4;
end
% find the nearest neighbours for each milestone
```

```
for j = 1:size(milestones)
    curr_pt = milestones(j, :);
    dist_xy = abs(milestones - curr_pt);
    dist_tot = sum(dist_xy, 2);
    indices = find(dist_tot < neigh_dist);
    indices = indices(indices > j);
    % for each nearest neighbour, compute an edge
    for k = 1:size(indices)
        curr_ind = indices(k);
        neigh_pt = milestones(curr_ind, :);
        edges = [edges; curr_pt neigh_pt];
        edges = [edges; neigh_pt curr_pt];
    end
end

% check edges for collision with walls
edges_removed = edges;
remove_ind = [];
for e = 1:size(edges)
    curr_edge1 = edges(e, 1:2);
    curr_edge2 = edges(e, 3:4);
    % compute all edge indices that contain colliding edges
    if (CheckCollision(curr_edge1, curr_edge2, map) == 1)  % if collision
        remove_ind = [remove_ind; e];
    end
end
% remove colliding edges
for r = size(remove_ind):-1:1  % start from the end of the indices list
    curr_remove = remove_ind(r);
    edges_removed(curr_remove, :) = [];  % remove
end
% set edges as the set with all colliding edges removed
edges = edges_removed;


% ------end of your PRM generation code -------
toc;

figure(1);
plot(milestones(:,1),milestones(:,2),'m.');
if (~isempty(edges))
    line(edges(:,1:2:3)', edges(:,2:2:4)','Color','magenta'); % line uses [x1
 x2 y1 y2]
end
str = sprintf('Q1 - %d X %d Maze PRM', row, col);
title(str);
drawnow;

print -dpng assignment1_q1.png


% =========================================================
% Question 2: Find the shortest path over the PRM graph
% =========================================================
```

```
%
% Using an optimal graph search method (Dijkstra's or A*) , find the
% shortest path across the graph generated.  Please code your own
% implementation instead of using any built in functions.

disp('Time to find shortest path');
tic;

% Variable to store shortest path
spath = []; % shortest path, stored as a milestone row index sequence


% ------insert your shortest path finding algorithm here-------


% A* algorithm is implemented as a function
% (functions are written at the end of the code)
spath = A_star(1, 2, milestones, edges);


% ------end of shortest path finding algorithm-------
toc;

% plot the shortest path
figure(1);
for i=1:length(spath)-1
    plot(milestones(spath(i:i+1),1),milestones(spath(i:i
+1),2), 'go-', 'LineWidth',3);
end
str = sprintf('Q2 - %d X %d Maze Shortest Path', row, col);
title(str);
drawnow;

print -dpng assingment1_q2.png


% ================================================================
% Question 3: find a faster way
% ================================================================
%
% Modify your milestone generation, edge connection, collision detection
% and/or shortest path methods to reduce runtime.  What is the largest maze
% for which you can find a shortest path from start to goal in under 20
% seconds on your computer? (Anything larger than 40x40 will suffice for
% full marks)


row = 42;
col = 42;
map = maze(row,col);
start = [0.5, 1.0];
finish = [col+0.5, row];
milestones = [start; finish];  % each row is a point [x y] in feasible space
edges = [];  % each row is should be an edge of the form [x1 y1 x2 y2]
```

```
h = figure(2);clf; hold on;
plot(start(1), start(2),'go');
plot(finish(1), finish(2),'rx');
show_maze(map,row,col,h); % Draws the maze
drawnow;

fprintf("Attempting large %d X %d maze... \n", row, col);
tic;
% ------insert your optimized algorithm here------


% uniform sampling (with distance of 1 unit in each direction)
dist_uni = 1;

for i = 1:col
    % for x coordinate
    % start from x = 0.5, and want center so +0.5
    pt_x = (0.5 + 0.5) + dist_uni * (i - 1);
    % for y coordinate
    for j = 1:row
        % start from y = 0.5, and want center so +0.5
        pt_y = (0.5 + 0.5) + dist_uni * (j - 1);
        pt = [pt_x pt_y];
        % ensure each milestone is at least 0.1 units away from maze walls
        if (MinDist2Edges(pt, map) > 0.1)
            milestones = [milestones;pt];
        end
    end
end

% ensure each milestone is unique
milestones = unique(milestones, 'rows', 'stable');

% nearest neighbour connection
neigh_dist = 2;  % nearest neighbour distance
% find the nearest neighbours for each milestone
for j = 1:size(milestones)
    curr_pt = milestones(j, :);
    dist_xy = abs(milestones - curr_pt);
    dist_tot = sum(dist_xy, 2);
    indices = find(dist_tot < neigh_dist);
    indices = indices(indices > j);
    % for each nearest neighbour, compute an edge
    for k = 1:size(indices)
        curr_ind = indices(k);
        neigh_pt = milestones(curr_ind, :);
        edges = [edges; curr_pt neigh_pt];
        edges = [edges; neigh_pt curr_pt];
    end
end

% check edges for collision with walls
edges_removed = edges;
```

```
remove_ind = [];
for e = 1:size(edges)
    curr_edge1 = edges(e, 1:2);
    curr_edge2 = edges(e, 3:4);
    % compute all edge indices that contain colliding edges
    if (CheckCollision(curr_edge1, curr_edge2, map) == 1)  % if collision
        remove_ind = [remove_ind; e];
    end
end
% remove colliding edges
for r = size(remove_ind):-1:1  % start from the end of the indices list
    curr_remove = remove_ind(r);
    edges_removed(curr_remove, :) = [];  % remove
end
% set edges as the set with all colliding edges removed
edges = edges_removed;

% find the shortest path, stored as a milestone row index sequence
spath = A_star(1, 2, milestones, edges);


% ------end of your optimized algorithm-------
dt = toc;

figure(2); hold on;
plot(milestones(:,1),milestones(:,2),'m.');
if (~isempty(edges))
    line(edges(:,1:2:3)', edges(:,2:2:4)','Color','magenta');
end
if (~isempty(spath))
    for i=1:length(spath)-1
        plot(milestones(spath(i:i+1),1),milestones(spath(i:i
+1),2), 'go-', 'LineWidth',3);
    end
end
str = sprintf('Q3 - %d X %d Maze solved in %f seconds', row, col, dt);
title(str);

print -dpng assignment1_q3.png

Time to create PRM graph
Elapsed time is 1.602603 seconds.
Time to find shortest path
```



Helper Functions

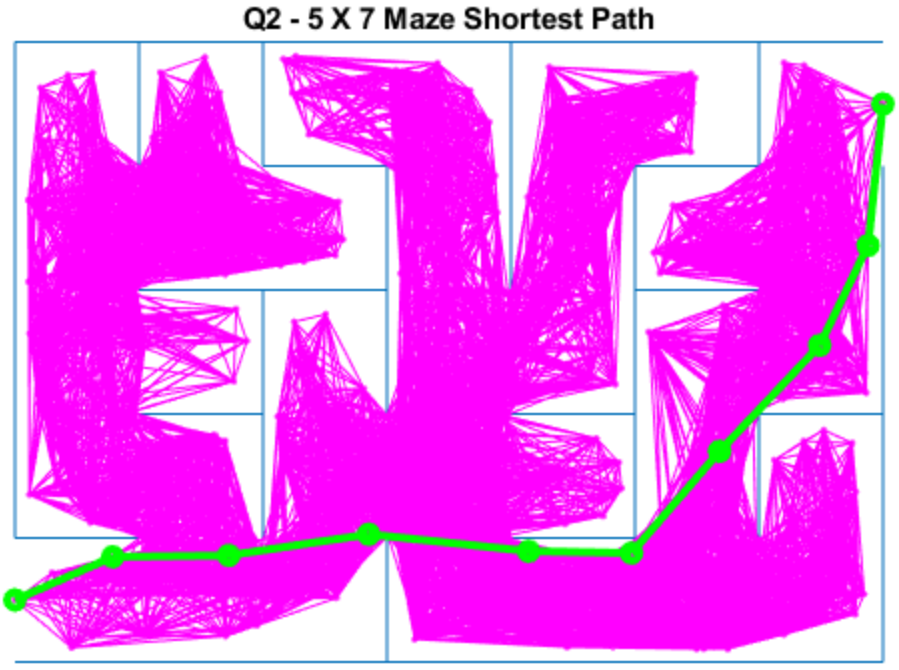
compute a list of milestone indices constructing the shortest path input "precedent" is a column vector of the precedent milestone for each milestone of the corresponding vector index

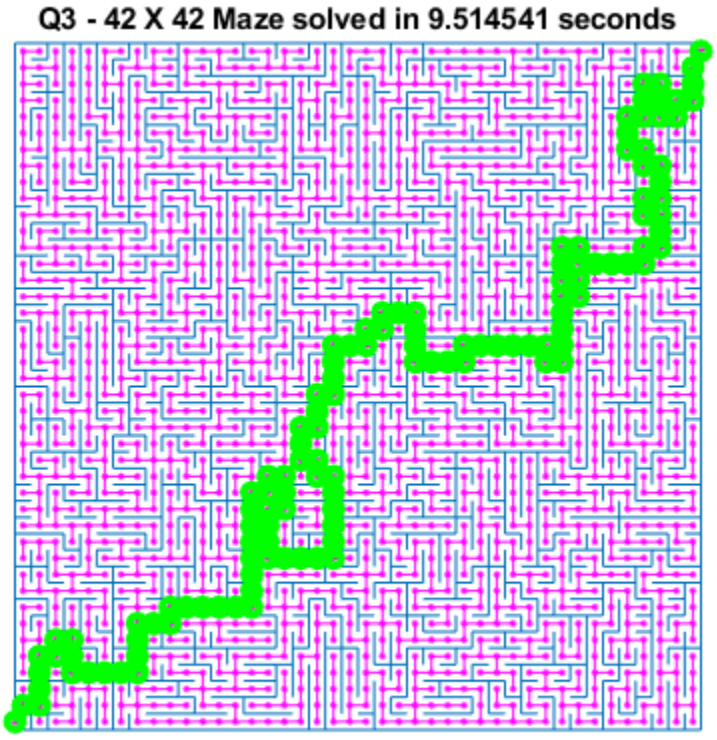
```
function path_tot = construct_path(precedent, current, start)
    path_tot = [current];
    while current ~= start
        current = precedent(current);
        path_tot = [current path_tot];
    end
end

% the heuristic function (using the Manhattan distance)
function hScore = heuristic(state, goal, milestones)
    x1 = milestones(state, 1);
    y1 = milestones(state, 2);
    x2 = milestones(goal, 1);
    y2 = milestones(goal, 2);
    hScore = abs(x2 - x1) + abs(y2 - y1);  % Manhattan distance
end

% the edge weight of each edge (using the edge distance in units)
function edge_w = edge_weight(state, neigh, milestones)
    x1 = milestones(state, 1);
    y1 = milestones(state, 2);
```

```
    x2 = milestones(neigh, 1);
    y2 = milestones(neigh, 2);
    edge_w = sqrt((x2 - x1)^2 + (y2 - y1)^2);
end

% compute the minimum fScore among only the currently available
% states/milestone indices
function curr_state = min_fScore(states, fScore)
    curr_fScore = [];
    % compute a list of fScore for all available states
    for i = 1:size(states, 2)
        curr_fScore = [curr_fScore fScore(states(i))];
    end
    % find the minimum fScore among the list computed
    ind = find(fScore == min(curr_fScore));
    % find the corresponding state(s) of the minimum fScore
    % (return only the first state of the corresponding minimum fScore
    % if multiple are found)
    for j = 1:size(ind)
        if (ismember(ind(j), states) == 1)
            curr_state = ind(j);
            return;
        end
    end
end

% compute the neighbours (which have an edge) of the current milestone
function neigh_list = neighbours(curr_mile, edges, milestones)
    neigh_list = [];
    % find indices in the edge list that has the current milestone
    % as a side of the edge
    edge_ind = find(edges(:, 1) == curr_mile(1) & edges(:, 2) ==
 curr_mile(2));
    % for each edge index, find the corresponding milestone from the
    % milestones list (i.e., the other side of the edge)
    for e = 1:size(edge_ind, 1)
        neigh_mile = edges(edge_ind(e), 3:4);  % neighbour milestone
        % find a milestone in the milestones list with the same
        % x and y coordinates of the neighbour milestone
        x_ind = find(milestones(:, 1) == neigh_mile(1));  % x coordinate
        y_ind = find(milestones(:, 2) == neigh_mile(2));  % y coordinate
        % compute the milestone that exist both in the x coordinate
        % and y coordinate indices list
        [neigh, ~] = intersect(x_ind, y_ind);
        neigh_list = [neigh_list neigh];
    end
end

% implement the A-star algorithm to find the shortest path
function sPath = A_star(start, goal, milestones, edges)
    states = [start];  % to store milestone indices/states
    % list of precedent milestone of each corresponding milestone index
    precedent = zeros(size(milestones, 1), 1);
    gScore = inf(size(milestones, 1), 1);  % gScore for milestones
```

```
    gScore(start) = 0;
    % fScore for milestones (g + h scores)
    fScore = inf(size(milestones, 1), 1);
    fScore(start) = heuristic(start, goal, milestones);
    % while there are states in the list
    while (~isempty(states))
        % pick the state with min fScore (from list of available states)
        curr_state = min_fScore(states, fScore);
        % finish if current state is the goal state
        if (curr_state == goal)
            sPath = construct_path(precedent, curr_state, start);
            return;  % return after path found
        end
        % remove the current state from the list of available states
        states = states(states~=curr_state);
        % compute list of neighbour states (i.e. have an edge with current)
        curr_mile = milestones(curr_state, :);
        neigh_list = neighbours(curr_mile, edges, milestones);
        % for each neighbour of current state
        for n = 1:size(neigh_list, 2)
            neigh = neigh_list(n);  % neighbour state
            % compute the tentative gScore (current + new edge score)
            gScore_ten = gScore(curr_state) + edge_weight(curr_state, neigh,
 milestones);
            % if tentative gScore < than the neighbour gScore
            if gScore_ten < gScore(neigh)
                % set current state as the precendent of current neighbour
                precedent(neigh) = curr_state;
                % set tentative gScore as its gScore
                gScore(neigh) = gScore_ten;
                % set its fScore (gScore + heuristic)
                fScore(neigh) = gScore_ten + heuristic(neigh, goal,
 milestones);
                % add the current neighbour to the list of available states
                % if it is not already in
                if ismember(neigh, states) == 0
                   states = [states neigh];
                end
            end
        end
    end
    sPath = [];
end
```

```
Elapsed time is 18.124164 seconds.
Attempting large 42 X 42 maze...
```


Q2 - 5 X 7 Maze Shortest Path


Q3 - 42 X 42 Maze solved in 9.514541 seconds

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