Introduction to the Mathematics of Finance. Take-Home Midterm.

Due March 14, 2025 11.59 p.m.

Please write a pledge that you did not copy solutions from the work of other students. You can consult TAs if you have any difficulties. This midterm uses content of lectures in the last two weeks of February. It also uses base matlab models that can be downloaded from Courseworks.

1.Matlab option model. Download from Courseworks matlab option model files BlackScholesStocks.m and BlackScholesGraph.m and put them in the same directory. BlackScholesStocks.m contains the function that calculates the Black Scholes price for options on non-dividend paying stocks. BlackScholesGraph.m is a script that makes a graph of option price as a function of stock price. Type at Matlab prompt
>BlackScholesGraph and the script will be executed, and the graph will appear. Now modify the file BlackScholesStocks.m so that the function now calculates the price of options on stocks paying continuous dividends at a rate q. Modify the file BlackScholesGraph.m so that it now plots graph of a call with the same parameters as before but with the dividend yield q=2% and the new strike price 11. Submit printouts of code and graph.

2 Download matlab Brownian motion model from the courseworks. Modify it to Geometric Brownian motion with starting value Xo=100, growth rate $\mu = 0.14$, volatility $\sigma = 0.28$ and 5000 trajectories. Check that the code works. Try out 50,000 trajectories. Try out 100,000 trajectories. Submit the code printout and the graph printout for 5000 trajectories.

3. Using arbitrage arguments explain why the price of an American call option on a stock paying no dividends should be the same as the price of a corresponding European call. Why American calls on a nondividend paying stock should not be exercised early.

4. Why when the stock pays dividends the argument of the problem No.3 can not be used. Give a numerical example (choosing x, k, r, $T - t$, σ) in which it is obvious (without any formulas) that American put price on a nondividend paying stock is larger than the corresponding European put price.

5. (a) The stock price is 40 the volatility of the stock is 20%. Assuming that the time to expiration is 3 months and the interest rate is 1% per annum calculate the price P of the
European call option with strike 41.
(b) Calculate Δ, Γ, ρ, Vega using formulas for these parameters. Calculate the same parameters approximately using the options calculator.
(c) Check that following relationship holds

$$\Theta + rx\Delta + \frac{1}{2}\sigma^2 x^2 \Gamma = rP$$

6. What are the parameters affecting prices European and American calls and puts. How do the prices change when one of the parameters changes with all the others remaining the same?

7. Suppose that we have three European calls with strikes 60, 65, and 70 and the same maturity 1 month. Their prices are 9.00, 7.00, 4.00. Is it possible to do an arbitrage?

8. Suppose that current stock price is 50 \$. Its annualized volatility is 30 % and annualized return 10 % i.e. we assume that the stock price follows $dX_t = 0.1\, X_t\, dt + 0.3\, X_t\, dW_t$. Write the probability density function for the stock in 1 year. What is the mean and standard deviation of the terminal stock price? (standard deviation of price, not of return)