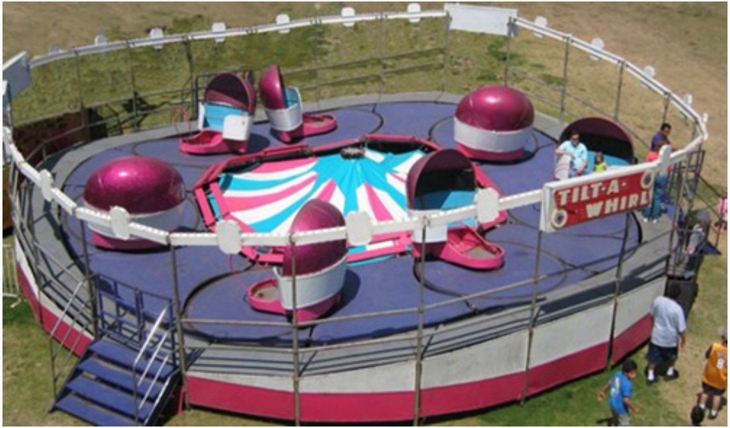
Project #3 Model

Reminder

Objective: Your objective is to maximize the standard deviation of the angular velocity of each car on the platform, $f = \sigma\left(\frac{d\phi}{dt}\right)$



Tilt-A-Whirl as Dynamical System

If we treat each car as a point mass, we can model the car's motion as a dynamical system.

For the equation of motion, refer to *"Chaos at the amusement park: Dynamics of the Tilt-A-Whirl"*, R. L. Kautz and B. M. Huggard, Am. J. Phys. 62 (1), January 1994.

- The equation we will use is (27) on page 63.

Tilt-A-Whirl as Dynamical System (cont.)

Equation (27) is a second-order in time ordinary differential equation (ODE), of the form

$$\frac{d^2\phi}{d\tau^2} = F\left[\phi, \frac{d\phi}{d\tau}\right],$$

where

- ϕ is the car's angle with respect to the beam, and;
- $\tau = 3\omega t$ is a nondimensional time.

Tilt-A-Whirl as Dynamical System (cont.)

To solve this numerically, rewrite it as a first-order system.

- Define the state vector

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{d\phi}{d\tau} \\ \phi \end{bmatrix}$$

or

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \phi \\ \frac{d\phi}{d\tau} \end{bmatrix}$$

- Add a new ODE to the system:

$$\frac{d\phi}{d\tau} = \frac{dy_2}{d\tau} = y_1$$

or

$$\frac{d\phi}{d\tau} = \frac{dy_1}{d\tau} = y_2$$

$$\frac{dy_1}{d\tau} = \frac{d}{d\tau}\left(\frac{d\phi}{d\tau}\right) = \frac{d^2\phi}{d\tau^2} = F\Big(\underbrace{\phi}_{y_2}, \underbrace{\frac{d\phi}{d\tau}}_{y_1}\Big)$$

Tilt-A-Whirl as Dynamical System (cont.)

Then, the first-order system of ODEs to be solved is

$$\frac{dy}{d\tau} = \frac{d}{d\tau}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} F(y_2, y_1) \\ y_1 \end{bmatrix}$$

- You can use Matlab's `ODE45` or other appropriate ODE solver to solve this system.

Initial conditions ?

Calculating the Objective

The objective is the standard deviation of $\frac{d\phi}{dt} = y_1$ over time:

$$f(x) = 3\omega\sqrt{\frac{1}{T}\int_0^T (y_1 - \bar{y}_1)^2 \ d\tau}$$

where

- $\bar{y}_1$ is the mean angular velocity, $\frac{1}{T}\int_0^T y_1 \ d\tau$, and;
- T is the total (non-dimensional) period of simulation.

The Matlab function `trapz` will come in handy for these calculations.

Verify, Verify, Verify!

You must provide evidence that your analysis is working.

- Do ϕ and $d\phi/dt$ behave as expected for $\omega << 1$?
- Do ϕ and $d\phi/dt$ behave as expected for $\omega >> 1$?
- Does your analysis agree with the results in the paper?

Some other Pointers

- Check that your integration period T is sufficiently long that the statistics are converged.
- ~~To reduce the influence of the initial conditions, you should run your analysis for a short "spin-up" period,~~ ~~T_{spin}~~~~, the final solution of which becomes the initial condition for the actual statistics-gathering run.~~

Beware of numerator + denominator

Eq (26) $\varepsilon = r_1 / 9 r_2 = \dfrac{r_1}{9 r_2}$