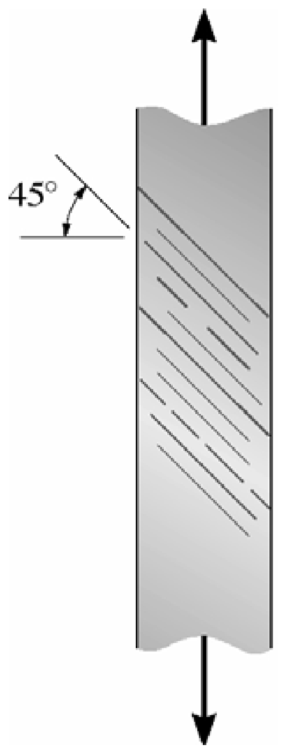
Theories of Failure

- **Fragile materials**: - Theory of maximum normal stress - Mohr Failure Criterion

- **Ductile Materials**: - Tresca Flow Criterion -Theory of Maximum Distortion Energy (von Mises)

- The flow of the ductile material occurs along the contact planes of the randomly oriented crystals that form the material. Maximum Shear Stress Theory or Tresca flow criteria are used to predict the failure stress of a ductile material subject to any type of load.

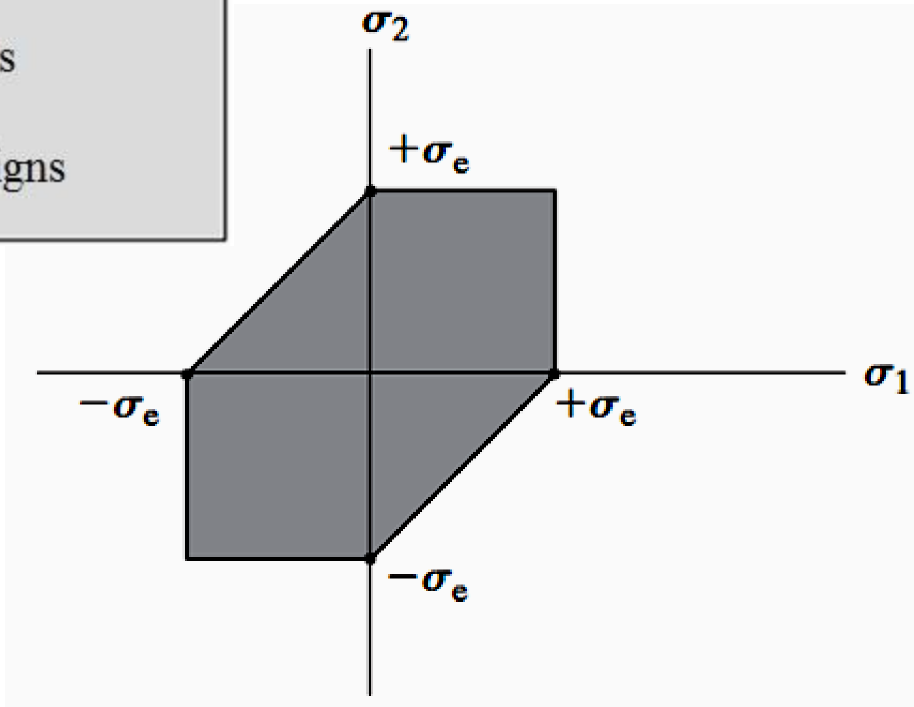


Tresca criterion

- In reference to the plane stress, the theory of the maximum shear stress for plane stress can be expressed by the principal stresses (σ_1 and σ_2).

$$\left.\begin{array}{l}|\sigma_1| \leq \sigma_e \\ |\sigma_2| \leq \sigma_e\end{array}\right\} \sigma_1, \sigma_2 \quad \text{same signs}$$

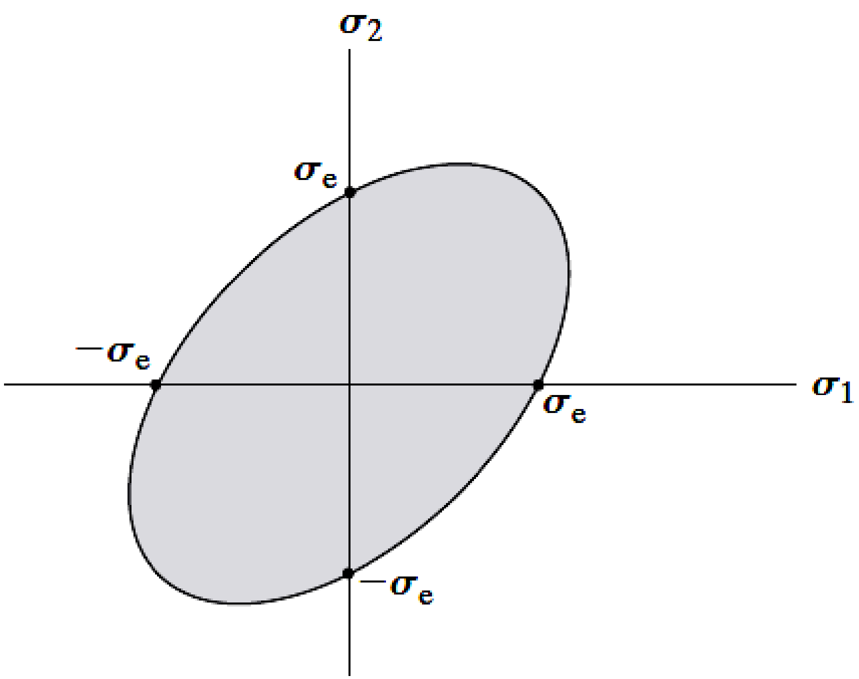
$$|\sigma_1 - \sigma_2| \leq \sigma_e \;\} \; \sigma_1, \sigma_2 \text{ opposite signs}$$



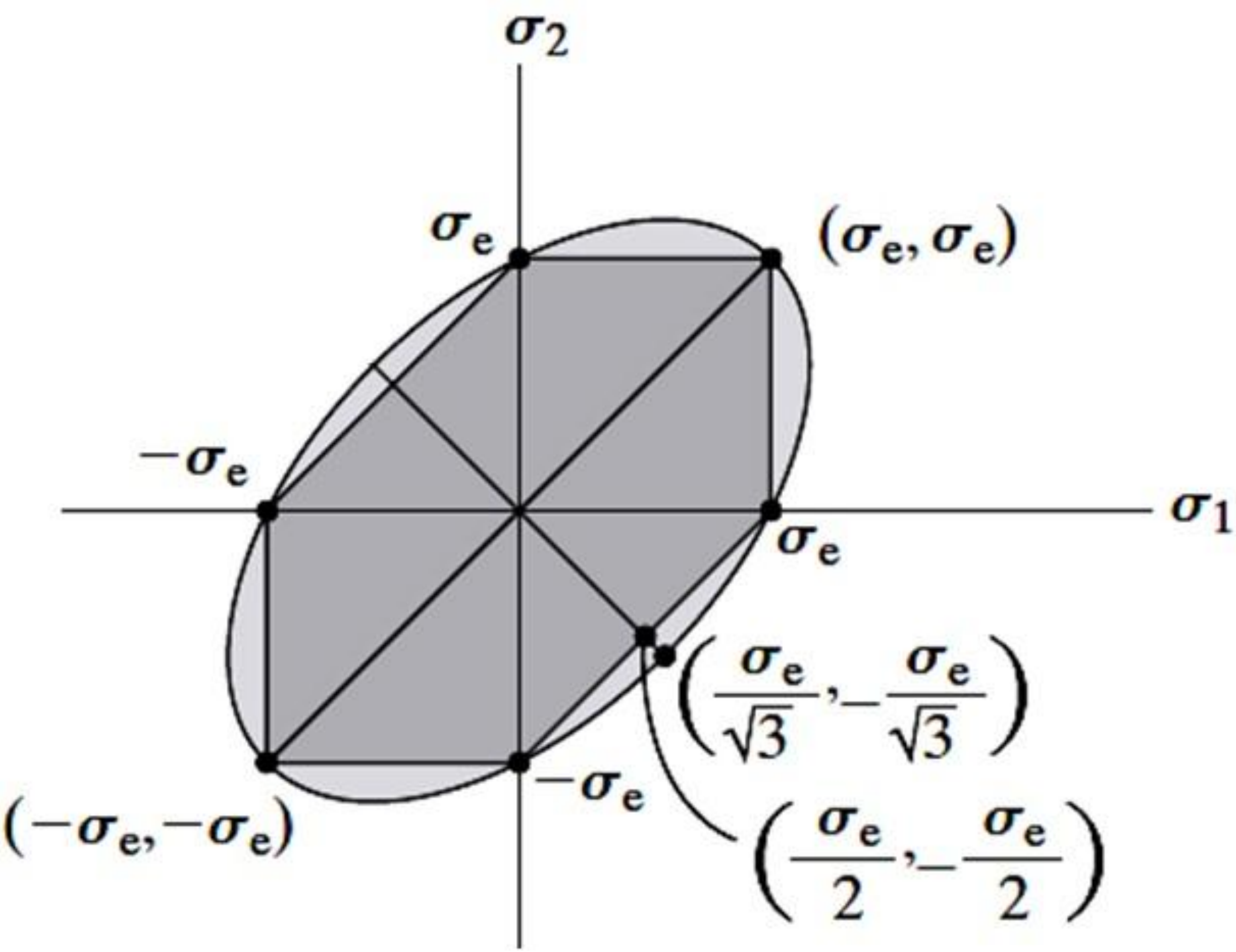
von Mises criterion

- Flow in a ductile material occurs when the distortion energy per unit volume of the material is equal or exceeds the distortion energy per unit volume of the same material.

$$\boxed{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \leq \sigma_e^2}$$

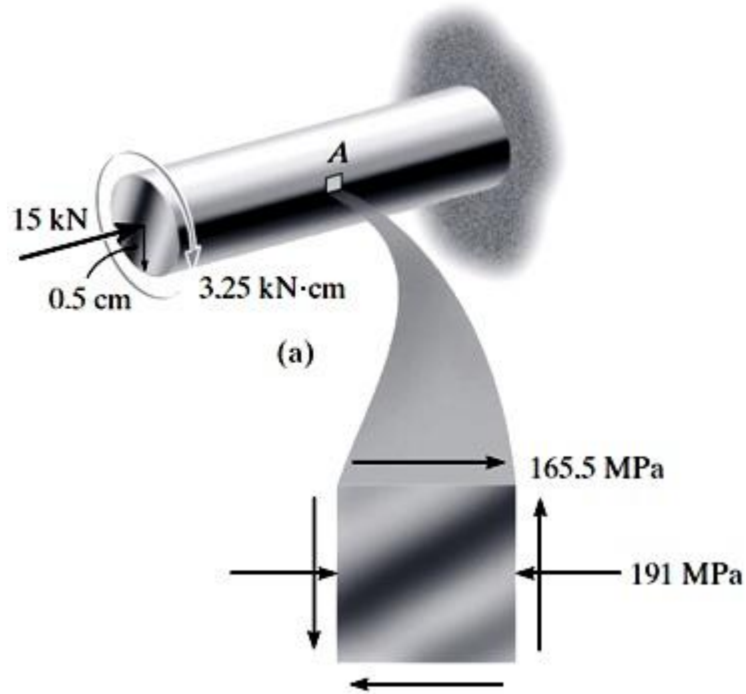


Tresca vs. Von Mises



Example

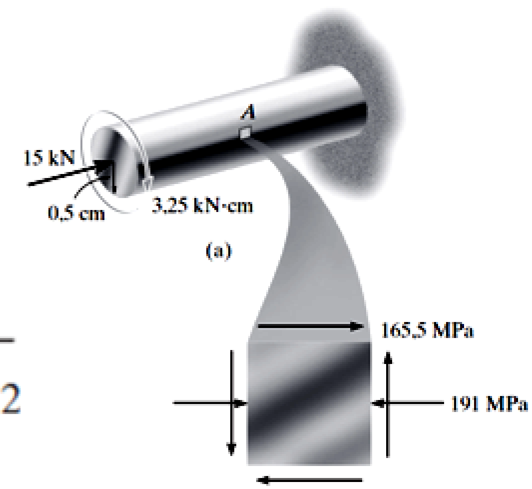
The solid shaft shown in the Figure has a radius of 0.5 cm and it was produced by steel (yield stress 360 MPa). Determine if the loads cause the axis to fail according to the theory of maximum shear stress (Tresca flow criterion) and the theory of maximum distortion energy (von Mises criterion).



Solution

$$\sigma_x = \frac{P}{A} = \frac{15\text{ kN}}{\pi(0.5\text{ cm})^2} = -19.10\text{ kN/cm}^2 = \text{-191MPa}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{3.25\text{ N}\cdot\text{m}\,(0.5\text{ cm})}{\frac{\pi}{2}(0.5\text{ cm})^4} = 16.55\text{ kN/cm}^2 = 165.5\text{ MPa}$$



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-191 + 0}{2} \pm \sqrt{\left(\frac{-191 - 0}{2}\right)^2 + (165.5)^2}$$

$$= -95.5 \pm 191.1$$

$$\sigma_1 = 95.6\text{ MPa}$$

$$\sigma_2 = -286.6\text{ MPa}$$

Prediction

Tresca:

$$|\sigma_1 - \sigma_2| \leq \sigma_e$$

$$|95.6 - (-286.6)| \overset{?}{\leq} 360$$

$$382.2 > 360$$

Failure occur.

von Mises:

$$\left(\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2\right) \leq \sigma_e{}^2$$

$$[(95.6)^2 - (95.6)(-286.6) - (-286.6)^2] \overset{?}{\leq} (360)^2$$

$$118\,677.9 \leq 129\,600$$

Failure does not occur.

Example on Scilab

Knowing that the yield stress of a shaft is σ = 270 MPa. Determine if a load causes fail to this shaft, according to the theory of maximum shear stress (Tresca criterion) and the theory of maximum distortion energy (von Mises criterion), being:

σ_x = 40 MPa

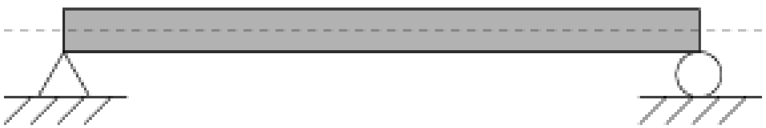
σ_y = -30 MPa

τ_{xy} = 100 MPa

```
tx = 40; ty = -30; txy = 100; te = 270; // MPa
tm = (tx+ty)/2; t = (tx-ty)/2;
r = sqrt(t^2+txy^2); tmax1 = tm+r; tmax2 = tm-r;
// Tresca: sigma 1 and sigma 2 same signs
if ((tmax1 > 0) & (tmax2 > 0)) | ((tmax1 < 0) & (tmax2 < 0))
     if (abs(tmax1) > te) | (abs(tmax2) > te)
    printf("Failure occur according to Tresca\n");
  else
    printf("Failure does not occur according to Tresca criterion\n");
  end;
// Tresca: sigma 1 and sigma 2 opposite signs
else //  ((tmax1 <= 0) && (tmax2 >= 0)) || ((tmax1 >= 0) && (tmax2 <= 0))
  tresca = abs(tmax1 - tmax2);
    if tresca > te
    printf("Failure occur according to Tresca criterion\n");
  else
    printf("Failure does not occur according to Tresca criterion\n");
 end;
end;
vm = tmax1^2-tmax1*tmax2+tmax2^2; // von Mises
if vm>=(te^2) then
    printf("Failure occur according to von Mises criterion\n");
  else
    printf("Failure does not occur according to von Mises criterion\n");
end;
```

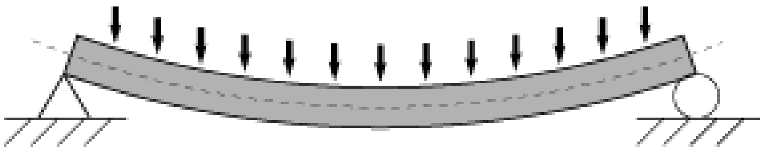
- - > Failure does not occur according to Tresca and von Mises criteria

Bending stresses



Beam before bending

- a *neutral surface* must exist that is parallel to the upper and lower surfaces and for which the length does not change.

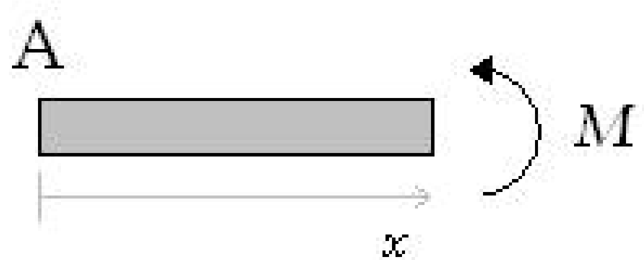


Beam after bending

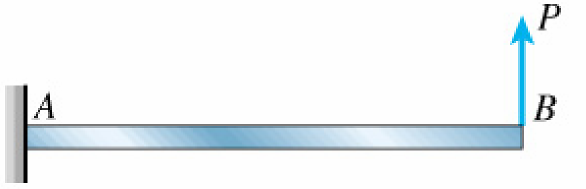
- stresses and strains are negative (compressive) above the neutral plane and positive (tension) below it.

Bending moment

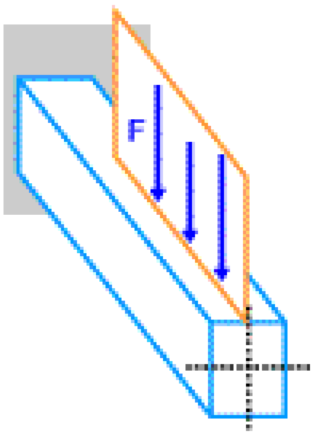
is the moment that results from the integrated portion over the other portion in the direction transverse to the axis of the cutting crossbar.



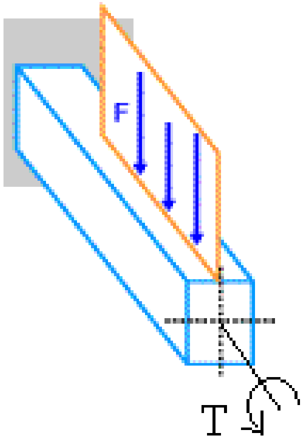
Bending: types



Pure bending (M)



Simple bending (M+V)

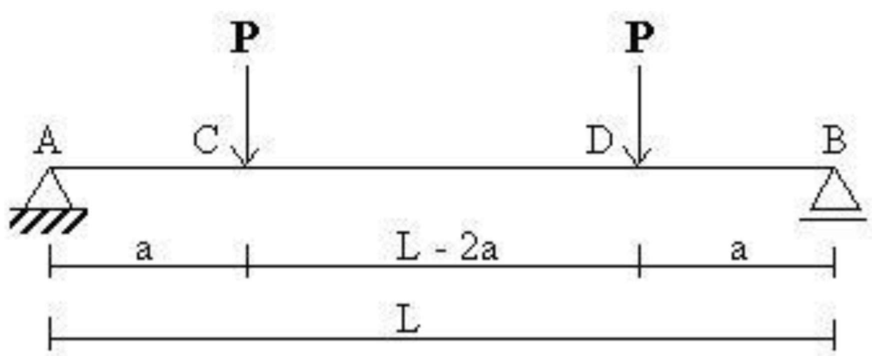


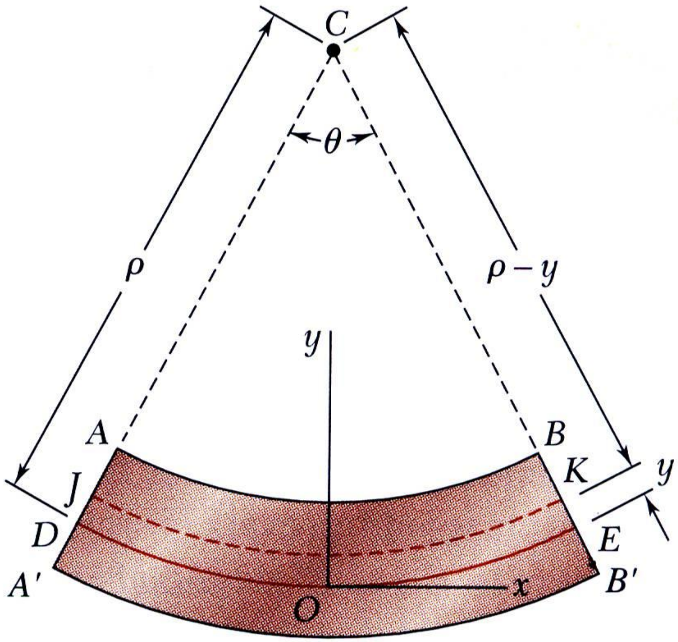
(M+V+T)

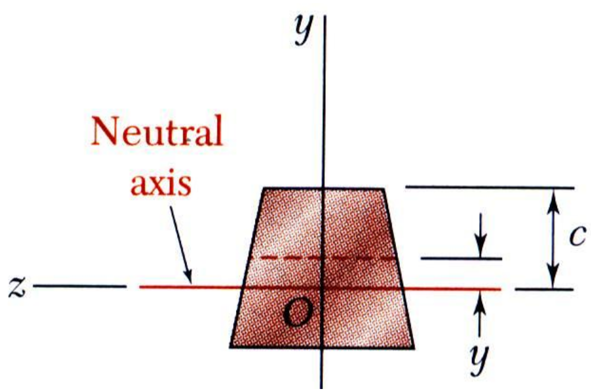
In addition to these, oblique flexion should be considered when the loading takes place by angle.

⇨ Pure bending

Pure flexion occurs when a structure or part of a structure is requested only by a bending moment. This is the case for the CD section of the beam showed in the Figure below. In this section, the shear force is zero and the bending moment is constant. It is noted that in order to avoid shear force in the CD section, the P forces are symmetrical and the structure's own weight, in the presence of the P loads.







Consider a beam segment of length *L*.

After deformation, the length of the neutral surface remains *L*. At other sections,

$$L' = (\rho - y)\theta$$

$$\delta = L' - L = (\rho - y)\theta - \rho\theta = -y\theta$$

$$\varepsilon_x = \frac{\delta}{L} = -\frac{y\theta}{\rho\theta} = -\frac{y}{\rho} \quad \text{(strain varies linearly)}$$

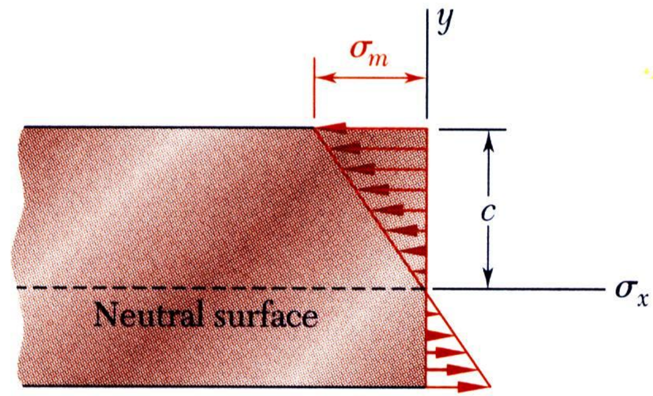
$$\varepsilon_m = \frac{c}{\rho} \quad \text{or} \quad \rho = \frac{c}{\varepsilon_m}$$

$$\varepsilon_x = -\frac{y}{c}\varepsilon_m$$

- For a linearly elastic material,

$$\sigma_x = E\varepsilon_x = -\frac{y}{c}E\varepsilon_m$$

$$= -\frac{y}{c}\sigma_m \quad \text{(stress varies linearly)}$$



- For static equilibrium,

$$F_x = 0 = \int \sigma_x \, dA = \int -\frac{y}{c}\sigma_m \, dA$$

$$0 = -\frac{\sigma_m}{c}\int y \, dA$$

First moment with respect to neutral plane is zero. Therefore, the neutral surface must pass through the section centroid.

- For static equilibrium,

$$M = \int(-y\sigma_x \, dA) = \int(-y)\left(-\frac{y}{c}\sigma_m\right) dA$$

$$M = \frac{\sigma_m}{c}\int y^2 \, dA = \frac{\sigma_m I}{c}$$

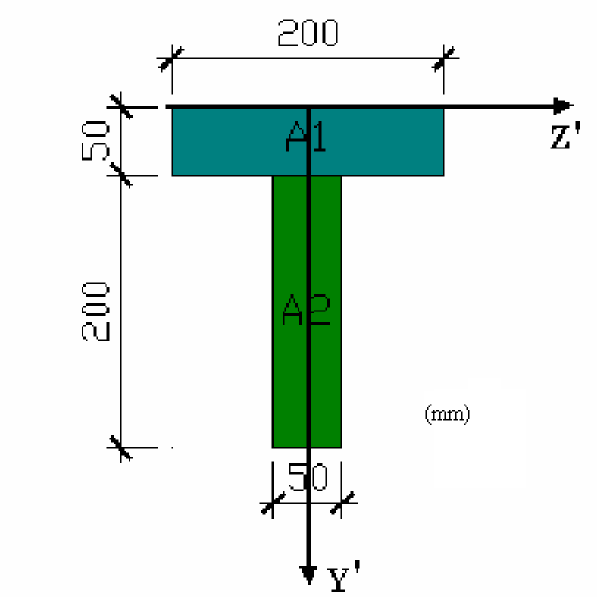
$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

Substituting $\sigma_x = -\frac{y}{c}\sigma_m$

$$\sigma_x = -\frac{My}{I}$$

EXAMPLE:

Cross section:



Max moment = 4,67x 10^7 N.mm

Load applied from bottom.

Centroid Position:

$z' = 0;$ A1 = A2 = 10000 mm^2:

$$y' = \frac{\sum A_i y_i}{\sum A_i} = \frac{A_1 \times 25 + A_2 \times 150}{A_1 + A_2} = 87{,}5mm$$

Moment of inertia :

$$I_z i = I' + A.d^2 (I'_{ret} = bh^3/12)$$

d = distance from the centroid position of each area to y.

$$I_{z1} = \frac{200 \times 50^3}{12} + 10000 \times 62{,}5^2 = 4{,}11 \times 10^7 mm^4$$

$$I_{z2} = \frac{50 \times 200^3}{12} + 10000 \times 62{,}5^2 = 7{,}24 \times 10^7 mm^4$$

$$I_z = I_{z1} + I_{z2} = (4{,}11 + 7{,}24) \times 10^7 = 11{,}35 \times 10^7 mm^4$$

Maximum tensile and compressile stresses

- Compressile

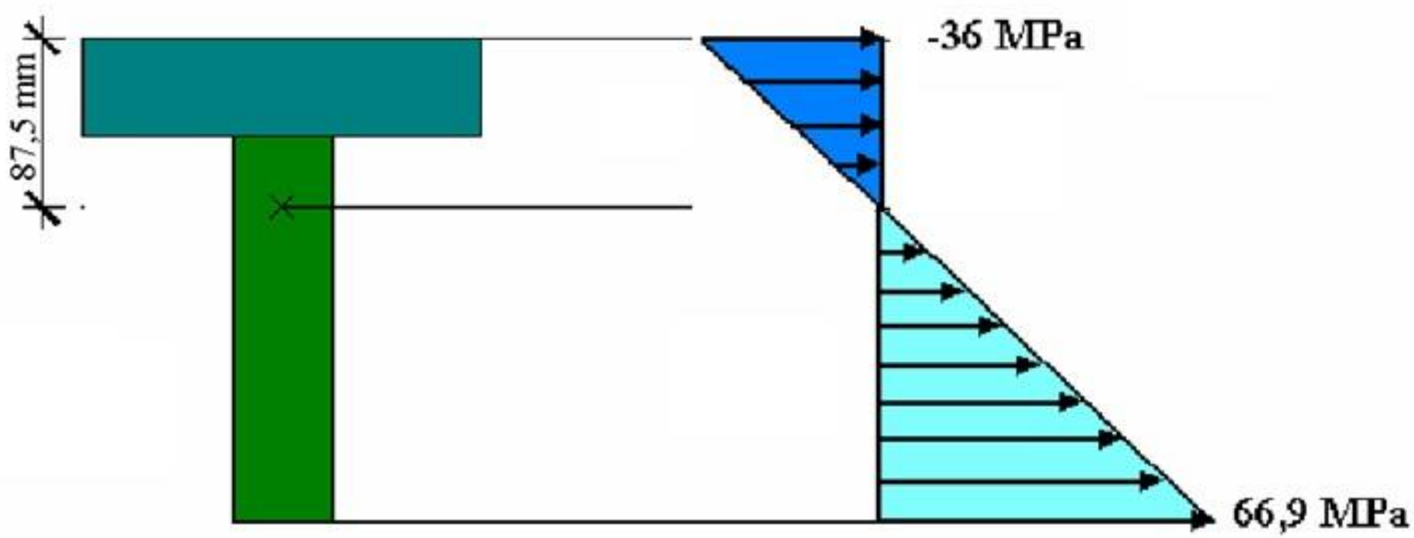
$$\sigma_x = \frac{M_{máx} \cdot y}{I_z} = -\frac{4{,}67\times10^7 \times 87{,}5}{11{,}35x10^7}$$

$$\boxed{\sigma_x = -36MPa}$$

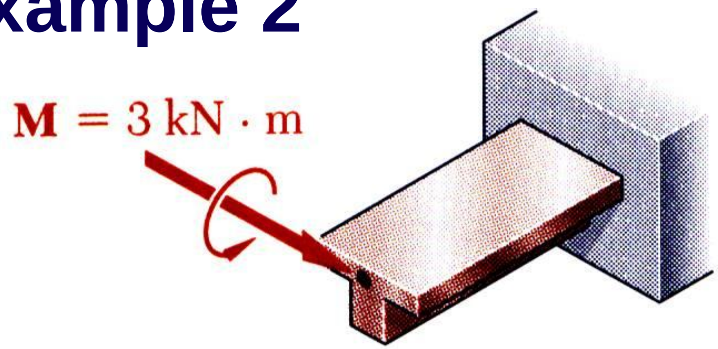
- Tensile

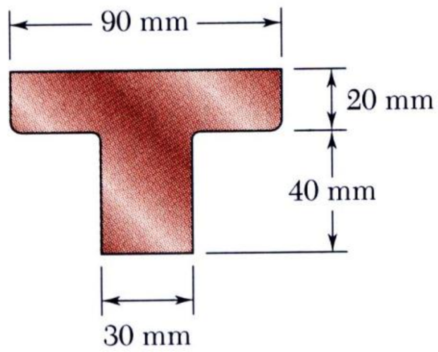
$$\sigma_x = \frac{M_{máx} \cdot y}{I_z} = \frac{4{,}67\times10^7 \times 162{,}5}{11{,}35x10^7}$$

$$\boxed{\sigma_x = 66{,}9MPa}$$



Example 2





A cast-iron machine part is acted upon by a 3 kN-m couple. Knowing *E* = 165 GPa and neglecting the effects of fillets, determine the maximum tensile and compressive stresses.

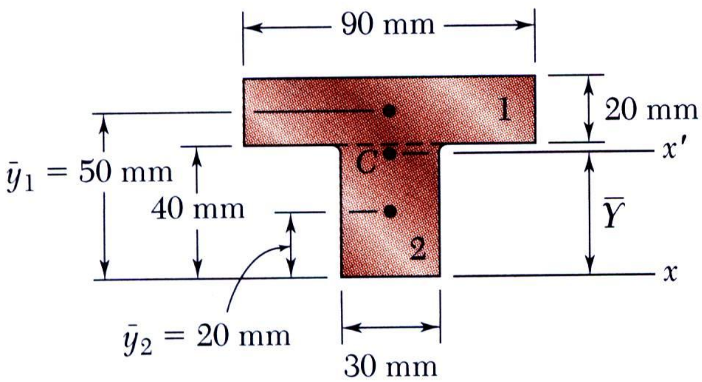
SOLUTION:

- Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} \qquad I_{x'} = \sum\left(\bar{I} + Ad^2\right)$$

- Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

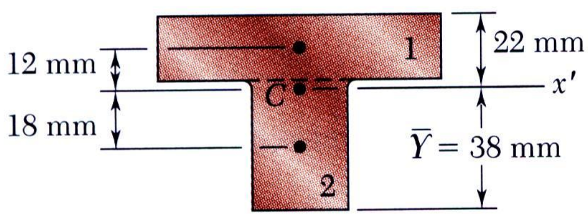
$$\sigma_m = \frac{Mc}{I}$$



SOLUTION:

Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

	Area, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$20 \times 90 = 1800$	50	90×10^3
2	$40 \times 30 = 1200$	20	24×10^3
	$\sum A = 3000$		$\sum \bar{y}A = 114 \times 10^3$

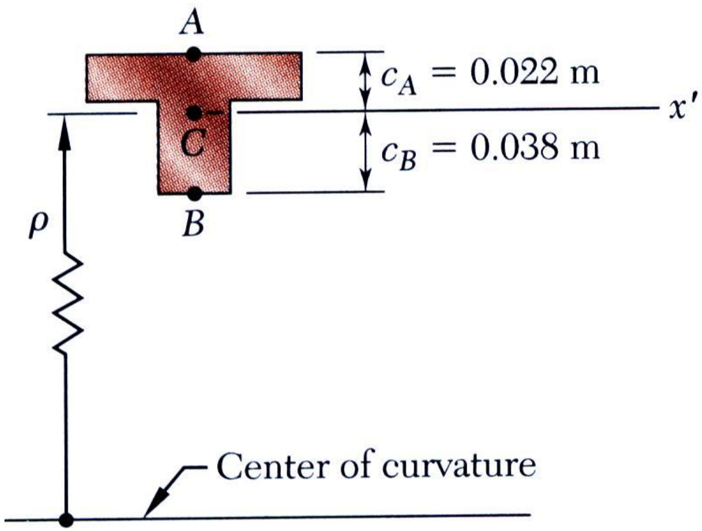


$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{114 \times 10^3}{3000} = 38 \text{ mm}$$

$$I_{x'} = \sum\left(\bar{I} + Ad^2\right) = \sum\left(\tfrac{1}{12} bh^3 + Ad^2\right)$$

$$= \left(\tfrac{1}{12} 90 \times 20^3 + 1800 \times 12^2\right) + \left(\tfrac{1}{12} 30 \times 40^3 + 1200 \times 18^2\right)$$

$$I = 868 \times 10^3 \text{mm}^4 = 868 \times 10^{-9} \text{m}^4$$



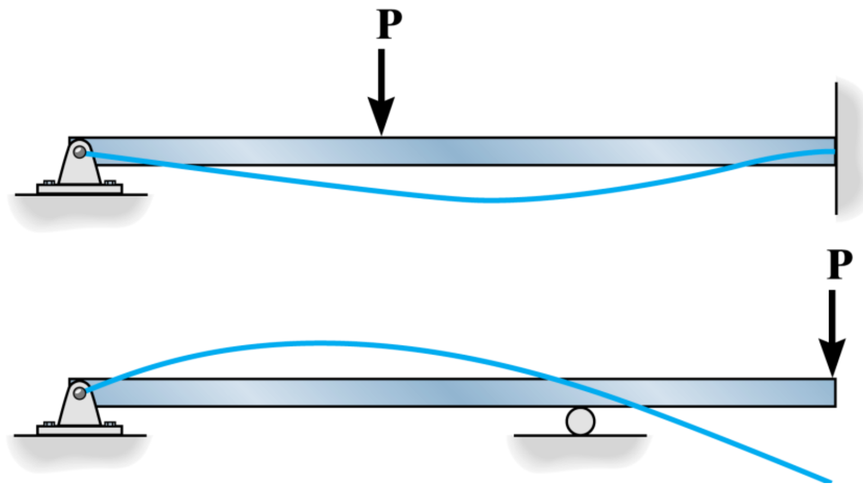
- Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m = \frac{Mc}{I}$$

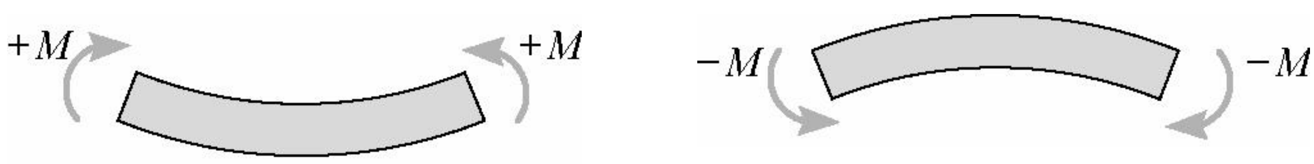
$$\sigma_A = \frac{M\,c_A}{I} = \frac{3\,\mathrm{kN}\cdot\mathrm{m}\times 0.022\,\mathrm{m}}{868\times 10^{-9}\,\mathrm{m}^4} \qquad \boxed{\sigma_A = +76.0\,\mathrm{MPa}}$$

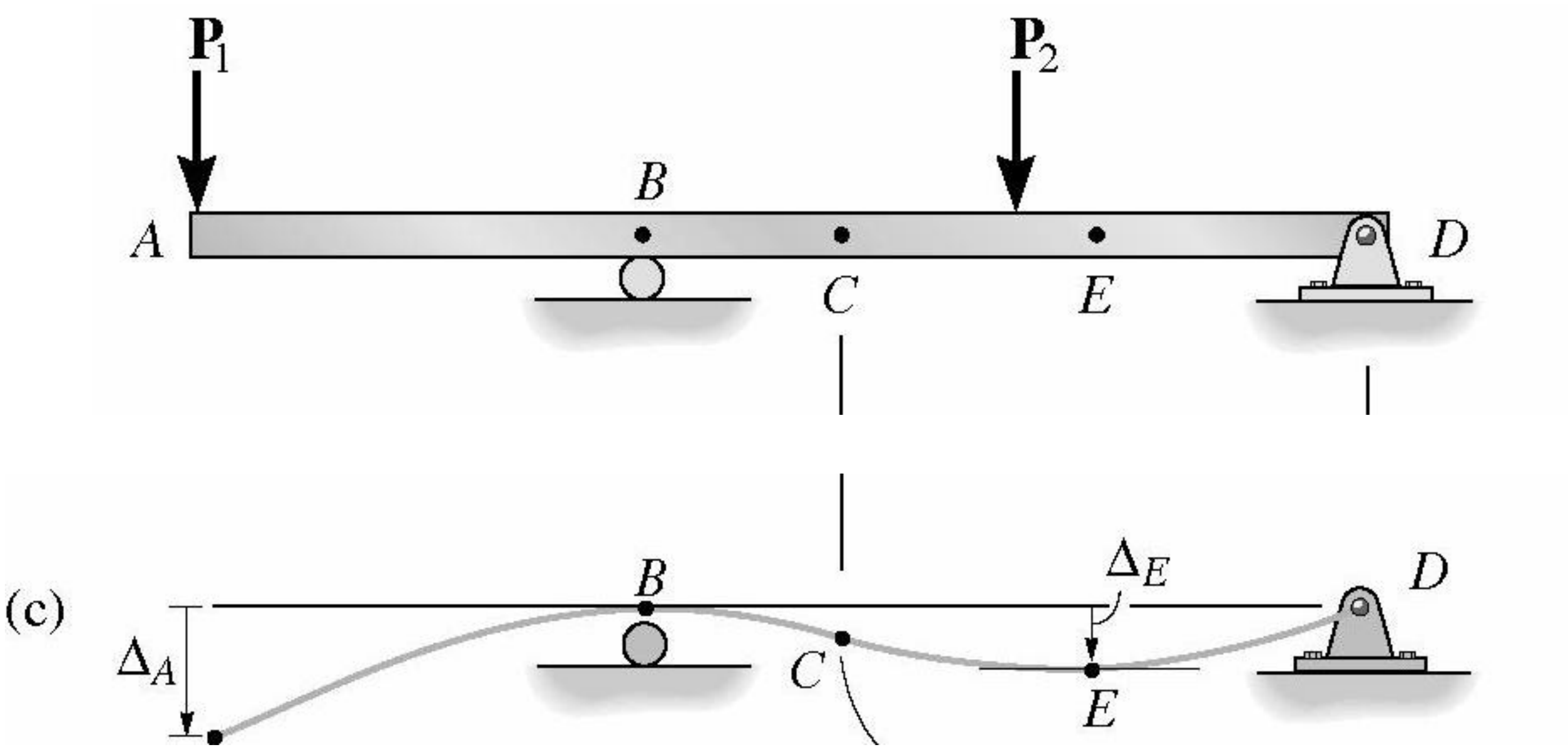
$$\sigma_B = -\frac{M\,c_B}{I} = -\frac{3\,\mathrm{kN}\cdot\mathrm{m}\times 0.038\,\mathrm{m}}{868\times 10^{-9}\,\mathrm{m}^4} \qquad \boxed{\sigma_B = -131.3\,\mathrm{MPa}}$$

Elastic curve



Moment signs convention





$$EI\frac{d^4v}{dx^4} = q(x)$$

Loading equation: **q(x) = 0**

$$EI_z \frac{d^4 v(x)}{dx^4} = 0$$

- First integration: transverse loading

$$V_y(x) = EI_z \frac{d^3 v(x)}{dx^3} = C_1$$

- Second integration: bending moment

$$M_z(x) = EI_z \frac{d^2 v(x)}{dx^2} = C_1 x + C_2$$

- Third integration: slope

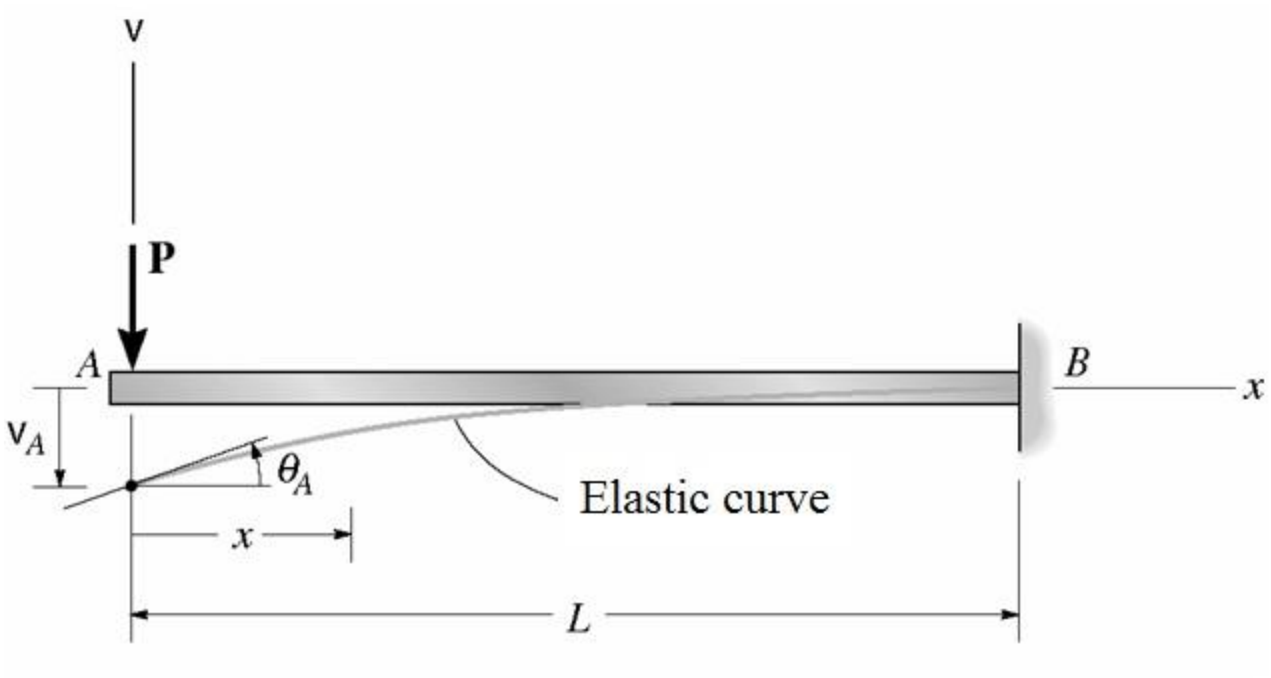
$$\theta_z(x) = EI_z \frac{dv(x)}{dx} = C_1 \frac{x^2}{2} + C_2 x + C_3$$

- **Fourth integration: vertical displacement**

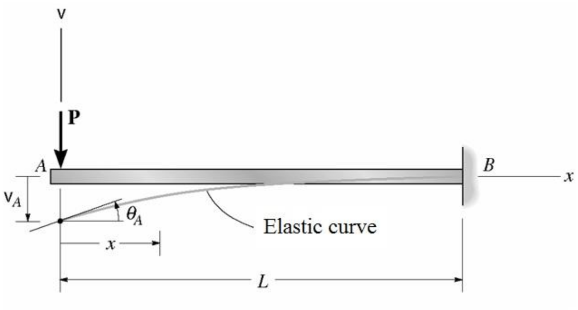
$$EI_z v(x) = C_1 \frac{x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$$

Example

The cantilever beam shown in the figure below is subject to a vertical load P at its end. Determine the elastic curve equation. Consider EI constant.



The load tends to cause deflection of the beam as shown in the figure. The internal bending moment can be represented in the entire beam using a single *x* coordinate:



$$M = -P \cdot x$$

Applying the equation $EI\dfrac{d^2v}{dx^2} = M(x)$ and integrating twice, we have:

$$EI\frac{d^2v}{dx^2} = -P \cdot x \qquad (1)$$

$$EI\frac{dv}{dx} = -\frac{P \cdot x^2}{2} + C_1 \qquad (2)$$

$$EI \cdot v = -\frac{P \cdot x^3}{6} + C_1 \cdot x + C_2 \qquad (3)$$

Considering the boundary conditions, we have :

- ✓ $\dfrac{dv}{dx} = 0 \quad \text{em} \quad x = L$

- ✓ $v = 0 \quad \text{em} \quad x = L$

Therefore:

$$0 = -\frac{P \cdot L^2}{2} + C_1$$

$$0 = -\frac{P \cdot L^3}{6} + C_1 \cdot L + C_2$$

So:

$$C_1 = \frac{P \cdot L^2}{2} \quad \text{e} \quad C_2 = -\frac{P \cdot L^3}{3}$$

Replacing C1 and C2 in the equations (2) and (3), where $\boxed{\theta = dv/dx}$, we have:

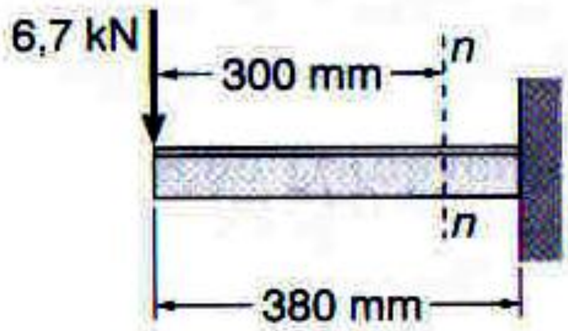
$$\theta = \frac{P}{2 \cdot EI}\left(L^2 - x^2\right) \qquad v = \frac{P}{6 \cdot EI}\left(-x^3 + 3 \cdot L^2 x - 2 \cdot L^3\right)$$

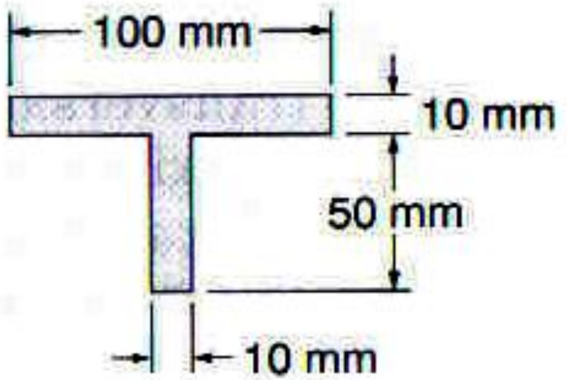
Maximum slope and displacement occur in A (x = 0):

$$\theta_A = \frac{PL^2}{2 \cdot EI} \qquad v_A = -\frac{PL^3}{3 \cdot EI}$$

Example on Scilab

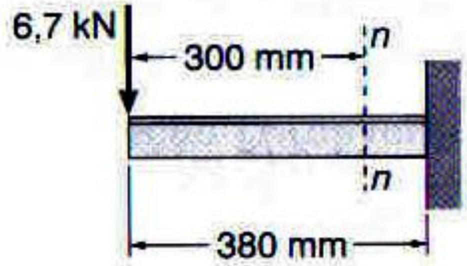
Considering the cantilever beam an its cross section (n-n) showed in the figure below and knowing that the material's modulus of elasticity is 210 GPa.

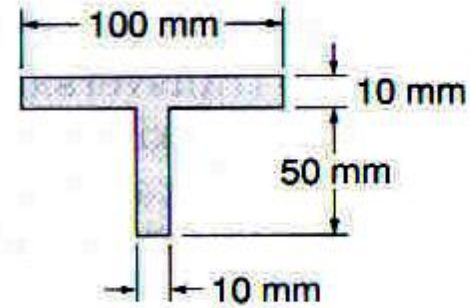




a) Determine the maximum slope and vertical displacement at the load application point;
b) Plot the stress graph at the cross section n-n.

a) Scilab code





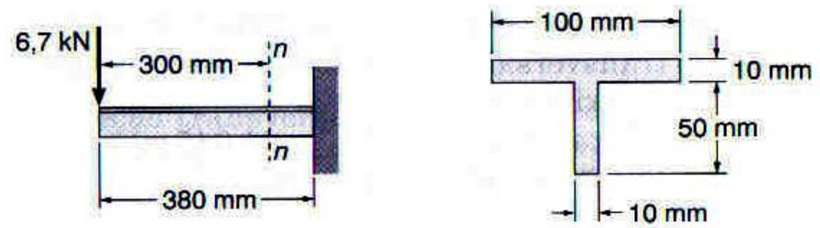

```
clc; clear
P = 6700; //N
E = 210000; //MPa
L1 = 380; // mm
L2 = 300; // mm
// Area from the bottom to the cross section top
b1 = 10;// base of fig 1 (mm)
h1 = 50;// height of fig 1 (mm)
b2 = 100;//base of fig 2 (mm)
h2 = 10;// height of fig 2 (mm)
y1 = h1/2; y2 = h1+h2/2; A1 = b1*h1;
A2 = b2*h2; A = [A1,A2]; Y = [y1;y2];
y_bar = sum(A*Y)/sum(A);
y_bar_comp = h1+h2-y_bar;
d1 = y_bar-y1;
d2 = y_bar-y2;
i1 = (b1*(h1^3))/12+A1*(d1^2);
i2 = (b2*(h2^3))/12+A2*(d2^2);
I = [i1,i2];
I = sum(I);
```

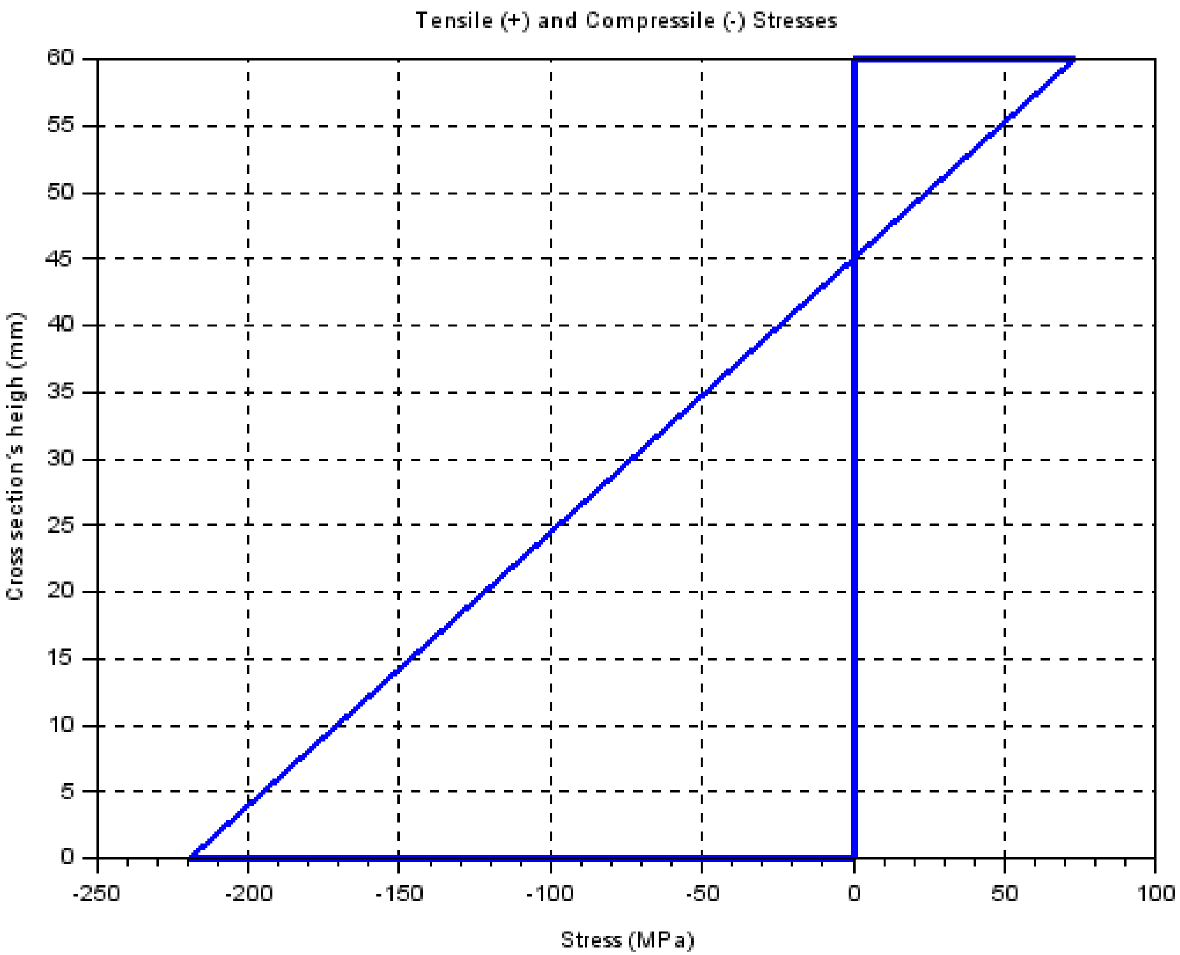
Solution

```
// a)
teta = (P*(L1^2))/(2*E*I); // radian
disp('teta (rad) = '); disp(teta)
v = (P*(L1^3))/(3*E*I); // mm
disp('Vertical displacement (mm) = '); disp(v)
// b) graph
Mmax = P*L2;// Max moment
compr = -(Mmax*y_bar)/I
disp('Compressile stress (MPa) = '); disp(compr)

y_bar_comp = (h1+h2)-y_bar;
tens = ((Mmax)*y_bar_comp)/I
disp('Tensile stress (MPa) = '); disp(tens)

plot([0,tens,0,0,0,compr,0],...
[y_bar,h1+h2,h1+h2,y_bar,0,0,y_bar],"b-","LineWidth",3);
title("Tensile (+) and Compressile (-) Stresses")
xlabel("Stress (MPa)")
ylabel("Heigh (mm)")
xgrid
```

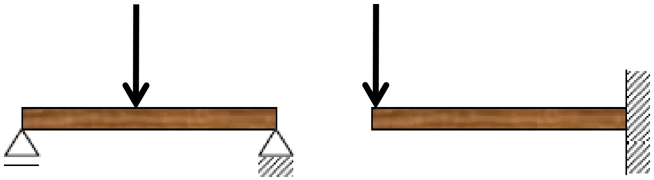
```
teta (rad) =

   0.0055843

Vertical displacement (mm) =

   1.4146894

Compressile stress (MPa) =

 - 219.27273

Tensile stress (MPa) =

   73.090909
```

b) Graph


6,7 kN
300 mm
n
n
380 mm
100 mm
10 mm
50 mm
10 mm


Tensile (+) and Compressile (-) Stresses
Cross section's heigh (mm)
Stress (MPa)
60
55
50
45
40
35
30
25
20
15
10
5
0
-250
-200
-150
-100
-50
0
50
100

Transverse loading



⇨ Necessary and sufficient conditions for the balance of a rigid body in space:

$$\sum F_x = 0 \qquad \sum M_x = 0$$

$$\sum F_y = 0 \qquad \sum M_y = 0$$

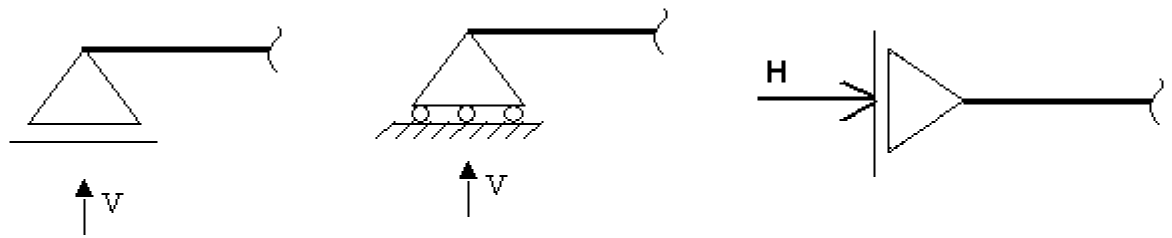
$$\sum F_z = 0 \qquad \sum M_z = 0$$

NOTE: These are the six universal equations of statics.

⇨ Necessary and sufficient conditions for the balance of a rigid body in the plane:

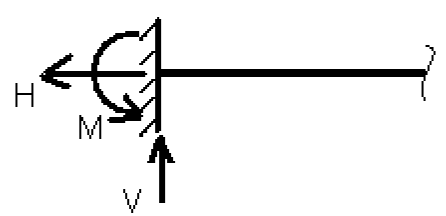
$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M = 0$$

- roller support $\rightarrow$ prevents translation;



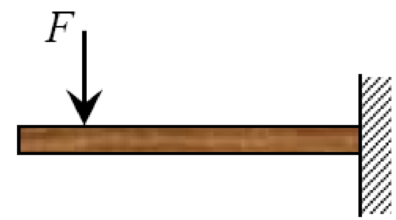
- articulated support $\rightarrow$ prevents two translations;

- fixed support $\rightarrow$ prevents two translations and one rotation.

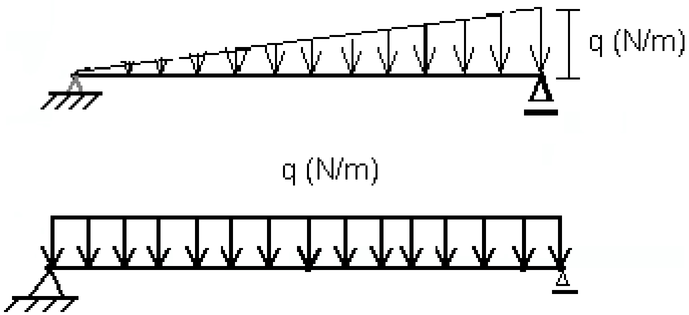


Ways of distribution

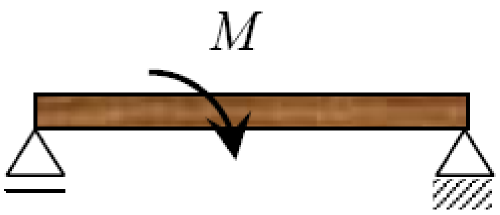
a) Concentrated loads - are an approximate way to treat loads distributed in very small areas. They are represented by loads applied punctually.



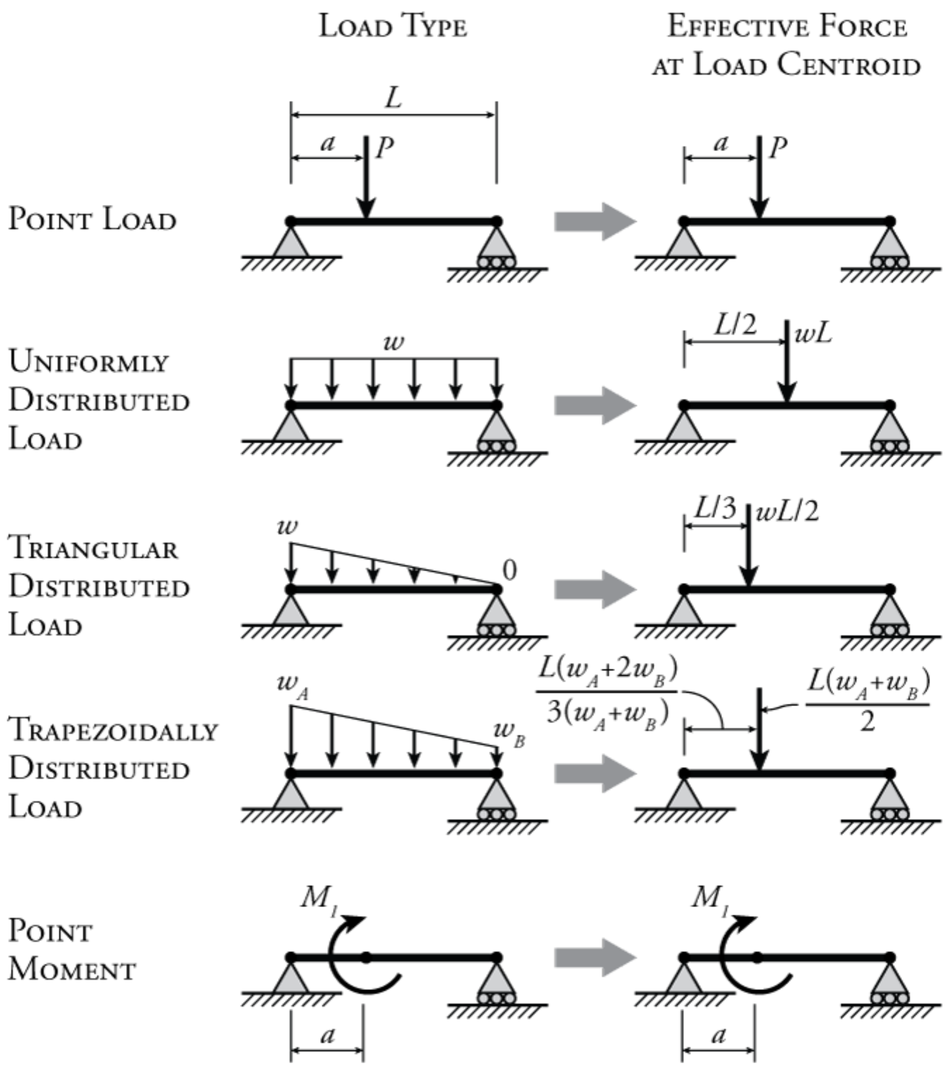
b) Distributed loads - the most common types are uniformly distributed loads and triangular loads.



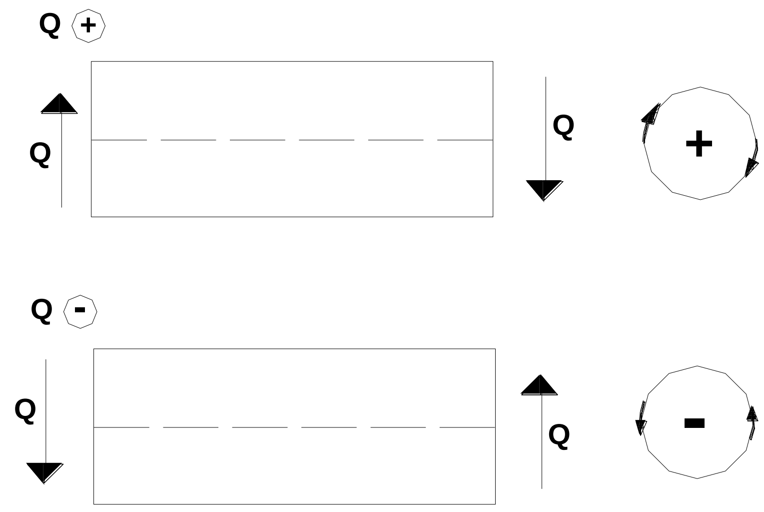
c) Moment - are loads of the type bending moment (or torsor) applied to any point of the structure.



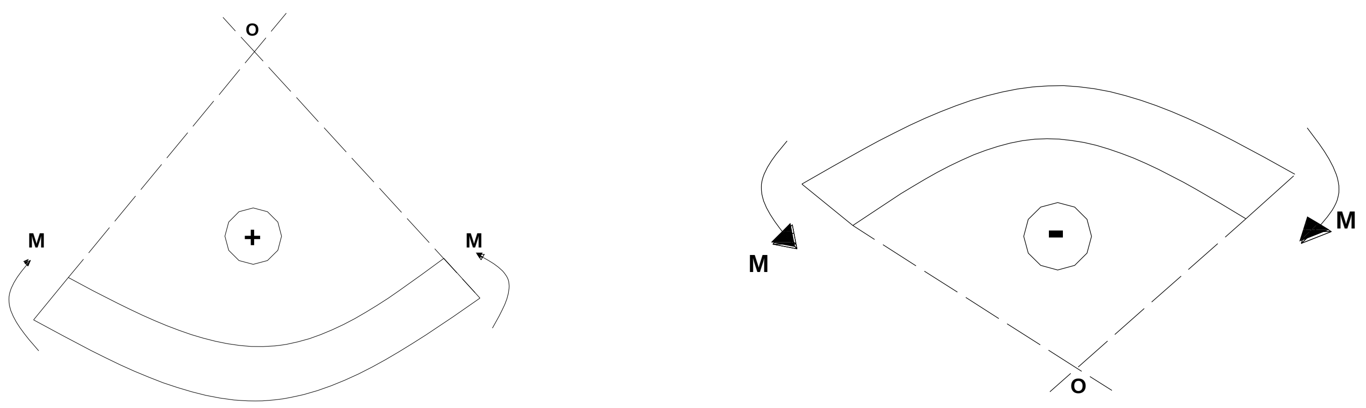
Loading types



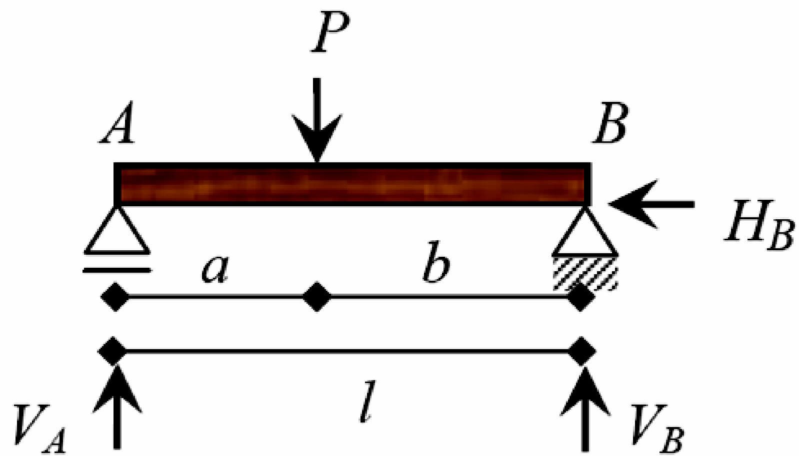
⇨ Sign Convention – Shear (Q)



⇨ Sign Convention – Moment (Q)



Example: Point load

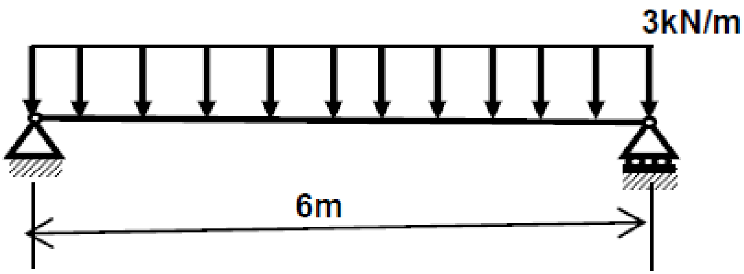


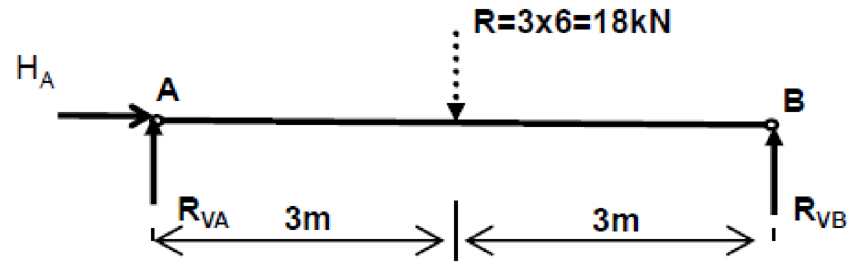
$$\sum F_x = 0 \Rightarrow H_B = 0$$

$$\sum F_y = 0 \Rightarrow V_A + V_B = P$$

$$\sum M_A = 0 \Rightarrow V_B \cdot l - P \cdot a = 0 \Rightarrow V_B = \frac{P \cdot a}{l} \Rightarrow V_A = \frac{P \cdot b}{l}$$

Example: Distributed load





$$\sum M_A = 0$$

$$6xR_{VB} - 3x18 = 0 \quad \therefore \quad R_{VB} = 9kN$$

$$\sum M_B = 0$$

$$-6xR_{VA} + 3x18 = 0 \quad \therefore \quad R_{VA} = 9kN$$

$$\sum Fx = 0$$

$$\therefore HA = 0$$

Shear and moment diagram

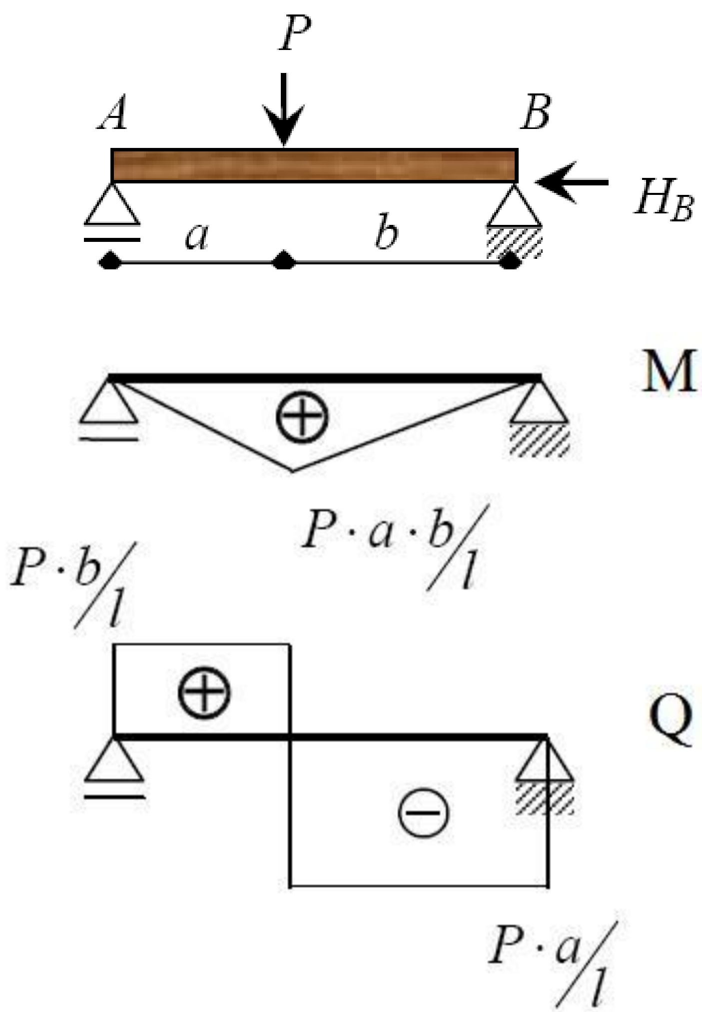
⇨ Obtaining Shear and moment diagram in the sections of the beam as a result of the active loading.

⇨ Stress Diagrams: graphical representation of the stresses in the sections along the entire element (perpendicular to the beam axis).

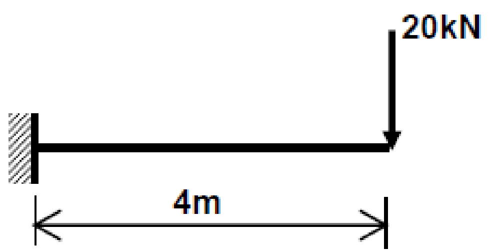
⇨ For a beam loaded in its own plane, we have:

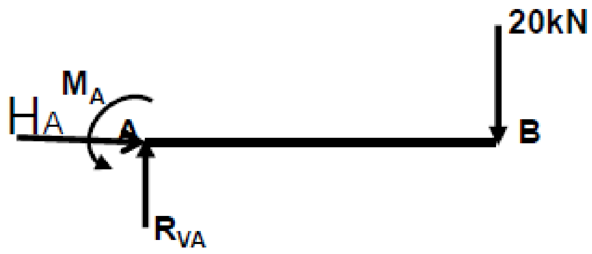
- **Shear diagram**
- **Moment diagram**

Shear and moment diagram: Point load



Cantilever beam: Point load





$$\sum Y = 0$$

$$R_{VA} - 20 = 0 \quad \therefore \quad R_{VA} = 20kN$$

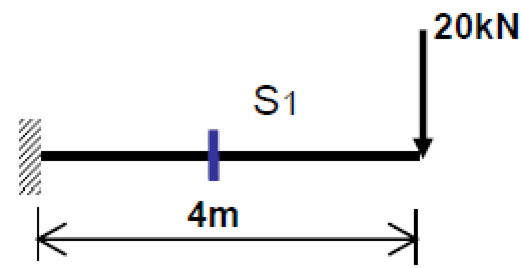
$$\sum Fx = 0$$

$$\therefore HA = 0$$

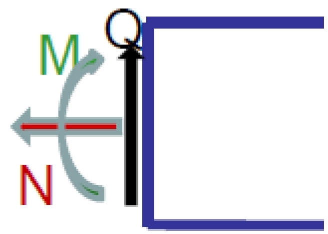
$$\sum M_A = 0$$

$$-4x20 + M_A = 0 \quad \therefore \quad M_A = 80kN.m$$

Cantilever beam: Point load



S1: $0 \leq x < 4$

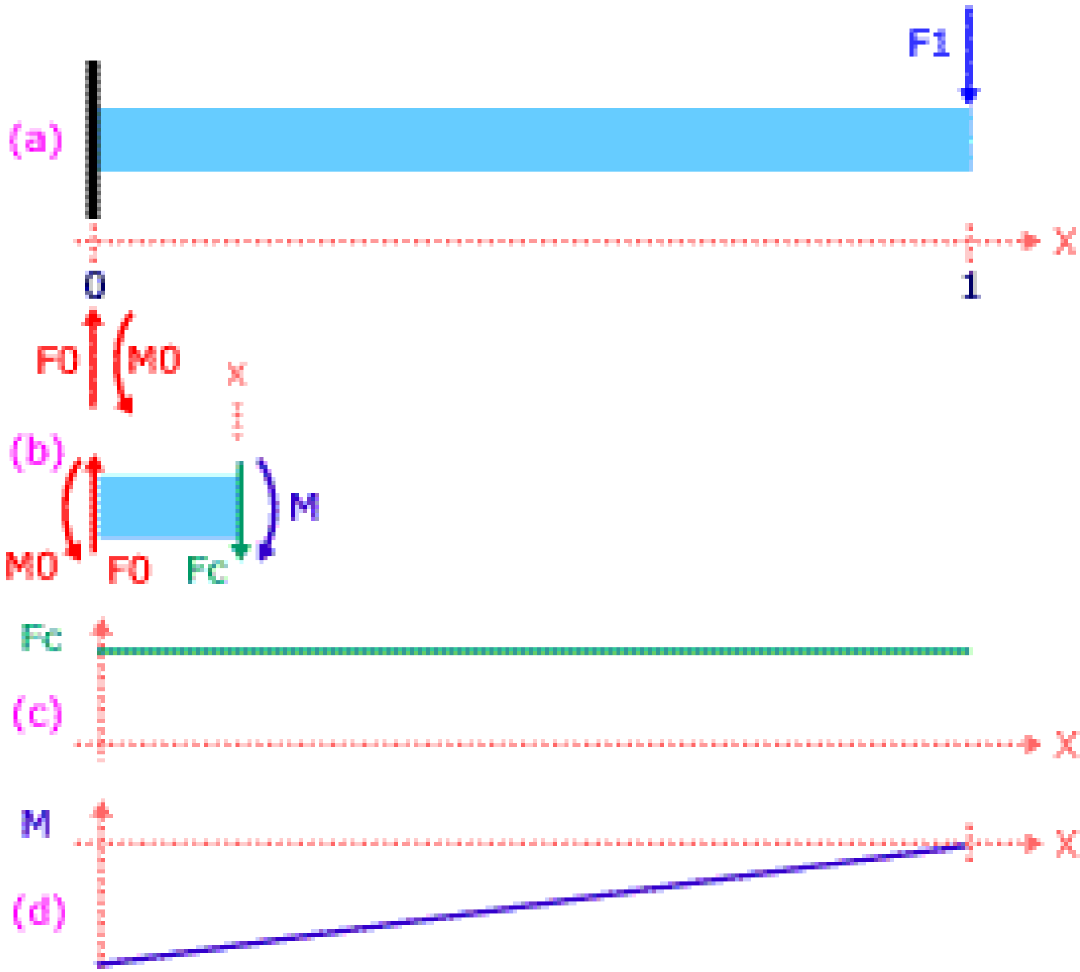


$N(x) = -H_A \longrightarrow N(x)=0$

$Q(x) = V_{RA} \longrightarrow V(x) = 20\ kN$

$M(x) = V_{RA} \times X - M_A \longrightarrow M(x) = 20.X - 80$

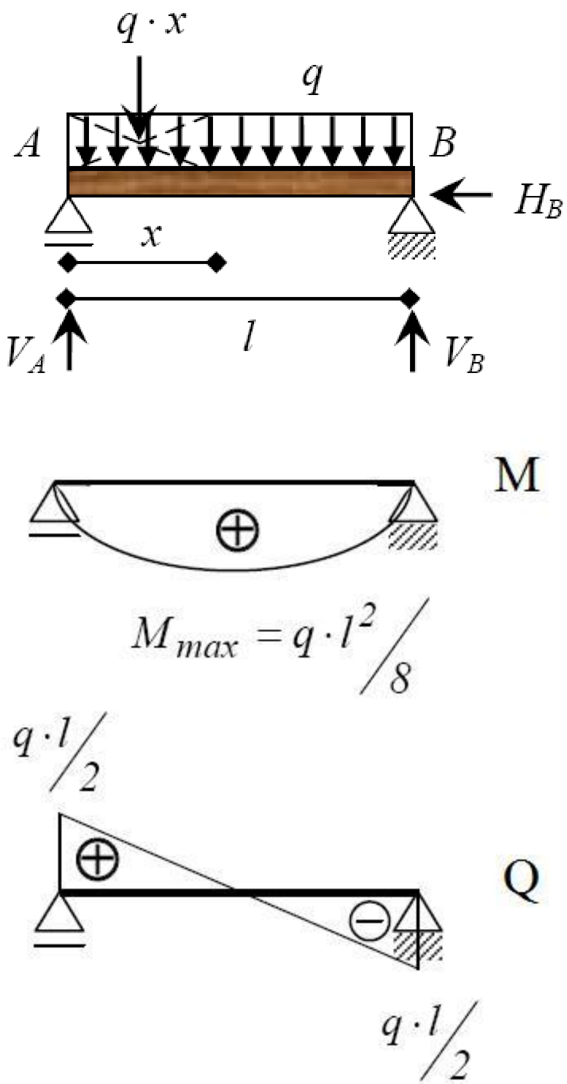
Shear and moment diagram



Shear

Moment

Shear and moment diagram: Distributed load

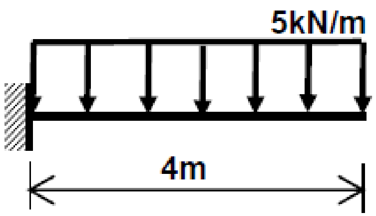


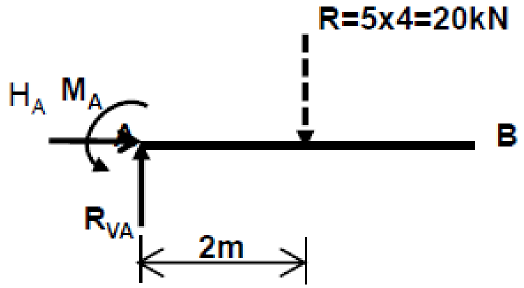
In a generic section S, we have:

$$M_s = \frac{q \cdot l}{2} \cdot x - q \cdot x \cdot \frac{x}{2} = q \cdot \frac{l^2}{2} \cdot \left(\frac{x}{l} - \frac{x^2}{l^2} \right)$$

$$Q_s = \frac{q \cdot l}{2} - q \cdot x$$

Cantilever: Distributed load





$$\sum Y = 0$$

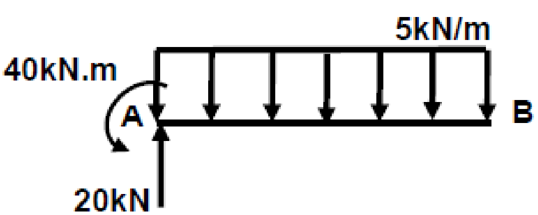
$$R_{VA} - 20 = 0 \quad \therefore \quad R_{VA} = 20kN$$

$$\sum Fx = 0$$

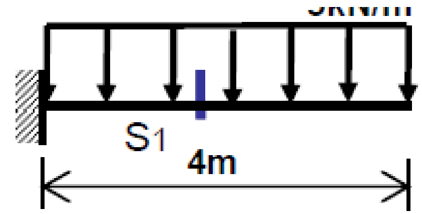
$$\therefore HA = 0$$

$$\sum M_A = 0$$

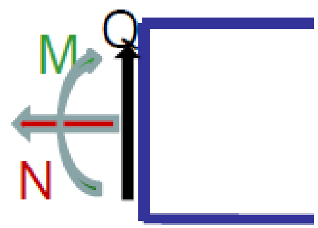
$$-20x2 + M_A = 0 \quad \therefore \quad M_A = 40kN.m$$



Shear and moment calculation



S1: $0 \leq x < 4$

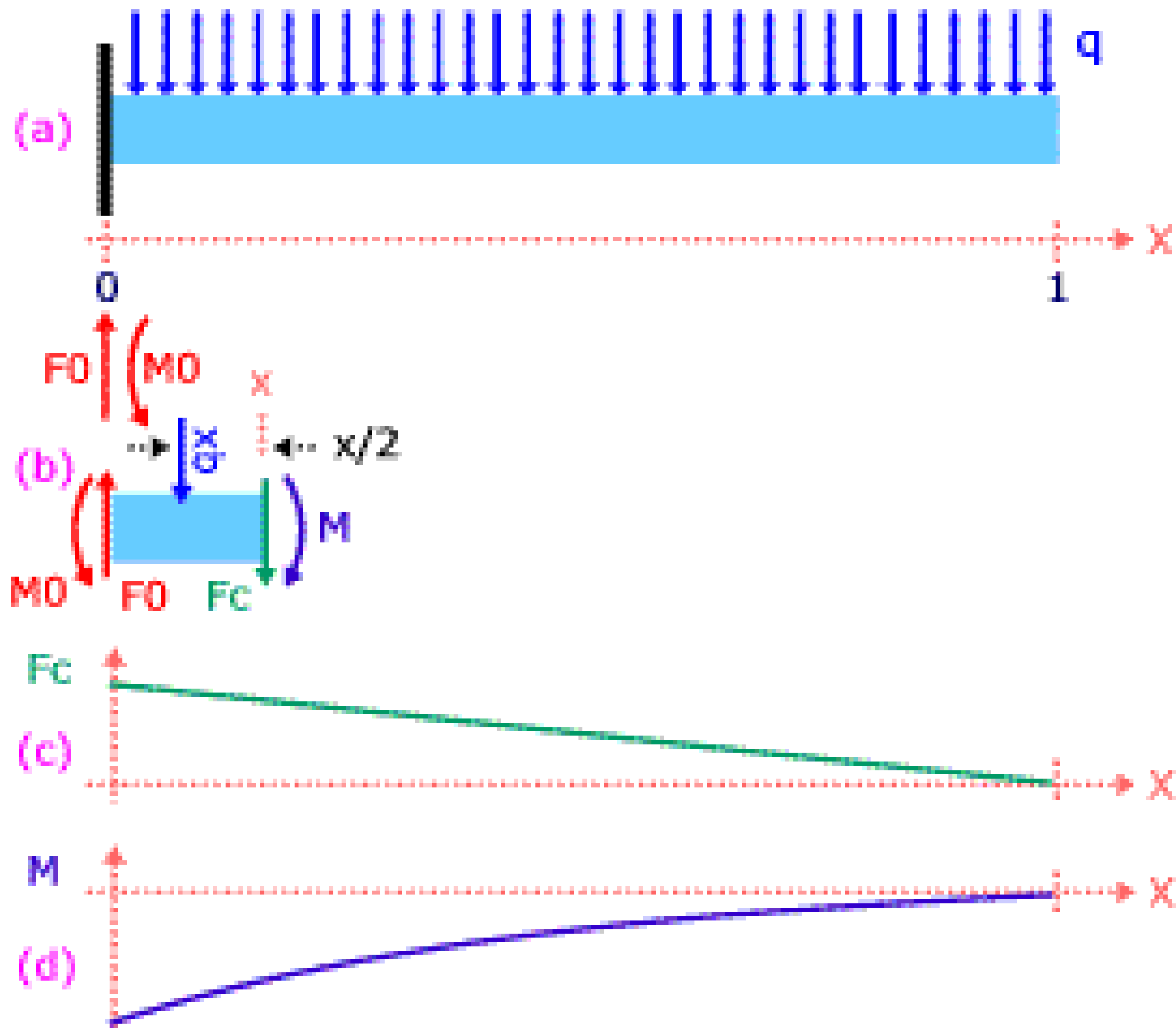


$N(x) = -H_A \longrightarrow N(x) = 0$

$Q(x) = V_{RA} - 5.X \longrightarrow V(x) = 20 - 5.X$

$M(x) = V_{RA} \times X - M_A - 5.(X^2)/2 \longrightarrow M(x) = 20.X - 40 - 5.(X^2)/2$

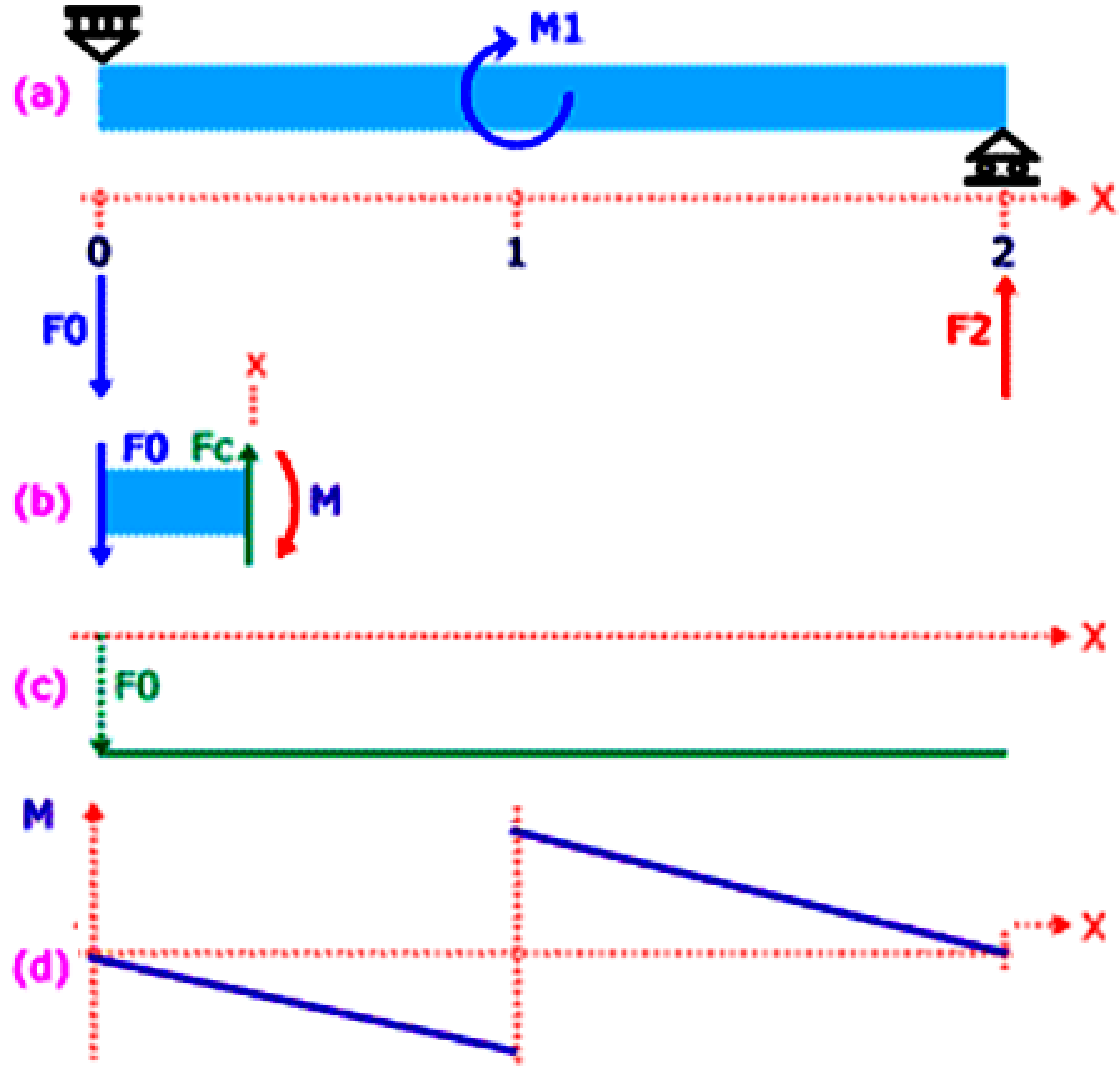
Diagrams



Shear

Moment

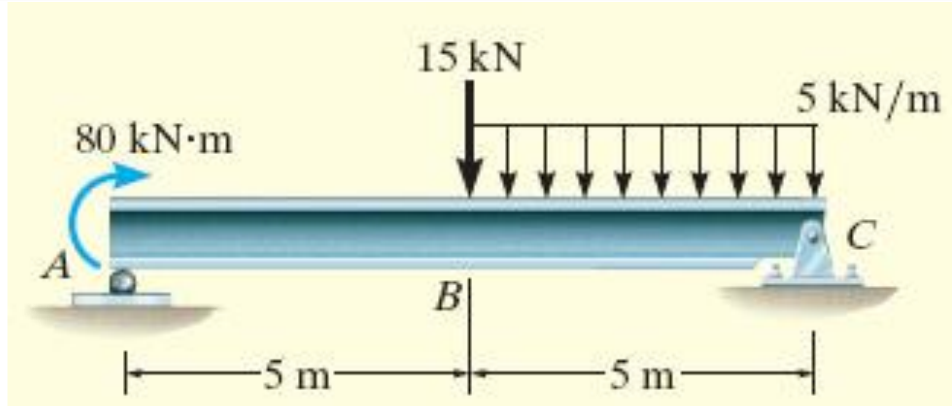
External moment



Shear

Moment

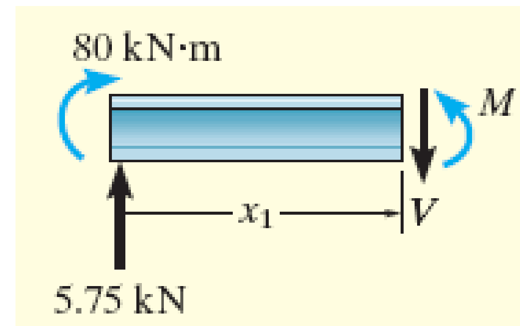
External moment: Example



$$0 \leq x_1 < 5\,\text{m},$$

$$+\uparrow \sum F_y = 0; \qquad 5{,}75 - V = 0 \Rightarrow V = 5{,}75\,\text{kN} \qquad (1)$$

$$+\sum M = 0; \qquad -80 - 5{,}75x_1 + M = 0 \Rightarrow M = (5{,}75x_1 + 80)\,\text{kNm} \qquad (2)$$

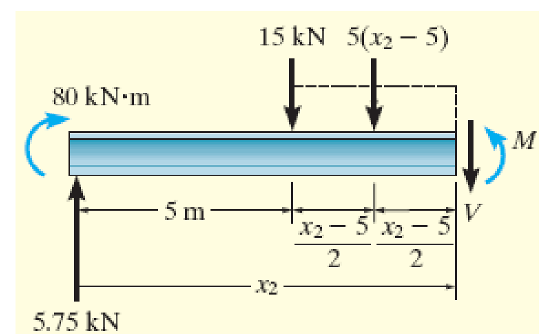


$$5\,\text{m} \leq x_1 < 10\,\text{m},$$

$$+\uparrow \sum F_y = 0; \qquad 5{,}75 - 15 - 5(x_2 - 5) - V = 0 \Rightarrow V = (15{,}75 - 5x_2)\,\text{kN}\ (3)$$

$$+\sum M = 0; \qquad -80 - 5{,}75x_1 + +15 + 5(x_2 - 5)\left(\frac{x_2 - 5}{2}\right) + M = 0$$

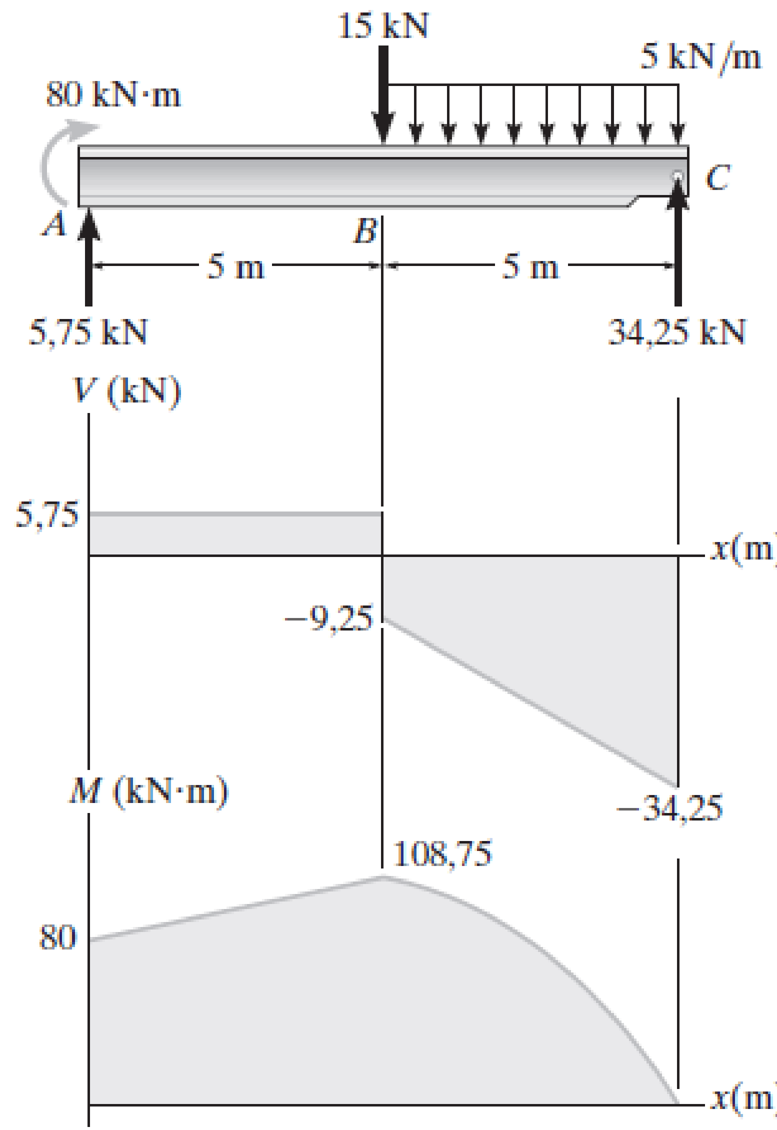
$$M = \left(-2{,}5x_2^2 + 15{,}75x_2 + 92{,}5\right)\text{kNm} \qquad (4)$$



Diagrams

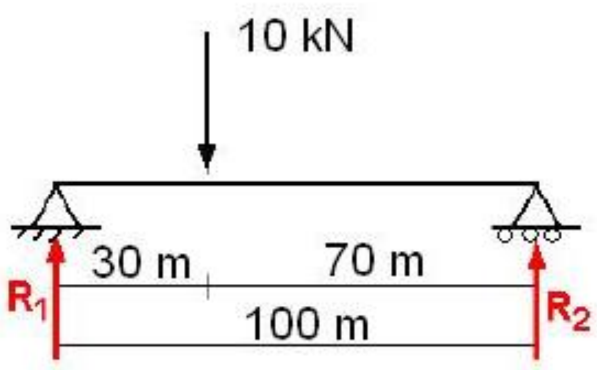
Shear ➔

Moment ➔

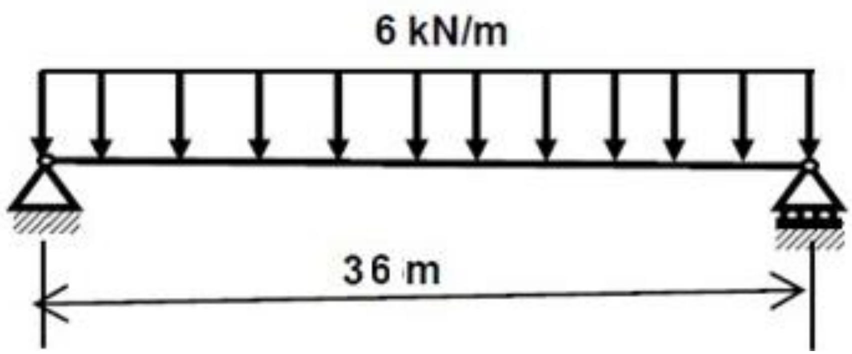


Examples on Scilab

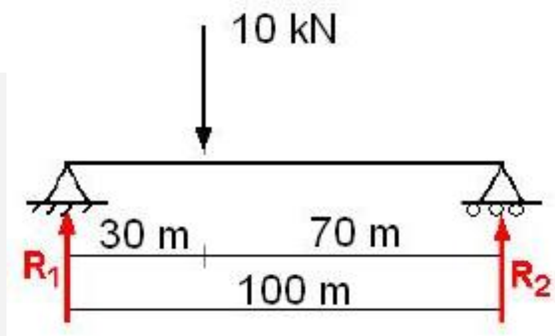
a)



b)

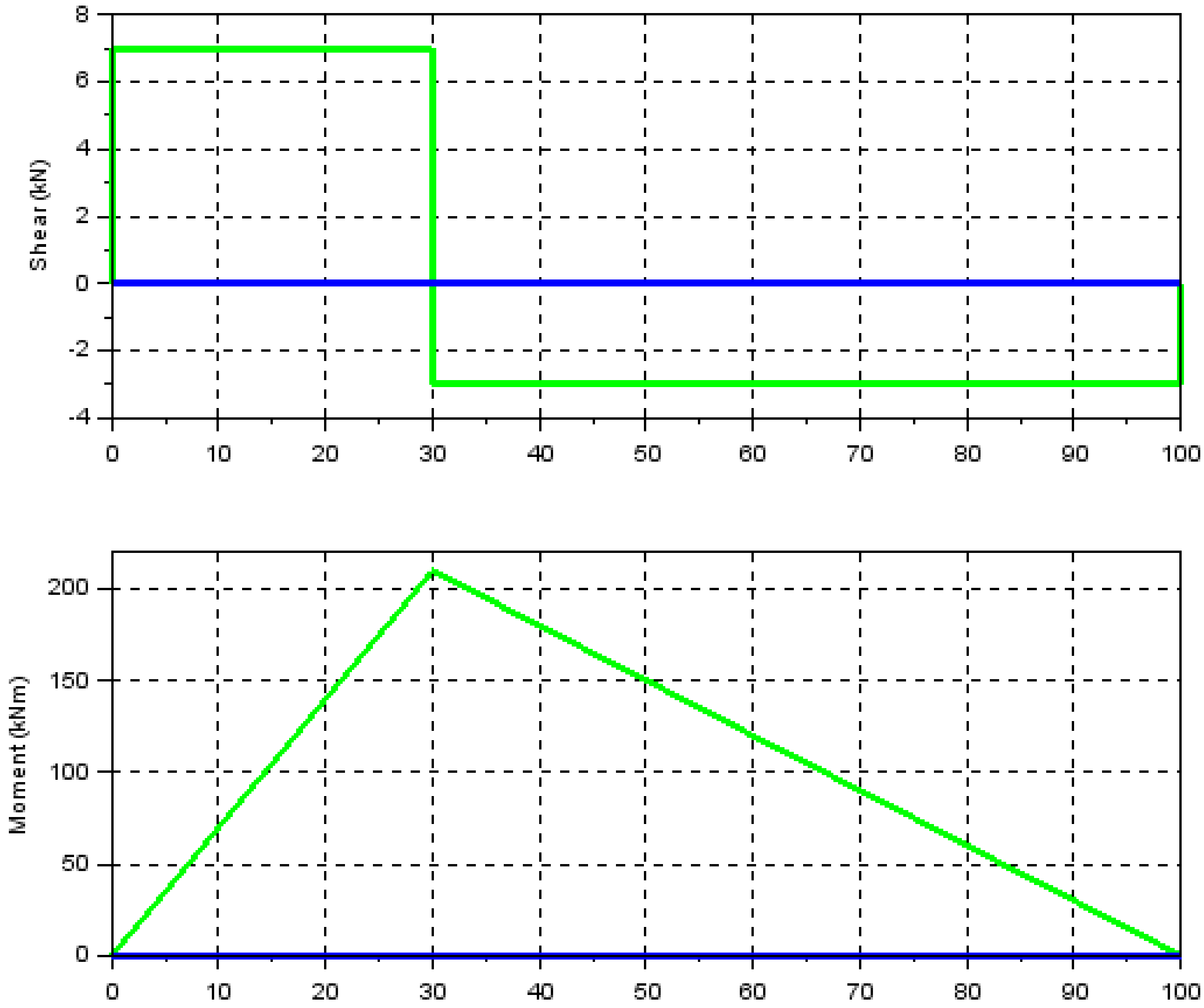


Solution (a)

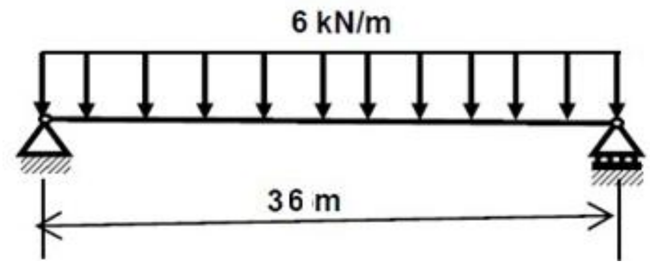



```
clc; clear;
F = 10; // kN
a = 30; // length of the 1st patch (m)
b = 70; // length of the 2nd patch (m)
L = a+b;// beam total length (m)
// Sum of the moments in the point 1: r2*L=F*a =>
r2 = (F*a)/L;
r1 = F-r2; // r1+r2=F;
// Moment (1st patch)
x = 0:0.01:a;
m = r1*x;
// Moment (2nd patch)
x = a:0.01:L;
m = (r1*x)-(F*(x-a));// Sum of the moments in the point 2
subplot(2,1,1);
plot([0,0,a,a,L,L],[0,r1,r1,-r2,-r2,0],"g-",[0,L],[0,0],"b-","LineWidth",3);
ylabel("Shear (kN)")
xgrid
subplot(2,1,2);
M_max = r1*a; // Max moment at x = a in the 1st patch
plot([0,a,L],[0,M_max,0],"g-",[0,L],[0,0],"b-","LineWidth",3);
ylabel("Moment (kNm)")
xgrid
```

Graph (a)

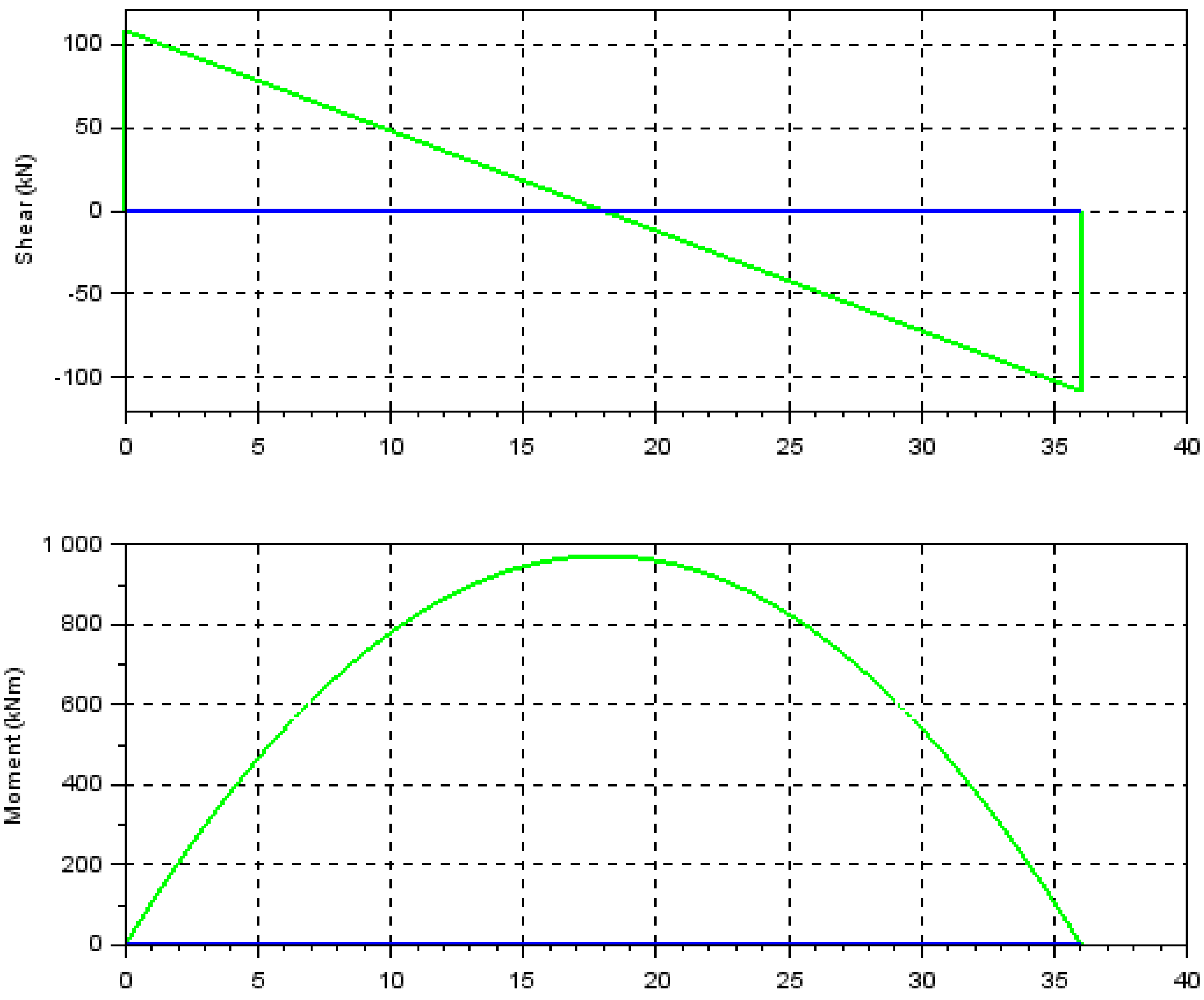

Shear (kN)
8
6
4
2
0
-2
-4
0
10
20
30
40
50
60
70
80
90
100
Moment (kNm)
200
150
100
50
0

Solution (b)




```
clc; clear;
Q = 6; //(kN/m)
L = 36; //(m)
ra = (Q*L)/2;
rb = ra;
x = 0:0.01:L; // for section S varying along L
m = (ra*x)-((Q*(x .^2))/2);
subplot(2,1,1);
plot([0,0,L,L],[0,ra,-rb,0],"g-",[0,L],[0,0],"b-","LineWidth",2);
ylabel("Shear (kN)")
xgrid
subplot(2,1,2);
plot(x,m,"g-",[0,L],[0,0],"b-","LineWidth",2);
ylabel("Moment (kNm)")
xgrid
```

Graph (b)



Shear (kN)
100
50
0
-50
-100
0
5
10
15
20
25
30
35
40
Moment (kNm)
1 000
800
600
400
200
0
0
5
10
15
20
25
30
35
40