

# Strength of materials review

## The stress-strain diagram

The engineering stress is determined by dividing the applied load  $P$  by the original cross-sectional area of the specimen,  $A_0$ .

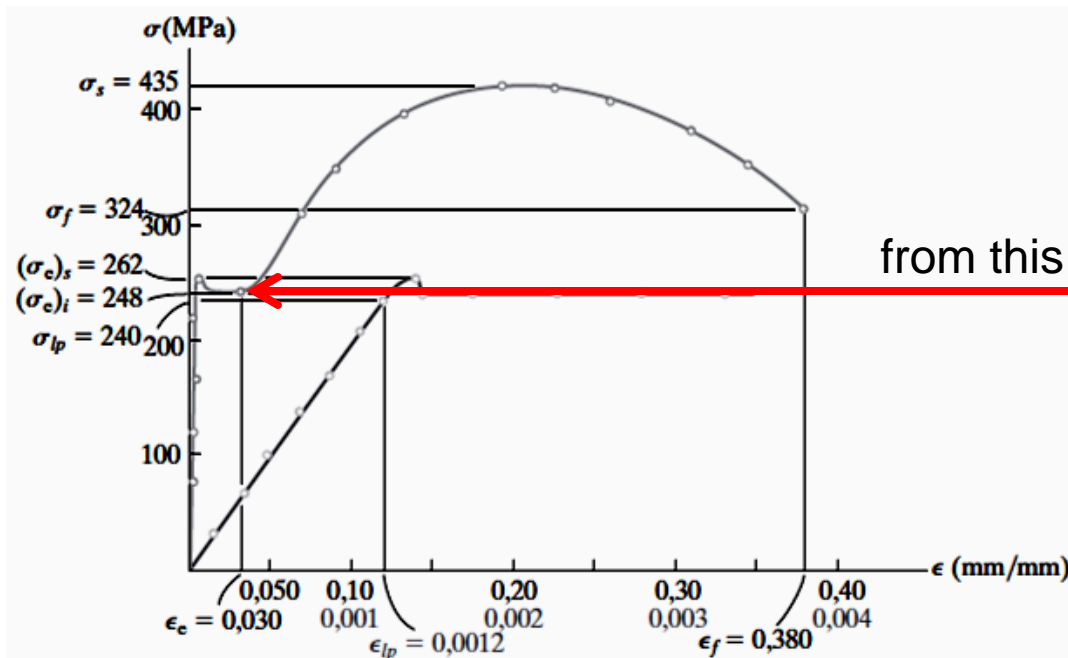
$$\sigma = \frac{P}{A_0}$$

The normal strain is determined by dividing the variation,  $\delta$ , in the reference specimen length, by the original specimen reference length,  $L_0$ .

$$\varepsilon = \frac{\delta}{L_0}$$

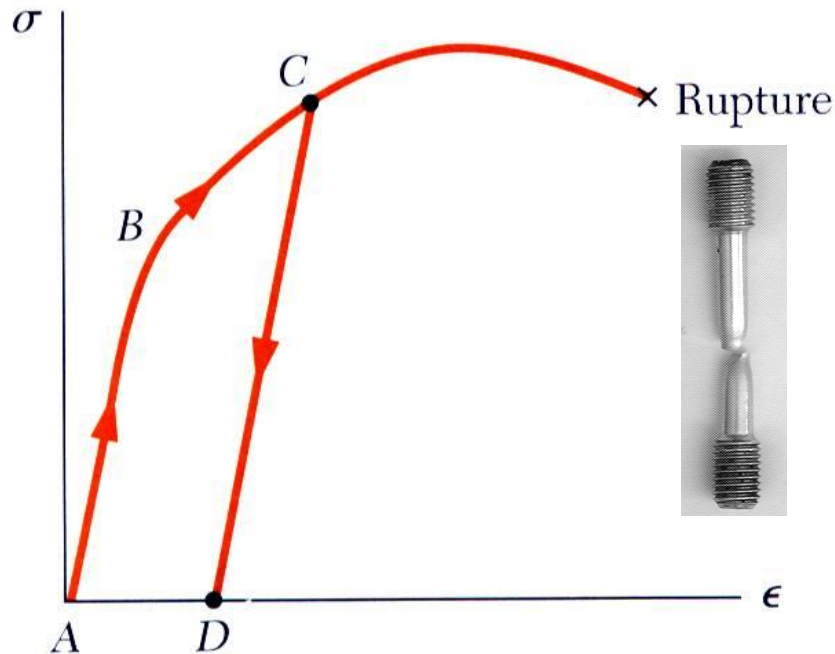
## Real stress – strain diagram

- The values of stress and normal strain are called real stress and real strain. This diagram is widely used in the industrial environment, since most engineering projects are done within the elastic range of the material.



from this point





- If the strain disappears when the stress is removed, the material is said to behave *elastically*.
- The largest stress for which this occurs is called the *elastic limit*.
- When the strain does not return to zero after the stress is removed, the material is said to behave *plastically*.

**Factor of Safety (SF):**

$$SF = \frac{\text{Expected Stress}}{\text{Stress at Component Failure}}$$

## Hooke's Law

- Defines the linear relationship between stress and normal strain within the elastic region.

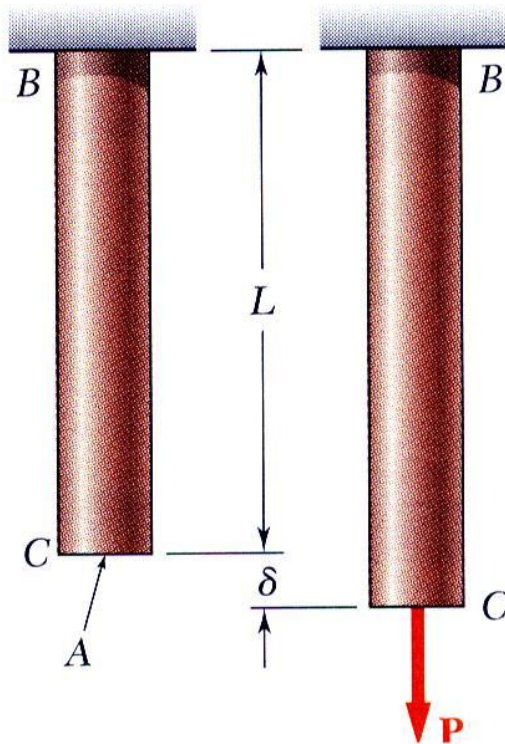
$$\sigma = E\varepsilon$$

$\sigma$  = stress (MPa = N/mm<sup>2</sup>),

$E$  = modulus of elasticity or Young's modulus (MPa)

$\varepsilon$  = normal strain (dimensionless).

## Deformations Under Axial Loading



- From Hooke's Law:

$$\sigma = E\varepsilon \quad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

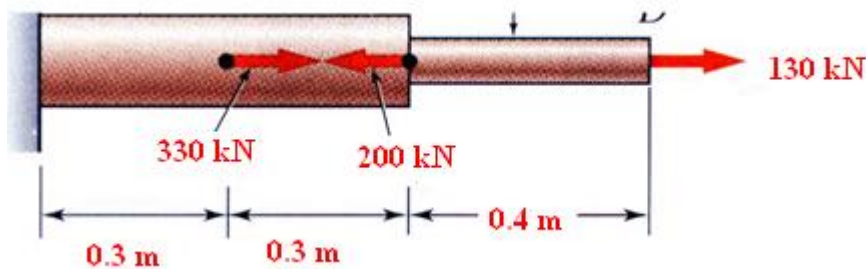
- Equating and solving for the deformation,

$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

## Example 1



$$E = 200 \text{ GPa}$$

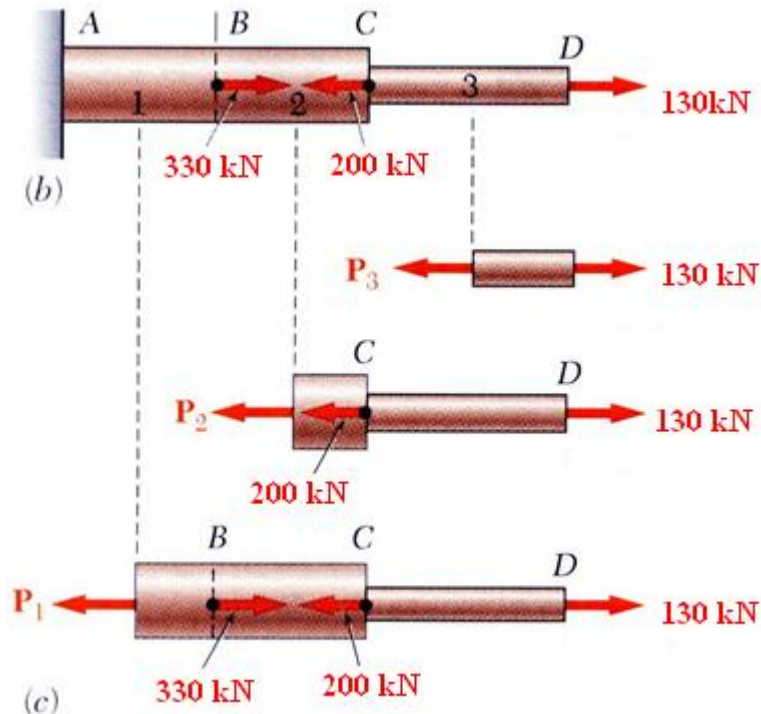
$$D = 27.64 \text{ mm. } d = 15.96 \text{ mm.}$$

Determine the deformation of the steel rod shown under the given loads.

### SOLUTION:

- Divide the rod into components at the load application points.
- Apply a free-body analysis on each component to determine the internal force
- Evaluate the total of the component deflections.

SOLUTION:



$$P_1 = 260 \times 10^3 \text{ N}$$

$$P_2 = -70 \times 10^3 \text{ N}$$

$$P_3 = 130 \times 10^3 \text{ N}$$

Evaluate total deformation,

$$\begin{aligned} \delta &= \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E} \left( \frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3} \right) \\ &= \frac{1}{2 \times 10^5} \left[ \frac{(260 \times 10^3) 300}{600} + \frac{(-70 \times 10^3) 300}{600} + \frac{(130 \times 10^3) 400}{200} \right] \\ &= 1.775 \text{ mm.} \end{aligned}$$

$$L_1 = L_2 = 0.3 \text{ m.}$$

$$L_3 = 0.4 \text{ m.}$$

$$A_1 = A_2 = 600 \text{ mm}^2$$

$$A_3 = 200 \text{ mm}^2$$

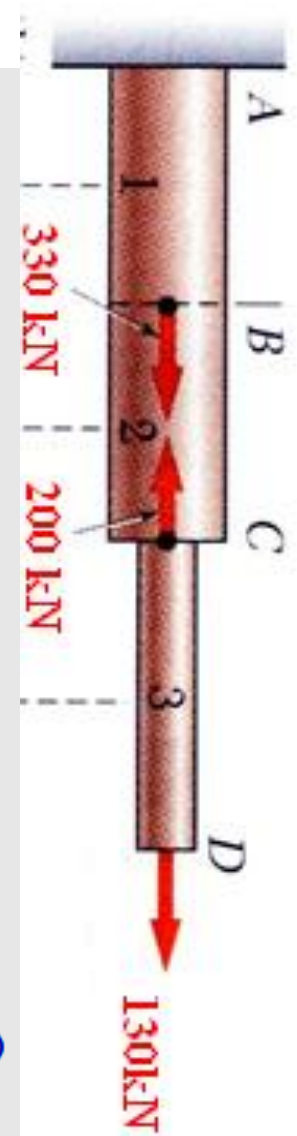
$$\delta = 1.775 \text{ mm.}$$

## Scilab implementation

```

1 clear; clc;
2 E = 200000; // MPa
3 L1 = 300; L2 = L1; L3 = 400; // mm
4 d1 = 27.64; d3 = 15.96; // mm
5 A1 = (%pi*(d1^2))/4; // (mm^2)
6 A3 = (%pi*(d3^2))/4; // (mm^2)
7 A2 = A1;
8
9 P3 = 130000; // (N)
10 P2 = P3-200000; // (N)
11 P1 = P3-200000+330000; // (N)
12
13 // Loads calculation
14 T1 = P1/A1; T2 = P2/A2; T3 = P3/A3; // MPa
15 disp('Load at the section 1 (N) = '); disp(P1)
16 disp('Load at the section 2 (N) = '); disp(P2)
17 disp('Load at the section 3 (N) = '); disp(P3)
18
19 // Total deflection
20 d1 = (P1*L1)/(E*A1); d2 = (P2*L2)/(E*A2); d3 = (P3*L3)/(E*A3)
21 d = d1 + d2 + d3;
22 printf('The deformation of the steel rod is %.3f mm', d);

```

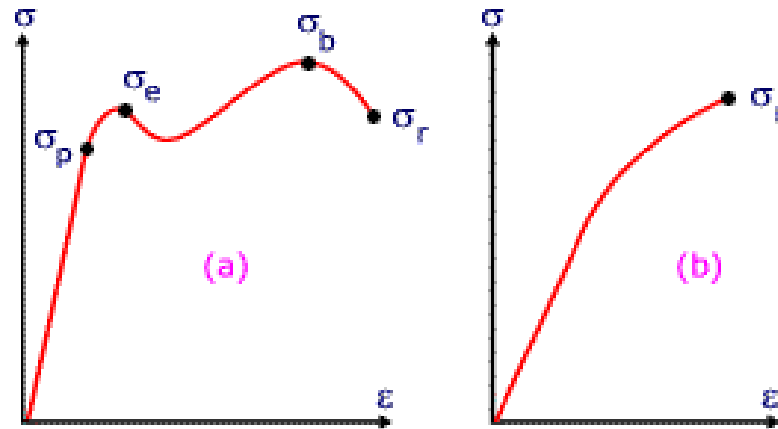




## Stress-strain behavior of ductile and fragile materials

### Ductile materials (a)

- Subjected to major deformations before rupture.



### Fragile materials (b)

- Materials that exhibit little or no flow before failure.

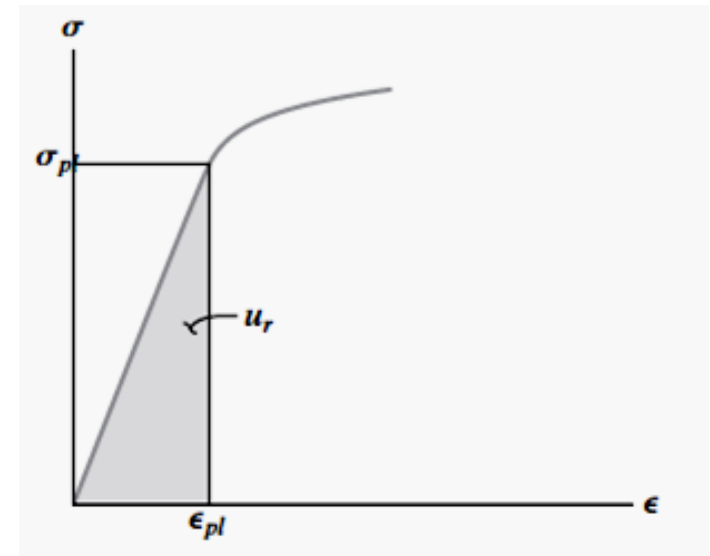
## Deformation energy

- When a material is deformed by an external load, it tends to store energy internally in its entire volume. This energy is related to deformations in the material and is called deformation energy.

### Resilience module

- When the stress reaches the proportionality limit, the strain energy density is called the resilience module,  $u_r$ .

$$u_r = \frac{1}{2} \sigma_{pl} \epsilon_{pl} = \frac{1}{2} \frac{\sigma_{pl}^2}{E}$$



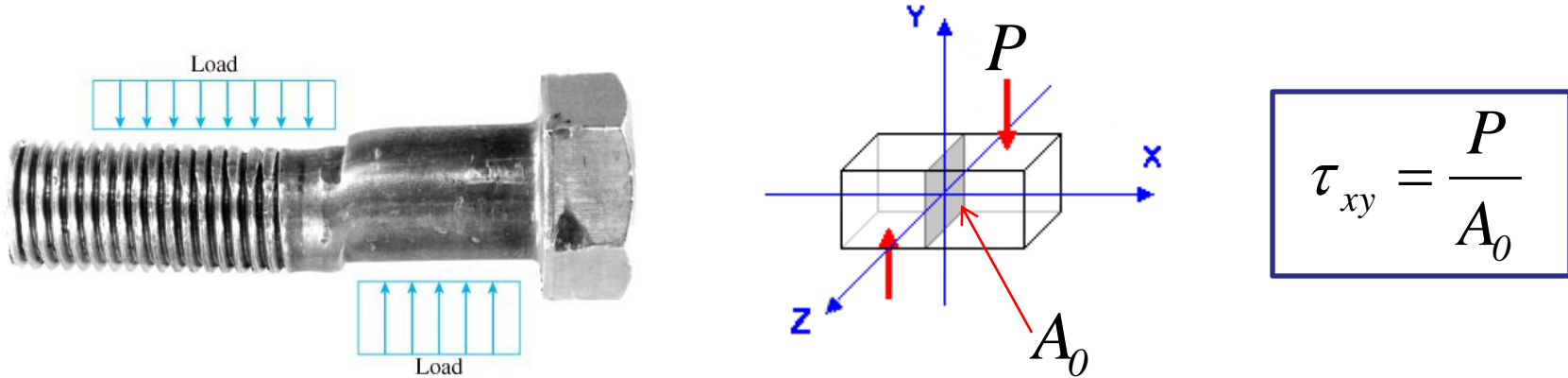
## Poisson' ratio

**Poisson' ratio**,  $\nu$ , establishes that, within the elastic range, the ratio between deformations is constant, since these are proportional.

$$\nu = - \frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

- The above equation has a negative sign because longitudinal elongation (positive deformation) causes lateral contraction (negative deformation) and vice versa.

## Shear stress



Shear stress is the ratio of the tangential force (P) to the cross sectional area (A<sub>0</sub>). If the material is homogeneous and isotropic, the shear stress distorts the element evenly.

## Shear strain and stiffness modulus

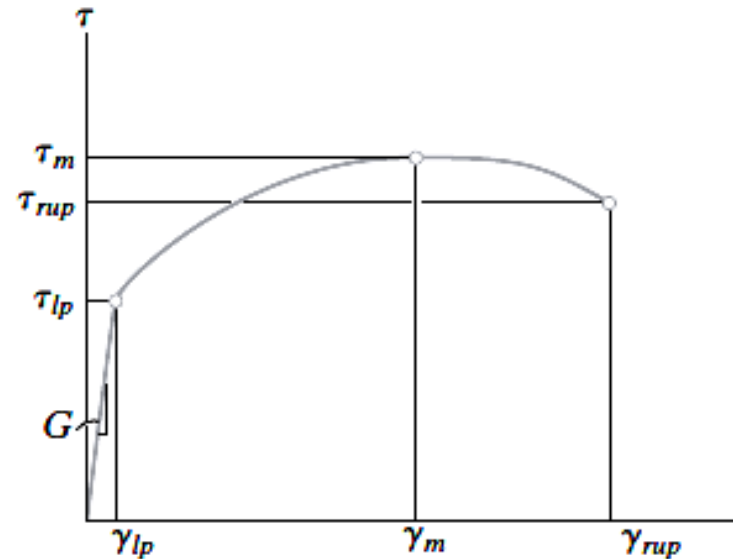
- Most engineering materials exhibit linear elastic behavior, so Hooke's law for shear can be expressed by the equation below.

$$\tau = G\gamma$$

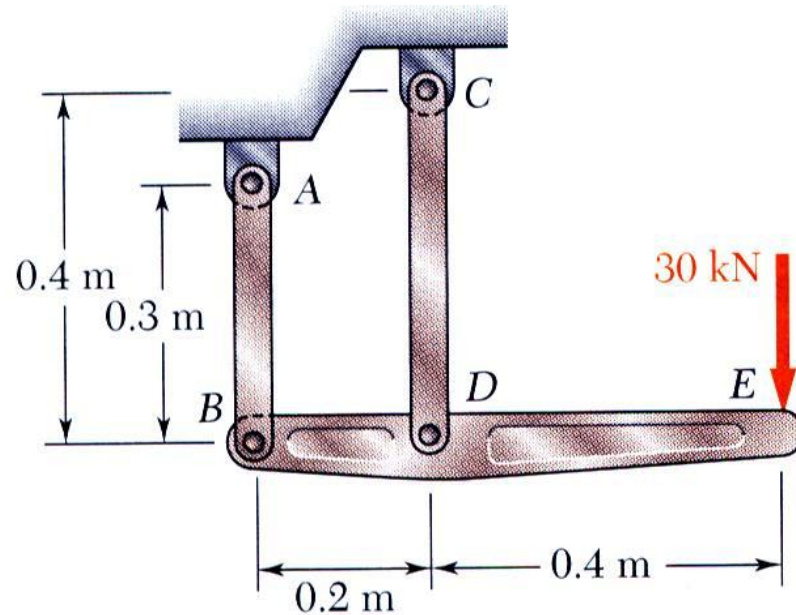
- Three material constants,  $E$ ,  $\nu$  and  $G$  are related by the equation:

$$G = \frac{E}{2(1 + \nu)}$$

$G$  = transverse elastic modulus or stiffness modulus.



## Example 2



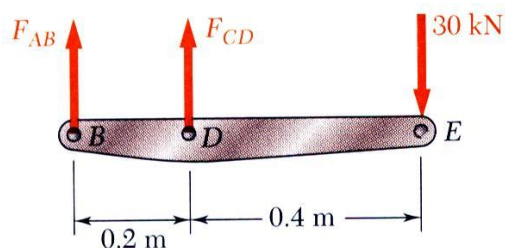
The rigid bar  $BDE$  is supported by two links  $AB$  and  $CD$ .

Link  $AB$  is made of aluminum ( $E = 70\text{ GPa}$ ) and has a cross-sectional area of  $500\text{ mm}^2$ . Link  $CD$  is made of steel ( $E = 200\text{ GPa}$ ) and has a cross-sectional area of  $(600\text{ mm}^2)$ .

For the  $30\text{ kN}$  force shown, determine the deflection of  $B$ ,  $D$  and  $E$ .

SOLUTION:

Free body: Bar *BDE*



$$+\circlearrowleft \sum M_B = 0$$

$$0 = -(30 \text{ kN} \times 0.6 \text{ m}) + F_{CD} \times 0.2 \text{ m}$$

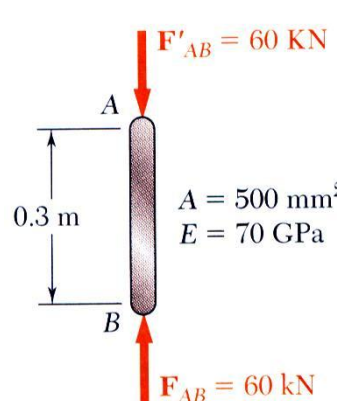
$$F_{CD} = +90 \text{ kN} \text{ tension}$$

$$+\circlearrowleft \sum M_D = 0$$

$$0 = -(30 \text{ kN} \times 0.4 \text{ m}) - F_{AB} \times 0.2 \text{ m}$$

$$F_{AB} = -60 \text{ kN} \text{ compression}$$

Displacement of *B*:



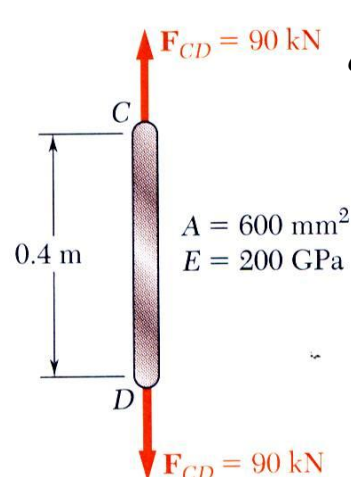
$$\delta_B = \frac{PL}{AE}$$

$$= \frac{(-60 \times 10^3 \text{ N})(0.3 \text{ m})}{(500 \times 10^{-6} \text{ m}^2)(70 \times 10^9 \text{ Pa})}$$

$$= -514 \times 10^{-6} \text{ m}$$

$$\delta_B = 0.514 \text{ mm} \uparrow$$

Displacement of *D*:

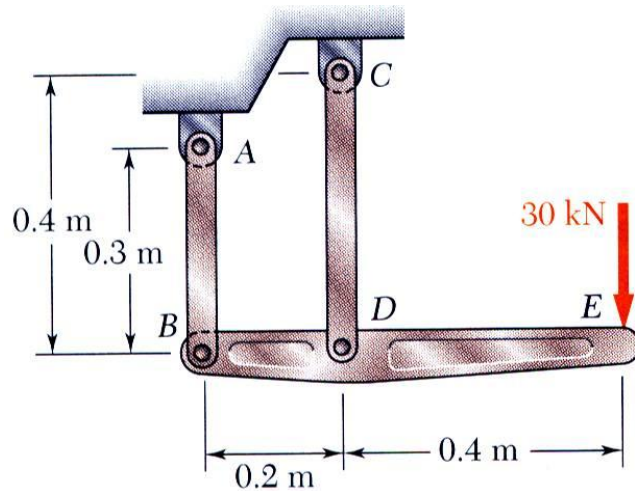


$$\delta_D = \frac{PL}{AE}$$

$$= \frac{(90 \times 10^3 \text{ N})(0.4 \text{ m})}{(600 \times 10^{-6} \text{ m}^2)(200 \times 10^9 \text{ Pa})}$$

$$= 300 \times 10^{-6} \text{ m}$$

$$\delta_D = 0.300 \text{ mm} \downarrow$$



Displacement of E:

$$\frac{BB'}{DD'} = \frac{BH}{HD}$$

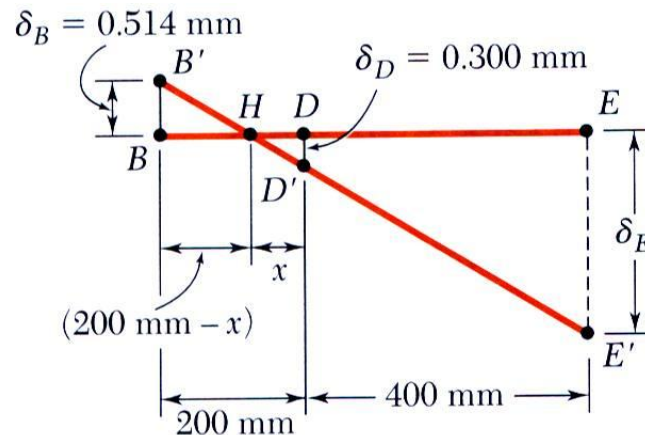
$$\frac{0.514 \text{ mm}}{0.300 \text{ mm}} = \frac{(200 \text{ mm}) - x}{x}$$

$$x = 73.7 \text{ mm}$$

$$\frac{EE'}{DD'} = \frac{HE}{HD}$$

$$\frac{\delta_E}{0.300 \text{ mm}} = \frac{(400 + 73.7) \text{ mm}}{73.7 \text{ mm}}$$

$$\delta_E = 1.928 \text{ mm}$$



$$\delta_E = 1.928 \text{ mm} \downarrow$$

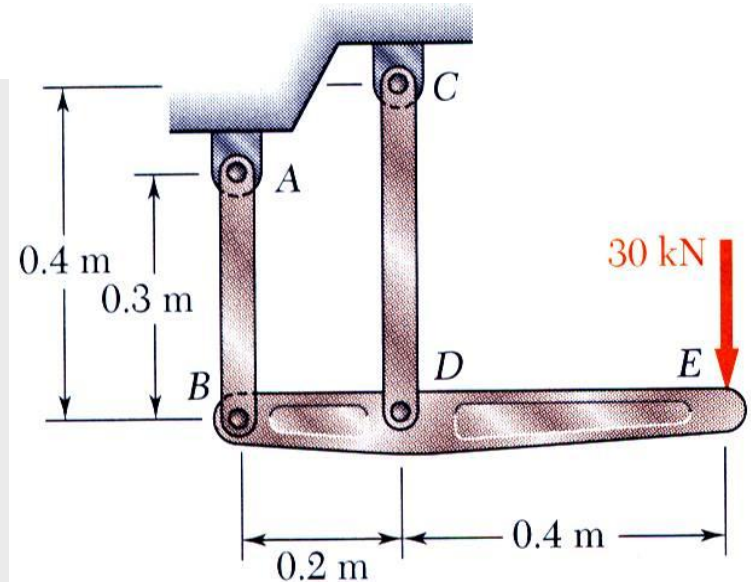


## Scilab implementation

```

1 clear; clc
2 // Link AB
3 E_AB = 70*1000; // Elasticity modulus (MPa)
4 A_AB = 500; // cross-sectional area (mm2)
5 Lab = 300; // length of link AB (mm)
6 // Link CD
7 E_CD = 200*1000; // Elasticity modulus (MPa)
8 A_CD = 600; // cross-sectional area (mm2)
9 Lcd = 400; // length of link CD (mm)
10
11 Fe = 30*1000; // Load applied to E (N)
12 Dde = 400; // Distance from D to E (mm)
13 Dbd = 200; // Distance from B to D (mm)
14 Dbe = Dde + Dbd; // Distance from B to E (mm)
15
16 // Sum of moments in B = 0 => Fe*Dbe - Fcd*Dbd = 0
17 Fcd = (Fe*Dbe) / Dbd; // N
18 disp(Fcd)
19 // Sum of moments in D = 0 => Fe*Dde + Fab*Dbd = 0
20 Fab = -(Fe*Dde) / Dbd; // N
21 disp(Fab)

```

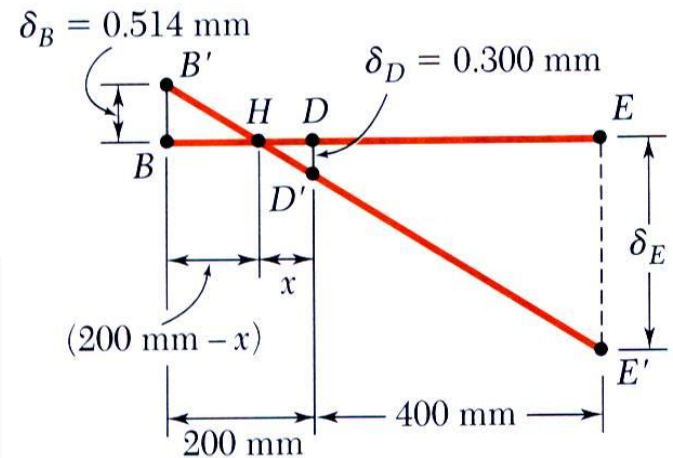


## Scilab implementation

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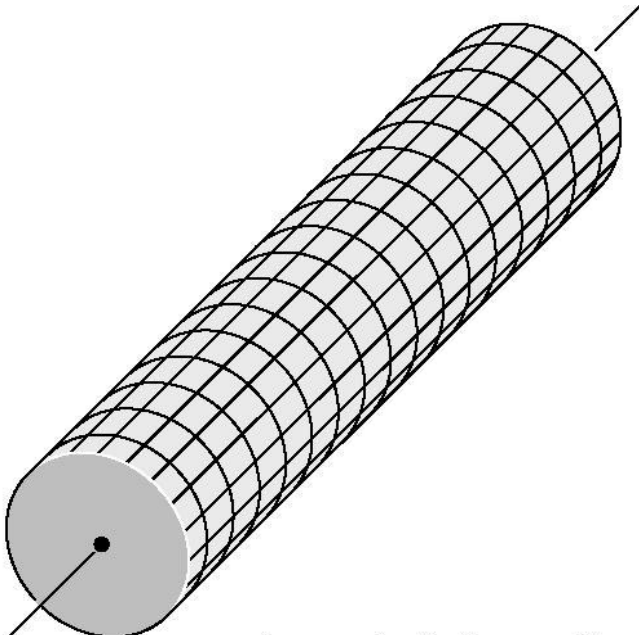
23 // Displacement of B (mm)
24 dispB = (Fab*Lab) / (A_AB*E_AB); // delta L = F*L/A*E
25 disp('dispB = '); disp(dispB)
26
27 // Displacement of D (mm)
28 dispD = (Fcd*Lcd) / (A_CD*E_CD); // delta L = F*L/A*E
29 disp('dispD = '); disp(dispD)
30
31 // Displacement of BE (mm) : dispB/dispD = (Dbd - x)/x and dispE/dispD = (Dde+x)/x
32 x = Dbd / ((abs(dispB)/dispD) + 1)
33 dispE = ((Dde+x)/x) * dispD
34 printf('The deformation of the steel rod is %.3f mm', dispE);

```

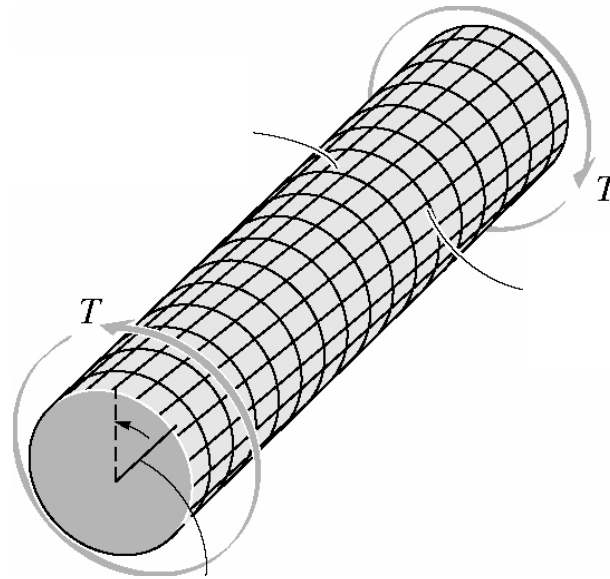


## Torsion

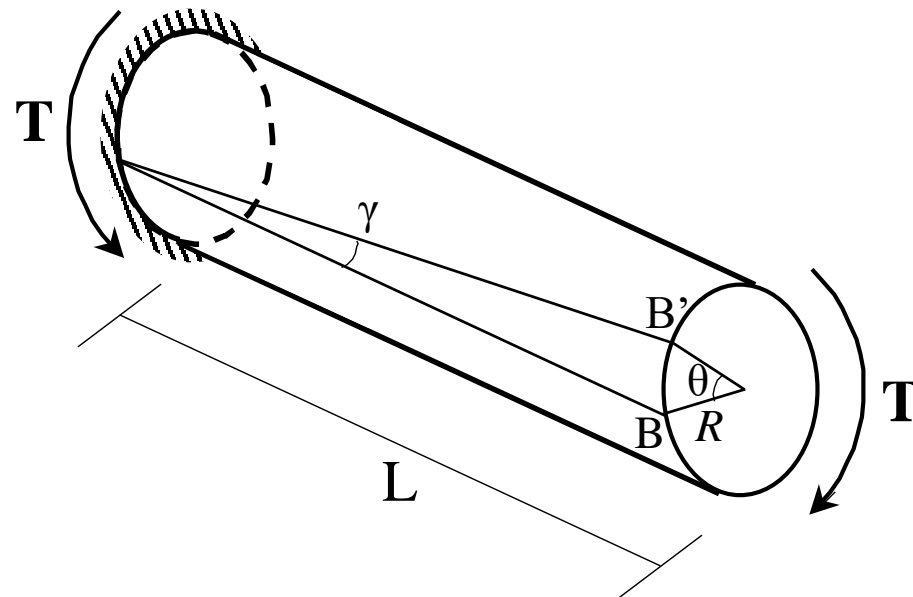
- before loading



- after loading 'T'



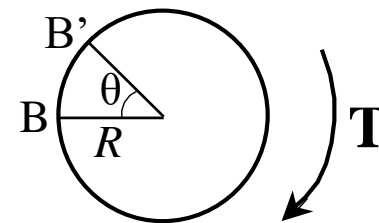
## Torsion angle



$\theta$  = torsion angle (rad)

$T$  = torque

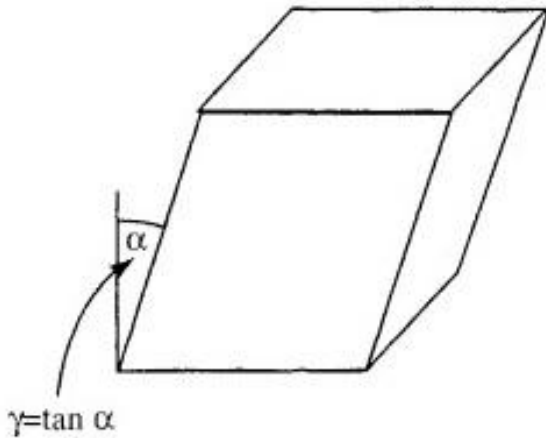
$$BB' = \theta \cdot R$$



$$\operatorname{tg} \gamma \cong \gamma = \frac{BB'}{L}$$

$$\operatorname{tg} \theta \cong \theta = \frac{BB'}{R}$$

## Shear deformation



In the case of torsion:

$$\operatorname{tg} \gamma \cong \gamma = \frac{BB'}{L}$$

$$BB' = \theta.R$$

$$\gamma = \frac{\theta.r}{L} \quad \gamma_{m\acute{a}x} = \frac{\theta.R}{L}$$

Isolating the term  $\theta/L$  in the two previous equations and achieving equality between them, we have:

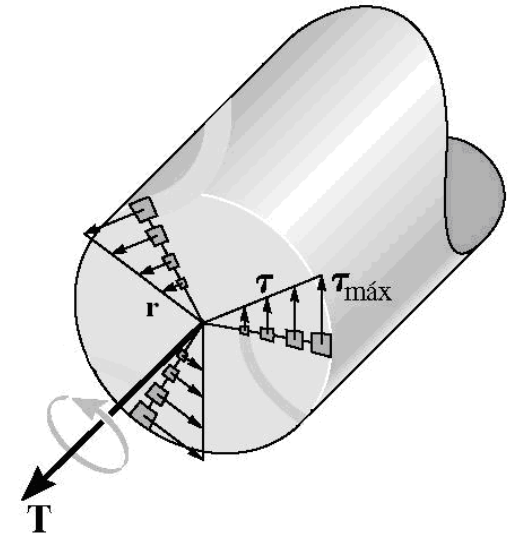
$$\gamma = \frac{r}{R} \gamma_{m\acute{a}x}$$

## Shear stress

Applying Hooke's law for shear ( $\tau = G \cdot \gamma$ ),

we have:

$$\tau = \frac{r}{R} \tau_{m\acute{a}x}$$



Knowing that :  $dF = \tau \cdot dA$  , we have:

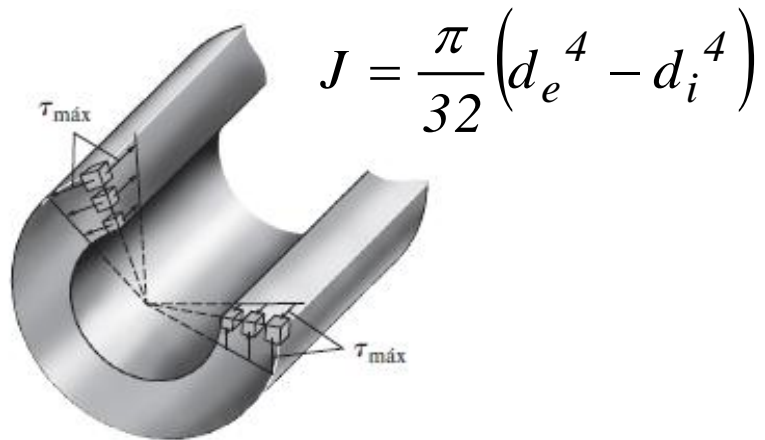
$$T = \int \tau r dA = \int \frac{r}{R} \tau_{m\acute{a}x} r dA = \frac{\tau_{m\acute{a}x}}{R} \int r^2 dA \quad \left( J = \int r^2 dA \right)$$

$$T = \frac{\tau_{m\acute{a}x}}{R} J \quad \Rightarrow \quad \tau_{m\acute{a}x} = \frac{T R}{J} \quad \Rightarrow$$

$$\tau = \frac{T r}{J}$$

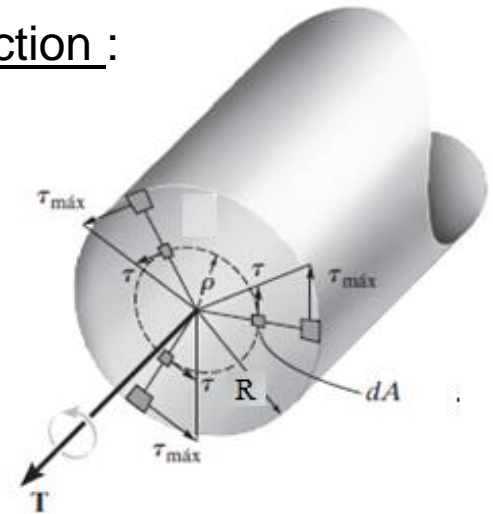
## Polar moment of inertia for shafts with circular cross section :

- Hollow shaft :



- Solid shaft:

$$J = \frac{\pi}{2} (r^4) = \frac{\pi}{32} (d^4)$$

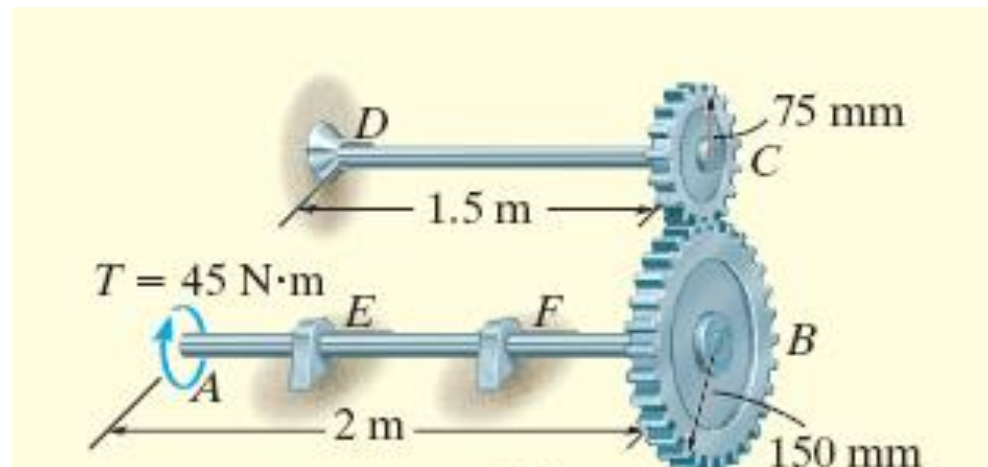


And applying Hooke's law for shear  $\tau = G \cdot \gamma$  to the Equation  $\tau = \frac{T r}{J}$

and knowing that  $\gamma = \frac{\theta \cdot r}{L}$ , we have:  $\theta = \frac{TL}{JG}$

## Example

Two steel shafts are interconnected by means of gears. Determine the torsion angle in C when a torque  $T$  is applied to A as shown in the picture. Consider that the AB axis is solid, non-deformable and free to rotate inside the E and F bearings, while the DC axis is hollow and fixed in D (external and internal diameters are 20 mm and 10 mm, respectively). Consider  $G = 80$  GPa.



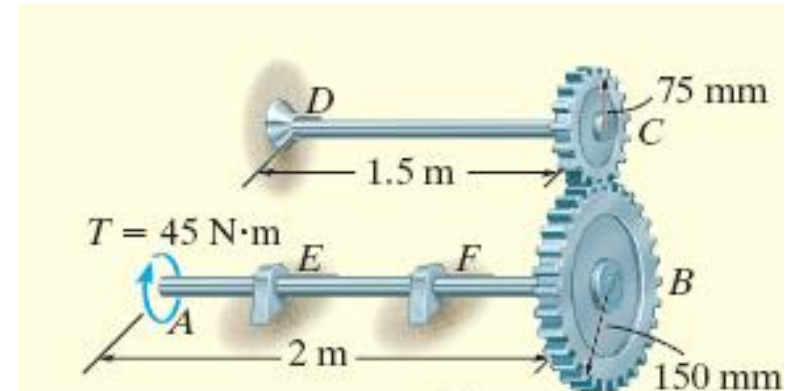


## Scilab implementation

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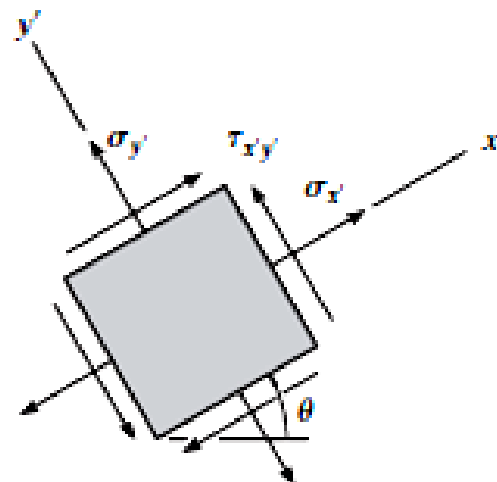
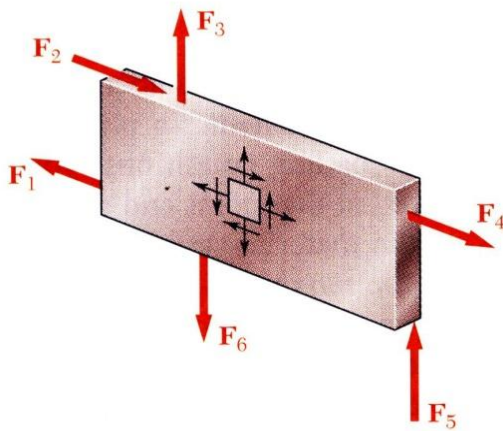
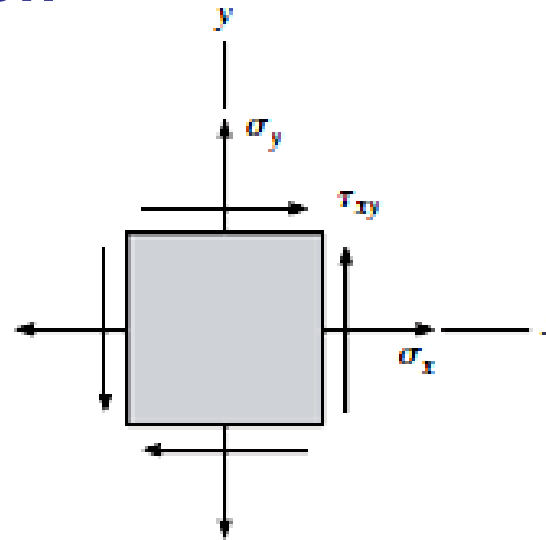
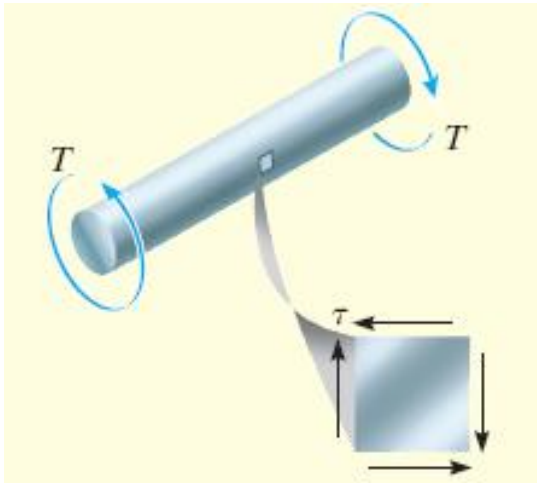
1  clc; clear
2  Ta = -45*1000; // Torque in A (Nmm)
3  G = 80*1000; // MPa
4  Ldc = 1500; // axis DC length (mm)
5  dB = 150*2; // gear B diameter (mm)
6  dC = 75*2; // gear C diameter (mm)
7  de = 20; // external axis DC diameter (mm)
8  di = 10; // internal axis DC diameter (mm)
9  Tb = Ta; // Torque in B (Nmm)
10 // Gear ratio: Tb/Tc = dB/dC =>
11 Tc = (Tb*dC)/dB; // gear C torque
12 J = (%pi/32) * ((de^4) - (di^4)); // mm4
13 teta = (Tc*Ldc)/(J*G); // rad
14 printf('The torsion angle at C is %.2f degrees', teta*(180/%pi))

```

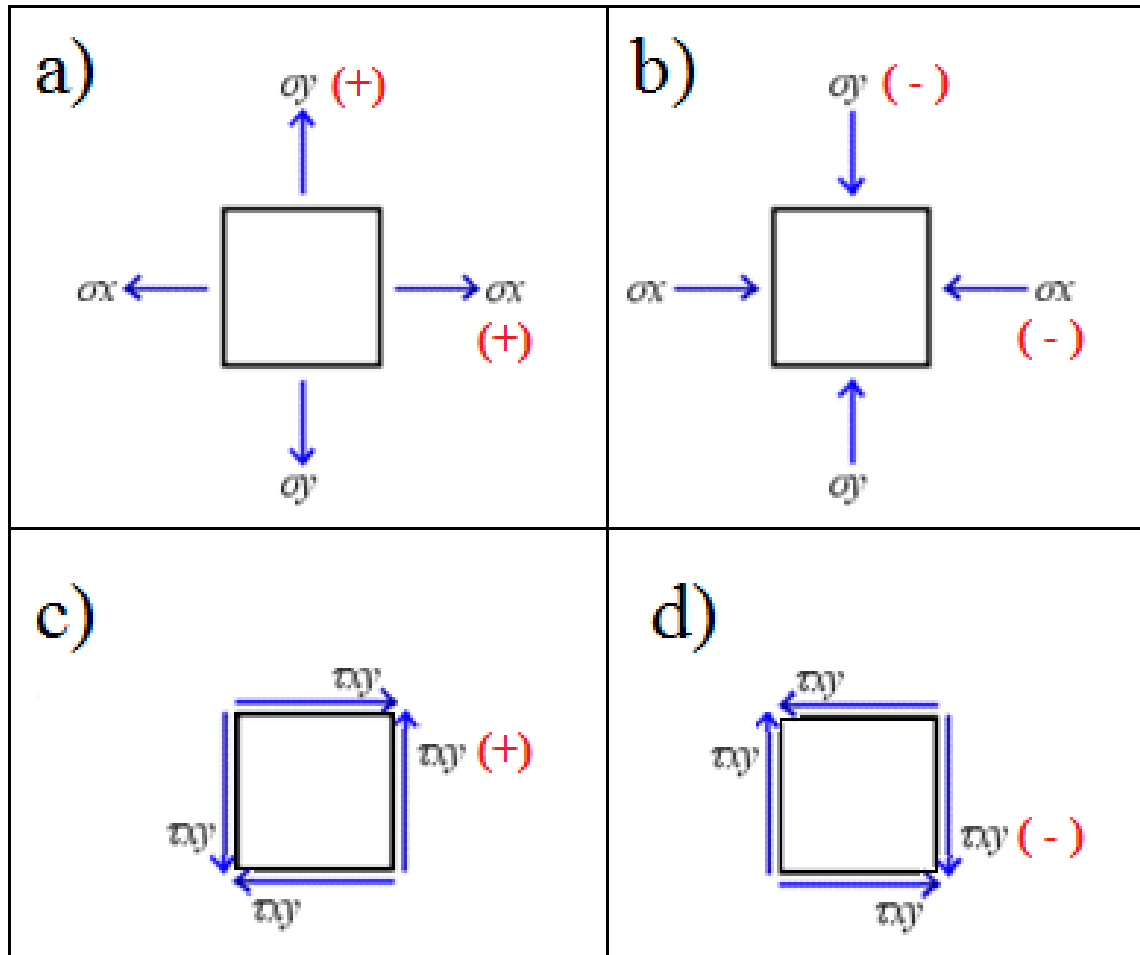


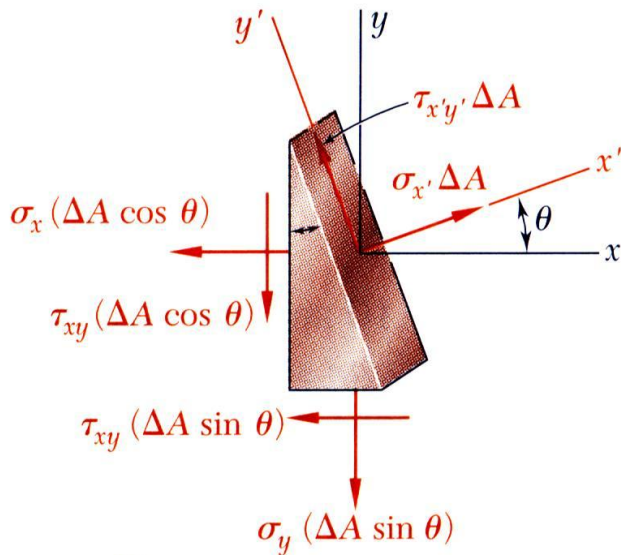
Answer: The torsion angle at C is 1.64 degrees

## Plane-Stress Transformation



## Sign Convention





- Consider the conditions for equilibrium of a prismatic element with faces perpendicular to the  $x$ ,  $y$ , and  $x'$  axes.

$$\sum F_{x'} = 0 = \sigma_{x'} \Delta A - \sigma_x (\Delta A \cos \theta) \cos \theta - \tau_{xy} (\Delta A \cos \theta) \sin \theta - \sigma_y (\Delta A \sin \theta) \sin \theta - \tau_{xy} (\Delta A \sin \theta) \cos \theta$$

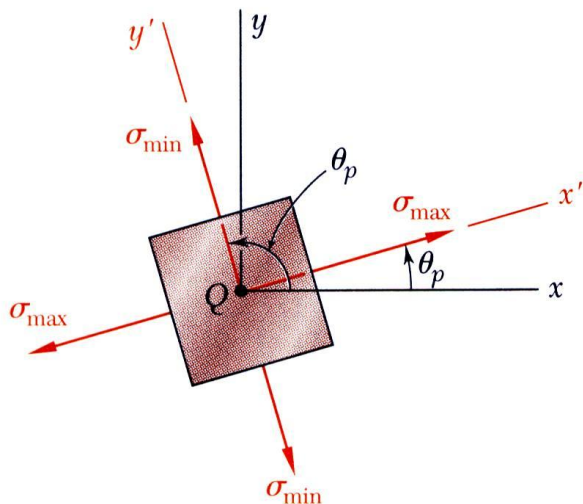
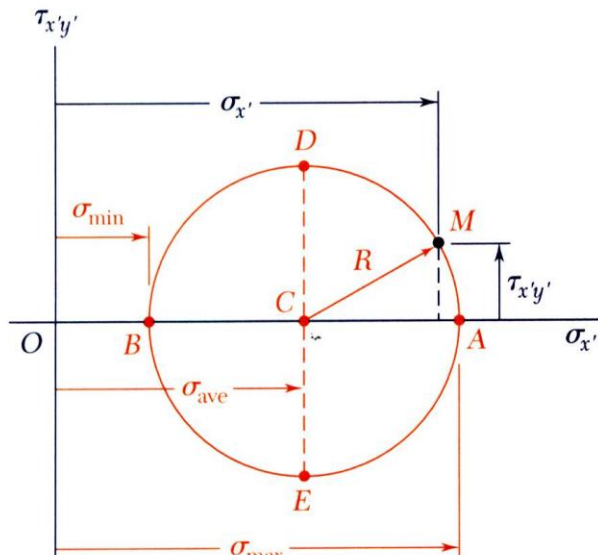
$$\sum F_{y'} = 0 = \tau_{x'y'} \Delta A + \sigma_x (\Delta A \cos \theta) \sin \theta - \tau_{xy} (\Delta A \cos \theta) \cos \theta - \sigma_y (\Delta A \sin \theta) \cos \theta + \tau_{xy} (\Delta A \sin \theta) \sin \theta$$

- The equations may be rewritten according to:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$



- The previous equations are combined to parametric equations for a circle,

$$(\sigma_{x'} - \sigma_{ave})^2 + \tau_{x'y'}^2 = R^2$$

where

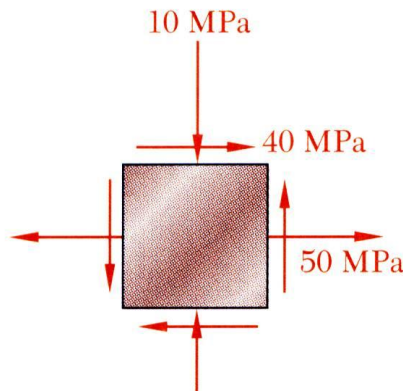
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} \quad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- Principal stresses* occur on the *principal planes of stress* with zero shearing stresses.

$$\sigma_{maxmin} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

## Example



For the state of plane stress shown, determine (a) the element orientation for the principal stresses, (b) the principal stresses, (c) the maximum shearing stress.

SOLUTION:

- Find the element orientation for the principal stresses from

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

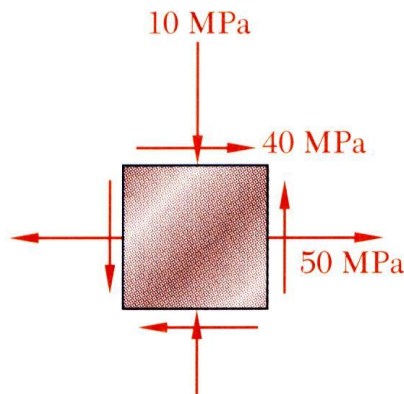
- Determine the principal stresses from

$$\sigma_{\max, \min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- Calculate the maximum shearing stress with

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma' = \frac{\sigma_x + \sigma_y}{2}$$



$$\sigma_x = +50 \text{ MPa} \quad \tau_{xy} = +40 \text{ MPa}$$

$$\sigma_y = -10 \text{ MPa}$$

SOLUTION:

- Find the element orientation for the principal stresses (a)

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(+40)}{50 - (-10)} = 1.333$$

$$2\theta_p = 53.1^\circ$$

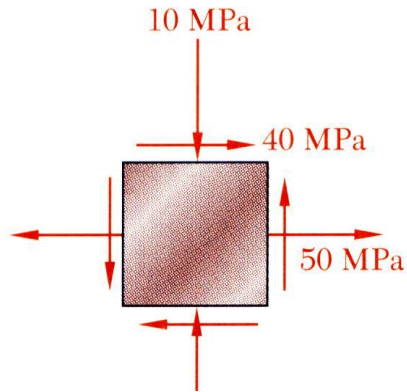
$$\theta_p = 26.6^\circ$$

- Determine the principal stresses (b)

$$\begin{aligned} \sigma_{\max, \min} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= 20 \pm \sqrt{(30)^2 + (40)^2} \end{aligned}$$

$$\sigma_{\max} = 70 \text{ MPa}$$

$$\sigma_{\min} = -30 \text{ MPa}$$

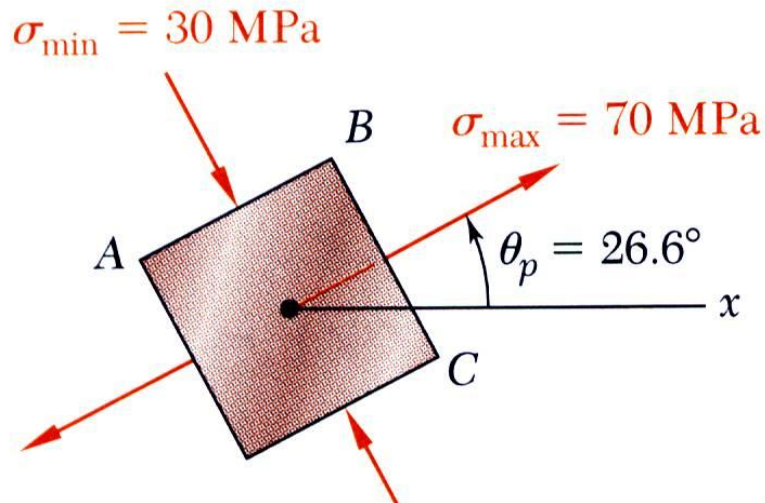


- Calculate the maximum shearing stress (c):

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

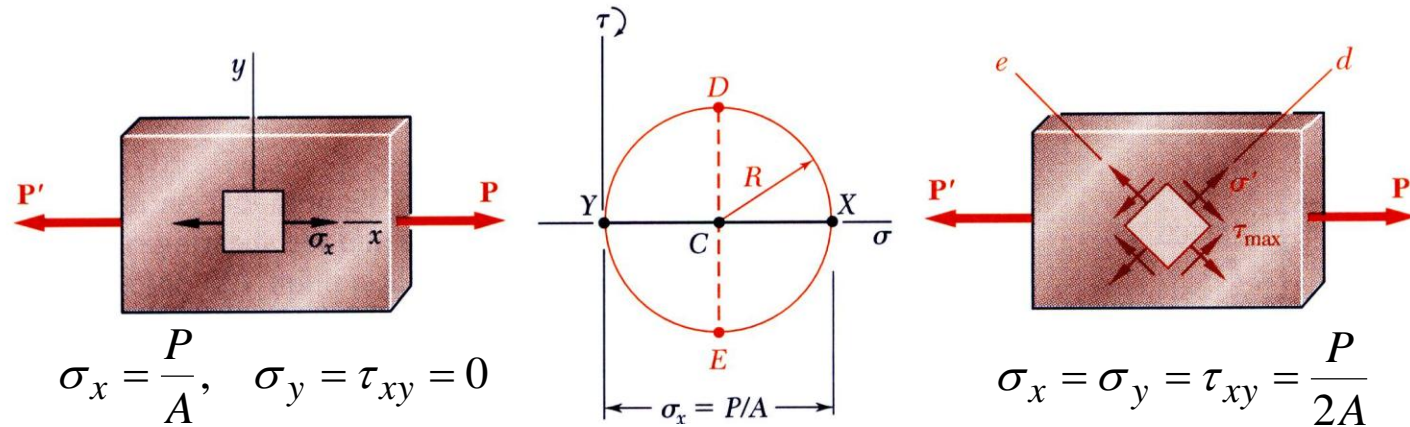
$$= \sqrt{(30)^2 + (40)^2}$$

$$\tau_{\max} = 50 \text{ MPa}$$

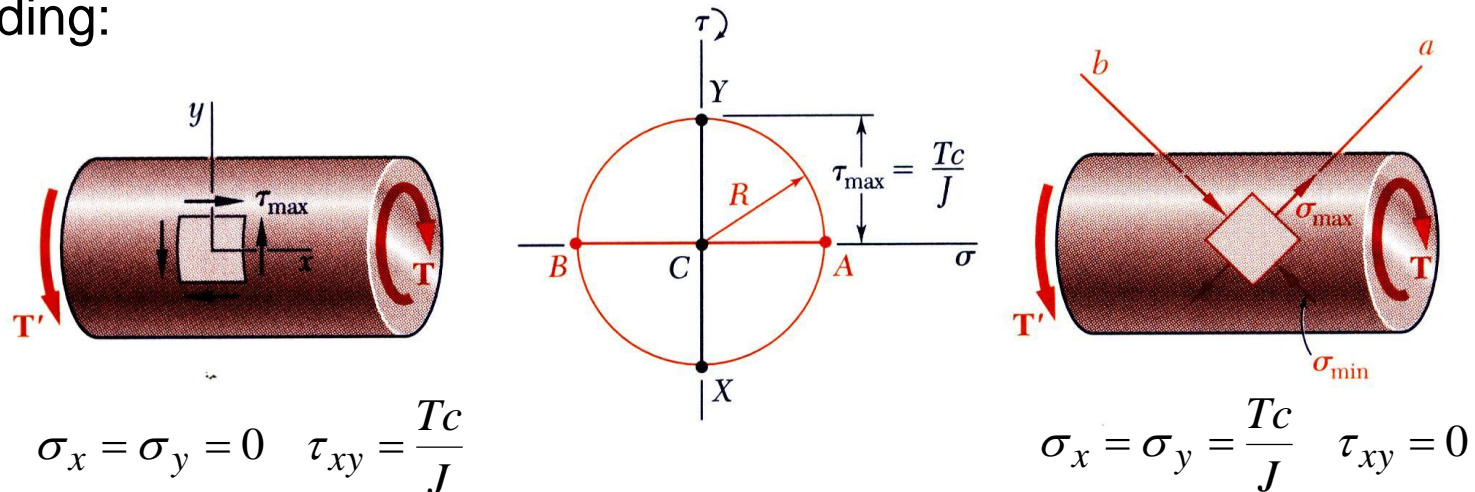




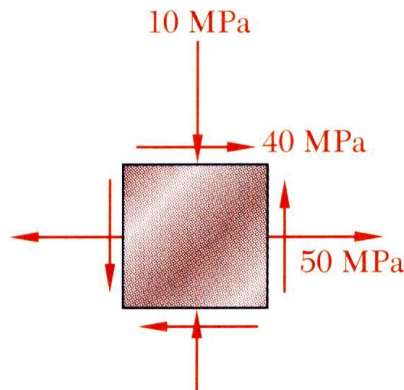
- Mohr's circle for centric axial loading:



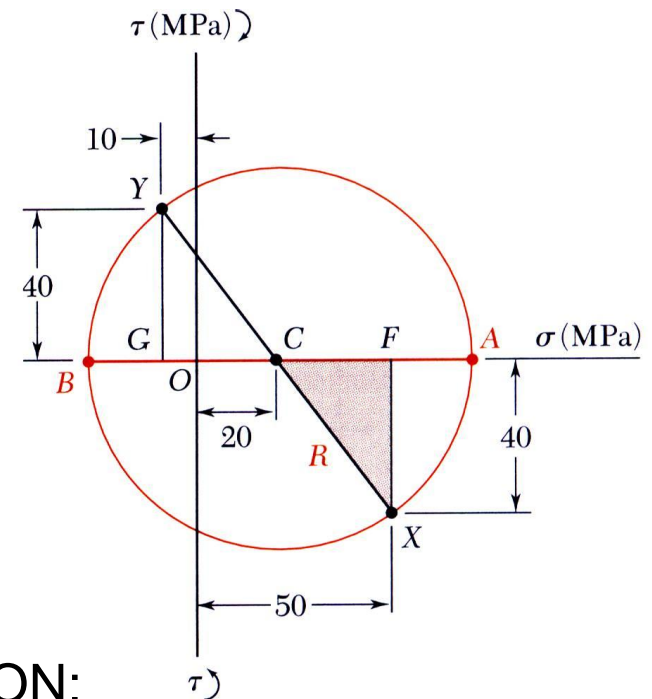
- Mohr's circle for torsional loading:



## Example



For the state of plane stress shown, construct Mohr's circle.



SOLUTION:

- Construction of Mohr's circle

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{(50) + (-10)}{2} = 20 \text{ MPa}$$

$$CF = 50 - 20 = 30 \text{ MPa} \quad FX = 40 \text{ MPa}$$

$$R = CX = \sqrt{(30)^2 + (40)^2} = 50 \text{ MPa}$$

- Principal planes and stresses

$$\sigma_{\max} = OA = OC + CA = 20 + 50$$

$$\sigma_{\max} = 70 \text{ MPa}$$

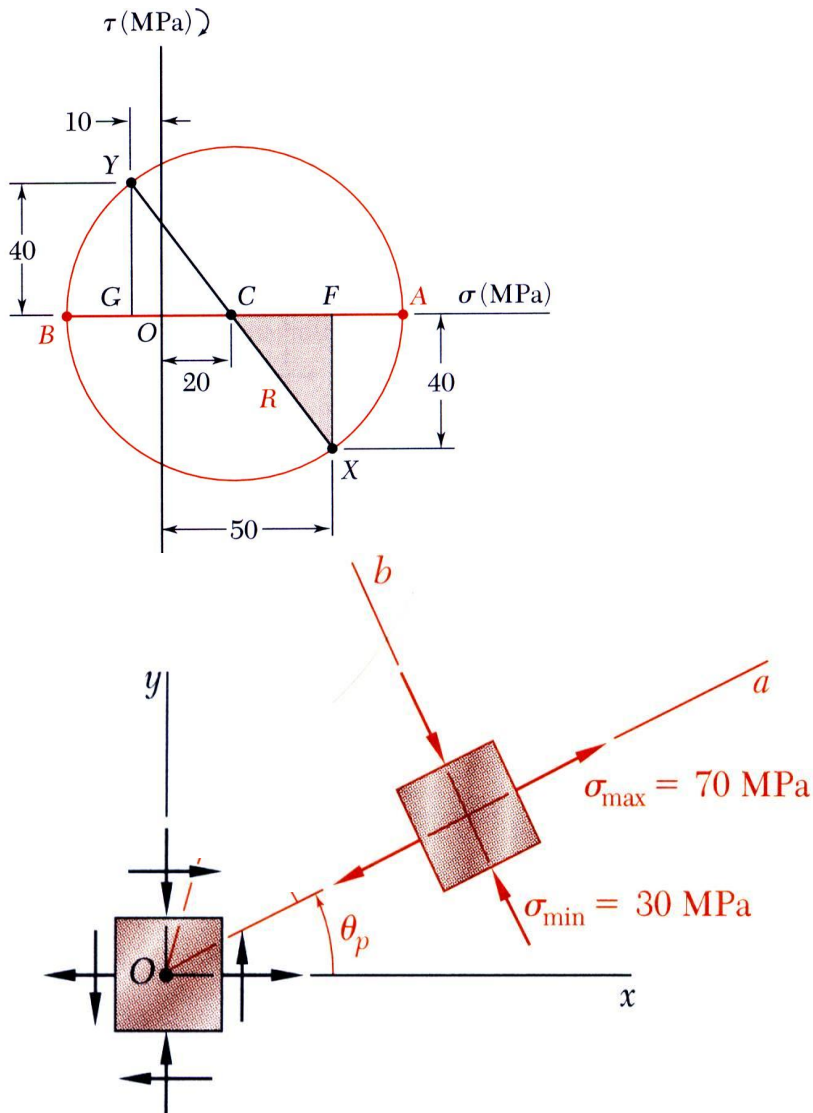
$$\sigma_{\min} = OB = OC - BC = 20 - 50$$

$$\sigma_{\min} = -30 \text{ MPa}$$

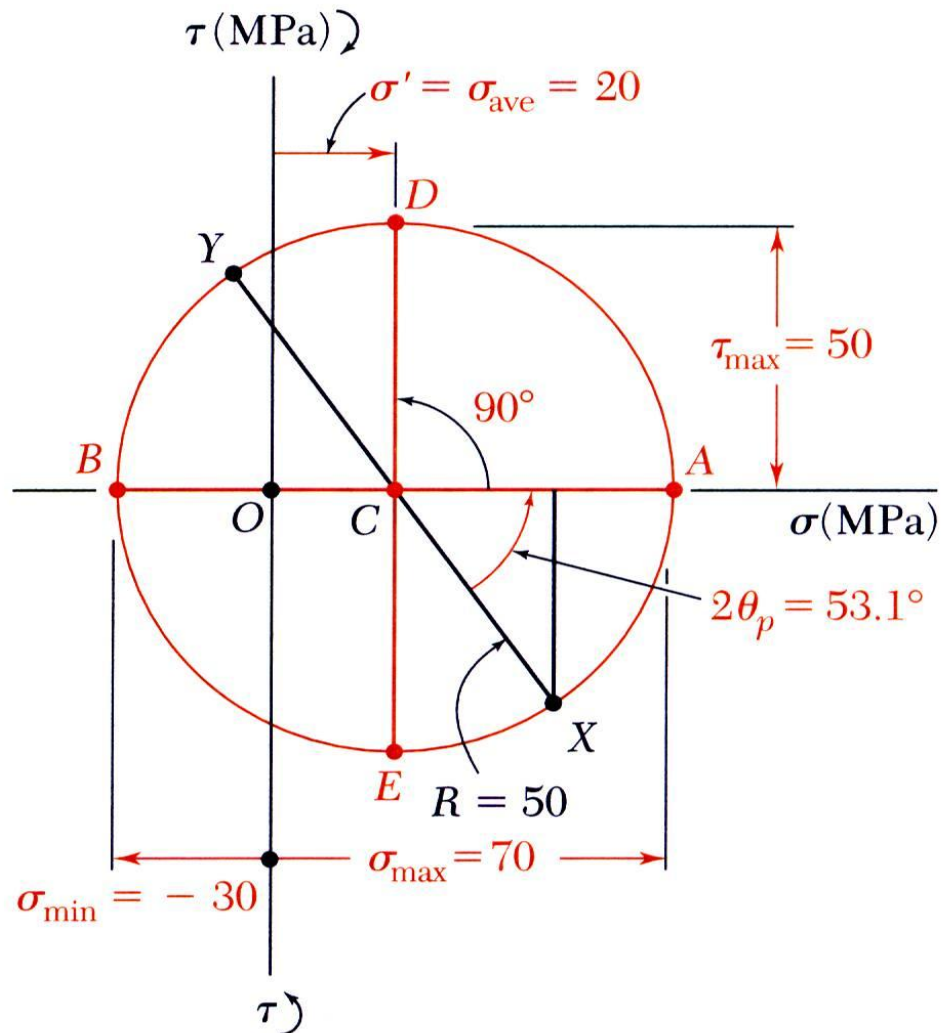
$$\tan 2\theta_p = \frac{FX}{CF} = \frac{40}{30}$$

$$2\theta_p = 53.1^\circ$$

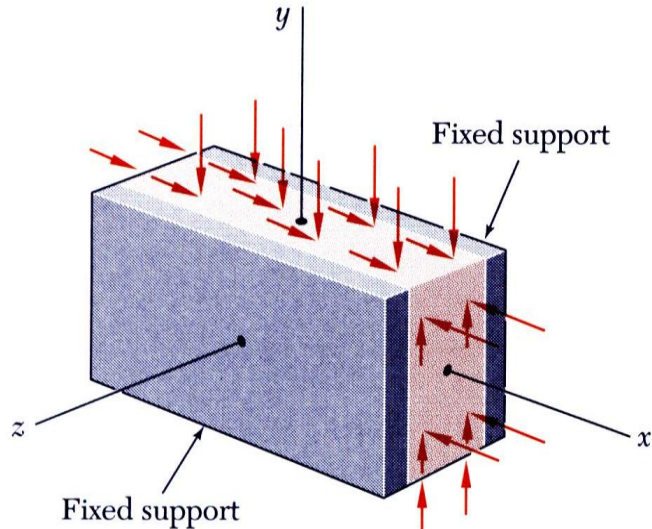
$$\theta_p = 26.6^\circ$$



## Mohr's circle



## Plane-strain Transformation



- *Plane strain* - deformations of the material take place in parallel planes and are the same in each of those planes.
  - Plane strain occurs in a plate subjected along its edges to a uniformly distributed load and restrained from expanding or contracting laterally by smooth, rigid and fixed supports
- components of strain :

$$\epsilon_x \quad \epsilon_y \quad \gamma_{xy} \quad \left( \epsilon_z = \gamma_{zx} = \gamma_{zy} = 0 \right)$$

- State of strain at the point Q results in different strain components with respect to the  $xy$  and  $x'y'$  reference frames.

$$\varepsilon(\theta) = \varepsilon_x \cos^2 \theta + \varepsilon_y \sin^2 \theta + \gamma_{xy} \sin \theta \cos \theta$$

$$\varepsilon_{OB} = \varepsilon(45^\circ) = \frac{1}{2}(\varepsilon_x + \varepsilon_y + \gamma_{xy})$$

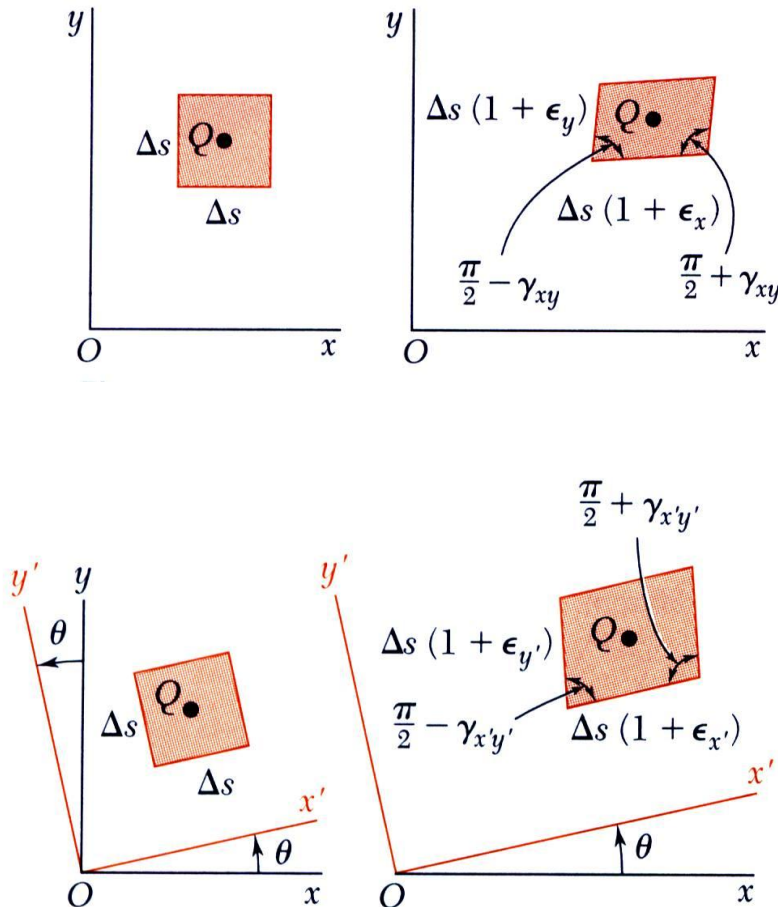
$$\gamma_{xy} = 2\varepsilon_{OB} - (\varepsilon_x + \varepsilon_y)$$

- Applying the trigonometric relations used for the transformation of stress,

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2} \cos 2\theta - \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2} \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta$$



- Abscissa for the center  $C$  and radius  $R$ ,

$$\epsilon_{ave} = \frac{\epsilon_x + \epsilon_y}{2} \quad R = \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

- The equations for the transformation of plane strain are of the same form as the equations for the transformation of plane stress - *Mohr's circle techniques apply.*

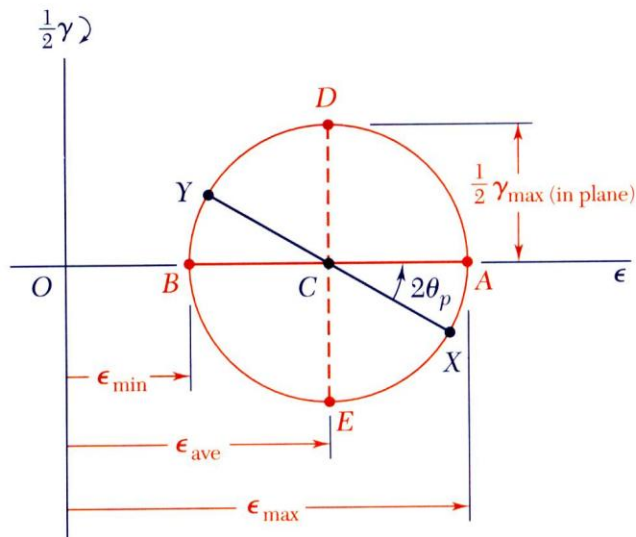
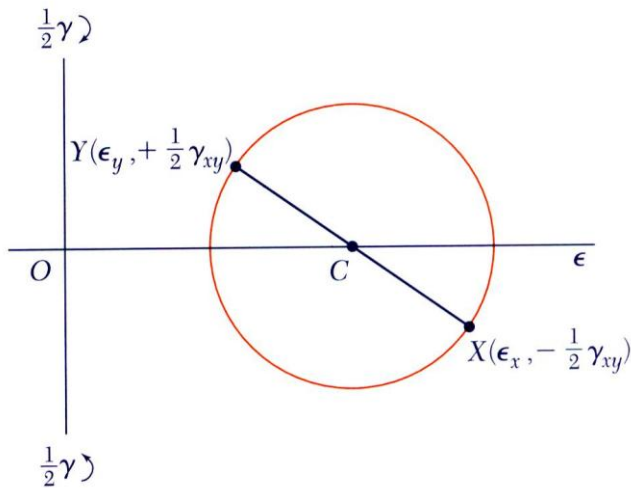
- Principal axes of strain and principal strains,

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

$$\epsilon_{max} = \epsilon_{ave} + R \quad \epsilon_{min} = \epsilon_{ave} - R$$

- Maximum in-plane shearing strain,

$$\gamma_{max} = 2R = \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$





## Example on Scilab

For a specific state of plane strain, where  $\varepsilon_x = 320\mu$ ,  $\varepsilon_y = 160\mu$  and  $\gamma_{xy} = 300\mu$ , determine the principal axes of strain, the principal strains and the maximum in-plane shearing strain. Generate the *Mohr's circle*.

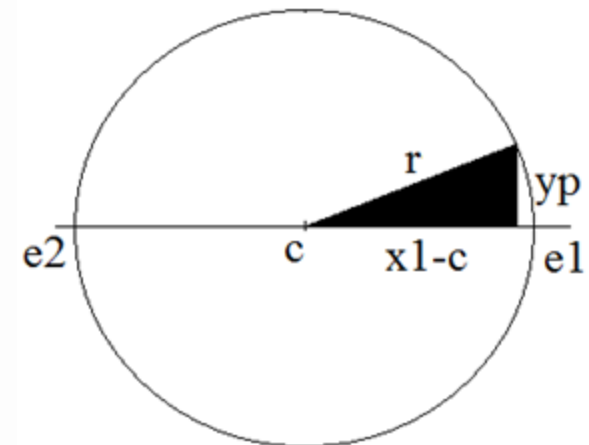


## Solution

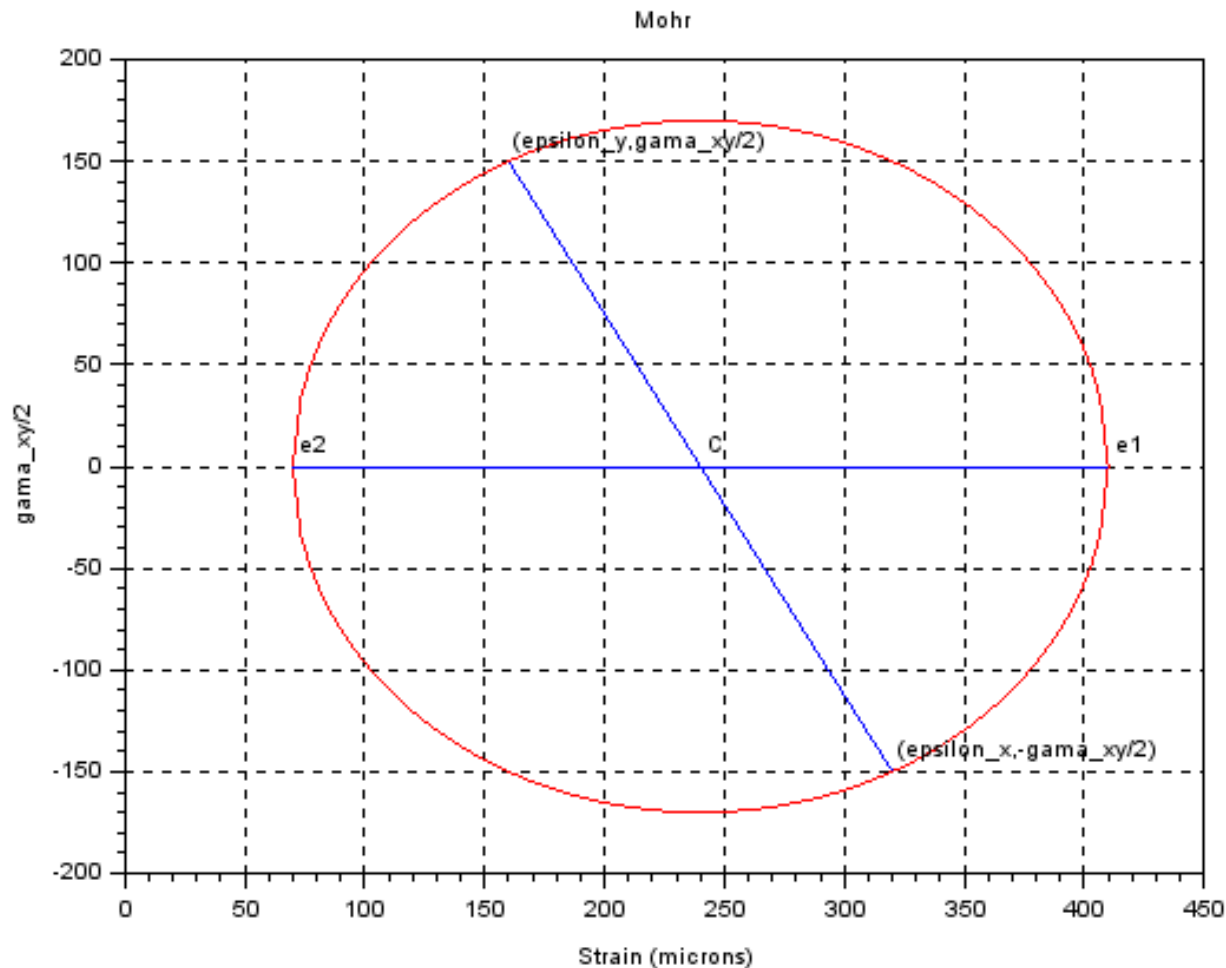
```

1 clear; clc
2 ex = 320; // microns
3 ey = 160; // microns
4 gama_xy = 300; // microns
5 r = sqrt(((ex-ey)/2)^2 + ((gama_xy/2)^2)); // circle radius
6 c = (ex+ey)/2; // center of the circle
7 // Principal strains
8 e1 = c+r; e2 = c-r;
9 teta = (atand((gama_xy)/(ex-ey)))/2; // Element orientation
10 // ***** Generating the circle vectors *****
11 x1 = e2:abs(e2-e1)/100:e1; // vector that starts at e2 and ends at e1
12 yp = sqrt(abs((r^2) - ((x1-c).^2))); // circumference equation
13 yn = -yp; // to complete the other half of the circle
14 plot(x1,yp,"r"); plot(x1,yn,"r");
15 plot([ex,ey],[-gama_xy/2,gama_xy/2]);
16 plot([e2,e1],[0,0],[0,0],[-r,r]);
17 title("Mohr")
18 xlabel("Strain (microns)")
19 ylabel("gama_xy/2")
20 xstring(c,0,"c") // to insert text in the chart
21 xstring(ex,(-gama_xy/2),"(epsilon_x,-gama_xy/2)")
22 xstring(ey,(gama_xy/2),"(epsilon_y,gama_xy/2)")
23 xstring(e2,0,"e2");
24 xstring(e1,0,"e1");
25 xgrid

```



## Graphic



$\epsilon_{max} = 410 \mu$   
 $\epsilon_{min} = 70 \mu$   
 $2\theta = 62^\circ$