Introduction to the Mathematics of Finance. HOMEWORK 1.
Due February 13, 2025, 11.59pm

Please write a pledge that homework solutions represent your own work and that you did not copy solutions from the work of other students. You can consult Teaching Assistants if you have difficulty understanding the problem.

1.(5pt) Suppose that the spot rate of EUR is 1.02395 USD for 1Euro. 1 year forward rate is 1.06280 USD for 1 Euro. Suppose that the 1 year USD interest rate is 4.21% annualized and Euro interest rate is 2.38% annualized. Rates are compounded annually that means that 1USD a year from now grows to 1*(1+0.0421) USD. Is there an arbitrage opportunity? If there is, describe it (what you borrow, what you invest etc.)

2.(5pt) Suppose that the spot exchange rate of EUR is 1.0240 USD for 1Euro. Suppose that the 3 months USD interest rate is 4.30% annualized and Euro interest rate is 2.59% annualized (now rates that used here are continuously compounded). What is the 3 month forward exchange rate?

3.(5pt) What does it mean for interest rate to be negative? Read the document from courseworks. Explain in 4 lines.

4.(5pt) What is the six-month forward price for a stock providing no income. The stock price is 100 and the continuously compounded interest rate is 4%? What is the forward price if the stock pays a 3%, 4%, 5% continuously compounded dividend yield?

5.(5pt) What is the difference between a forward contract and a futures contract?

6.(10pt) Make futures margin table similar to the class handout using oil futures prices spreadsheet Oil_CLH25.xls on courseworks. Suppose that initial margin is 5,720 USD and a maintenance margin 5,200 USD per contract and margins are constant through the life of the contract. You buy 2 contracts at the close on September 13, 2023 and sell at the close on January 15, 2025. Oil futures point value is 1000$. When are the margin calls?

7. (10pt) This problem should be done in Excel and not in other programs. Go to http://finance.yahoo.com, chose Chart and type MSFT in the box. Click Historical Quotes below the chart. Input dates: Start January 24, 2014, End January 15, 2025. Click download spreadsheet format in the bottom of the page. Data is in the Excel format Date, Open, High, Low, Close, Volume, Adjusted Close. Make and submit the printouts of 2 plots: cumulative distribution functions of returns and approximate probability density function of returns using 0.2% horizontal intervals. Calculate mean, variance, standard deviation, mean absolute deviation, kurtosis and skewness of daily returns in Excel. You should use Excel and not other programs. If you need help with excel graphing talk to TAs. Use only adjusted close prices.

8. a) (5pt) Calculate and plot 100day, 50 day, 30day and 15 day Moving Average of SPY Close from January 2, 2008 to January 15, 2025 in a spreadsheet similar to HW1SPYMovingAve.xls. Add data from Yahoo as needed. Submit printout of plots and last 20 days of data.

b) (5pt) Calculate and plot 15 day, 30 day, 50 day and 100 day volatility of SPY Adjusted Close from January 2, 2008 to January 15, 2025 in a spreadsheet similar to the HW1SPYvol.xls. Submit a printout.

9. (5pt) Let X be a continuous random variable taking values between 0 and 4 with probability density function $p(x) = 0.25$. Find E(X), Var(X) and Stdev(X). Plot its Cumulative Distribution Function.

10. (5pt) Suppose that X and Y are two normally distributed random variables. X has mean 1 and standard deviation 2. Y has mean 4 and standard deviation 3. Their correlation is 0.3. What is the mean and standard deviation of X + Y? What is the distribution of X + Y? What if X and Y are jointly normally distributed? What if they are not jointly normally distributed? Explain your answer.

11. (5pt) Suppose you are applying to graduate schools. Your chances to be admitted to each one school are 10% and are the same for any school. To how many different schools you need to apply if you want your chances to be admitted to at least one school to be above 95%.