

Project Proposal, MATH 5010

Zongyi Liu

Feb 28, 2025

Topic

My intended topic is **Comparison of Monte Carlo methods in American Option Pricing**

Using Monte Carlo methods to calculate the value of an option is a important and quite recent topic in mathematical finance. It deals with multiple sources of uncertainty or with complicated features. The first application to option pricing was by Phelim Boyle in 1977 (for European options). In 1996, M. Broadie and P. Glasserman from Columbia Business School showed how to price Asian options by Monte Carlo. An important development was the introduction in 1996 by Carriere of Monte Carlo methods for options with early exercise features.

American options, unlike their European counterparts, allow the holder to exercise the option at any time before expiration. This additional flexibility makes American options more valuable but significantly more complex to price. Traditional numerical methods like binomial trees and finite difference methods face challenges in high-dimensional settings.

There are various kinds of methods utilizing Monte Carlo, and in this project, I hope to:

- Implement and compare different Monte Carlo simulation approaches for pricing American options.
- Evaluate the performance of each method in terms of accuracy, computational efficiency, and scalability.
- Investigate the applicability of machine learning techniques to improve the efficiency of option pricing models.
- Rediscover the pros and cons of Monte Carlo simulation.

Methodology

Monte Carlo Simulation Techniques

- Longstaff-Schwartz Method: Use least-squares regression to approximate the continuation value at each exercise date.
- Stochastic Mesh Method: Create a random mesh of points to approximate the continuation value using weighted averages.
- Parametric Exercise Boundaries: Fit parametric models to the optimal exercise boundary.

Programming Language to Realize the Goal

Mainly Python, and if time permits, I will rewrite them in C++.

Evaluation Metrics

- Pricing accuracy compared to benchmark solutions.
- Computational efficiency (execution time and memory usage).
- Robustness under varying market conditions (volatility, interest rates, etc.)

Data Sources

- Barchart – Comprehensive options data.
- Nasdaq Options – Options chain and pricing.
- CBOE – Official site for US options markets.
- Polygon.io – Real-time and historical options data.
- Interactive Brokers API – Market data with trading API.

Settings

Many tests made by previous scholar set similar parameters, in which the expiration date will be denoted by T , the strike price by K , the interest rate by r , the volatility by σ , and the stock price at time t by S_t (starting price S_0), with an argument $S_t(k)$ indicating a particular sample path k and superscripts S_t^i when there are multiple stocks. Here the interest rate and volatility are assumed constant, but these can easily be made stochastic for all the procedures with little difficulty, a usual advantage of Monte Carlo methods. Under the scenario, the price of an American-style option can be written as the solution to the following optimal stopping problem:

$$\sup_{\tau} E^Q [e^{-r\tau} L_{\tau} (\{S_t : t \in [0, \tau]\})]$$

where Q denotes the appropriate risk-neutral (martingale) measure, $L_t(\cdot)$ represents the payoff at time t , which itself could be path dependent (as in an Asian option), and the supremum is over all stopping times $\tau \leq T$. All cases will have a finite number of discrete exercise opportunities, at points $\{t_0 = 0, t_1, \dots, t_N = T\}$, where $N + 1$ gives the total number of exercise opportunities.

Scenario 1: Call Option with Discrete Dividends

Here a single stock that distributes dividend D_j at time $t_j = \sum_{i=1}^j \tau_i$ ($\tau_i > 0$), $j = 1, \dots, N$, where τ_i is the time between the distribution of dividend $i - 1$ and i , and N is the number of dividends distributed during the lifetime of the call contract, the last dividend distributed on the expiration date. Thus,

$$L_t = (S_t - K)^+$$

The dividend amounts $\{D_j\}$ can be deterministic or stochastic

Scenario 2: Call Option with Continuous Dividends

In this scenario, a single stock pays continuous dividends at a rate δ , but early exercise is restricted to the discrete points $\{t_j, j = 0, 1, \dots, N\}$. We assume that the stock price net of the present value of escrowed dividends changes continuously according to

$$S_{t_j} = S_{t_j^-} = S_{t_j^+} = h(Z; S_{t_{j-1}}, \tau_j, \mu - \delta, \sigma), \quad j = 1, \dots, N.$$

Scenario 3: Max Option Multiple Underlying Assets

Here the payoff function upon exercise at time t is

$$L_t = \left(\max_{i=1, \dots, n} S_t^i - K \right)^+$$

where again early exercise is restricted to the discrete points $\{t_j, j = 0, 1, \dots, N\}$. The individual stock prices are assumed to follow the correlated geometric Brownian motion processes

$$dS_t^i = S_t^i [(r - \delta_i) dt + \sigma_i dZ_t^i]$$

where Z_t^i is a standard Brownian motion process and the instantaneous correlation of Z^i and Z^j is ρ_{ij} .

This part mainly follows Broadie and Glasserman's paper.

To be Expanded