Matrix Algebra Review
Assignment 4
Points – 30

4.1

Eigenset

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

a. Find det(A). **(2 pts)**
b. Find the eigenvalues of A. **(3 pts)**
c. What is the relationship of det(A) to the eigenvalues of A? **(1 pt)**
d. Is A positive definite, positive semi-definite or non-negative definite or none of these? **(1 pt)**
e. Find the normalized eigenvectors corresponding to the eigenvalues. **(4 pts)**
f. Show how A is formed by its spectral decomposition. **(4 pts)**

4.2

Eigenset of a 3 x 3 exchangeable correlation matrix

An "exchangeable" correlation matrix is one in which each covariate has the same correlation with each other. This represents a very simple correlation structure that is exploited in such applications as generalized estimating equations ("GEE") that are used to model populations instead of subjects when random effects are present. Such a matrix is, e.g.,

$$A = \begin{bmatrix} 1 & 0.2 & 0.2 \\ 0.2 & 1 & 0.2 \\ 0.2 & 0.2 & 1 \end{bmatrix}$$

with eigenvalues λ = 1.4, 0.8, 0.8
a. Is A positive definite, positive semi-definite or non-negative definite or none of these? **(3 pts)**
b. The normalized eigenvectors corresponding to the eigenvalues 1.4, 0.8 and 0.8, resp., are

$$V_1 = \begin{bmatrix} 0.57735 \\ 0.57735 \\ 0.57735 \end{bmatrix} \qquad V_2 = \begin{bmatrix} 0.66057 \\ 0.08532 \\ -0.74590 \end{bmatrix} \qquad V_3 = \begin{bmatrix} 0.47991 \\ -0.81203 \\ 0.33212 \end{bmatrix}$$

Show how A is formed by its spectral decomposition. **(12 pts)**