

## Assignment 1A: Introduction to Bayesian Statistics

### Q1 (5 points)

There are two events A and B.  $P(A) = .5$  and  $P(B) = .3$ . The events A and B are independent.

- Find  $P(A')$
- Find  $P(A \cap B)$
- Find  $P(A \cup B)$

**Ans:**

### Q2 (6 points)

Two fair dice, one red and one green, are rolled. Let the event A be “the sum of the faces showing is equal to seven”. Let the event B be “the faces showing on the two dice are equal”.

- List out the sample space of the experiment.
- List the outcomes in A, and find  $P(A)$ .
- List the outcomes in B, and find  $P(B)$ .
- List the outcomes in  $(A \cap B)$ , and find  $P(A \cap B)$ .
- Are the events in A and B independent? Explain why or why not.
- How would you describe the relationship between event A and event B?

**Ans:**

### Q3 (4 points)

Suppose there is a medical screening procedure for a specific cancer that has sensitivity = 0.90, and specificity = 0.95. Suppose the underlying rate of the cancer in the population is 0.001. Let B be the event “the person has that specific cancer,” and let A be the event “the screening procedure gives a positive result.”

- What is the probability that a person has the disease given the results of the screening is positive?
- Does this show that screening is effective in detecting this cancer?

**Ans:**

**Q4 (6 points)**

- A. You are playing blackjack with a newly shuffled deck that consists of four decks of 52 cards each shuffled together. You have been dealt a ten and a six. The dealer has been dealt a jack face up, and another card face down. If you ask to be dealt another card, what is the probability you do not get "busted"
- B. Suppose the deck isn't being shuffled between every hand played. The cards from previous hands are in the discard pile. In the previous hands you have observed 4 cards that are five or below, and 20 cards that are over five. On this hand you have been dealt a ten and a six. The dealer has been dealt a jack face up, and another card face down. If you ask to be dealt another card, what is the probability you do not get "busted"

**Ans:**

**Q5 (6 points)**

A discrete random variable  $Y$  has discrete distribution given in the table on page 97 of the Second Edition (page 92 of the First Edition) of the text.(see exercise 5.2 in the text)

- Calculate  $P(0 < Y < 2)$ .
- Calculate  $E(Y)$ .
- Calculate  $\text{Var}(Y)$ .
- Let  $W = 3Y - 1$ . Calculate  $E(W)$ .
- Calculate  $\text{Var}(W)$ .

**Ans:**

**Q6 (4 points)**

Let  $Y$  have the Poisson ( $\mu=3$ ) distribution.

- Calculate  $P(Y = 3)$
- Calculate  $P(Y \leq 3)$
- Calculate  $P(1 \leq Y < 5)$

**Ans:**

**Q7 (5 points)**

Let  $X$  have a beta (12, 4) distribution.

- Find  $E(X)$ .
- Find  $\text{Var}(X)$ .

**Ans:**

**Q8 (4 points)**

Let  $Z$  have the standard normal distribution.

- a. Find  $P(0 \leq Z \leq 1.52)$ .
- b. Find  $P(Z \geq 2.11)$ .
- c. Find  $P(-1.45 \leq Z \leq 1.74)$ .

**Ans:**

**Q9 (3 points)**

Let  $Y$  be normally distributed with mean  $\mu = 860$  and variance  $\sigma^2 = 576$ .

- a. Find  $P(Y \leq 900)$ .
- b. Find  $P(Y \geq 825)$ .
- c. Find  $P(840 \leq Y \leq 890)$ .

**Ans:**

**Q10 (5 points)**

Let  $Y$  be distributed according to the beta (15, 10) distribution.

- a. Find  $E(Y)$ .
- b. Find  $\text{Var}(Y)$ .
- c. Find  $P(Y < .5)$  using the normal approximation

**Ans:**

**Q11 (5 points)**

Let  $Y$  be distributed according to the gamma (26,5) distribution.

- a. Find  $E(Y)$ .
- b. Find  $\text{Var}(Y)$ .
- c. Find  $P(Y > 5)$ .

**Ans:**

Updated on 27<sup>th</sup> June 2014