CONTROL OF AIRCRAFT
PRACTICAL WORK

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CONTROL OF AIRCRAFT

Choose an operating point for the aircraft modeling and the controllers synthesis (a different flight point for each group).
Subject number as a function of operating point:

Mach / Alt (ft)	0.71	0.95	1.22	1.33	1.52	1.71	1.92
511	11	12	13	14	15	16	17
2775	21	22	23	24	25	26	27
5835	31	32	33	34	35	36	37
9285	41	42	43	44	45	46	47
11812	51	52	53	54	55	56	57
15855	61	62	63	64	65	66	67
18585	71	72	73	74	75	76	77
21230	81	82	83	84	85	86	87

Control of aircraft

AIRCRAFT CHARACTERISTICS

The considered aircraft is a fighter aircraft of MIRAGE III class.

Total length	$\ell_t = \frac{3}{2}\ell_{ref}$
Mass	m=8400 kg
Aircraft centering (center of gravity position) (as % of total length)	c=52 %
Reference surface (Wings)	S=34 m^2
Radius of gyration	r_g=2,65 m
Reference length	ℓ_{ref}=5,24 m $= \frac{2}{3}\ell_t$

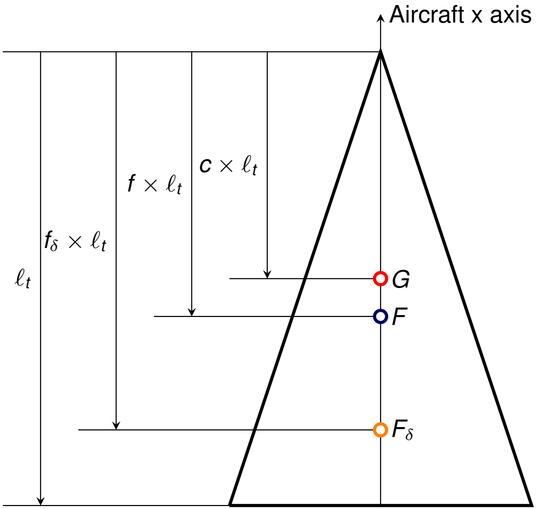
For the calculus of air density and speed of sound as a function of altitude, we will use the US 76 standard atmosphere model.

HYPOTHESIS

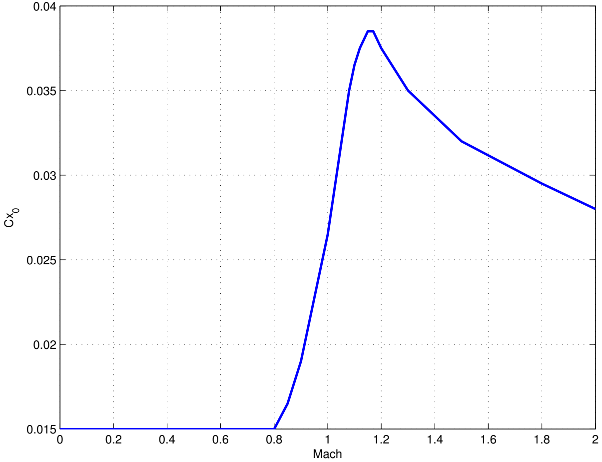
- Symmetrical flight, in the vertical plane (null sideslip and roll)
- Thrust axis merged with aircraft longitudinal axis
- Inertia principal axis = aircraft transverse axis (diagonal inertia matrix)
- Fin control loop: its dynamics will be neglected for the controller synthesis
- The altitude sensor is modeled by a 1^{st} order transfer function with a time constant $\tau = 0.65\ s$

AIRCRAFT AERODYNAMIC MODEL

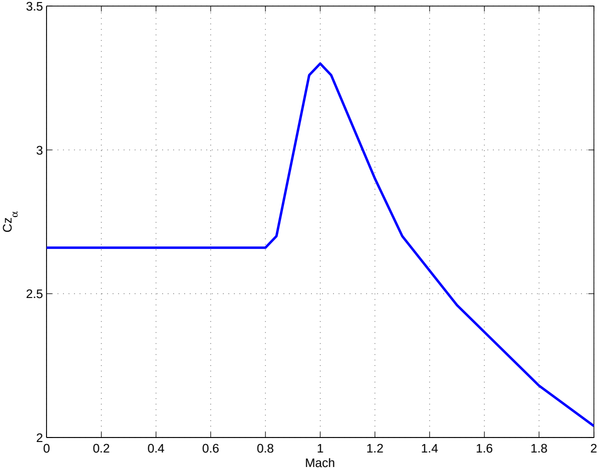
The aircraft aerodynamic coefficients for the longitudinal motion (drag, gradient of drag and lift, aerodynamic center for body and fins, polar coefficient and damping coefficient) are given on the following slides as functions of Mach number.


Aircraft x axis
$c \times \ell_t$
$f \times \ell_t$
$f_\delta \times \ell_t$
ℓ_t
G
F
F_δ

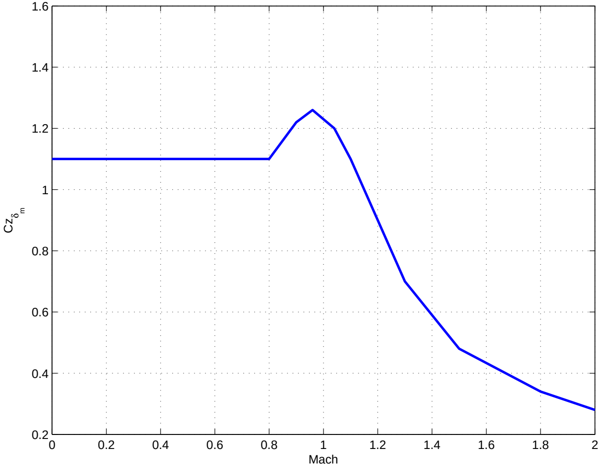
DRAG COEFFICIENT FOR NULL INCIDENCE Cx_0



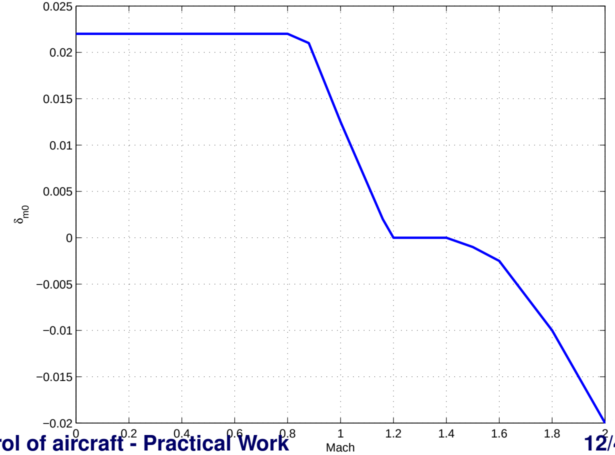
LIFT GRADIENT COEFFICIENT WRT α Cz_α (rad^{-1})



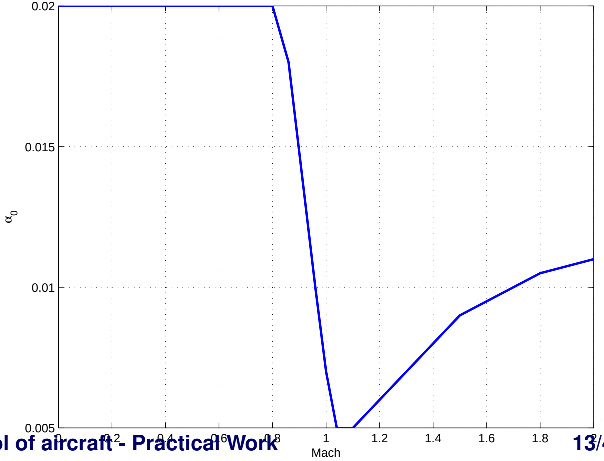
Lift gradient coefficient wrt δ_m Cz_{δ_m} (rad^{-1})



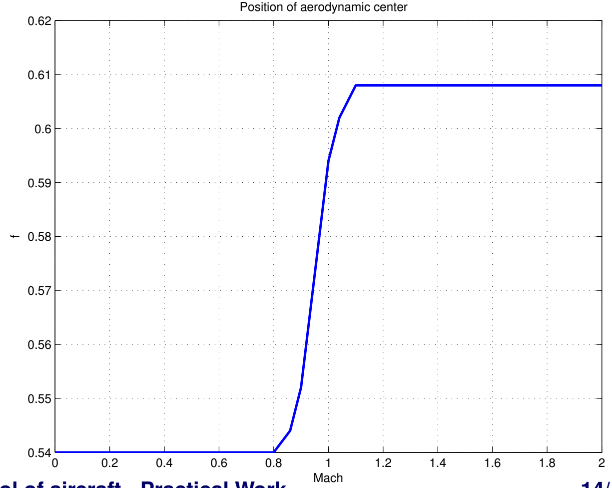
EQUILIBRIUM FIN DEFLECTION FOR NULL LIFT δ_{m_0} (rad)



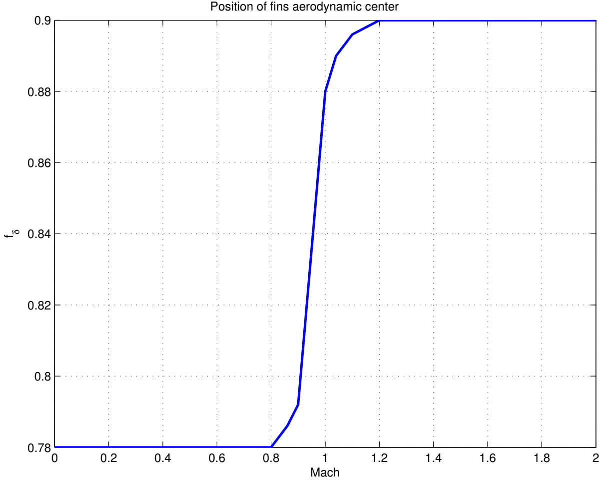
INCIDENCE FOR NULL LIFT AND NULL FIN DEFLECTION α_0 (rad)



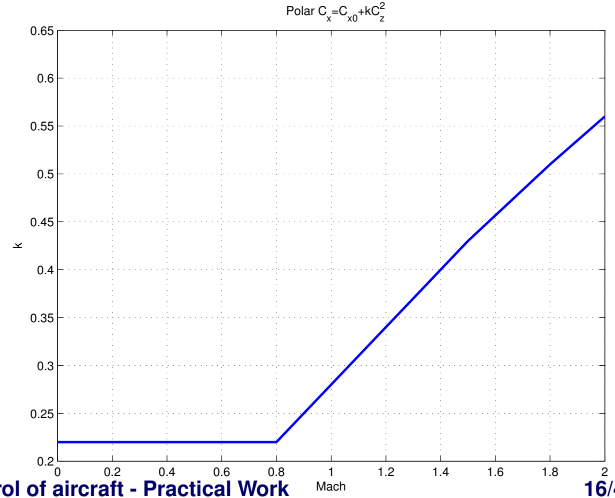
Aerodynamic center of body and wings f



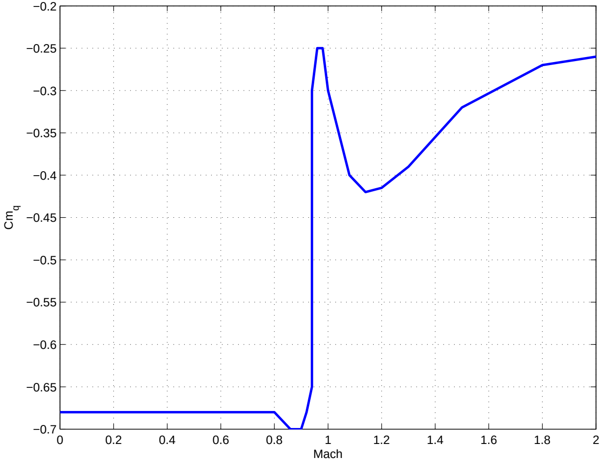
Aerodynamic center of fins (pitch axis) f_δ



POLAR COEFFICIENT k



DAMPING COEFFICIENT Cm_q (s/rad)



CONTROL OF AIRCRAFT

Python installation

- Windows
  - download and install Anaconda 64 bit
  - open an Anaconda prompt and type
  - `conda install -c conda-forge slycot control`
- MacOS or Linux
  - download and install Anaconda
  - open a terminal and type
  - `conda install -c conda-forge slycot control`

Download `sisopy31.py` on moodle and save it in your directory

STUDY OF THE UNCONTROLLED AIRCRAFT

- Determine the equilibrium conditions around the chosen operating point (the slide 49 of the presentation on aircraft longitudinal dynamics for the algorithm to find the equilibrium point is recalled on next slide),
- Build a small signals model: give the state space representation (A, B, C, D) around this equilibrium point, considering the following state vector, with 6 variables:
  $X = \begin{pmatrix} V & \gamma & \alpha & q & \theta & z \end{pmatrix}^T$ and as the command vector, only $U = \begin{pmatrix} \delta_m \end{pmatrix}$.
  Look at the pages 68 to 70 of the presentation on aircraft longitudinal dynamics.
- Study of open loop modes: give the values of the modes, their damping ratio and their proper pulsation.

Algorithm for computing the equilibrium point

Initialization $\alpha_{eq_0} = 0,\ F_{p_{x_{eq_0}}} = 0$

$$C_{z_{eq}} = \frac{1}{QS}\left(mg_0 - F_{p_{x_{éq}^i}} \sin\alpha_{eq_i}\right)$$

$$C_{x_{eq}} = C_{x_0} + kC_{z_{eq}}^2$$

$$\delta_{m_{eq}} = \delta_{m_0} - \frac{C_{x_{eq}}\sin\alpha_{eq_i} + C_{z_{eq}}\cos\alpha_{eq_i}}{C_{x_{\delta_m}}\sin\alpha_{eq_i} + C_{z_{\delta_m}}\cos\alpha_{eq_i}}\,\frac{X}{Y-X}$$

$$C_{x_{\delta_m}} = 2kC_{z_{eq}}C_{z_{\delta_m}}$$

$$\alpha_{eq_{i+1}} = \alpha_0 + \frac{Cz_{eq}}{C_{z_\alpha}} - \frac{Cz_{\delta_m}}{C_{z_\alpha}}\delta_{m_{eq}}$$

$$F_{p_{x_{eq_{i+1}}}} = \frac{QSC_{x_{eq}}}{\cos\alpha_{eq_{i+1}}}$$

$$\left|\alpha_{eq_{i+1}} - \alpha_{eq_i}\right| < \varepsilon$$

No

Yes

- Study the transient phase of the uncontrolled aircraft (short period and phugoid oscillation modes):
  - Give the poles associated with each mode;
  - Give their damping ratio and their proper pulsation;
  - Give the state space representation for each mode;
  - Give the transfer function associated with each variable associated with each mode;
  - Plot the step response for each variable associated with each mode.

STATE SPACE MODEL

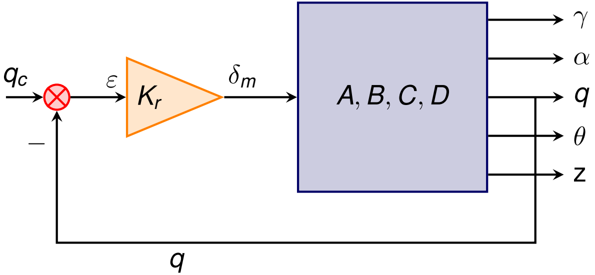
$$\begin{pmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} -X_V & -X_\gamma & -X_\alpha & 0 & 0 & 0 \\ Z_V & 0 & Z_\alpha & 0 & 0 & 0 \\ -Z_V & 0 & -Z_\alpha & 1 & 0 & 0 \\ 0 & 0 & m_\alpha & m_q & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & V_{éq} & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} V \\ \gamma \\ \alpha \\ q \\ \theta \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ Z_{\delta_m} \\ -Z_{\delta_m} \\ m_{\delta_m} \\ 0 \\ 0 \end{pmatrix} (\delta_m)$$

- We will now consider that the speed is controlled with an auto-throttle which is perfect (with an instantaneous response). The speed V can be removed from the state vector. We will now consider the following 5×1 state vector $X = \begin{pmatrix} \gamma & \alpha & q & \theta & z \end{pmatrix}^T$ and $U = (\delta_m)$ as the command vector.

CONTROL OF AIRCRAFT

q FEEDBACK LOOP

We are beginning to build an autopilot by adding a gyrometric feedback loop (with q as the measured variable).



- With the help of sisotool (see sisopy31.py), choose the gain K_r of q feedback loop such as the closed loop damping ratio is $\xi = 0.65$. Justify the choice.
- give the closed loop state space representation (A_k, B_k, C_k, D_k), with q as the output, (see slide 81 of the longitudinal autopilot presentation);

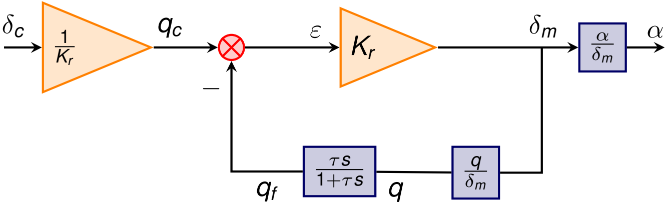
$$A_k = A - K_r B C_q$$

$$B_k = K_r B$$

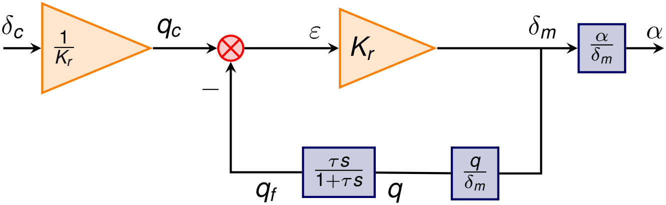
$$C_k = C_{out} = C_q$$

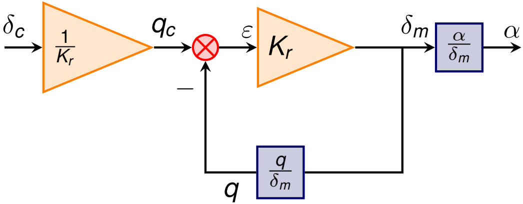
$$D_k = K_r D$$

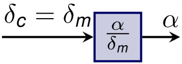
- give the transfer function of the closed loop;
- give the poles of the closed loop, their damping ratio, their proper pulsation;
- plot the step response of the closed loop.



- Choose the time constant τ of the washout filter $\left(\frac{\tau s}{1+\tau s}\right)$ allowing to have the same steady state gain for α with or without the q feedback loop (see slide 147 of the longitudinal autopilot presentation).
- Plot the open loop response, the closed loop response without filter and the closed loop response with the washout filter: these are the step responses of the 3 systems described on next slide. For this question, use the feedback and series commands of the control toolbox. In the following of this study, this filter will not be taken into account.

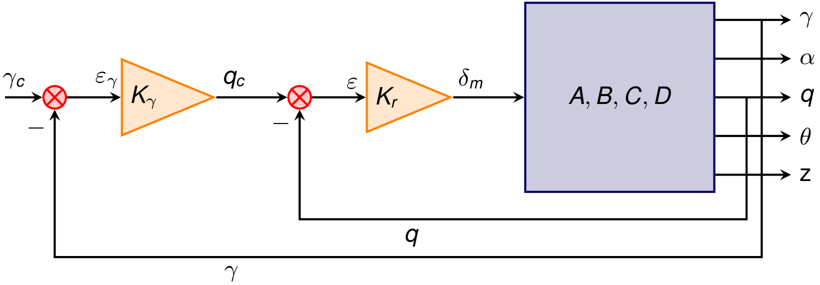

δ_c
$\frac{1}{K_r}$
q_c
ε
K_r
δ_m
$\frac{\alpha}{\delta_m}$
α
$-$
$\frac{\tau s}{1+\tau s}$
q_f
q
$\frac{q}{\delta_m}$


δ_c
$\frac{1}{K_r}$
q_c
ε
K_r
δ_m
$\frac{\alpha}{\delta_m}$
α
$-$
$\frac{q}{\delta_m}$
q


$\delta_c = \delta_m$
$\frac{\alpha}{\delta_m}$
α

γ FEEDBACK LOOP

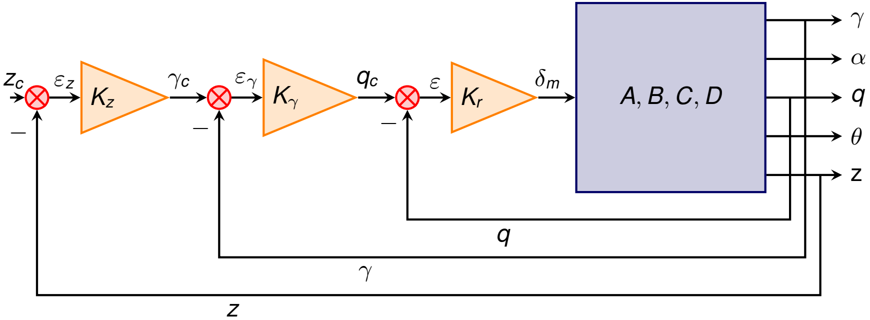
We consider that the auto-throttle perfectly ensures that the speed is constant, so that $\dot{v} = \frac{dv}{dt} = 0$.



A flight path angle feedback loop is added to the preceding controlled system (with the q feedback loop, keeping the preceding K_r tuning).

- Choose the gain K_γ of this flight path angle control loop with the help of sisotool (use the closed loop state space representation with q, and choose γ as output);
- Propose a first choice of a gain allowing a gain margin ≥ 7 dB and a phase margin $\geq 30°$ and an optimized settling time (to within a 5 % threshold). Comment;
- Choose a second tuning (that will be kept for going on with the study), with the following requirements:
    - an overshoot $D_1 \leq 5\%$;
    - a settling time to within 5% $t_{r5\%}$ for a step response that must be optimized (meaning minimized);
    - the pseudo-periodic modes must be correctly damped ($\xi \geq 0.5$).
- Give the closed loop state space representation (γ is the output);
- Give the transfer function of the closed loop;
- Give the poles of the closed loop, their damping ratio, their proper pulsation;
- Plot the step response of the closed loop.

Z FEEDBACK LOOP

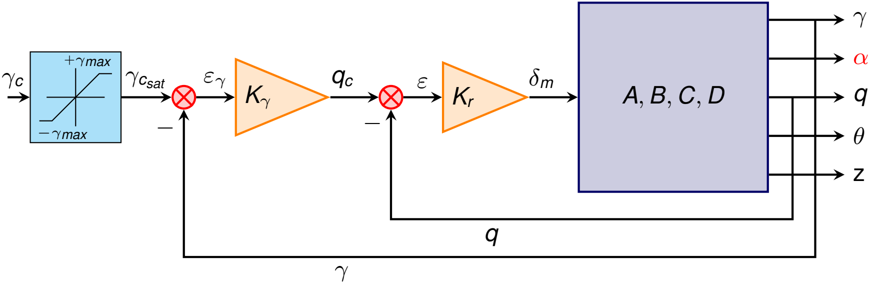


We add another control loop, using the measurement of the altitude z to the previous controlled system (aircraft + q feedback loop + γ feedback loop, while keeping the K_r and K_γ tuning)

- Choose the gain K_z (using sisotool from sisopy31) of this superior mode, whose damping depends on flight angle control loop and rotation speed control loop (do not forget the transfer function of the altitude sensor);
- The expected performances are:
  - an overshoot $D_1 \leq 5\%$;
  - a settling time (to within 5%) $t_{r5\%}$ that must be optimized (meaning minimized);
  - the pseudo-periodic modes must be correctly damped ($\xi \geq 0.5$).
- Give the closed loop state space representation (z is the output);
- Give the transfer function of the closed loop;
- Give the poles of the closed loop, their damping ratio, their proper pulsation;
- Plot the step response of the closed loop.

ADDITION OF A SATURATION IN THE γ CONTROL LOOP

A saturation is added at the input of the γ feedback loop.
In this question, we are going to determine the value of $\gamma_{c_{sat}}$, but we will not implement the non linear simulation of the saturated autopilot.



- Build the state space representation of the closed loop between $\gamma_{c_{sat}}$ and α (this state space representation includes the q feedback loop and the γ feedback loop).
- We want a maximum transverse load factor of $\Delta n_z = 3\ g$. Using the formula on next slide, evaluate α_{max} knowing Δn_z;
- Build a Python function f which associates $\gamma_{c_{sat}}$ to the difference $\max(\alpha(t)) - \alpha_{max}$, $\alpha(t)$ being the response of the transfer between $\gamma_{c_{sat}}$ and α to step of a value of $\gamma_{c_{sat}}$
- Determine the value γ_{max} of the flight path angle (at input of flight path angle control loop) such as the maximum incidence α equals α_{max}, meaning find the zero of the function f;

  You will use a bisection method for the function f in order to find the maximum flight path angle corresponding to the maximum load factor Δn_z.
  The obtained value of γ is the value of the γ_{max}.
- What other methods could be used? Propose a simpler one.

FIND THE SATURATION VALUE FOR γ (CONTINUATION)

As the load factor is generated by the incidence α, you will use the following simplified relations, in order to determine the maximum incidence α corresponding to the load factor Δn_z:

$$mg \cdot n_z = \frac{1}{2}\rho S V_e^2 Cz_\alpha(\alpha - \alpha_0)$$

$$mg = \frac{1}{2}\rho S V_e^2 Cz_\alpha(\alpha_{éq} - \alpha_0)$$

$$n_z = \frac{\alpha - \alpha_0}{\alpha_{éq} - \alpha_0} = 1 + \frac{\alpha - \alpha_{éq}}{\alpha_{éq} - \alpha_0}$$

$$\Delta n_z = \frac{\alpha - \alpha_{éq}}{\alpha_{éq} - \alpha_0}$$

α_{max} VALUE

$$\alpha_{max} = \alpha_{éq} + (\alpha_{éq} - \alpha_0)\Delta n_z$$

Flight Management

Create a script allowing to link the following control modes, by taking the previous control loops:

- an ascent phase with a constant flight path angle (in steady state) using a γ hold control loop
- a cruise flight at constant altitude (of about 100 s) using a z hold control loop
- a descent phase with a constant flight path angle using a γ hold control loop
- a final flare and a small phase of level flight at constant altitude using a z hold control loop

FLIGHT MANAGEMENT

The choice of the initial, cruise and final altitude, as well as initial and final flight path angle is free, but has to be consistent with synthesized controller and representative of a fighter aircraft capability.

For this question, use the step command of the control toolbox in special mode where you can obtain the state vector at each time step `Xout` and specify the initial state vector `Xinitial`
using the following syntax

```
y,t,Xout = control.matlab.step(sys, X0=Xinitial, \
return_x=False)
```

Flight management: alternative final flare

- Instead of a step command for the final flare, use a ramp command and plot the outputs.
- Then, instead of ramp for the final flare, use an exponential function of altitude, and plot the outputs.

CONTROL OF AIRCRAFT

Some pieces of advice

- Minutes (report) is expected at the end of the last session dedicated to this mini project, and must imperatively be transmitted at the end of the last session of practical work. It will be supplied as a computer file under pdf format plus the original format (e.g. .doc, .odt or .tex) and all the Python script files written during the practical work must be provided.
  English will be used;
- This report must showcase your work. It must be clear, easily workable, full, correctly written and present relevant conclusions;
- A graph must have its caption (and don't forget the units);
- The python code must be commented, and prefer international standard units for the calculus (and change to the desired units for plots and outputs only)
- The tunings must be justified (curves illustrating the obtained results) and the results have to be analyzed.