Computer Graphics

CHAPTER 2
PART 1

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Graphics Output Primitives

Plotting Point

Point **is the fundamental element of the picture representation.**

- **It is nothing but the position in a plan defined as either pairs or triplets of number depending on whether the data are two or three dimensional.**
- **Thus, (x1,y1) or (x1,y1,z1) would represent a point in either two three dimensional space.**
  - **Two points would represent a line or edge,**
  - **A collection of three or more points a polygon.**

- **The *representation of curved lines* is usually accomplished by approximating them by *short straight line segments*.**

Graphics Output Primitives

If the two points used to specify a line are (x1,y1) and (x2,y2), then an equation for the line is given as.

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) + y_1$$

$$Y = mx + b$$

Where $m = \frac{y_2 - y_1}{x_2 - x_1}$ **And** $b = y_1 - mx_1$

- **The above equation is called *the slop intercept form of the line*.**

- **The slope m is the change in height (y_2 - y_1) divided by the change in the width (x_2 - x_1) for two points on the line.**

- **The intercept b is the height at which the line crosses the y-axis.**

Example :

Write an equation in slope-intercept form of the line that passes through the points (2,2) and (3,4).

- **'m' represents the slope**

4 is y_2 and 2 is y_1
3 is x_2 and 2 is x_1

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \frac{4-2}{3-2} \qquad m = \frac{2}{1} \qquad m = 2$$

- **Use the slope-intercept form of a line and the values we know to solve for 'b', which is the y-intercept**

We can plug in 2 for m, and we take one of our ordered pairs and plug it in for x and y we solve b

$$Y = mx + b \qquad Y = 2\,x + b \qquad 4 = 2\,(3) + b \qquad 4 = 6 + b \qquad b = 4 - 6 \qquad b = -2$$

'b' is equal to -2, our y-intercept

Plug the values for slope and y-intercept into the slope-intercept form to find the equation of our line

$$Y = 2x - 2$$

Write the slope-intercept form of the equation of the line that passes through the two points.

1. (2, 3), (6, 11)

$\boxed{y = 2x - 1}$

2. (1, -7), (3, -15)

$\boxed{y = -4x - 3}$

Write an equation of the line in point-slope form that passes through the point and has the given slope. Then rewrite the equation in slope-intercept form.

3. (-1, 1), $m = \frac{2}{3}$

$\boxed{y = \frac{2}{3}x + \frac{5}{3}}$

4. (6, -3), $m = -\frac{1}{2}$

$\boxed{y = -\frac{1}{2}x}$

Write the equation in standard form of the line that passes through the two points.

5. (5, 8), (3, 2)

$\boxed{3x - y = 7}$

6. (-4, -5), (-2, 5)

$\boxed{5x - y = -15}$

- We can determine whether two lines are crossing or parallel.
- When two lines cross, they share some point in common. Of course, that point satisfies equations for the two lines. Let us see how to obtain this point.

Consider two lines having
slops m_1 and m_2 and
y-intercepts b_1 and b_2 respectively.
Then we can write the equations for these two lines as,

Line 1 : $y = m_1 x + b_1$
Line 2: $y = m_2 x + b_2$
If point (x_p, y_p) is shared by both lines, then it should satisfy the last two equations

And
$$y_p = m_1 x_p + b_1$$
$$y_p = m_2 x_p + b_2$$

Equating the above equations gives,

$$m_1 x_p + b_1 = m_2 x_p + b_2$$

$$x_p(m_1 - m_2) = b_2 - b_1$$

$$x_p = \frac{b_2 - b_1}{m_1 - m_2}$$

Substituting this into equation for line 1 or for line 2 gives,

$$y_p = m_1 \left[\frac{b_2 - b_1}{m_1 - m_2}\right] + b_1$$

$$y_p = \frac{m_1\,(b_2 - b_1) + b_1\,(m_1 - m_2)}{m_1 - m_2}$$

$$y_p = \frac{m_1\,b_2 - m_1 b_1 + b_1\,m_1 - b_1 m_2)}{m_1 - m_2}$$

$$y_p = \frac{m_1\,b_2 - b_1 m_2}{m_1 - m_2}$$

Therefore, the point $\left[\frac{b_2 - b_1}{m_1 - m_2}, \frac{m_1\,b_2 - b_1 m_2}{m_1 - m_2}\right]$ **is the point of intersection.**

If the two lines are parallel, they have the same slope. In the expression for the point of intersection, $m_1 = m_2$. So the denominator becomes zero. So the expression results in a divide by zero. This means, there is no point of intersection for the parallel lines.

Line Drawing Algorithms

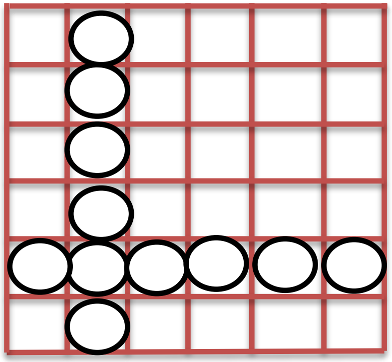
The process of 'turning on' the pixels for a line segment is called *vector generation* or *line generation*, and the algorithms for them are known as *vector generation algorithms* or *line drawing algorithms*.

Before discussing specific line drawing algorithms it is useful to note the general requirements for such algorithms. These requirements specify the desired characteristics of line.

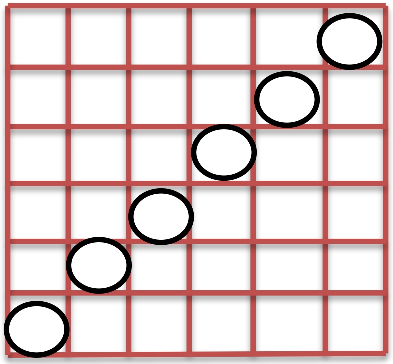
- **The line should appear as a straight line and it should start and end accurately.**
- **The line should be displayed with constant brightness along its length independent of its length and orientation.**
- **The line should be drawn rapidly.**

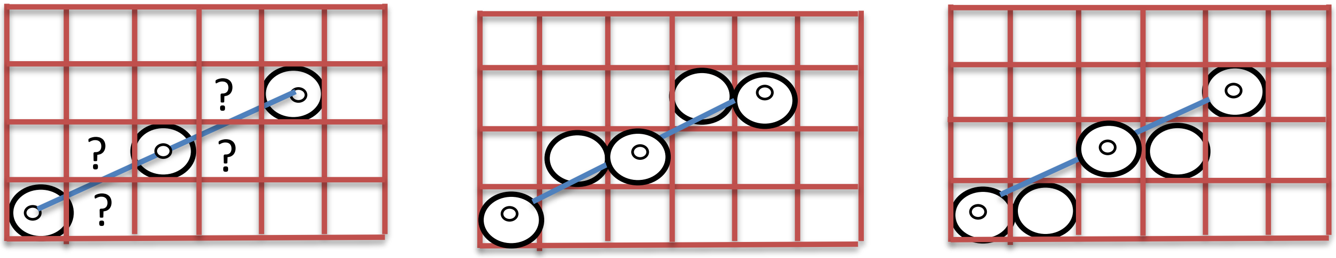
Let us see the different lines drawn in fig bellow,

Vertical and Horizontal line



45◦ line





Line with other orientation

- **As shown in the above figure, horizontal and vertical lines are straight and have same width.**
- **The 45° line is straight, but its width is not constant.**
- **The line with any other orientation is neither straight nor has same width. Such cases are due to the finite resolution of display and we have to accept approximate pixels in such situation.**
- **The brightness of the line is dependent on the orientation of the line. We can observe that the effective spacing between pixels for the 45° line is greater than for the vertical and horizontal lines. This will make the vertical and horizontal lines appear brighter than the 45° line.**
- **Complex calculations are required to provide equal brightness along lines of varying length and orientation.**

Vector Generation/Digital Differential Analyzer (DDA) line Algorithm

The DDA algorithm generates lines from their differential equations.

- **We calculate the length of the line in the X direction (number of pointes) by the equation:**

  ABS (x2-x1)

- **And calculate the length of the line in the Y direction (number of pointes) by the Equation:**

  ABS (y2-y1)

Where ABS is a function takes the positive of the arguments.

The increment steps (Δx and Δy) are used to increment the x and y coordinates for the next pointes to be plotted.

$$\Delta x = (x2 - x1)/length\,, \qquad \Delta y = (y2 - y1)/length$$

Algorithm DDA

1. Read the line end points ($x1$, y1) and ($x2$, y2) such that they are not equal. [If equal then plot that point and exit]
2. $\Delta x = | x2 - x1|$ and $\Delta y = | y2 - y1|$
3. If $(\Delta x \geq \Delta y)$ *then*
   Length=Δx
   Else
   Length=Δy
   End if
4. $\Delta x = (x2 - x1)/length$
   $\Delta y = (y2 - y1)/length$

[This makes either Δx or Δy equal to 1 because length is either $x2 - x1$ or $y2 - y1$ .Therefore, the incremental value for either x or y is one.]

5. $x = x1 + 0.5$ sign (Δx)
   $Y = y1 + 0.5$ sign (Δy)

[Here, sign function makes the algorithm work in all quadrants. It returns -1, 0, 1 depending on whether its argument is <0, =0, >0, respectively.]

6 . i=1 [begins the loop, in this points are plotted]
   While (i ≤ length)
   Begin
   Plot (integer (x), integer (y))
   $x = x + \Delta x$
   $y = y + \Delta y$
   $i = i + 1$
   End.

Stop

Note :
1- Sign function returns
: -1 if its argument is < 0
: 0 if its arguments is = 0
: +1 if its arguments is > 0
Ex. Sign(-10) = -1 , Sign (5) = 1
Using the Sign function makes the algorithm work in all quadrants.

Example :

consider the line from (0,0) to (6,6). Use the simple DDA algorithm to rasterize this line

Solution:

x1=0 , y1=0 , x2=6 , y2=6

Length = $y2 - y1 = x2 - x1$ =6

$\Delta x = (x2 - x1)/length$

= 6/6 = 1 and

$\Delta y = (y2 - y1)/length$

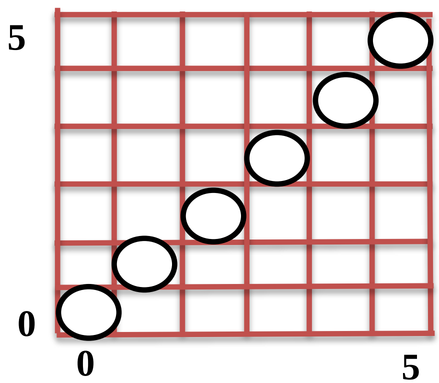
=6/6 = 1

Initial value for

$x = 0 + 0.5\ (1) = 0.5$

$Y = 0 + 0.5\ (1) = 0.5$

i	plot	x	y
		0.5	0.5
1	(0,0)		
		1.5	1.5
2	(1,1)		
		2.5	2.5
3	(2,2)		
		3.5	3.5
4	(3,3)		
		4.5	4.5
5	(4,4)		
		5.5	5.5
6	(5,5)		
		6.5	6.5



Example 2 :

consider the line from (0,0) to (4,6). Use the simple DDA algorithm to rasterize this line

Solution:

x1=0 , y1=0 , x2=4 , y2=6

Length = $y2 - y1$ = 6

$\Delta x = (x2 - x1)/length$

= 4/6 and

$\Delta y = (y2 - y1)/length$

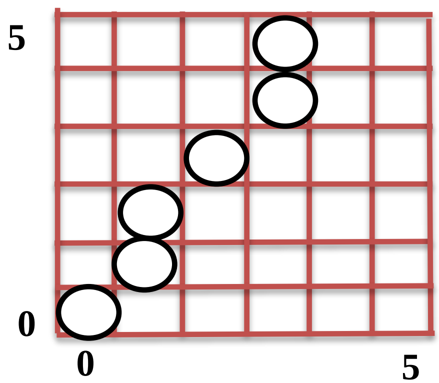
=6/6 = 1

Initial value for

x = 0 + 0.5 (4/6) =0.5

Y = 0 + 0.5 (1) = 0.5

The result are plot ted as shown



i	plot	x	y
		0.5	0.5
1	(0,0)		
		1.167	1.5
2	(1,1)		
		1.83	2.5
3	(1,2)		
		2.5	3.5
4	(2,3)		
		3.167	4.5
5	(3,4)		
		3.833	5.5
6	(3,5)		
		4.5	6.5

Advantages of DDA Algorithm

1. It is the simplest algorithm and it does not require special skills for implementation.

2. It is a faster method for calculating pixel positions than the direct use of equation

$$y = mx + b.$$

It eliminates the multiplication in the equation by making use of raster characteristics, so that appropriate increments are applied in the x or y direction to find the pixel positions along the line path.

Disadvantages of DDA Algorithm

1. Floating point arithmetic in DDA algorithm is still time-consuming.

2. The algorithm is orientation dependent. So the end point accuracy is poor.