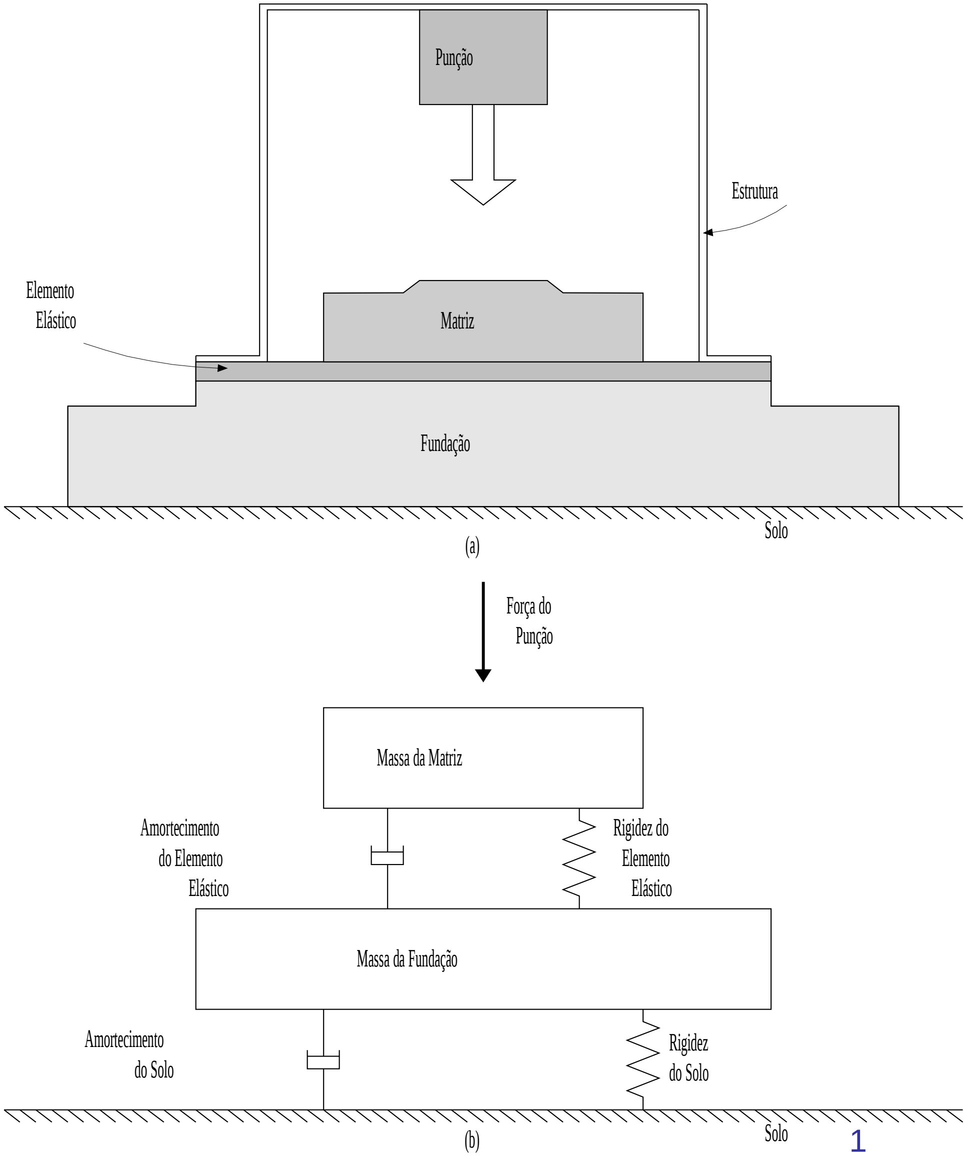
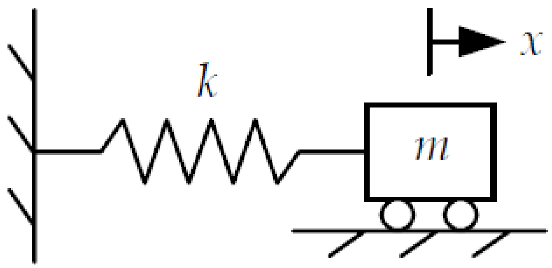
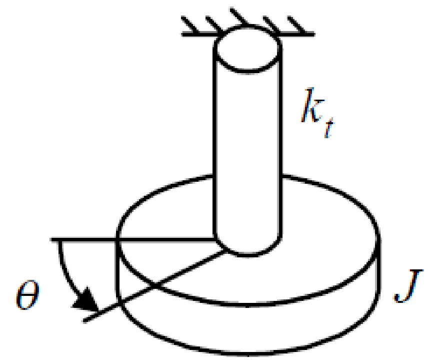
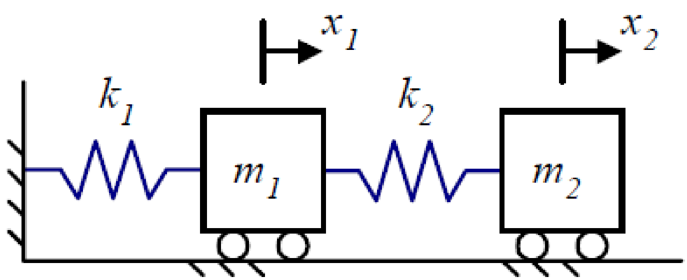
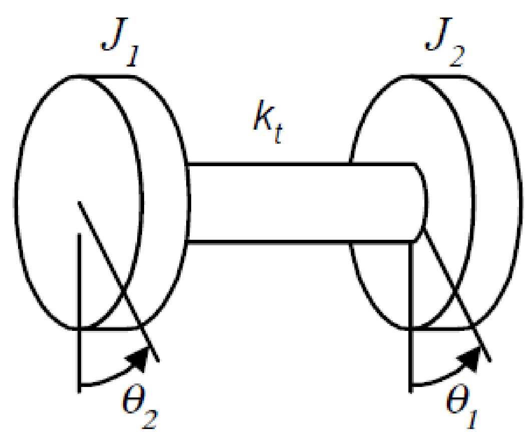
Rigidez mecânica

Sistema vibratório:

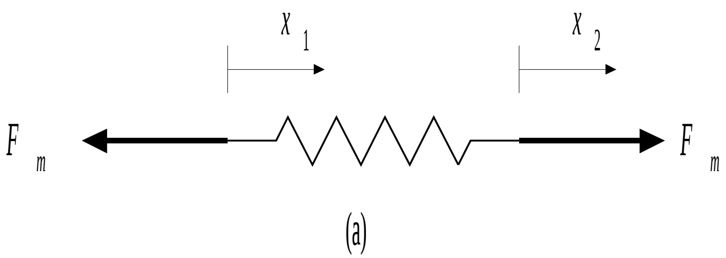
- massa ou inércia (armazena energia cinética)
- mola (armazena energia potencial)
- amortecedor (dissipa energia)

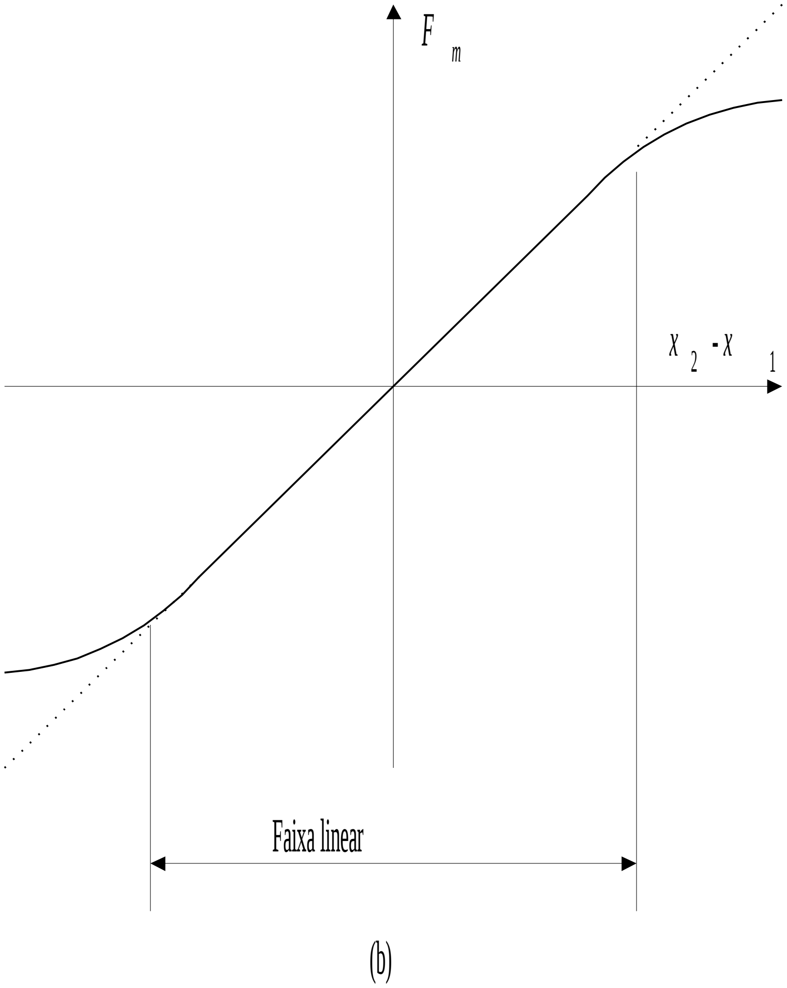


Graus de liberdade

<table>
<tr>
<td>Sistemas de 1 grau de liberdade</td>
<td>
</td>
<td>
</td>
</tr>
<tr>
<td>Sistemas de 2 graus de liberdade</td>
<td>
</td>
<td>
</td>
</tr>
</table>

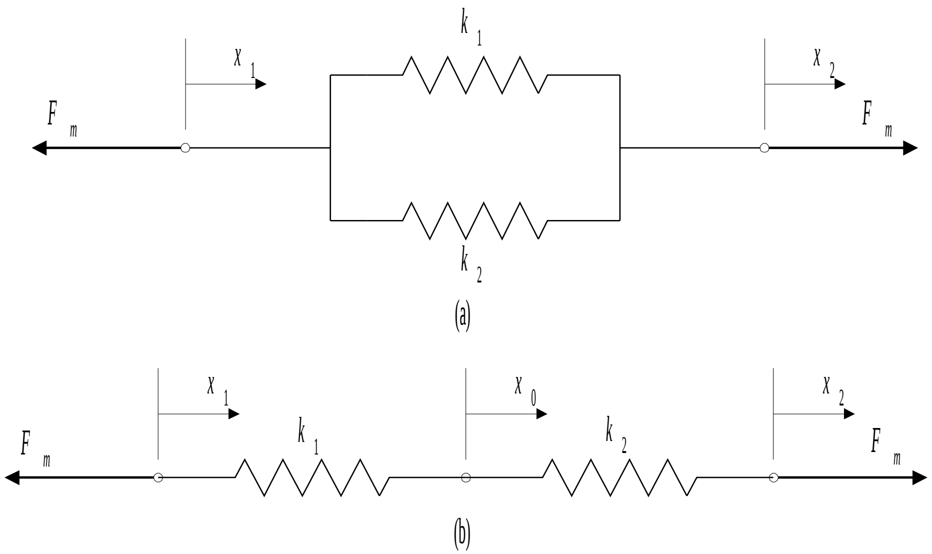
Elemento mola





$$F_m = k\left(x_2 - x_1\right)$$

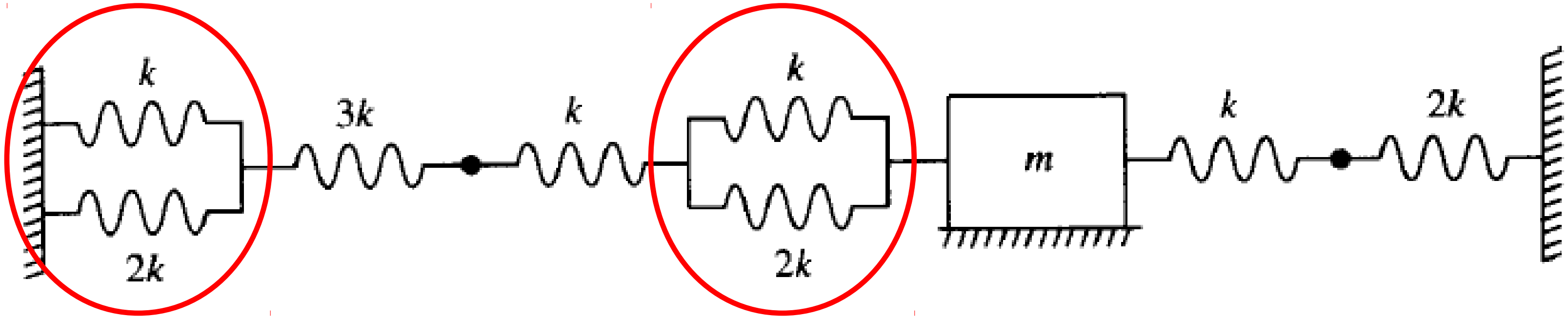
Associação de molas

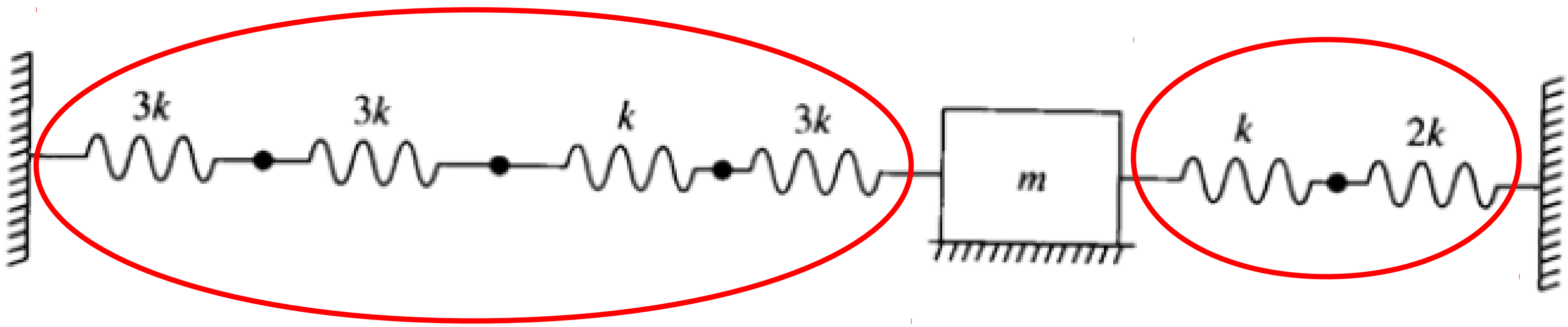


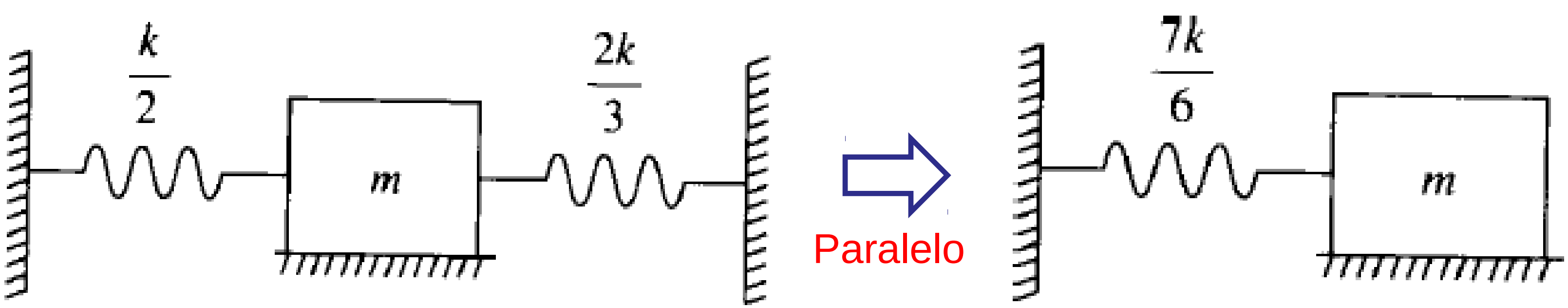
$$k_{eq} = k_1 + k_2 \qquad k_{eq} = \sum_{i=1}^{n} k_i$$

$$k_{eq} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} \qquad k_{eq} = \frac{1}{\sum_{i=1}^{n} \frac{1}{k_i}}$$

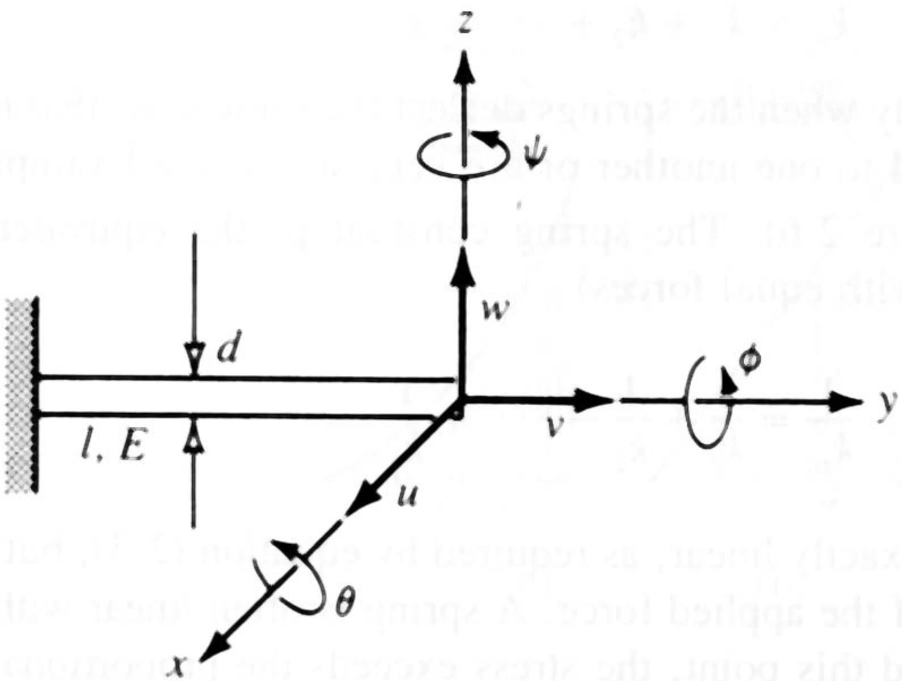
Exemplo







Sistemas contínuos



$$F_v = \frac{EAv}{L} \qquad F_u = \frac{3EI_z u}{L^3} \qquad F_w = \frac{3EI_x w}{L^3}$$

$$k_{vv} = \frac{EA}{L} \qquad k_{uu} = \frac{3EI_z}{L^3} \qquad k_{ww} = \frac{3EI_x}{L^3}$$

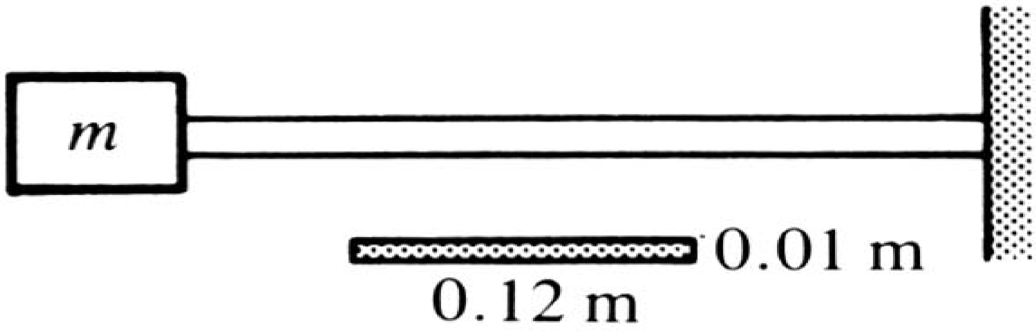
$$M_\varphi = \frac{GI_y \varphi}{L} \qquad M_\psi = \frac{EI_z \psi}{L} \qquad M_\theta = \frac{EI_x \theta}{L}$$

$$k_{\varphi\varphi} = \frac{GI_y}{L} \qquad k_{\psi\psi} = \frac{EI_z}{L} \qquad k_{\theta\theta} = \frac{EI_x}{L}$$

$$u = \frac{FL^3}{3EI} \qquad w = \frac{F_w L^3}{3EI_x} \qquad v = \frac{F_v L}{EA}$$

$$\theta = \frac{M_\theta L}{EI_x} \qquad \psi = \frac{M_\psi L}{EI_z} \qquad \varphi = \frac{M_\varphi L}{GI_y}$$

Viga engastada



$$m_{viga} = 100 \text{ kg}$$

$$m_{eff} = 3,12 \text{ kg}$$

$$L = 1\text{m}$$

$$E = 2,1\, x\, 20^{11}\, N/m^2$$

$$I = \frac{bh^3}{12} = \frac{0,12 \times 0,01^3}{12} = 1 \times 10^{-8} \text{ m}^4$$

$$k = \frac{3EI}{L^3} = \frac{3 \times 2,1 \times 10^{11} \times 10^{-8}}{1^3} = 6300 \text{ N/m}$$

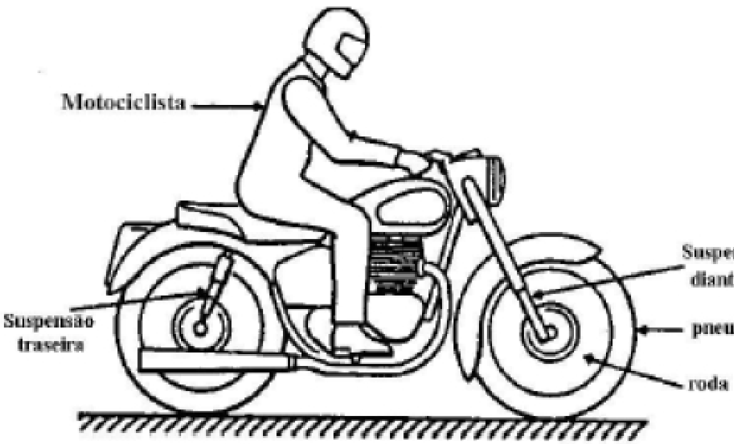
Sem massa da viga

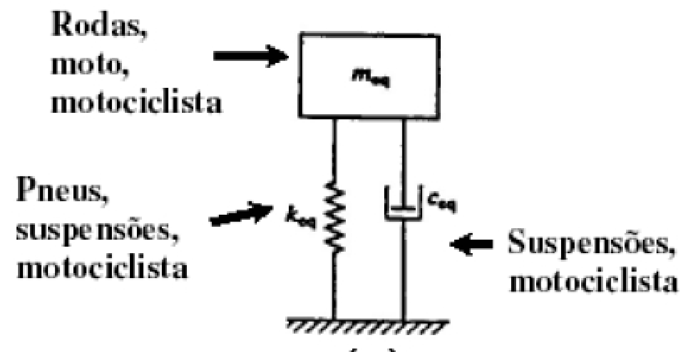
$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{6300}{100}} = 7,94 \text{ rad/s}$$

Com massa da viga

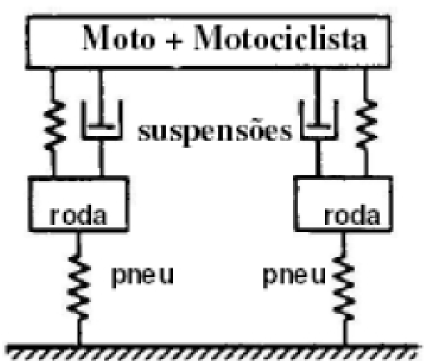
$$\omega_n = \sqrt{\frac{k}{m_{eq}}} = \sqrt{\frac{6300}{100 + 3,12}} = 7,82 \text{ rad/s}$$

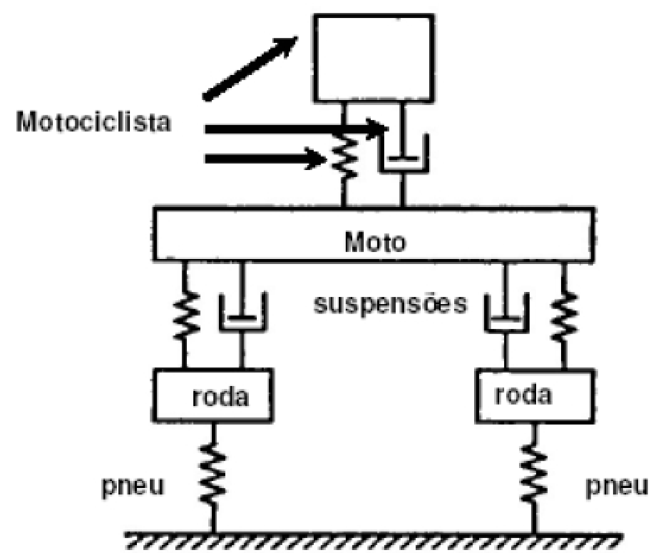
Exemplo prático - Motocicleta



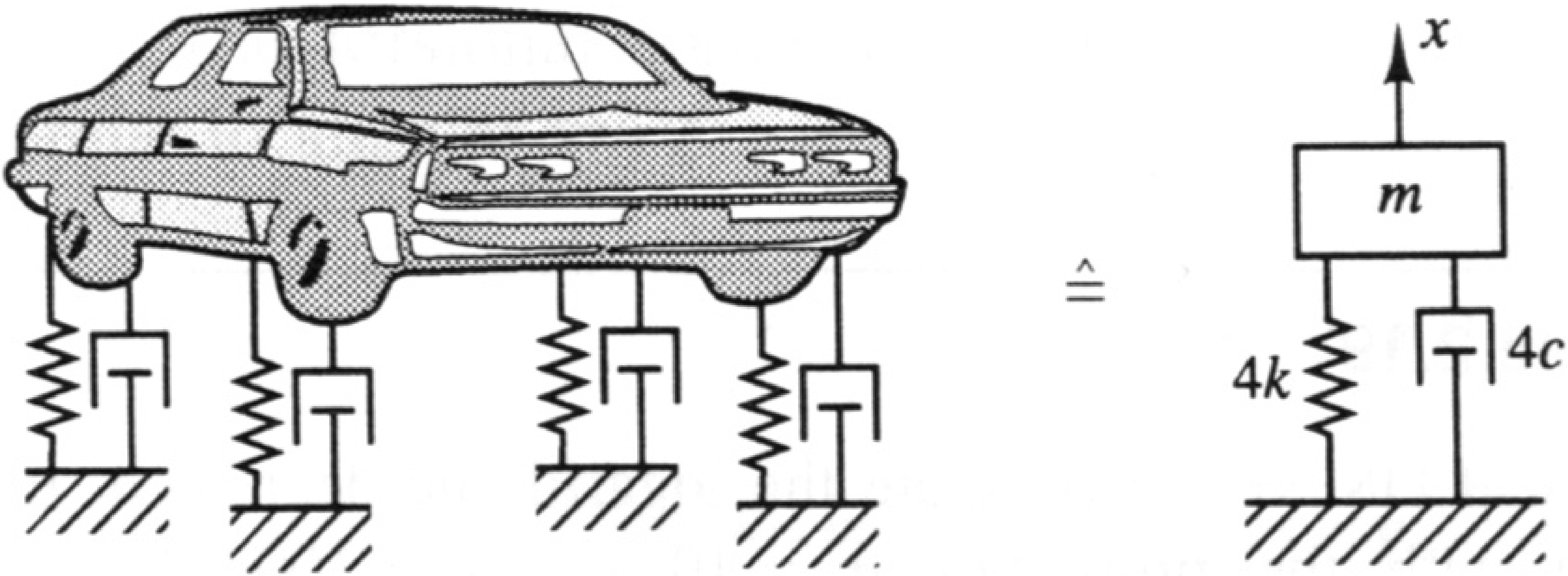


(a)

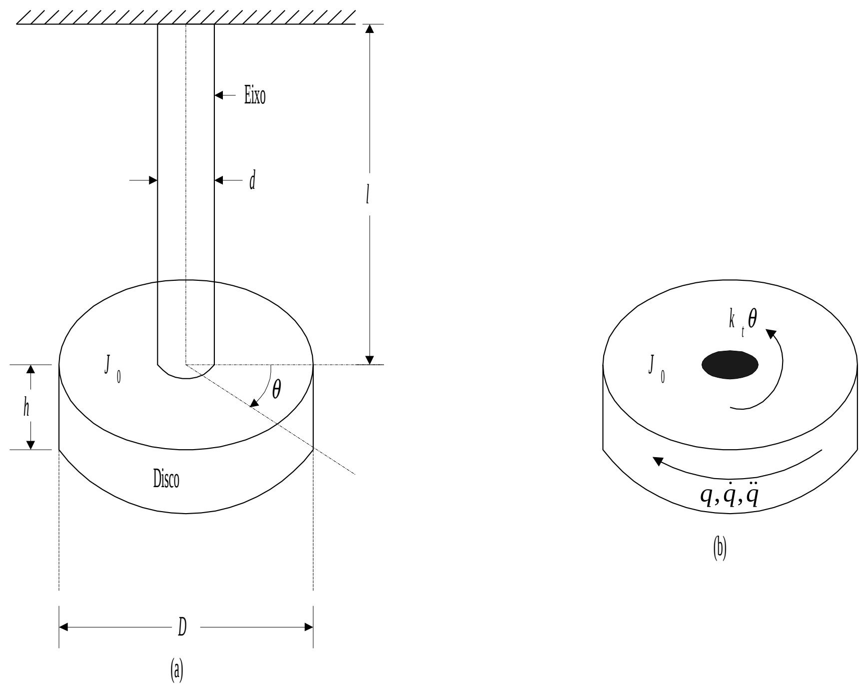




Exemplo prático – Veículo automotor



Rigidez torcional



- Coordenada angular θ
- Momentos
- Torção de eixos circulares

$$\frac{M_t}{q} = \frac{GJ}{l}$$

- Rigidez torcional

$$\boxed{k_t = \frac{M_t}{\theta} = \frac{GJ}{l}}$$

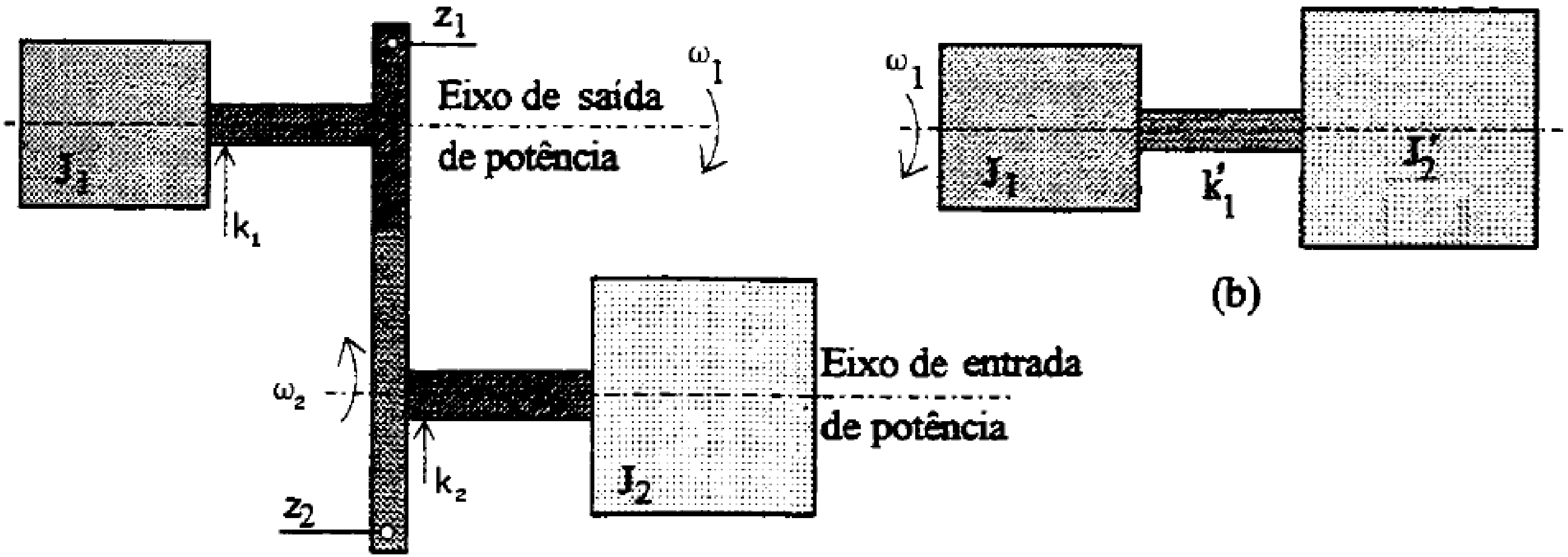
- Eixo vazado:

$$J = \frac{\pi}{32}\left(d_e^4 - d_i^4\right)$$

- Eixo maciço:

$$J = \frac{\pi}{2}\left(r^4\right) = \frac{\pi}{32}\left(d^4\right)$$

Acoplamento de rotores



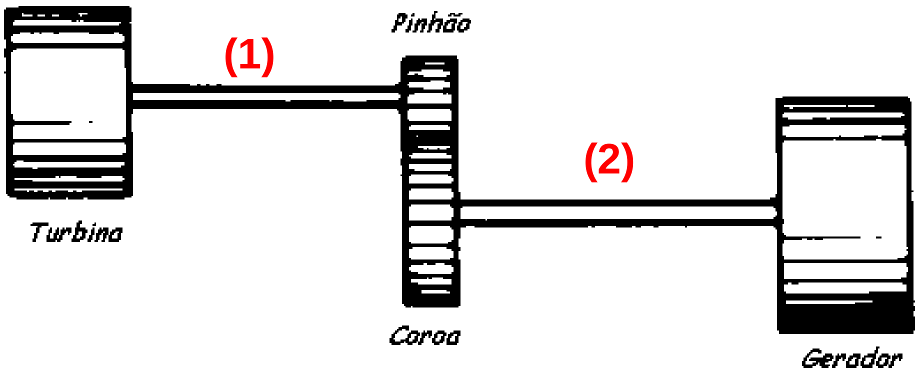
Princípio da Conservação da Energia Potencial elástica

$$\boxed{k_1 = k_2 \frac{\omega_2^2}{\omega_1^2}}$$

Exemplo: Uma turbina, de inércia J1 = 1 kg.m², aciona um gerador, de inércia J2 = 2,4 kg.m² através de um redutor de velocidades cuja inércia é desprezível. Calcular a rigidez do sistema (G = $8{,}4\times10^{10}$ N/m²) em relação ao eixo 2. Dados: eixo 1-Diâmetro 40 mm, comprimento 1m; eixo 2 - Diâmetro 50 mm, comprimento 0,75m;

Veloc. rotação do eixo 2 (gerador) = 1/2 da veloc. do eixo 1 (turbina).

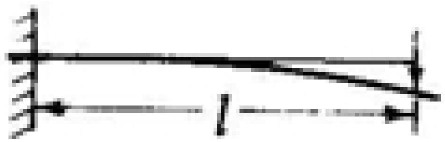
$$K_1 = \frac{8{,}4\times10^{10}\,\dfrac{\pi \times 0{,}040^4}{32}}{1} = 21110\ \text{Nm/rad}$$



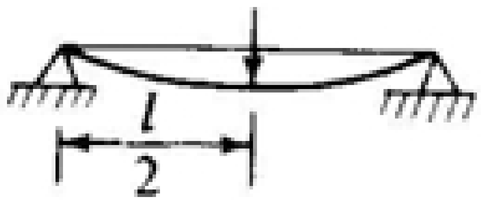
$$K_2 = \frac{8{,}4\times10^{10}\,\dfrac{\pi \times 0{,}050^4}{32}}{0{,}75} = 68720\ \text{Nm/rad}$$

$$K_{2eq} = K_2 + K_1\left(\frac{\omega_1}{\omega_2}\right)^2 = 68720 + 21110 \times \left(\frac{\omega_1}{\omega_1/2}\right)^2 = 153200\ \text{Nm/rad}$$

Cálculo de rigidez mecânica mais comuns



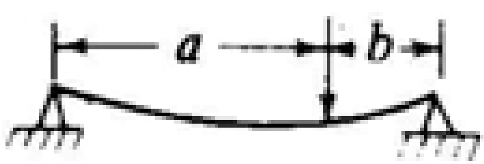
$$k = \frac{3EI}{l^3}$$



$$k = \frac{48EI}{l^3}$$

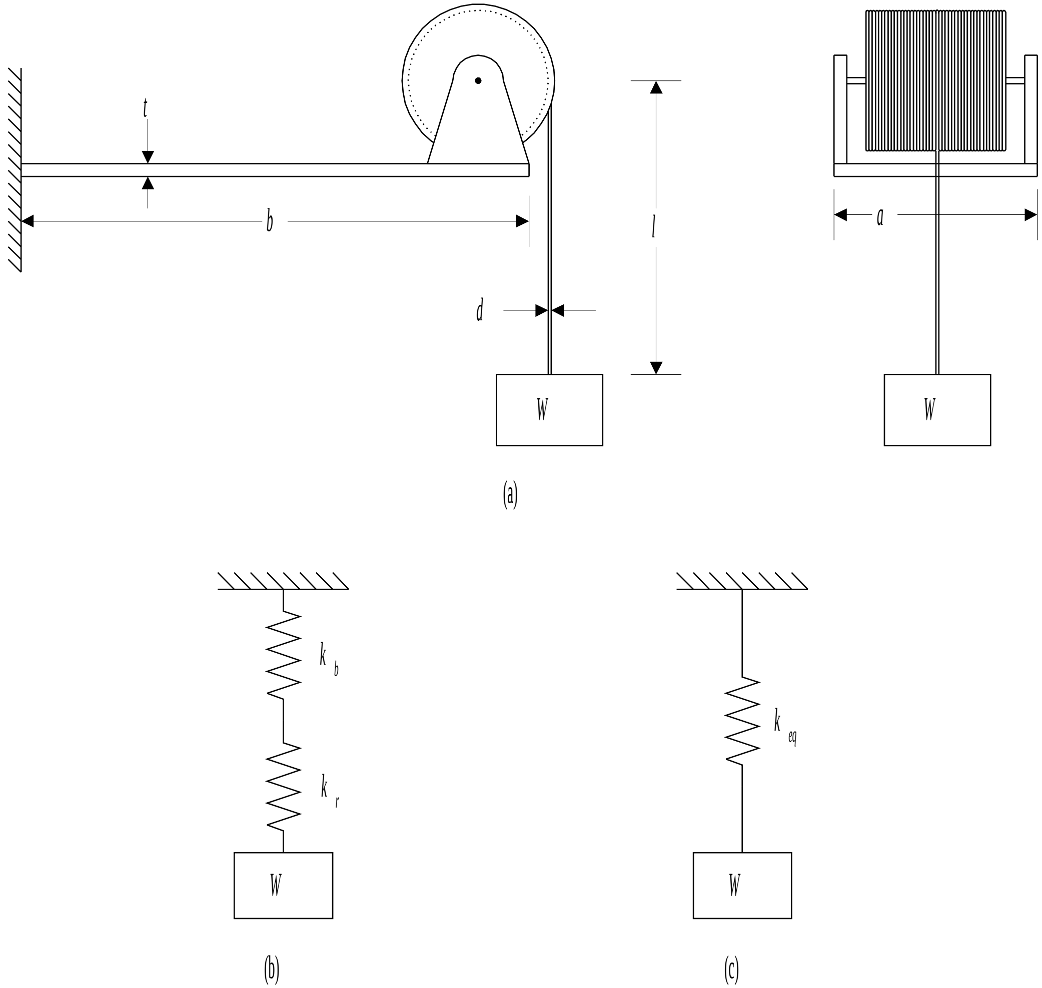
$$k = \frac{192\,EI}{l^3}$$

$$k = \frac{768EI}{7l^3}$$



$$k = \frac{3EIl}{a^2b^2}$$

Sistema de elevação

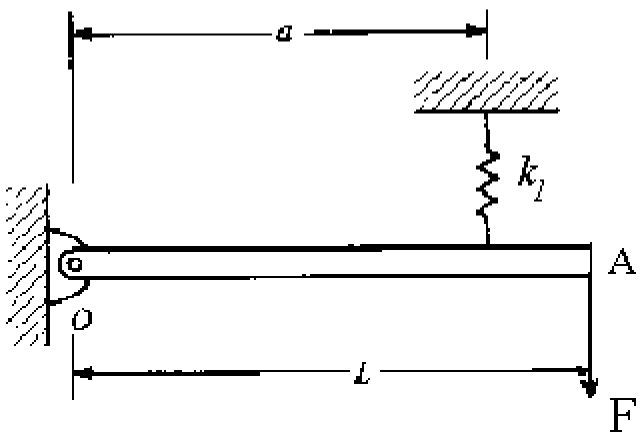


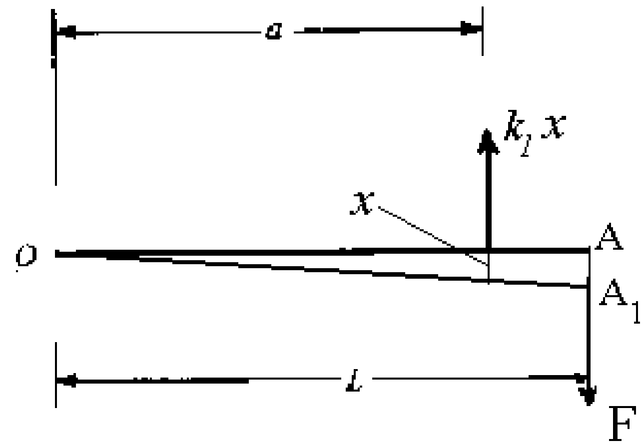
$$k_b = \frac{3EI}{b^3}$$

$$k_r = \frac{EA}{l}$$

$$k_{eq} = \frac{k_b k_r}{k_b + k_r}$$

Sistema com alavanca





Tomando momentos em relação ao ponto O: $FL = k_1 xa$ **(1)**

Sendo k a rigidez da mola equivalente: $F = kx = k\,AA_1$ **(2)**

Por outro lado, a semelhança de triângulos permite escrever: $\frac{x}{AA_1} = \frac{a}{L}$ **(3)**

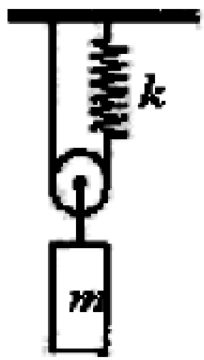
Combinando as expressões acima, chegamos a:

Isola-se o valor de k na eq.(2) = F/AA_1, e substitui o valor de F e AA_1 - eqs. (1) e (3)

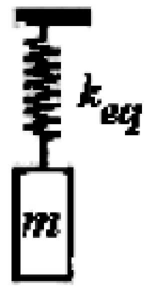
$$k = (a/L)^2 k_1$$

$$\boxed{k_{eq} = \sum_{i=1}^{n} \left(\frac{a}{L}\right)^2 k_i}$$

Rigidez mecânica utilizando polias

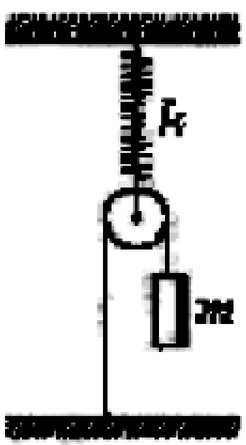


Polia Móvel

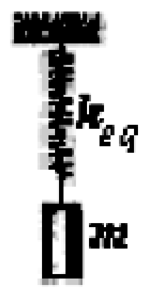


$$\frac{F}{2} = k.2x \Rightarrow F = 4kx$$

$$k_{eq} = 4k$$



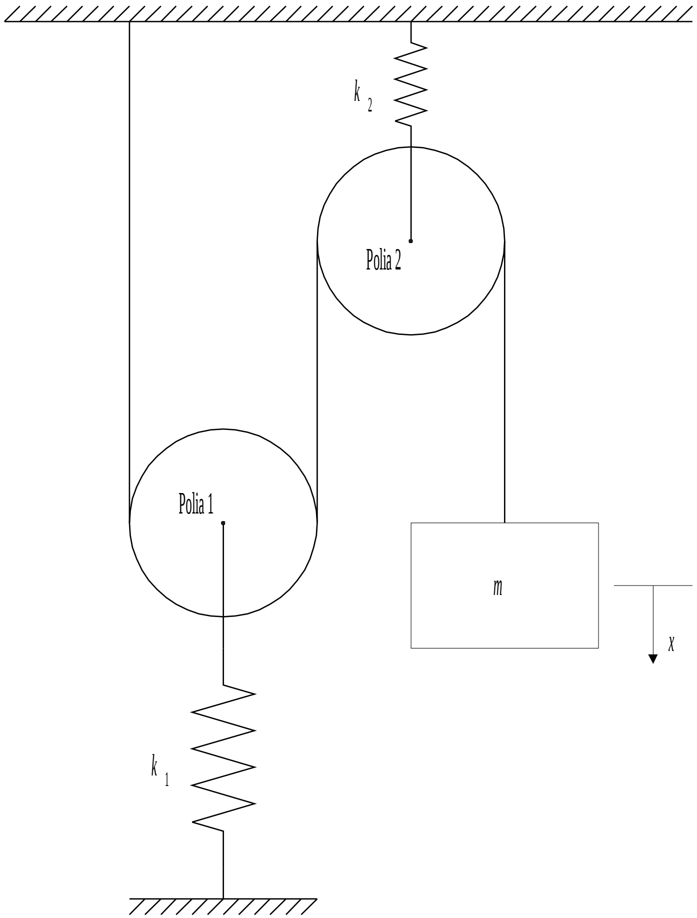
Polia Fixa



$$k_{eq} = \frac{k}{4}$$

$$2F = k\frac{x}{2} \Rightarrow F = \frac{k}{4}x$$

Sistema de elevação com polias



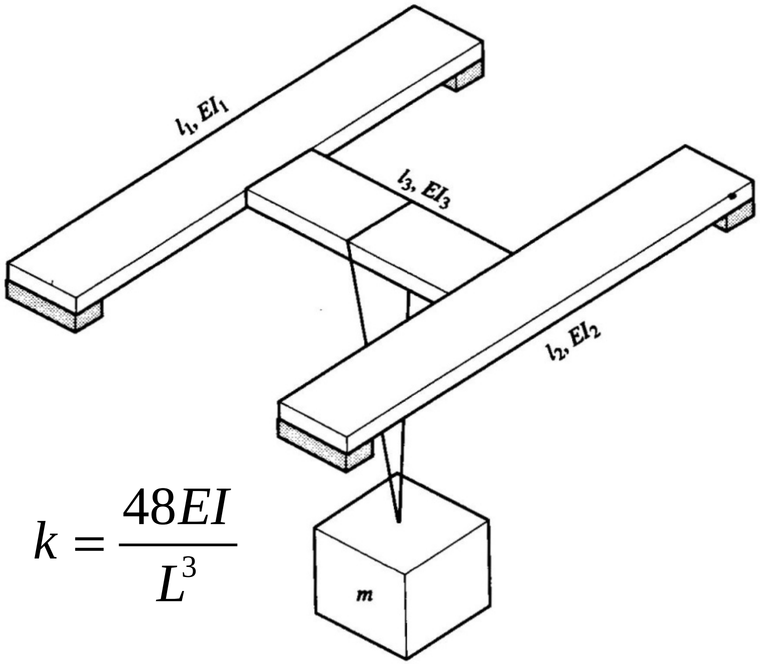
$$k_{1eq} = \frac{k_1}{4}$$

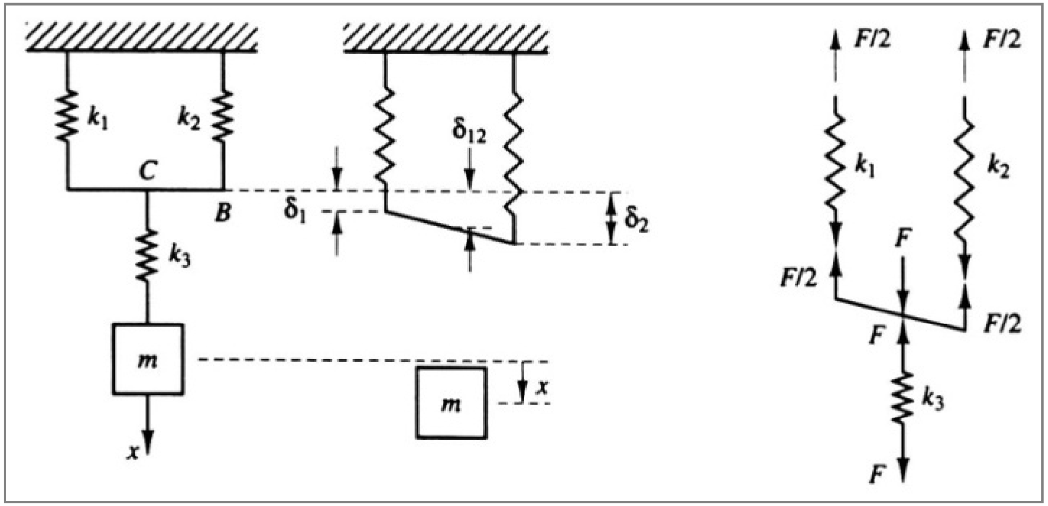
$$k_{2eq} = \frac{k_2}{4}$$

$$k_{eq} = \frac{k_{1eq} \cdot k_{2eq}}{k_{1eq} + k_{2eq}}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k_1 k_2}{4\mathrm{m}\left(k_1 + k_2\right)}} \quad \text{rad/seg}$$

Exemplo 4: Estruturas compostas



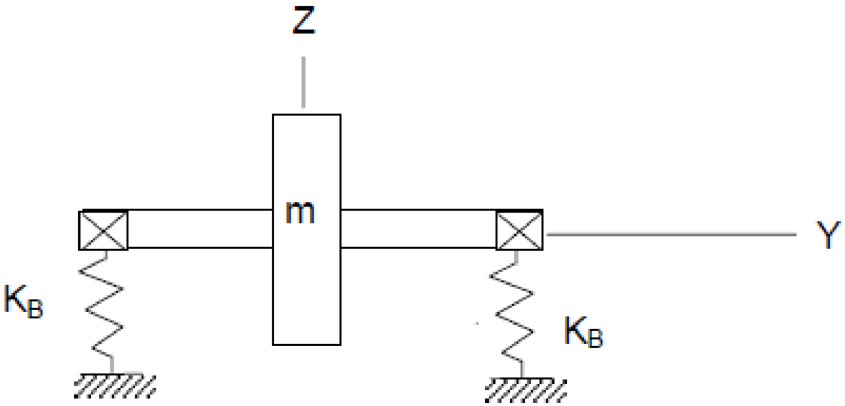


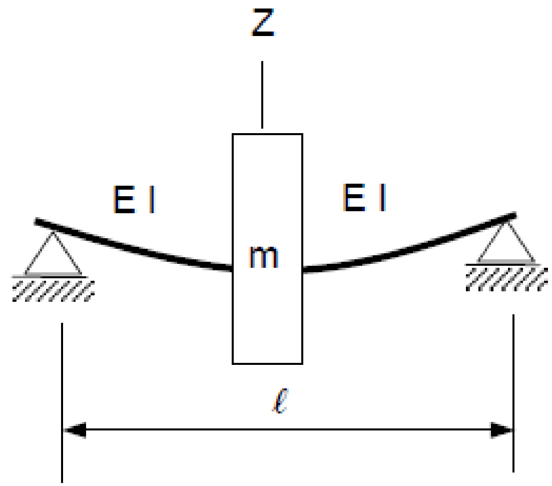
$$k = \frac{48EI}{L^3}$$

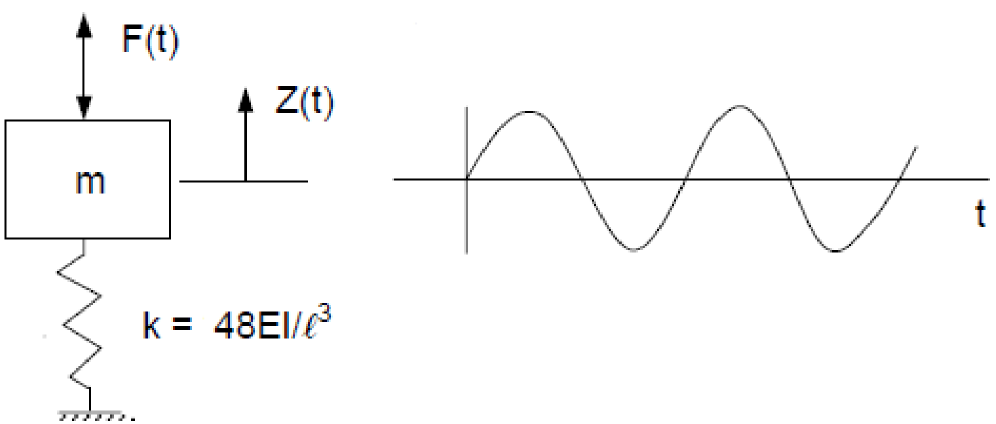
$$\delta_{12} = \frac{\delta_1 + \delta_2}{2}, \qquad \delta_1 = \frac{F/2}{k_1}, \qquad \delta_2 = \frac{F/2}{k_2}, \qquad \delta_3 = \frac{F}{k_3} \qquad \omega_n = \sqrt{\frac{\frac{1}{1/4k_1 + 1/4k_2 + 1/k_3}}{m}}$$

$$x = \delta_{12} + \delta_3 = \frac{\delta_1 + \delta_2}{2} + \delta_3 = \frac{F}{4k_1} + \frac{F}{4k_2} + \frac{F}{k_3} = F\left(\frac{1}{4k_1} + \frac{1}{4k_2} + \frac{1}{k_3}\right)$$

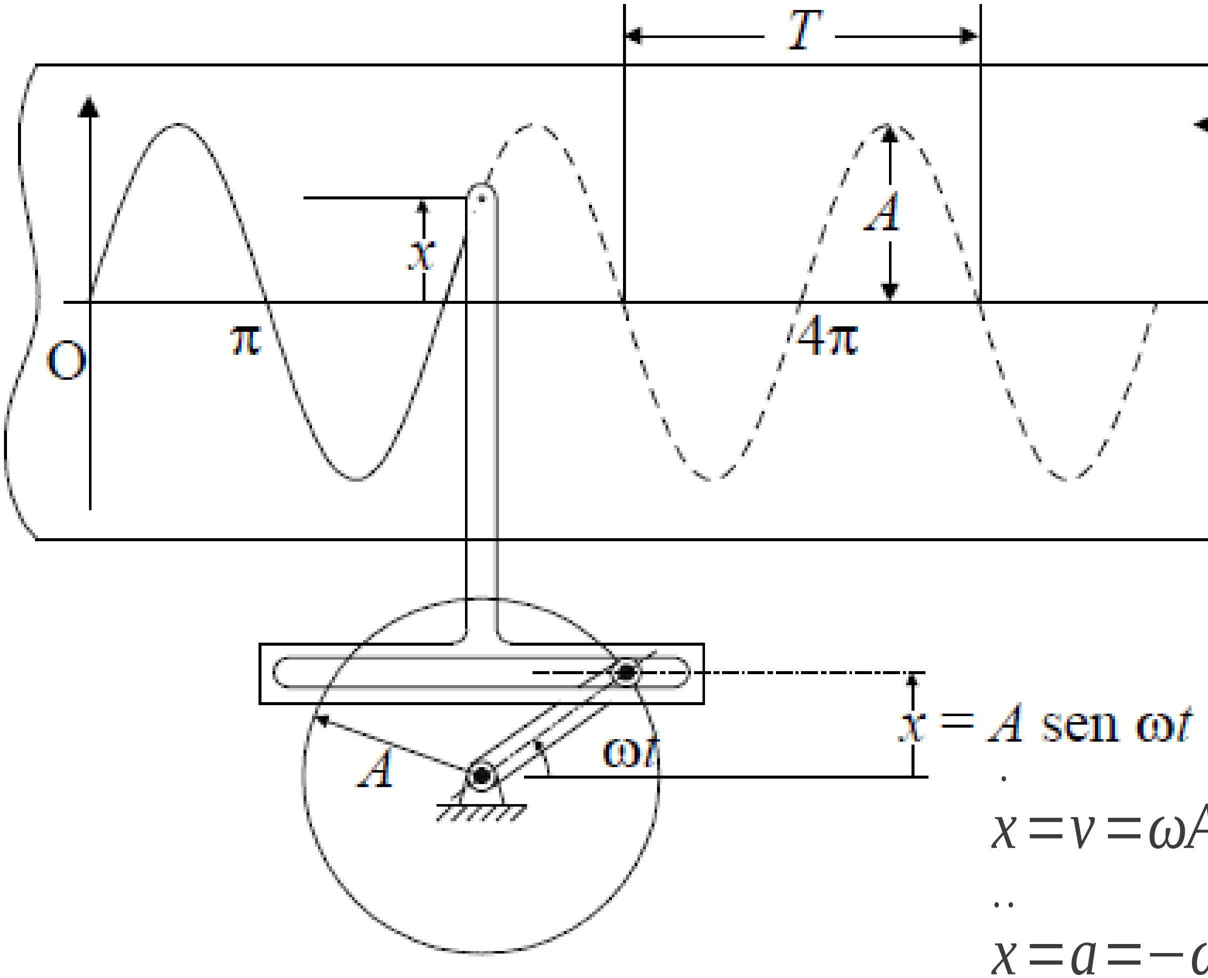
Exemplo: Rigidez de rotores







Movimento Harmônico

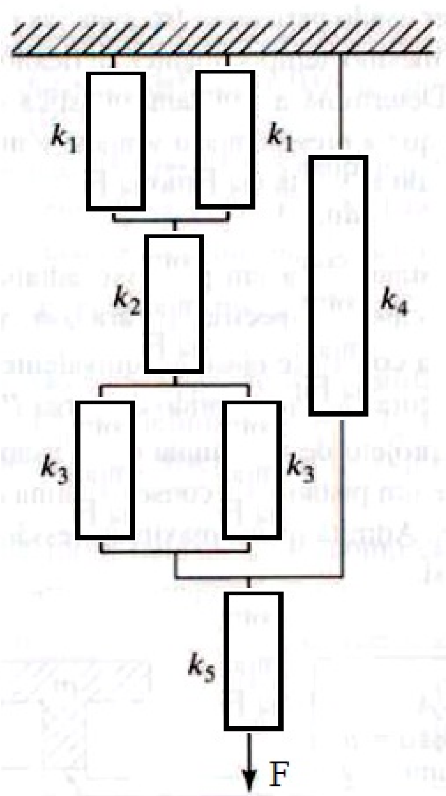


$$\dot{x} = v = \omega A \cos \omega t$$

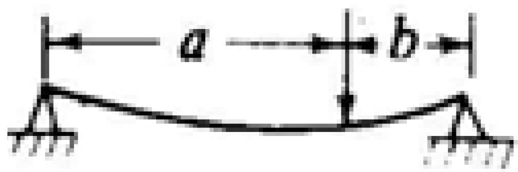
$$\ddot{x} = a = -\omega^2 A sen\, \omega t$$

Exercícios: Determinar a rigidez dos sistema abaixo.

a)



b)



Viga retangular abaixo (10x5 mm) e comprimento 1 m, sendo b = 300 mm e E = 210 GPa.

c)