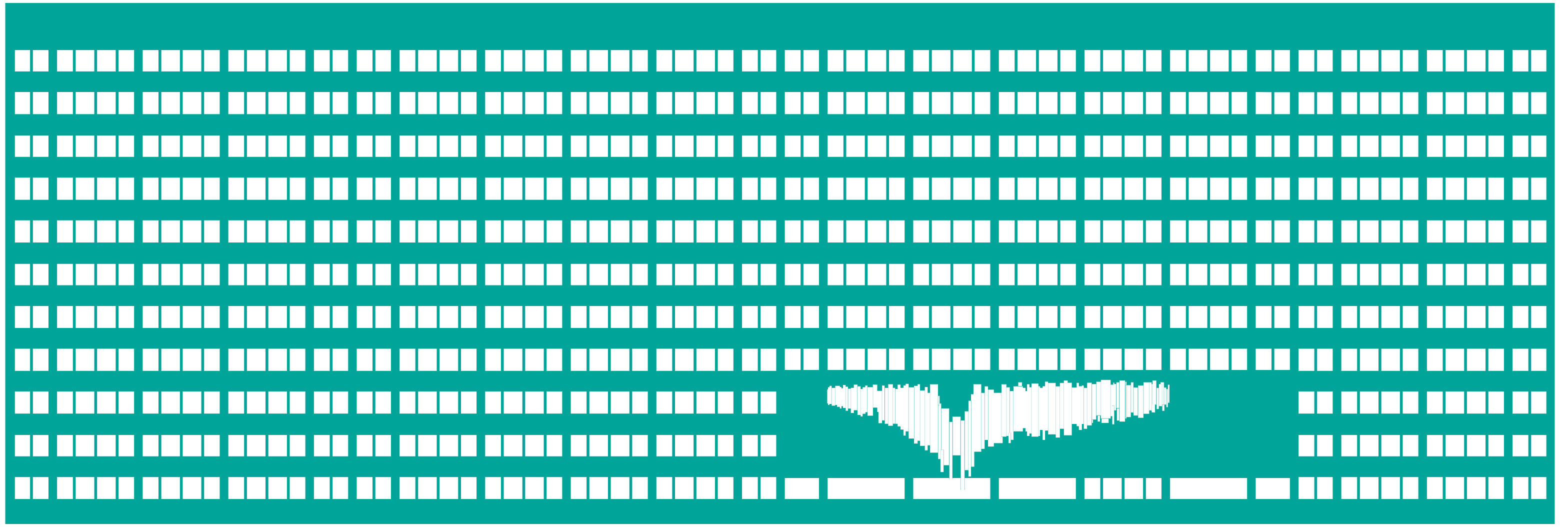
VŠB TECHNICKÁ UNIVERZITA OSTRAVA

VSB TECHNICAL UNIVERSITY OF OSTRAVA



www.vsb.cz

VSB TECHNICAL UNIVERSITY OF OSTRAVA | FACULTY OF MECHANICAL ENGINEERING | DEPARTMENT OF CONTROL SYSTEMS AND INSTRUMENTATION

Digital Control System Design

Z transform

Z transform

- Definition formulas

$$x(kT) = Z^{-1}\{X(z)\} = \frac{1}{2\pi j}\oint_r X(z) z^{k-1} dz$$

Inverse Z transform

$$X(z) = Z\{x(kT)\} = \sum_{k=0}^{\infty} x(kT) z^{-k}$$

- $t_k = kT$ – discrete time, T – sampling period, k=0,1,2,…
- Relative discrete time

$$k = \frac{t_k}{T}$$

Conditions for time function

- Time function must be zero for negative time

$$x(kT) = \begin{cases} x(kT) & \text{for} \quad k \geq 0 \\ 0 & \text{for} \quad k < 0 \end{cases}$$

- Time function must be exponential order

$$\left| x(kT) \right| \leq M e^{\alpha_0 kT}$$

Basic input signals

Discrete Heaviside step (unit step)

$$\eta(kT) = \begin{cases} 1 & \text{for} \quad k \geq 0 \\ 0 & \text{for} \quad k < 0 \end{cases}$$

$$Z\{\eta(kT)\} = \frac{z}{z-1}$$

Discrete Dirac impulse (unit impulse)

$$\delta(kT) = \begin{cases} 1 & \text{for} \quad k = 0 \\ 0 & \text{for} \quad k \neq 0 \end{cases}$$

$$Z\{\delta(kT)\} = 1$$

Basic properties of Z transform

Linearity

$$Z\{a_1 x_1(kT) \pm a_2 x_2(kT)\} = a_1 X_1(z) \pm a_2 X_2(z)$$

Shift in time

$$Z\{x[(k-m)T]\} = z^{-m} X(z)$$

$$Z\{x[(k+m)T]\} = z^{m}\left[X(z) - \sum_{i=0}^{m-1} x(iT) z^{-i}\right]$$

Forward differential

$$Z\{\Delta x(kT)\} = (z-1)X(z) - zx(0)$$

$$Z\{\Delta^n x(kT)\} = (z-1)^n X(z) - z\sum_{i=0}^{n-1} (z-1)^{n-i-1} \Delta^i x(0)$$

Basic properties of Z transform

Backward differential

$$Z\{\nabla x(kT)\} = \frac{z-1}{z} X(z)$$

$$Z\{\nabla^n x(kT)\} = \left(\frac{z-1}{z}\right)^n X(z)$$

Summation

$$Z\left\{\sum_{i=0}^{k-1} x(iT)\right\} = \frac{1}{z-1} X(z)$$

$$Z\left\{\sum_{i=0}^{k} x(iT)\right\} = \frac{z}{z-1} X(z)$$

Finding time function from Z transform

- **Using power series** $$X(z)=\sum_{k=0}^{\infty}x(kT)z^{-1}=x(0)+x(T)z^{-1}+x(2T)z^{-2}+x(3T)z^{-3}+\cdots$$

- **Decomposition into partial fraction**
  It is recommended to work with expression $$\frac{X(z)}{z}$$
  Then result is multiplied by *z*

- **Method of residues** $$x(kT)=\sum_i\left\{\frac{1}{(r_i-1)!}\lim_{z\to z_i}\frac{d^{r_i-1}}{dt^{r_i-1}}\left[(z-z_i)^{r_i}X(z)z^{k-1}\right]\right\}$$

Relation between Z and L transforms

Definition formula of L- transform

$$X(s) = \int_0^\infty x(t) e^{-st} dt$$

Where the continuous time is substitute by discrete time

$$X(s) \approx T \sum_{k=0}^{\infty} x(kT) e^{-skT}$$

Comparison to definition formula of Z transform

$$z = e^{sT} \quad \text{resp.} \quad s = \frac{1}{T} \ln z$$

Solving difference equations

Let suppose difference equations in recurrent form

$$a_n y[(k+n)T] + \cdots + a_1 y[(k+1)T] + a_0 y(kT) = b_m u[(k+m)T] + \cdots + b_1 u[(k+1)T] + b_0 u(kT)$$

With initial conditions
$$y(0), y(T), y(2T), \ldots, y[(n-1)T]$$
$$u(0), u(T), u(2T), \ldots, u[(m-1)T]$$

Z- transform
$$a_n z^n Y(z) + \cdots + a_1 z Y(z) + a_0 Y(z) - L(z) = b_m z^m U(z) + \cdots + b_1 z U(z) + b_0 U(z) - R(z)$$

Modifying it is obtained
$$\left(a_n z^n + \cdots + a_1 z + a_0\right) Y(z) = \left(b_m z^m + \cdots + b_1 z + b_0\right) U(z) - R(z) + L(z)$$

Solution
$$Y(z) = \frac{b_m z^m + \cdots + b_1 z + b_0}{a_n z^n + \cdots + a_1 z + a_0} U(z) - \frac{R(z) - L(z)}{a_n z^n + \cdots + a_1 z + a_0}$$

Solving difference equations

Solution in close form

$$y(kT) = Z^{-1}\left\{Y(z)\right\}$$

Solution in open form

$$y(kT) = \sum_{k=0}^{\infty} y(kT)$$

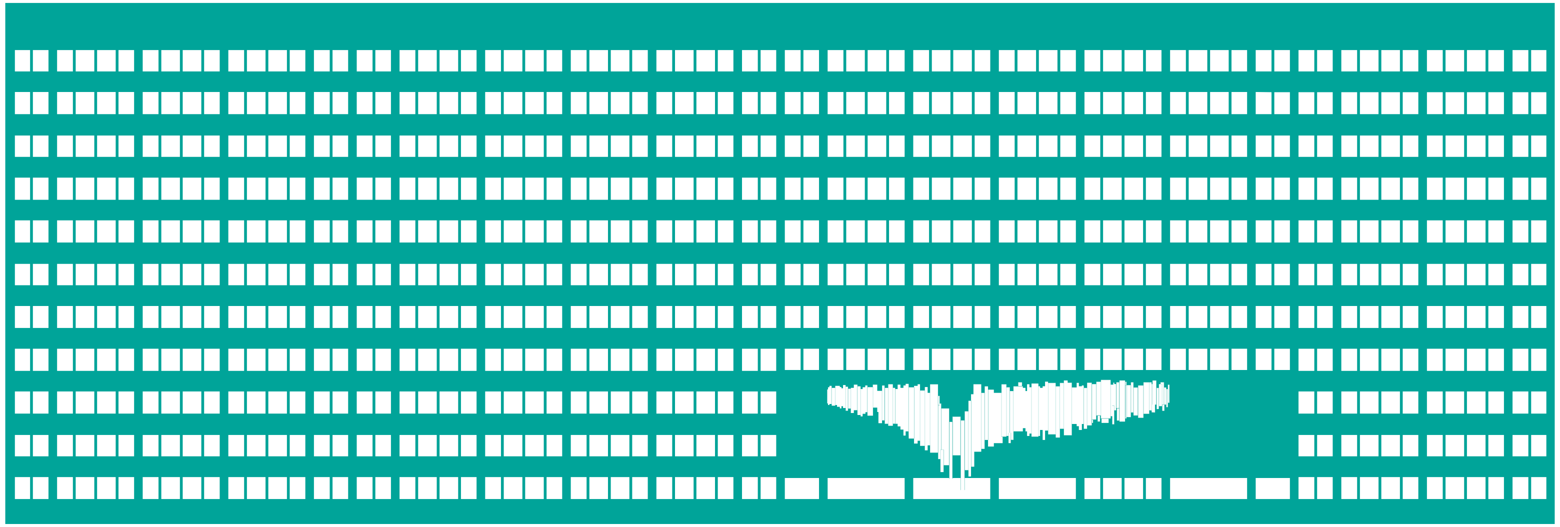
Direct solution of difference equations

$$y\left[(k+n)T\right] = \frac{b_m u\left[(k+m)T\right] + \cdots + b_1 u\left[(k+1)T\right] + b_0 u(kT) - a_{n-1} y\left[(n-1)T\right] - \cdots - a_1 y\left[(k+1)T\right] - a_0 y(kT)}{a_n}$$

Thank you for your attention

VŠB TECHNICKÁ UNIVERZITA OSTRAVA

VSB TECHNICAL UNIVERSITY OF OSTRAVA



www.vsb.cz

Digital Control System Design
Mathematical Models

Renata Wagnerová

Mathematical models

- A way how the properties of dynamic systems can be described
- For each mathematical model it is necessary to know
  - Name
  - Domain
  - General form
  - Physical conditions for feasibility
  - Conditions for steady-state (state in which all inputs and output are constant)
  - Responses (characteristic) – if exists
- Initial conditions are zero to guarantee equivalence of mathematical models

1. Difference equation

- Describes properties in **time** domain
- General form

$$a_n y\left[(k+n)T\right]+\cdots+a_1 y\left[(k+1)T\right]+a_0 y(kT)=b_m u\left[(k+m)T\right]+\cdots+b_1 u\left[(k+1)T\right]+b_0 u(kT)$$

- Physical condition for feasibility

n>m strong

n=m weak

n<m does not exist

(**n** is a shift in time of output variable,
m is a shift in time of input variable)

1. Difference equation

- General form

$$a_n y[(k+n)T] + \cdots + a_1 y[(k+1)T] + a_0 y(kT) = b_m u[(k+m)T] + \cdots + b_1 u[(k+1)T] + b_0 u(kT)$$

- Conditions for steady state

  u(∞) is steady state of input variable

  y(∞) is steady state of output variable

$$y(kT) = y[(k+1)T] = \cdots = y[(k+n)T] = y(\infty)$$
$$u(kT) = u[(k+1)T] = \cdots = u[(k+m)T] = u(\infty)$$

$$\Rightarrow \qquad y(\infty) = \frac{b_m + \cdots + b_1 + b_0}{a_n + \cdots + a_1 + a_0} u(\infty) = \frac{\sum_{i=0}^{m} b_i}{\sum_{j=0}^{n} a_j} u(\infty)$$

2. Transfer function G(z)

- Describes properties in **complex variable** domain
- A ratio of output variable to input variable
- General form

$$G(z) = \frac{Y(z)}{U(z)} = \frac{b_m z^m + \cdots + b_1 z + b_0}{a_n z^n + \cdots + a_1 z + a_0}$$

- Physical condition for feasibility

n>m strong

n=m weak

n<m does not exist

(**n** is a polynomial order in denominator,
m is a polynomial order in numerator)

2. Transfer function G(z)

- General form

$$G(z) = \frac{Y(z)}{U(z)} = \frac{b_m z^m + \cdots + b_1 z + b_0}{a_n z^n + \cdots + a_1 z + a_0}$$

- Condition for steady state

$$y(kT) = y[(k+1)T] = \cdots = y[(k+n)T] = y(\infty)$$
$$u(kT) = u[(k+1)T] = \cdots = u[(k+m)T] = u(\infty)$$

$$\Rightarrow \qquad y(\infty) = \left[\lim_{z \to 1} G(z)\right] u(\infty) = \frac{b_m + \cdots + b_1 + b_0}{a_n + \cdots + a_1 + a_0} u(\infty) = \frac{\sum_{i=0}^{m} b_i}{\sum_{j=0}^{n} a_j} u(\infty)$$

3. Step function h(kT)

- Describes properties in **time** domain
- A response of the system to input signal in form of discrete Heaviside step (unit step)
- General form

$$h(kT) = Z^{-1}\left\{\frac{z}{z-1}G(z)\right\}$$

- Physical condition for feasibility

$$h(0) = 0 \qquad \text{strong}$$

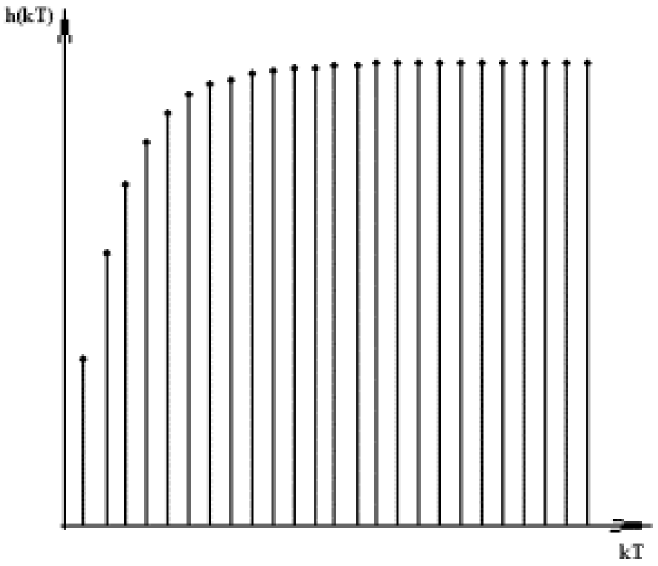
$$h(0) \neq 0 \qquad \text{weak}$$

3. Step function h(kT)

- Condition for steady state

$$y(\infty)=\left[\lim_{k\to\infty} h(kT)\right]u(\infty)=\left[\lim_{z\to 1}\frac{z-1}{z}G(z)\right]u(\infty)=\frac{\sum_{i=0}^{m} b_i}{\sum_{j=0}^{n} a_j}u(\infty)$$

- Step response – a graph of step function



4. Impulse function g(kT)

- Describes properties in **time** domain

- A response of the system to input signal in form of discrete Dirac impulse (unit impulse)
- General form

$$g(kT) = Z^{-1}\left\{G(z)\right\}$$

- Physical condition for feasibility

$$g(0) = 0 \qquad \text{strong}$$

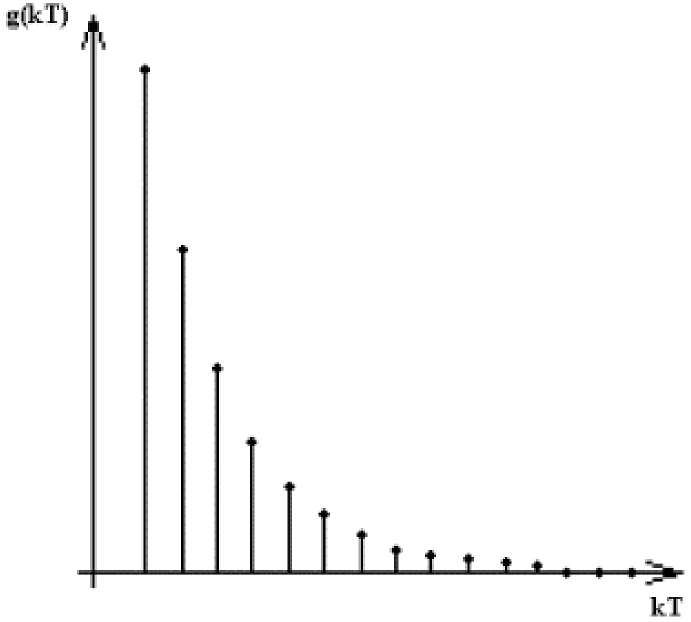
$$g(0) \neq 0 \qquad \text{weak}$$

4. Impulse function g(kT)

- Condition for steady state

$$y(\infty)=\left[\lim_{k\to\infty}\sum_{i=0}^{k} g(iT)\right]u(\infty)=\left[\lim_{z\to 1} G(z)\right]u(\infty)=\frac{\sum_{i=0}^{m} b_i}{\sum_{j=0}^{n} a_j}u(\infty)$$

- Impulse response – a graph of impulse function



Relation between step and impulse functions

In complex variable domain

$$H(z) = \frac{z}{z-1} G(z)$$

$$G(z) = \frac{z-1}{z} H(z)$$

In time domain

$$h(kT) = \sum_{i=0}^{k} g(iT)$$

$$g(kT) = \nabla h(kT) = h(kT) - h\left[(k-1)T\right]$$

5. Frequency transfer function

Description in frequency domain

$$G(j\omega) = G(z)\Big|_{z = e^{jT\omega}} = \frac{b_m e^{jmT\omega} + \cdots + b_1 e^{jT\omega} + b_0}{a_n e^{jnT\omega} + \cdots + a_1 e^{jT\omega} + a_0} =$$

$$P(\omega) + jQ(\omega) = A(\omega) e^{j\varphi(\omega)}$$

$$0 \le \omega \le \frac{\pi}{T}$$

Conditions of feasibility:

n>*m* … strong
n=*m* … weak
n<*m* … does not exist

Steady state:

$$y = \left[\lim_{\omega \to 0} G(j\omega)\right] u == \frac{\sum_{i=0}^{m} b_i}{\sum_{j=0}^{n} a_j} u$$

Do not use!

6. State space representation

- Describes properties in **time** domain
- General form

$$\mathbf{x}[(k+1)T] = \mathbf{A}\mathbf{x}(kT) + \mathbf{b}u(kT) \ldots \qquad \text{equation for state variable}$$

$$y(kT) = \mathbf{c}^{\mathrm{T}}\mathbf{x}(kT) + du(kT) \qquad \ldots \qquad \text{equation of output variable}$$

x(*kT*) –vector of state variables dimension (n,1), ***A*** – state matrix of system dimension (n,n), ***b*** – state vector of input dimension (n,1), ***c*** – state vector of output dimension (n,1),*d* – constant.

- Physical condition for feasibility

$$d = 0 \quad \text{strong} \qquad d \neq 0 \quad \text{weak}$$

- Condition for steady state

$$y = \left[\boldsymbol{c}^T(\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b} + \boldsymbol{d}\right]u$$

Transform from state space representation into transfer function

$$\mathbf{x}\left[(k+1)T\right] = \mathbf{A}\mathbf{x}(kT) + \mathbf{b}u(kT)$$
$$y(kT) = \mathbf{c}^{\mathrm{T}}\mathbf{x}(kT) + du(kT)$$

Description of state space equation in Z domain

$$z\boldsymbol{X}(z) = \mathbf{A}\boldsymbol{X}(z) + \mathbf{b}U(z)$$
$$Y(z) = \mathbf{c}^{\mathrm{T}}\boldsymbol{X}(z) + dU(z)$$

Formula for X(z)

$$\left(z\boldsymbol{I} - \boldsymbol{A}\right)\boldsymbol{X}(z) = \boldsymbol{b}U(z)$$
$$\boldsymbol{X}(z) = \left(z\boldsymbol{I} - \boldsymbol{A}\right)^{-1}\boldsymbol{b}U(z)$$

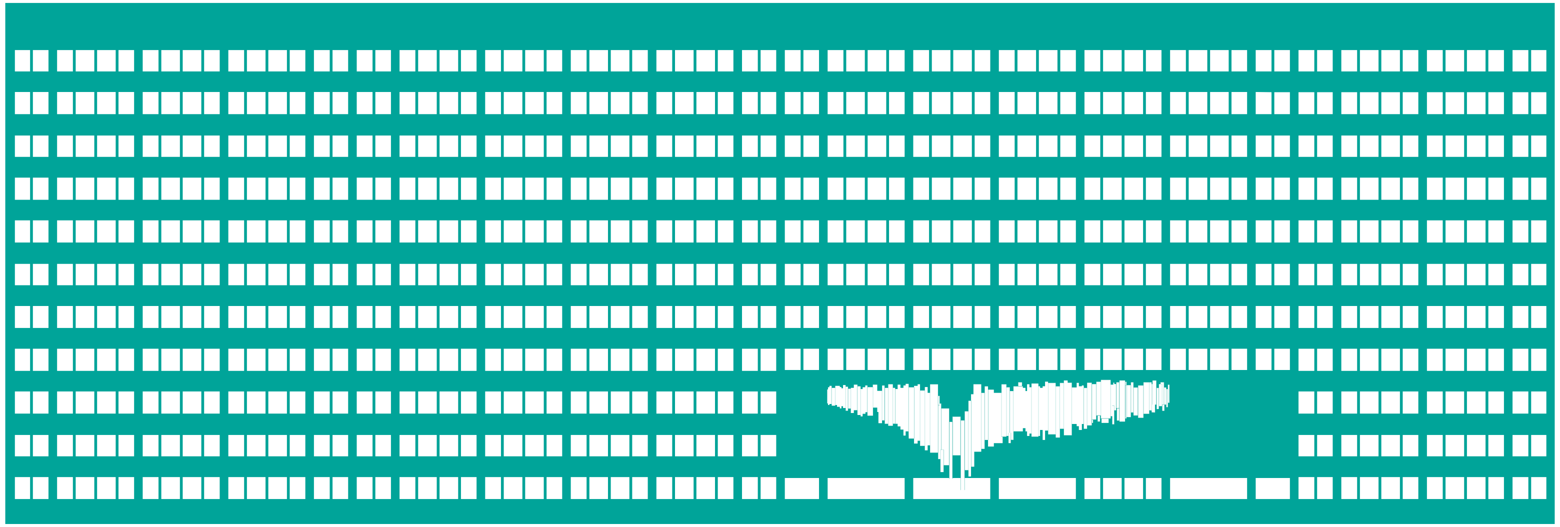
Substituting X(z) into output equation

$$Y(z) = \mathbf{c}^{T}\left(z\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{b}U(z) + \mathbf{d}U(z)$$
$$G(z) = \frac{Y(z)}{U(z)} = \mathbf{c}^{T}\left(z\mathbf{I} - \mathbf{A}\right)^{-1}\mathbf{b} + \mathbf{d}$$

Thank you for your attention

VŠB TECHNICKÁ
UNIVERZITA
OSTRAVA

VSB TECHNICAL
UNIVERSITY
OF OSTRAVA



www.vsb.cz

VSB TECHNICAL UNIVERSITY OF OSTRAVA | FACULTY OF MECHANICAL ENGINEERING | DEPARTMENT OF CONTROL SYSTEMS AND INSTRUMENTATION

Digital Control Systems Design
Mathematical Models
Examples

Renata Wagnerova

Example 1

For system described by difference equation it is necessary to determine

- Transfer function
- Step function in close form and step response (5 values)
- Impulse function in close form and impulse response (5 values)

$$y(k) + 0{,}5\,y(k-1) = u(k) + 2u(k-1)$$

Example 2

For system described by transfer function it is necessary to determine

- Step function and step response (5 values)
- Impulse function and impulse response (5 values)
- difference equation

$$G(z) = \frac{z-1}{z^2 + 0,7z + 0,1}$$

Example 3

For system described by difference equation it is necessary to determine

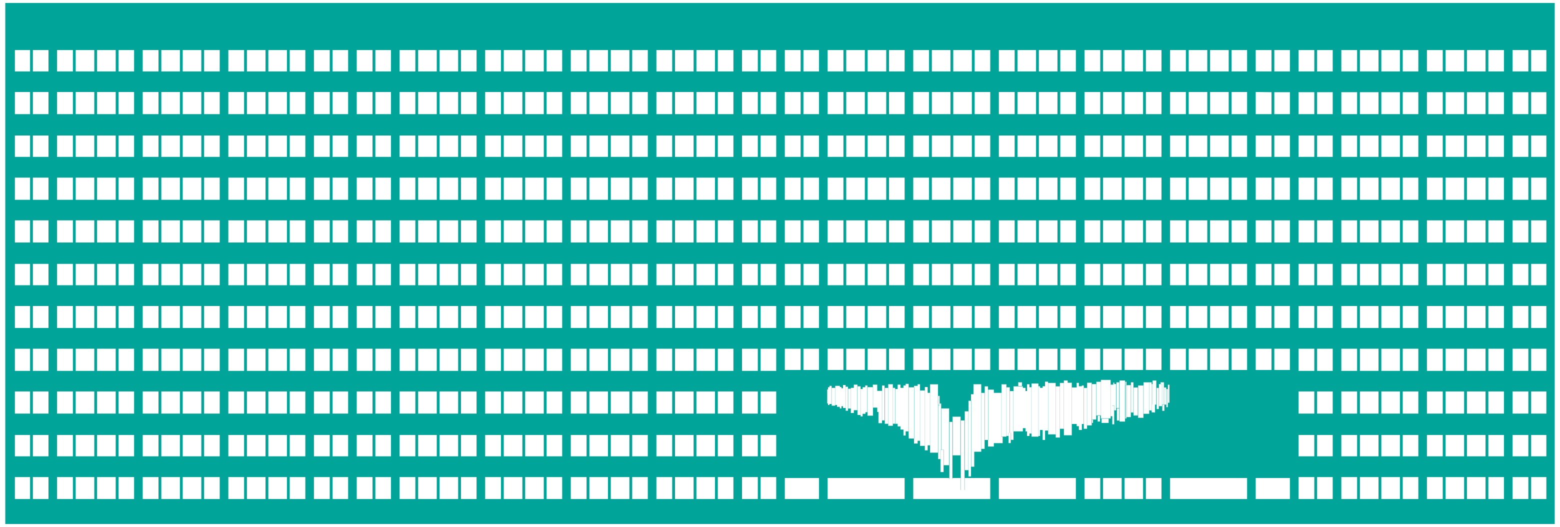
- transfer function
- step function and step response (5 values)
- impulse function and impulse response (5 values)

$$y(k+2)+1{,}2y(k+1)+0{,}36y(k)=2u(k)$$

Thank you for your attention

VŠB TECHNICKÁ UNIVERZITA OSTRAVA

VSB TECHNICAL UNIVERSITY OF OSTRAVA



www.vsb.cz

VSB TECHNICAL UNIVERSITY OF OSTRAVA | FACULTY OF MECHANICAL ENGINEERING | DEPARTMENT OF CONTROL SYSTEMS AND INSTRUMENTATION

Digital Control Systems Design
Basic type of systems

Renata Wagnerová

Basic types of discrete systems

Proportional systems

Steady state

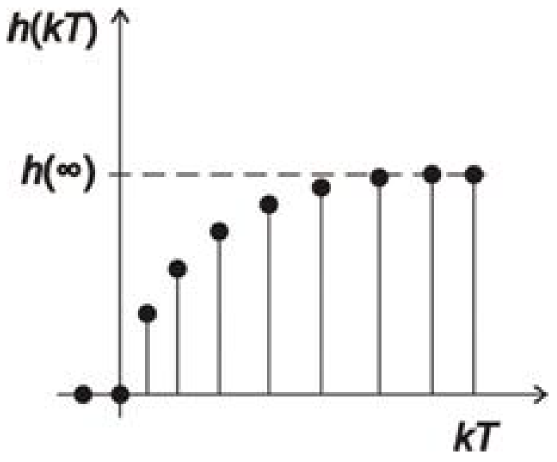
$$y = k_1 u$$

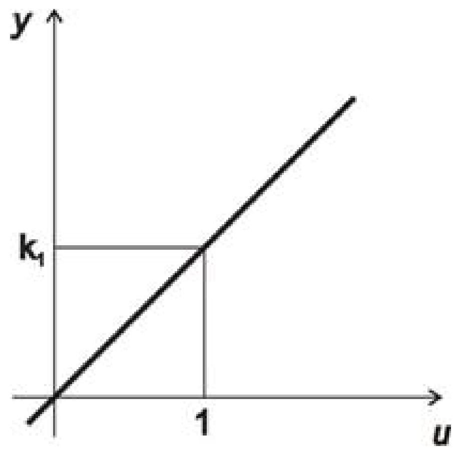
$$k_1 = \frac{\sum_{i=0}^{m} b_i}{\sum_{j=0}^{n} a_j}, \qquad \sum_{j=0}^{n} a_j \neq 0$$

Transfer function

$$G(z) = \frac{b_m z^m + \cdots + b_1 z + b_0}{a_n z^n + \cdots + a_1 z + a_0} z^{-d}$$

Step response





Basic types of discrete systems

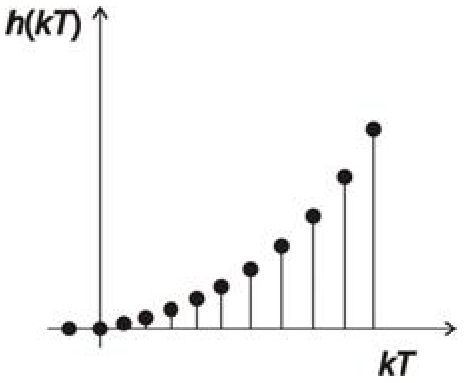
Summing systems

Steady state – does not exist $k_1 = \infty \quad \Leftrightarrow \quad \sum_{j=0}^{n} a_j = 0$

Transfer function

$$G(z) = \frac{b_m z^m + \cdots + b_1 z + b_0}{(z-1)^q \left(a_n z^n + \cdots + a_1 z + a_0\right)} z^{-d}$$

Step response

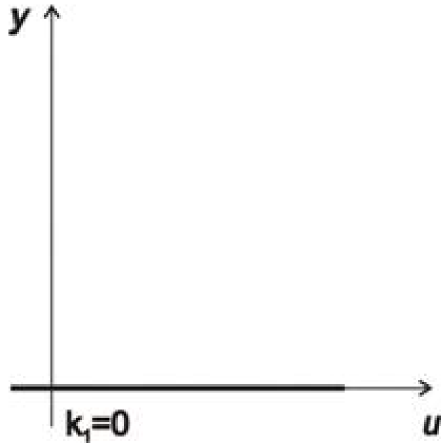


Basic types of discrete systems

Difference systems

Steady state – exists and is zero

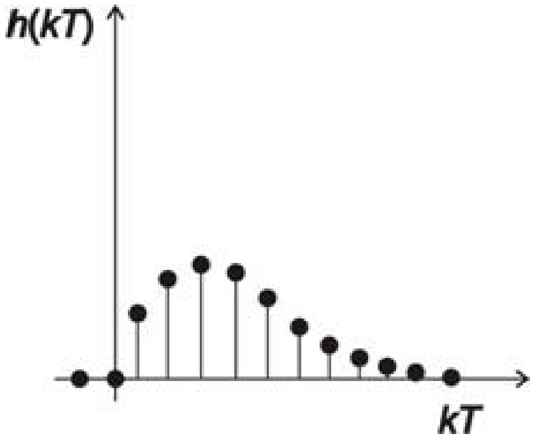
$$k_1 = 0 \quad \Leftrightarrow \quad \sum_{i=0}^{m} b_i = 0$$



Transfer function

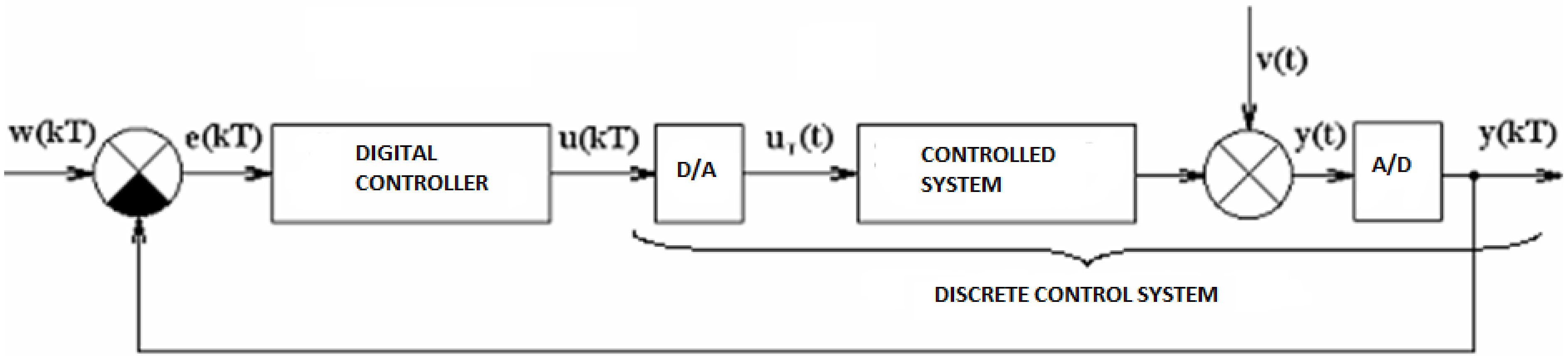
$$G(z) = \frac{(z-1)^r \left(b_m z^m + \cdots + b_1 z + b_0\right)}{a_n z^n + \cdots + a_1 z + a_0} z^{-d}$$

Step response



Digital Control System

Controlled system is always continuous. But if sampling period is too big D/A and A/D converters are part of controlled system and that is why it is possible to describe it as a discrete system.



Discretization of continuous systems

In case that A/D and D/A converters are zero order, the continuous controlled system can be transferred to discrete controlled system.

$$G_S(z) = \frac{z-1}{z} Z\left\{ L^{-1}\left\{ \frac{G_S(s)}{s} \right\} \bigg|_{t=kT} \right\}$$

Basic transfer functions of control system

$$G_{wy}(z) = \frac{Y(z)}{W(z)} = \frac{G_C(z)G_S(z)}{1+G_C(z)G_S(z)G_M(z)}$$

$$G_{vy}(z) = \frac{Y(z)}{V(z)} = \frac{G_P(z)}{1+G_C(z)G_S(z)G_M(z)}$$

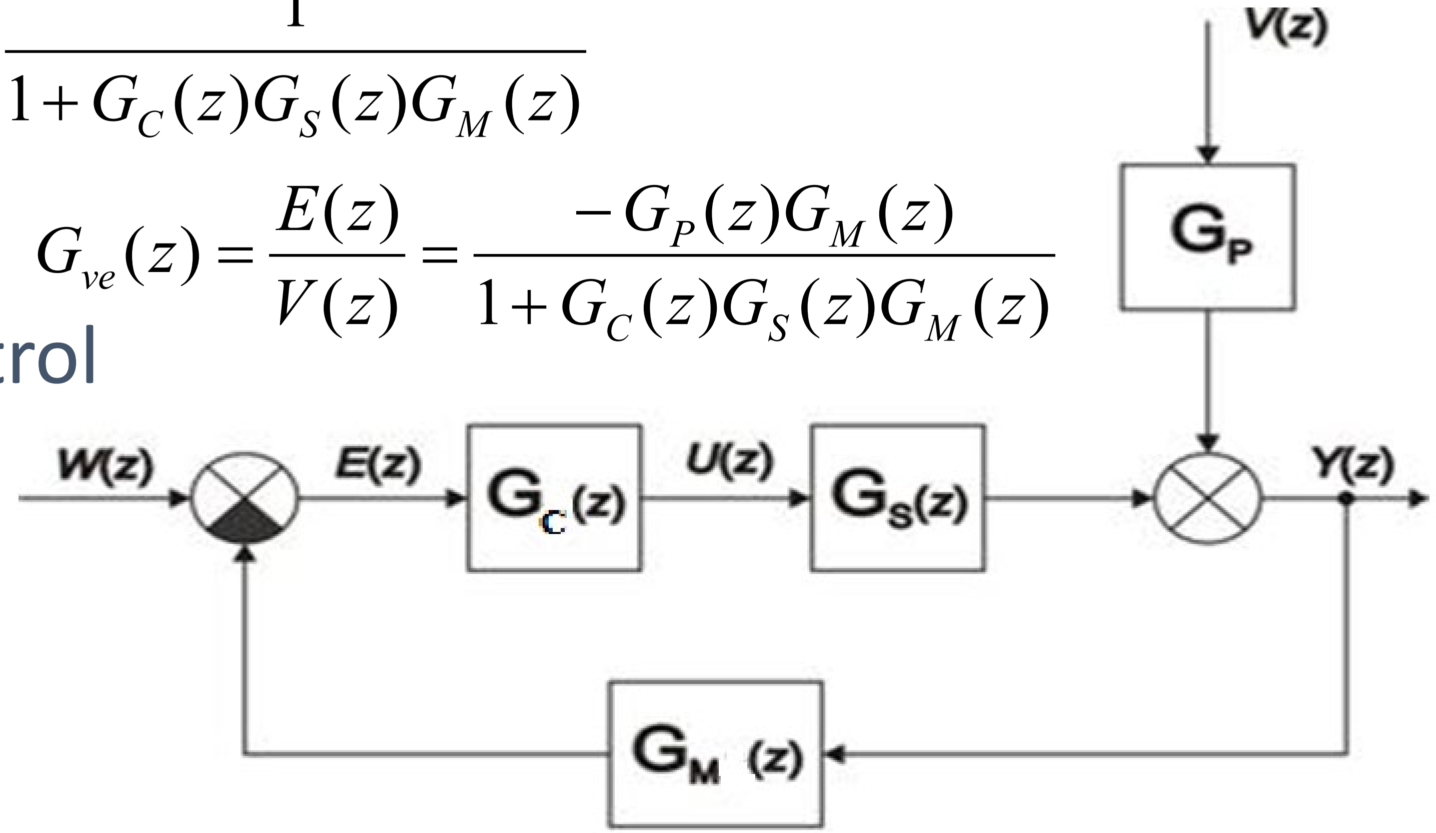
$$G_{we}(z) = \frac{E(z)}{W(z)} = \frac{1}{1+G_C(z)G_S(z)G_M(z)}$$

$$G_{ve}(z) = \frac{E(z)}{V(z)} = \frac{-G_P(z)G_M(z)}{1+G_C(z)G_S(z)G_M(z)}$$

Goal of control

$$Y(z) \to W(z)$$

$$E(z) \to 0$$



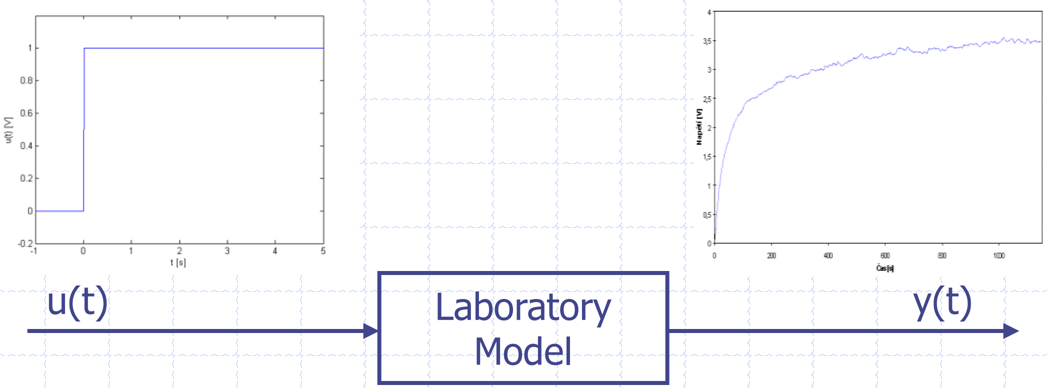
Thank you for your attention

Identification

Control of Rotation Frequency

System Identification

- **Analytical identification** – mathematical and physical analysis of plant.
- **Experimental identification** – measuring responses of real system.

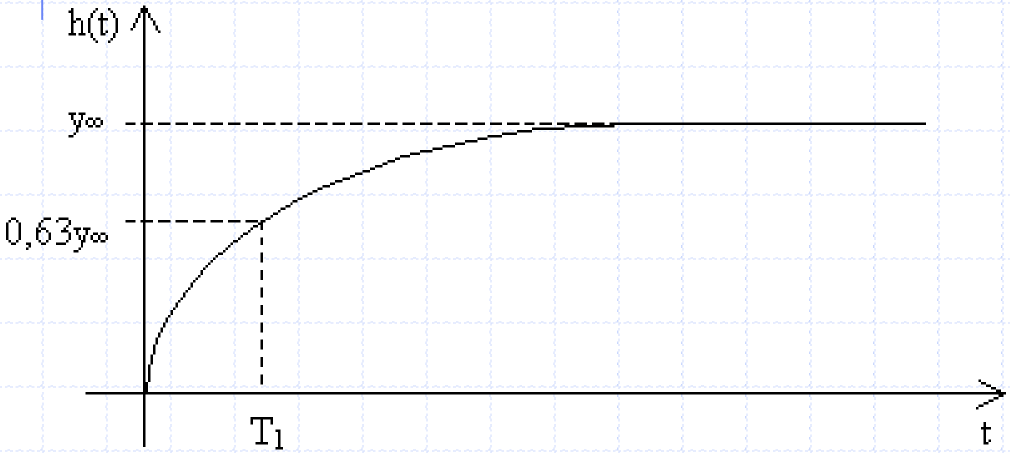


- **Let´s measure the step response of fan.**

System Identification

Using approximation method of step responses.

One Point Approximation – first order system



Transfer function:

$$G(s) = \frac{k_1}{T_1 s + 1}$$

where:

$$0{,}63\,y(\infty) \Rightarrow t_{0,63}$$

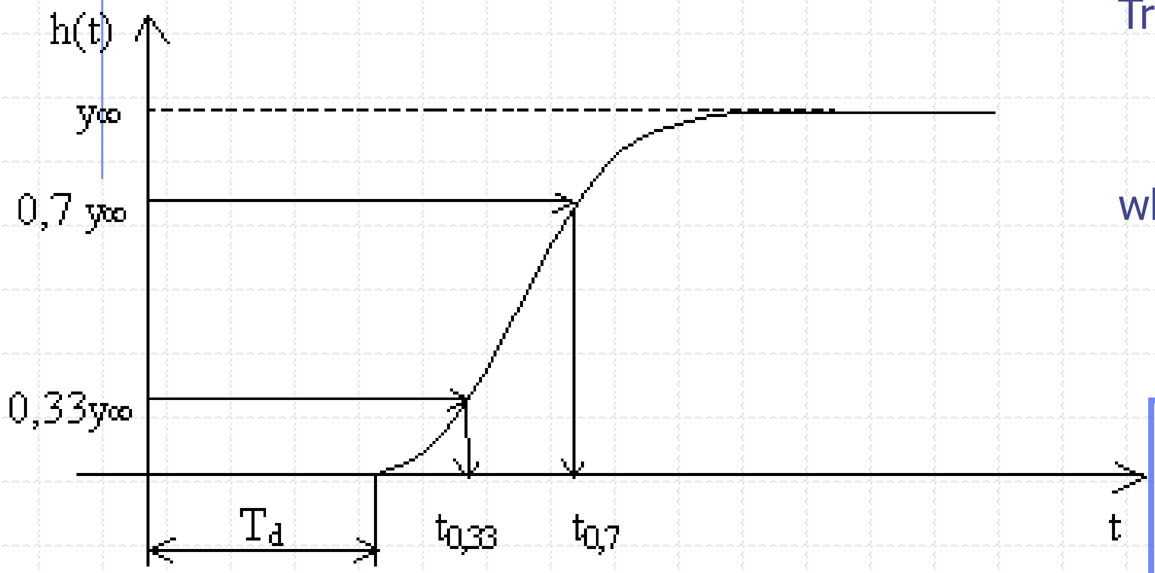
$$T_1 = t_{0,63}$$

$$k_1 = \frac{y(\infty)}{u(\infty)}$$

System Identification

Using approximation method of step responses.

Two Points Approximation – first order system



Transfer function:

$$G(s) = \frac{k_1}{T_1 s + 1} \cdot e^{-T_{d1} s}$$

where:

$$y_{0,7} = 0{,}7 \cdot y(\infty) \quad \Rightarrow \quad t_{0,7}$$

$$y_{0,33} = 0{,}33 \cdot y(\infty) \quad \Rightarrow \quad t_{0,33}$$

$$T_{d1} = 1{,}498 t_{0,33} - 0{,}498 t_{0,7}$$

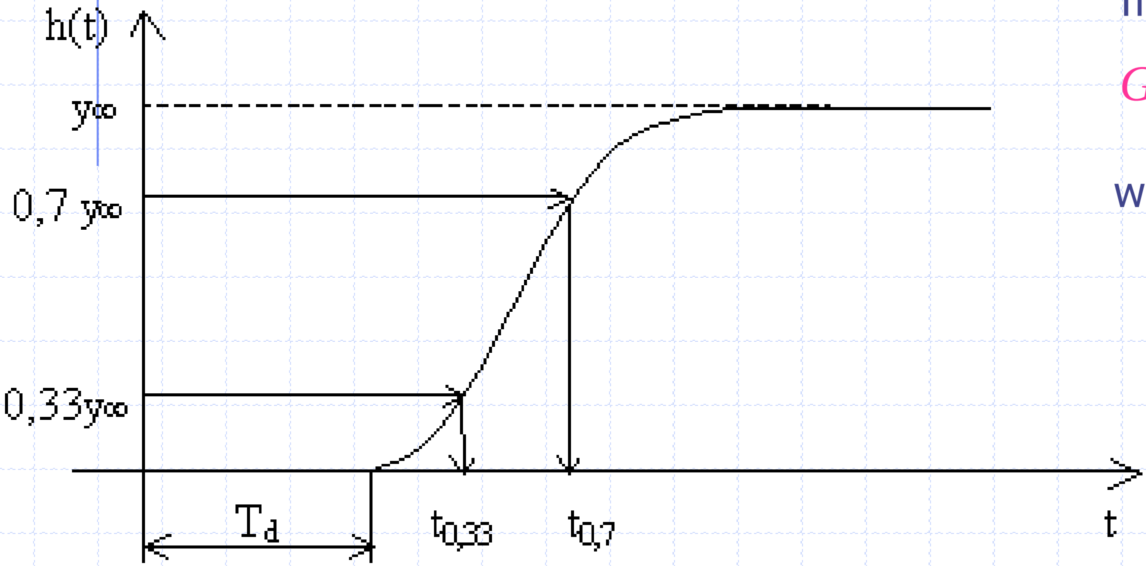
$$T_1 = 1{,}245 \cdot (t_{0,7} - t_{0,33})$$

$$k_1 = \frac{y(\infty)}{u(\infty)}$$

System Identification

Using approximation method of step responses.

Two Points Approximation – second order system



Transfer function:

$$G(s)=\frac{k_1}{(T_2 s+1)^2}\cdot e^{-T_{d2}s}$$

where:

$$y_{0,7}=0,7\cdot y(\infty) \quad \Rightarrow \quad t_{0,7}$$

$$y_{0,33}=0,33\cdot y(\infty) \quad \Rightarrow \quad t_{0,33}$$

$$T_{d2}=1,937.t_{0,33}-0,937.t_{0,7}$$

$$T_2=0,794\cdot(t_{0,7}-t_{0,33})$$

$$k_1=\frac{y(\infty)}{u(\infty)}$$

Laboratory model description
Experimental identification

1 Introduction

The goal of this examination is practicing and checking theoretical knowledge obtained during lecture and putting theoretical blocks together and finding their connection. This goal will be reached by applying knowledge on the Air-fan model. We will measure step response, do system identification to step response, tune PID regulator and discover possibility of on/off control.

2 Air-fan model

2.1 Hardware description

In *Figure 1* we can see hardware part of Air-fan model. There is an 11cm air fan with accelerometer, the National Instruments USB-6009 data acquisition and signal forming electronic circuit. The NI USB-6009 is in role of analog/digital converter with maximal Sampling Rate 48kS/s (kilo samples per second) there. This A/D converter connects the Air-fan software to the air fan, reads the value of rotations and generates control signals to the air fan. Voltage from USB is used as power supply.

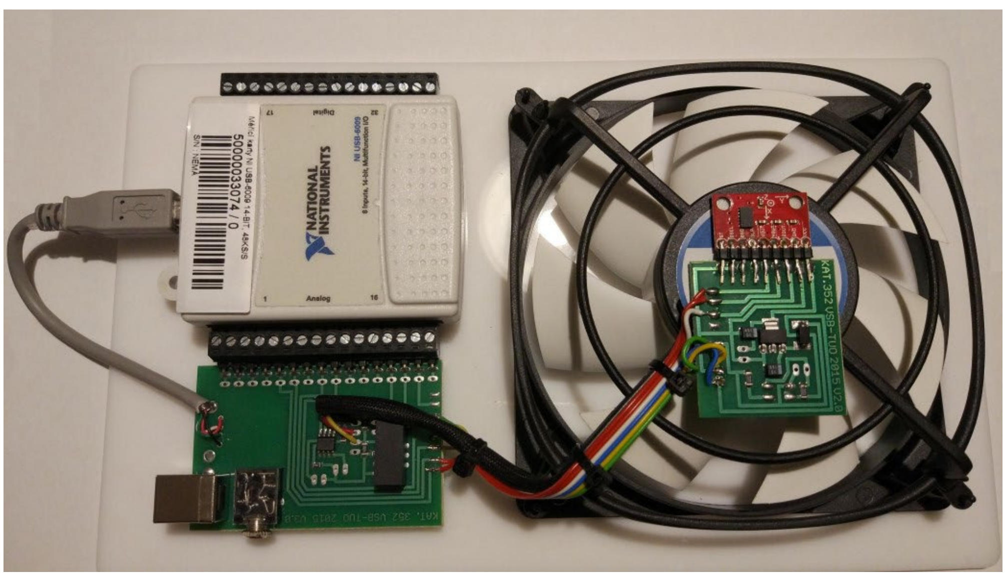


Figure 1 Air-fan model

2.2 Connection to PC, software setup and troubleshooting

Model connects to PC by USB interface. The Air-fan model application is available on PC in PC lab F232,3 When you connect the model to PC, system will find proper drivers and you will be able to work with the model. For using English version of "***Fan - control***" change language by switch, located on top center part of application.

When something does not go smoothly and the Air-fan control application shows the message shown in *Figure 2*. Try connecting the model to different USB port and if it does not help, reinstall the application.

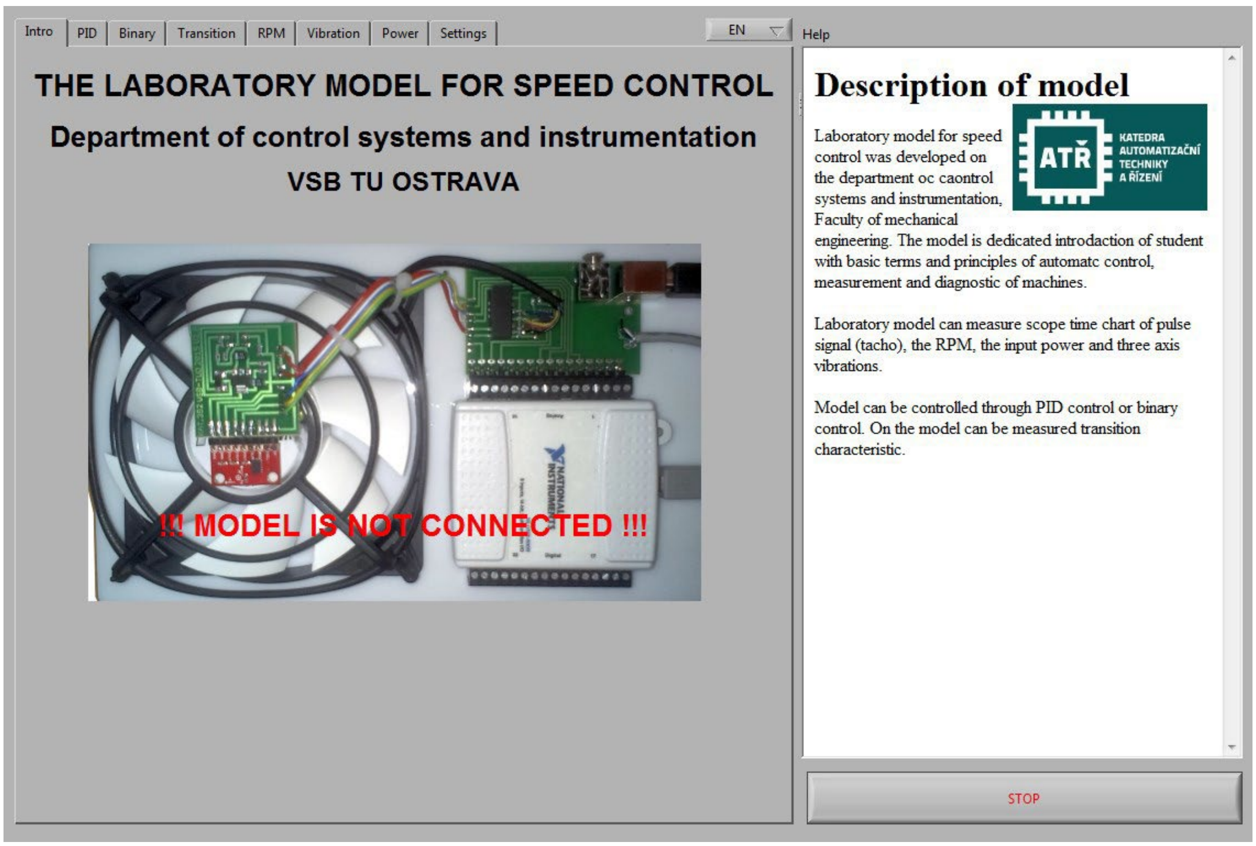


Figure 2 - Error message - model is not connected

There could be a problem if you have more than one NI USB-6009 connected to PC. Control application will be not able to decide which NI USB-6009 is dedicated for Air-fan model and you have to select the proper device in “*Setting*” tab – *Figure 3*.

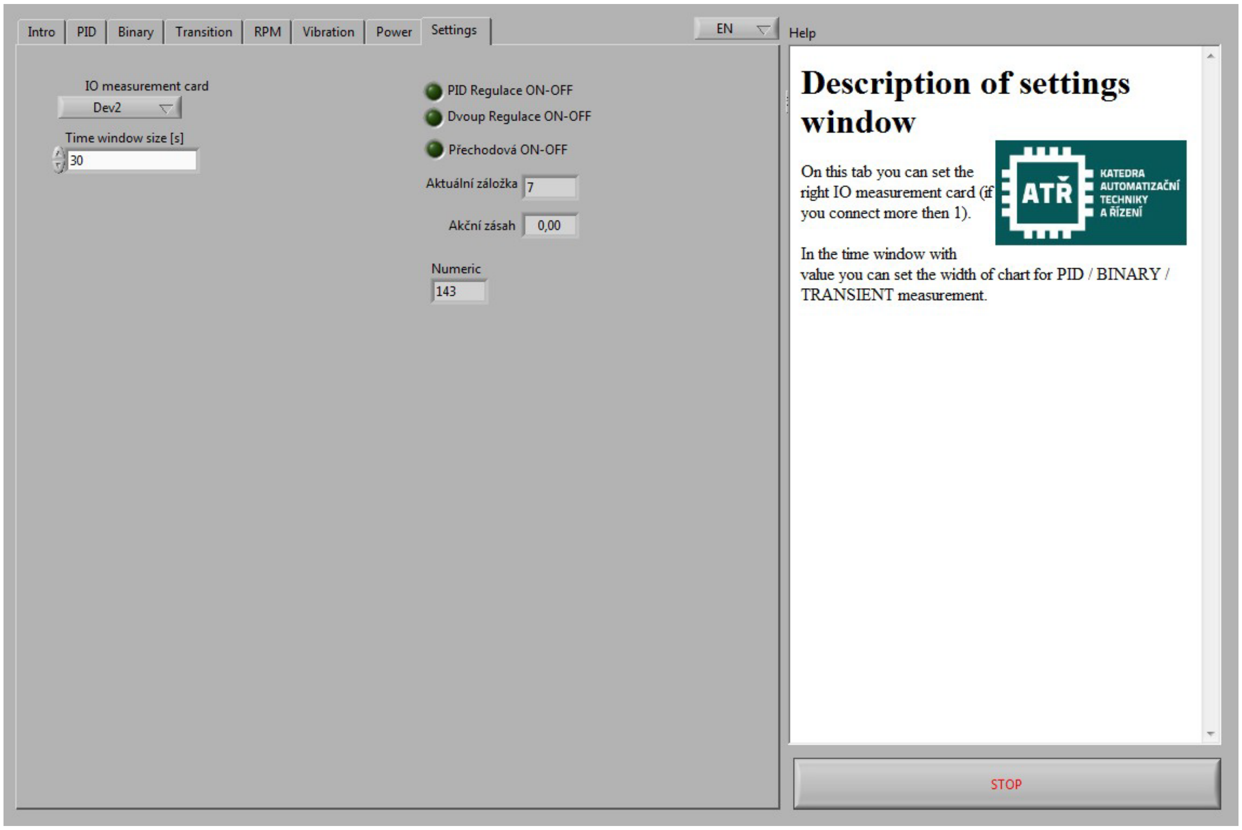


Figure 3 - Control application settings

It is possible to choose time axis range for graph of PID and on/off regulation, and the measuring of step response. There are notifications for running PID regulation, on/off regulation and measuring of step response, too.

3 Measuring of step response

3.1 Theory

Transfer function needed for getting the step function is given by Equation (1):

$$G(s) = \frac{Y(s)}{U(s)} \quad (1)$$

where $Y(s)$ is Laplace input function and $U(s)$ Laplace output function

Step response is graph of step function which is evaluated according Equation (2):

$$h(t) = L^{-1}\left\{\frac{G(s)}{s}\right\} \quad (2)$$

3.2 Measurement

Measurement will be done by "Fan - Control" application in tab "*Transition*".

There are following measurement parameters:

- „*Starting output value [V]*" – initial the level of input voltage from step on input (minimal value is 1,5V, for lower levels is model not operating) which will be done
- „*Ouptut value jump [V]* " – final level of input voltage to which step will be done
- „*Time of jump [s]*" – it determines the time when step of input voltage begins

Steps of measurement:

- start "*Fan - Control*" application
- check there is not the message shown at Figure 2
- switch to tab „*Transion*"
- setup parameters – default is step from 0 to 5 V at time 10 sec
- press button „*START*"
- wait for steady level of rotations
- press button „*STOP*"
- save measured step response by pressing „*save file*" button
- repeat measurement (it's recommended 5 times)

Measured values are saved in .CSV file, that can be opened in Excel or similar software.

"Fan – Control" application measures values of step response each 0.32sec, but Windows is not fully focused on measurement and it gives variance in gate time (multiples of 0.32sec). We made

several measurements for that case. We have to calculate the average value of time and value for each gate time.

The result should look like this measured step response:

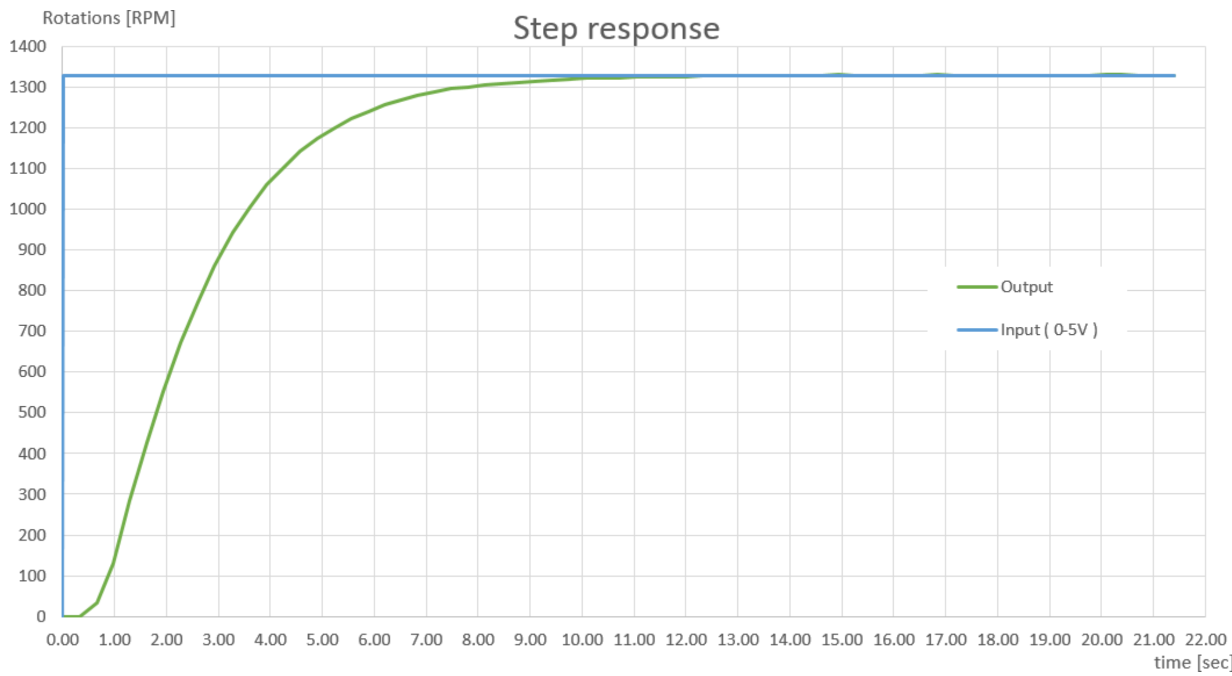


Figure 4 - Example of step response

4 Step response identification

Step response identification is a deterministic method, based on analysis of response of proportional control systems. Use of these methods is the best if output is not noisy signal. Use of these methods expects knowledge of response of basic system type to deterministic test inputs (ex. unit step input).

Figure 4 shows measured step response of Air-fan model, input step is set to level 5V.

When we realize physical basis of air fan, we will get equation of rotation (3):

$$I\ddot{\alpha} = \sum \tau \tag{3}$$

when we transform this equation to complex variable by Laplace transformation, we will obtain the transfer function of the nonoscillatory SOPTD (second order system plus time delay) plant.

The mathematical model can be obtained by identification of the transfer function of the FOPTD (first order plus time delay) plant:

$$G_p(s) = \frac{k_1}{T_1 s+1} e^{-T_{d1} s} \tag{4}$$

where T_1 is the time constant, T_{d1} – the time delay, k_1 – the plant gain. The plant gain k_1 equals the level of step input divided by output level in steady state:

$$k_1 = \frac{y(\infty)}{u(\infty)} \tag{5}$$

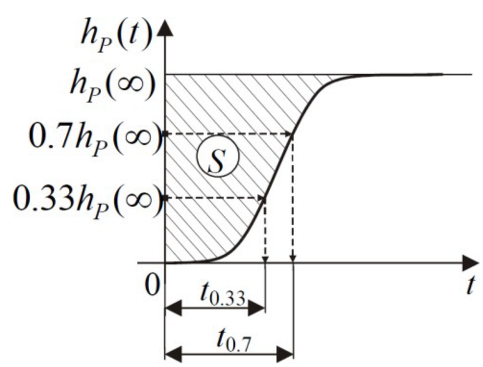


Figure 5 - Plant transfer function determination on the basis of times $t_{0.7}$ and $t_{0.33}$

Determination of the transfer function in the form (4) is uses the times $t_{0.7}$ and $t_{0.33}$ in accordance with Figure 5 and the following formulas:

$$T_1 = 1.245(t_{0.7} - t_{0.33}) \tag{6}$$

$$T_{d1} = 1.498t_{0.33} - 0.498t_{0.7} \tag{7}$$

Using discretization to obtain plant transfer function, we will need to calculate time between two obtained values t_x. We can use following formula for linear approximation:

$$t_x = \frac{y_x - y_1}{y_2 - y_1}(t_2 - t_1) + t_1 \tag{8}$$

The result can be verified in Matlab by following commands:

```
k1 = <value of k1>
T1 = <value of T1>
Td = <value of Td>
numerator = [k1]
denominator = [T1 1]

P = tf(numerator, denominator,'InputDelay',Td)
step(P)
```

On the basis of the times $t_{0.7}$ and $t_{0.33}$ it is possible to obtain the transfer function of the nonoscillatory SOPTD (**s**econd **o**rder system **p**lus **t**ime **d**elay) plant:

$$G_p(s) = \frac{k_1}{(T_2 s+1)^2} e^{-T_{d2} s} \tag{9}$$

where

$$T_2 = 0.794(t_{0.7} - t_{0.33}) \tag{10}$$

$$T_{d2} = 1.937t_{0.33} - 0.937t_{0.7} \tag{11}$$

Result can be verified in Matlab by following commands:

```
k1 = <value of k1>
T2 = <value of T2>
Td = <value of Td>
numerator = [k1]
denominator = [T2^2 2*T2 1]

P = tf(numerator, denominator,'InputDelay',Td)
step(P)
```

The best approximation of the step response course of the nonoscillatory SOPTD plant can be obtained for the transfer function with two different time constants:

$$G_p(s) = \frac{k_1}{(T_1 s+1)(T_2 s+1)} e^{-T_d s} \tag{12}$$

Using the values $y_{0.7} = 0.7h_p(\infty)$, $y_{0.26} = 0.26h_p(\infty)$, $y_{0.09} = 0.09h_p(\infty)$ similar with Figure 5 it is possible to obtain times $t_{0.7}$, $t_{0.26}$, $t_{0.09}$.

Where:

$$T_d = 2t_{0.09} - t_{0.26} \tag{13}$$

$$B = 0.83t_{0.7} - 0.24t_{0.26} + 0.48t_{0.09} - T_d \tag{14}$$

$$B = 4(t_{0.26} - t_{0.09})^2 \tag{15}$$

$$T_1 = \frac{B+\sqrt{B^2-4C}}{2} \tag{16}$$

$$T_2 = \frac{B-\sqrt{B^2-4C}}{2} \tag{17}$$

Result can be verified in Matlab by following commands:

```
k1 = <value of k1>
T1 = <value of T1>
T2 = <value of T2>
Td = <value of Td>

numerator = [k1]
denominator = [T1*T2 T1+T2 1]

P = tf(numerator, denominator,'InputDelay',Td)
step(P)
```

5 Synthesis

It’s possible to choose several taught tuning methods that will fit to our transfer function with equations (4), (9) a (12).

There will be shown an example of controller tuning on SIMC method and Desired model method for obtained the transfer function of the nonoscillatory SOPTD plant with equation (12).

5.1 Desired model method

The DMM (desired model method), formerly also known as inverse dynamics method, was developed at the Faculty of Mechanical Engineering, Technical University of Ostrava. This method is very simple, enables to tune both analog and digital controllers, the overshoot can be from range 0% to 50%.

Table 1 - Controller adjustable parameters for the desired model method (DMM)

	Plant transfer function	Controller (analog $T = 0$; digital $T > 0$)			
		Typ	k_P^*	T_I^*	T_D^*
1	$\frac{k_1}{s}\mathrm{e}^{-T_d s}$	P	$\frac{1}{(\alpha T + \beta T_d)k_1}$	–	–
2	$\frac{k_1}{T_1 s + 1}\mathrm{e}^{-T_d s}$	PI	$\frac{T_I^*}{(\alpha T + \beta T_d)k_1}$	$T_1 - \frac{T}{2}$	–
3	$\frac{k_1}{s(T_1 s + 1)}\mathrm{e}^{-T_d s}$	PD	$\frac{1}{(\alpha T + \beta T_d)k_1}$	–	$T_1 - \frac{T}{2}$
4	$\frac{k_1}{(T_1 s + 1)(T_2 s + 1)}\mathrm{e}^{-T_d s}$, $T_1 \geq T_2$	PID	$\frac{T_I^*}{(\alpha T + \beta T_d)k_1}$	$T_1 + T_2 - T$	$\frac{T_1 T_2}{T_1 + T_2} - \frac{T}{4}$
5	$\frac{k_1}{T_0^2 s^2 + 2\xi_0 T_0 s + 1}\mathrm{e}^{-T_d s}$, $0{,}5 < \xi_0 \leq 1$	PID	$\frac{T_I^*}{(\alpha T + \beta T_d)k_1}$	$2\xi_0 T_0 - T$	$\frac{T_0}{2\xi_0} - \frac{T}{4}$

For our plant with equation (9) or (12) the method suggests PID controller (line 5) with transfer function:

$$G_C(s) = k_p\left(1 + \frac{1}{T_I s} + T_D s\right) \tag{19}$$

Table 2 - Values of coefficients β' and β for given relative overshoot κ

κ	0	0,05	0,10	0,15	0,20	0,25	0,30	0,35	0,40	0,45	0,50
α	1,282	0,984	0,884	0,832	0,763	0,697	0,669	0,640	0,618	0,599	0,577
β	2,718	1,944	1,720	1,561	1,437	1,337	1,248	1,172	1,104	1,045	0,992

The overshoot is chosen 5%, that is why $\beta = 1{,}944$ from Table 2.

The tunable controller´s parameters are

$$T_D^* = \frac{T_1 T_2}{T_1 + T_2}$$
$$T_I^* = T_1 + T_2$$
$$k_p^* = \frac{T_I^*}{\beta k_1 T_d}$$

The result can be verified in Simulink by following simulation model:

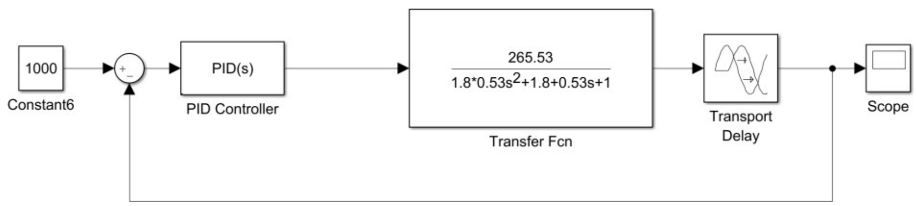


Figure 6 – Circuit for verification of tuning

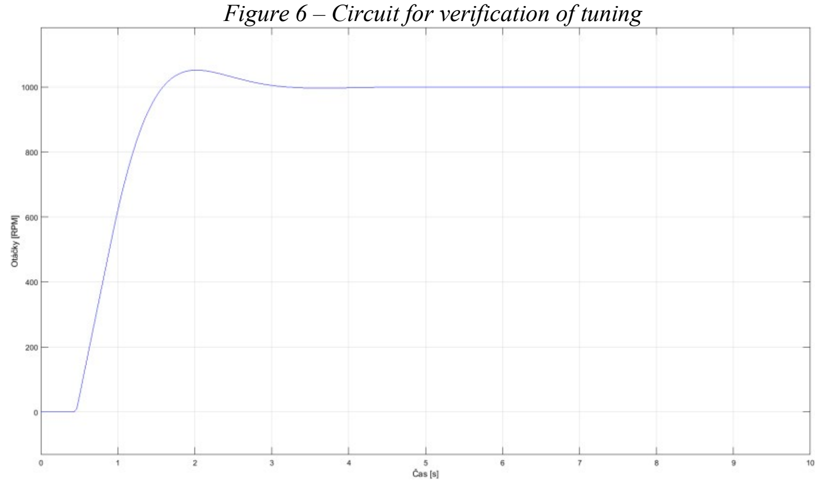


Figure 7 – Output signal of plant

5.2 Verification on model

Practical verification of tuning of controller will be done in “*Fan – control*” application, tab. “*PID*”.

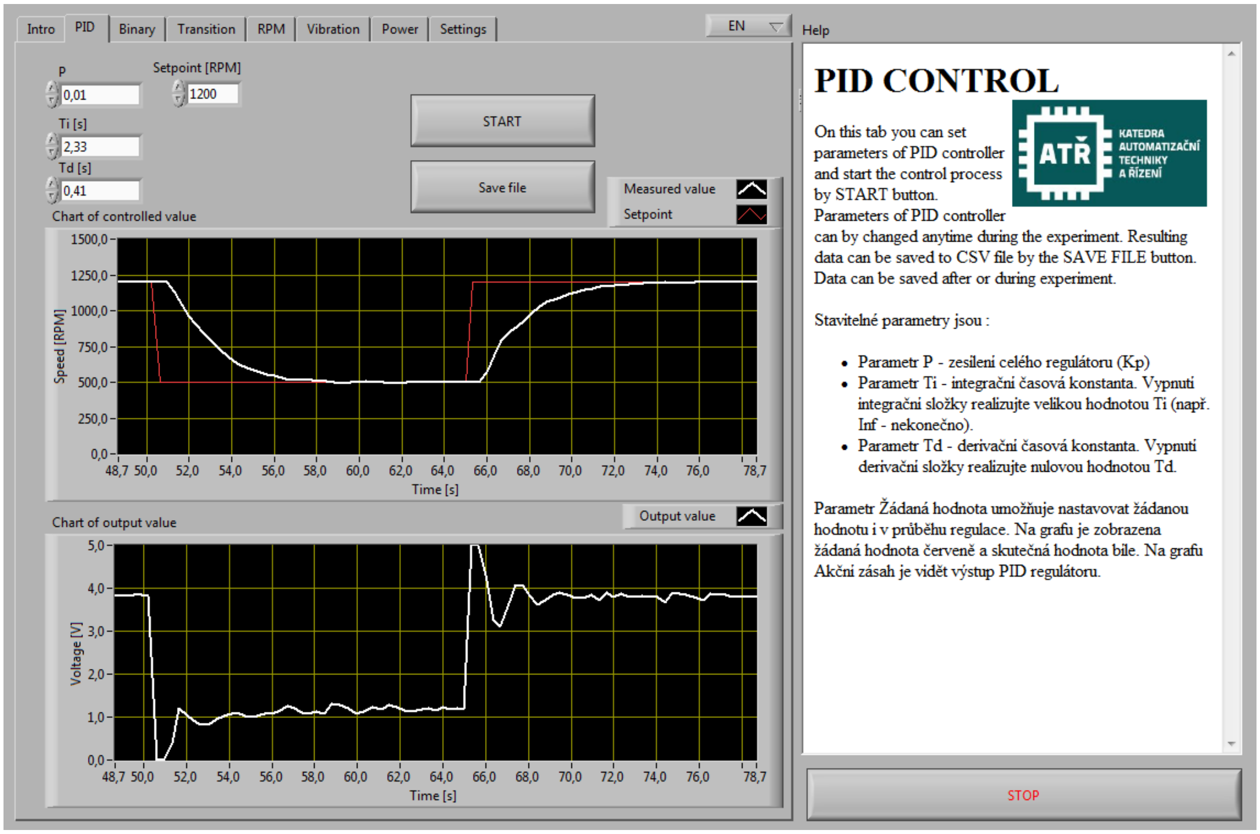


Figure 8 - PID regulation

There we can set up following parameters:

- P – gain of PID controller - k_P parameter
- Ti – integration time value - T_I parameter
- Td – derivation time value - T_D parameter
- Setpoint [RPM] – requested rotations

These parameters will be set according to the tuned values of PID controller. When we have PI controller, we have to set T_D time to value 0 and when we have PD controller, we have to set T_I time to value inf (infinity).

6 Two-state regulation (ON/OFF regulation)

6.1 Theory

Two-state regulation is the most widespread and simplest type of regulation at all. It can also be found in domestic appliances - iron temperature control, refrigerators, etc. The two-state control is the maintenance of a controlled quantity between the upper and lower limits. It can be imagined on the time course of the controlled variable y.

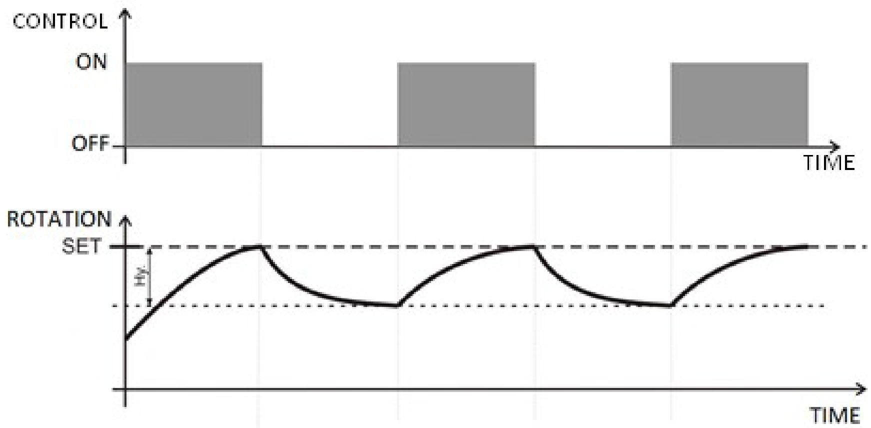


Figure 9 - How does 2-state regulation work

6.2 Verification on model

Practical exercise of two-state regulation will be done by “Fan - Control” application in tab “*Binary*”.

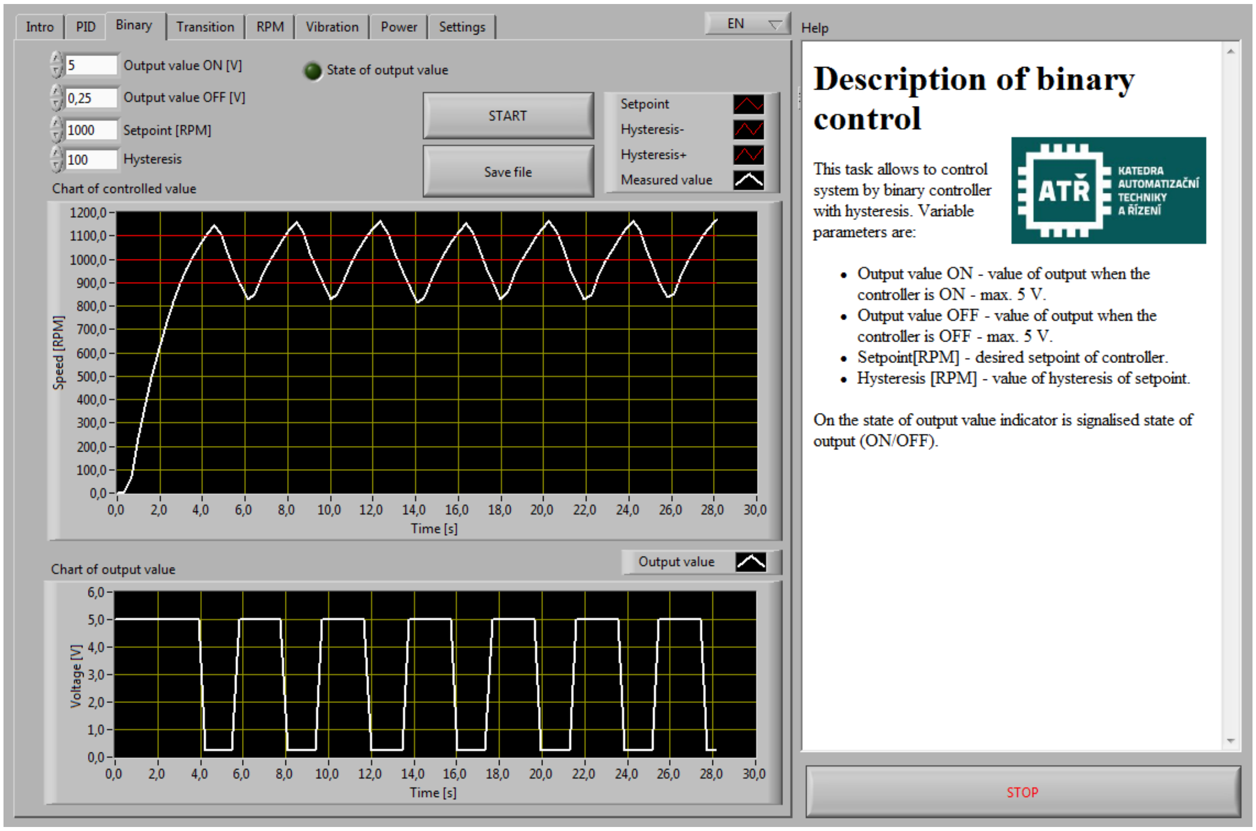


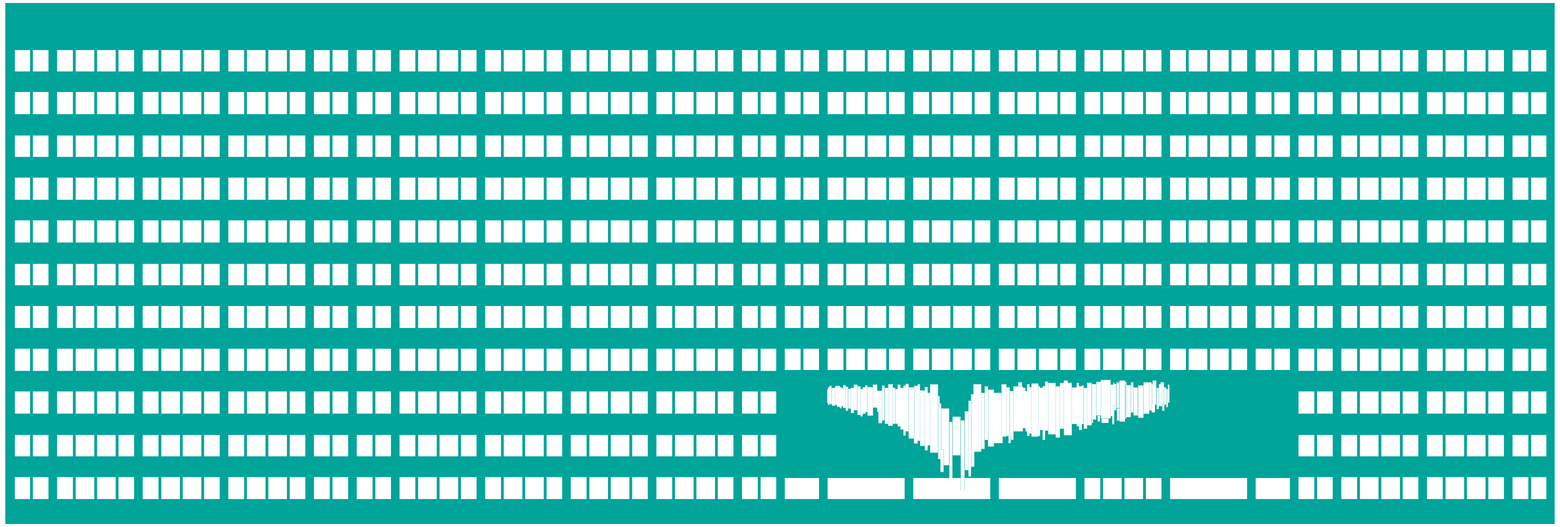
Figure 10 - Two-state regulation

There we can setup following values:

- "*Output value ON*" - value of output when the controller is ON - max. 5 V
- "*Output value OFF*" - value of output when the controller is OFF - max. 5 V
- *"Setpoint[RPM]"* - desired setpoint of controller
- *"Hysteresis [RPM]*" - value of hysteresis of setpoint

VŠB TECHNICKÁ UNIVERZITA OSTRAVA

VSB TECHNICAL UNIVERSITY OF OSTRAVA



www.vsb.cz

VSB TECHNICAL UNIVERSITY OF OSTRAVA
FACULTY OF MECHANICAL ENGINEERING
DEPARTMENT OF CONTROL SYSTEMS AND INSTRUMENTATION

DIGITAL CONTROL SYSTEMS DESIGN
SAMPLING, CONTROLLERS

Renata Wagnerová

Sampling

- It is operation which changes continuous signal into series of values
- For choosing sampling period T, sampling frequency f_s resp., there is no exact rule
- Choice of sampling period affects quality of control process and its stability

Choice of sampling period

Sampling period T	Process
• (10 – 500) µs	exact control and modelling, electric and energy systems, robot control
• (0,5 – 20) µs	simulators, (flight, drive,…)
• (10 – 100) ms	image processing, virtual reality, artificial vision

Choice of sampling period

Sampling period T	Process
• (0,5 – 1) s	objects monitoring and control, chemical processes, power station
• (1 – 3) s	flow control
• (1 – 5) s	pressure control
• (5 – 10) s	level control
• (10 – 20) s	temperature control

Recommendation for choice of sampling frequency

From the known step response

$$T = \left(\frac{1}{15} \div \frac{1}{6}\right) t_{0,95}$$

For dominant time delay

$$T = \left(\frac{1}{8} \div \frac{1}{3}\right) T_d$$

Digital Control Systems – difference elements should fulfill condition

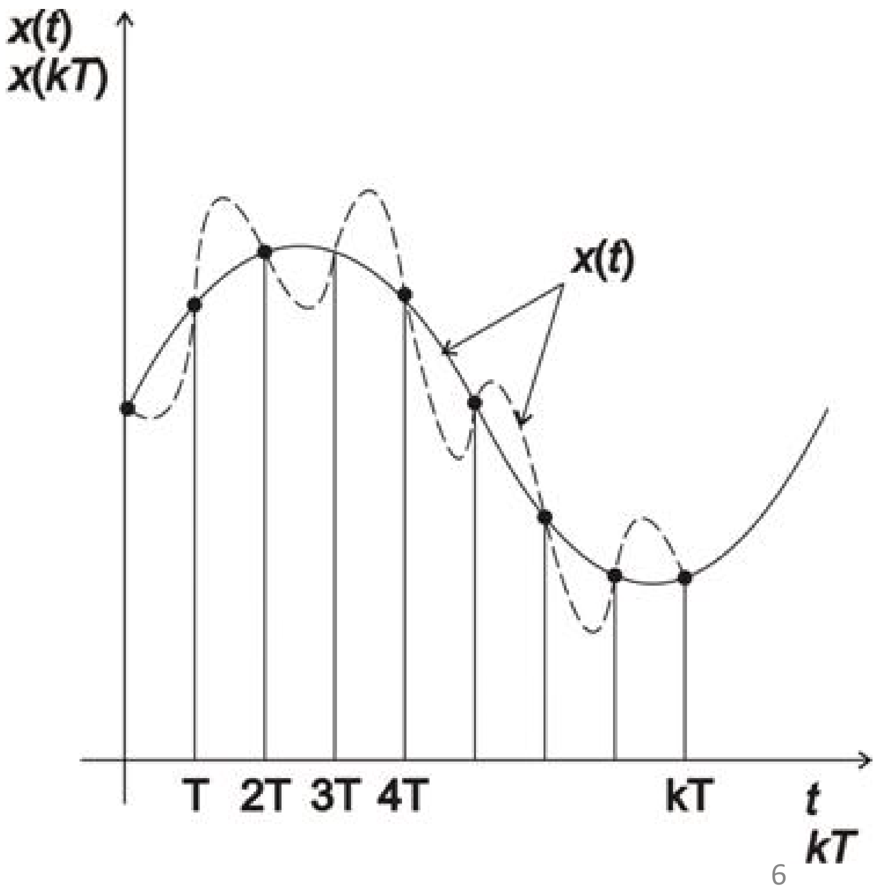
$$T = (0{,}1 \div 0{,}5) T_D$$

Discrete time function (series of values) corresponds to infinite continuous time function!!!

Shannon-Kotelnikov sampling theorem:

$$\omega \geq 2\omega_m$$

$$T \leq \frac{\pi}{\omega_m}$$



Digital PID Controllers

Continuous PID controller – differential equation

$$u(t) = k_P \left[e(t) + \frac{1}{T_I} \int_0^t e(\tau) d\tau + T_D \frac{de(t)}{dt} \right]$$

Let introduce sum and difference

$$u(kT) = k_P \left[e(kT) + \frac{T}{T_I} \sum_{i=0}^{k} e(iT) + \frac{T_D}{T} \nabla e(kT) \right]$$

Digital PID controller – PSD (proportional-sum-difference) controller.

$$G_R(z) = k_P \left(1 + \frac{T}{T_I} \frac{z}{z-1} + \frac{T_D}{T} \frac{z-1}{z} \right)$$

Digital PID Controllers

P	$u(kT)=k_P e(kT)$	$G_R(z)=k_P$
S	$u(kT)=\frac{T}{T_I}\sum_{i=0}^{k}e(iT)$	$G_R(z)=\frac{T}{T_I}\frac{z}{z-1}$
PS	$u(kT)=k_P\left[e(kT)+\frac{T}{T_I}\sum_{i=0}^{k}e(iT)\right]$	$G_R(z)=k_P\left(1+\frac{T}{T_I}\frac{z}{z-1}\right)$
PD	$u(kT)=k_P\left[e(kT)+\frac{T_D}{T}\nabla e(kT)\right]$	$G_R(z)=k_P\left(1+\frac{T_D}{T}\frac{z-1}{z}\right)$
PSD	$u(kT)=k_P\left[e(kT)+\frac{T}{T_I}\sum_{i=0}^{k}e(iT)+\frac{T_D}{T}\nabla e(kT)\right]$	$G_R(z)=k_P\left(1+\frac{T}{T_I}\frac{z}{z-1}+\frac{T_D}{T}\frac{z-1}{z}\right)$

Digital PID Controllers

Incremental algorithm of PSD:

actual value of control variable $u(kT)$ is not calculated, but its increment against previous value $u[(k\text{-}1)T]$ is determined

$$u(kT) = k_P\left[e(kT) + \frac{T}{T_I}\sum_{i=0}^{k} e(iT) + \frac{T_D}{T}\nabla e(kT)\right]$$

$$u[(k-1)T] = k_P\left\{e[(k-1)T] + \frac{T}{T_I}\sum_{i=0}^{k-1} e(iT) + \frac{T_D}{T}\nabla e[(k-1)T]\right\}$$

$$u(kT) - u[(k-1)T] = k_P\left\{\begin{aligned} & e(kT) - e[(k-1)T] + \frac{T}{T_I}e(kT) + \\ & + \frac{T_D}{T}\{e(kT) - 2e[(k-1)T] + e[(k-2)T]\} \end{aligned}\right\}$$

Incremental algorithm of PSD

$$\nabla u(kT) = k_P \left[\nabla e(kT) + \frac{T}{T_I} e(kT) + \frac{T_D}{T} \nabla^2 e(kT) \right]$$

Actual value of control is:

$$u(kT) = u[(k-1)T] + q_0 e(kT) + q_1 e[(k-1)T] + q_2 e[(k-2)T]$$

$$G_R(z) = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}}$$

$$q_0 = k_P \left(1 + \frac{T}{T_I} + \frac{T_D}{T} \right)$$

$$q_1 = -k_P \left(1 + 2\frac{T_D}{T} \right)$$

$$q_2 = k_P \frac{T_D}{T}$$

Filters in digital PID controllers

- Filters are put before PSD controller or algorithm is changed:

$$e_f(kT) = (1-a)e_f\left[(k-1)T\right] + ae(kT) \qquad a = \frac{1}{1+\frac{T_f}{T}}$$

- a – coefficient of filtration
- a=1 ...no filtering
- a=0 ... no input signal

Filters in digital PID controllers

Control algorithm is changed:

$$u(kT)-u[(k-1)T]=k_P\left\{\begin{array}{l} e(kT)-e[(k-1)T]+\dfrac{T}{T_I}e(kT)+ \\ +\dfrac{T_D}{T}\{e(kT)-2e[(k-1)T]+e[(k-2)T]\} \end{array}\right\}$$

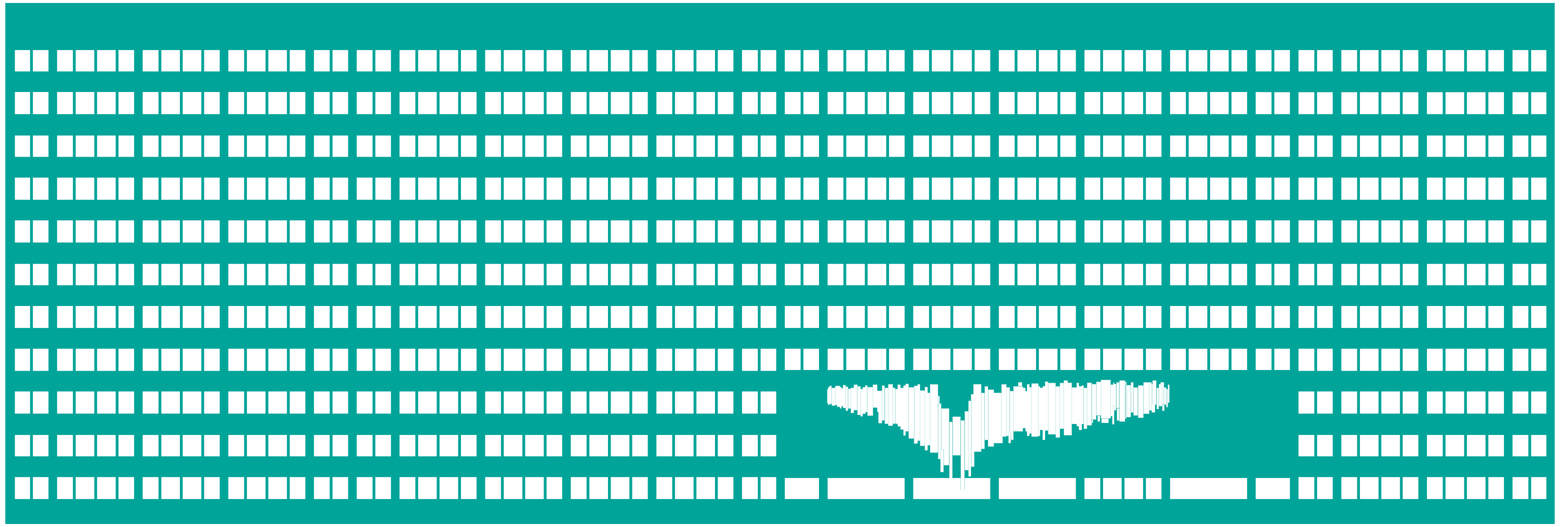
In case of control to constant value error signal *e* is substitute by *y*

$$u(kT)-u[(k-1)T]=k_P\left\{\begin{array}{l} -y(kT)+y[(k-1)T]+\dfrac{T}{T_I}[w-y(kT)]+ \\ +\dfrac{T_D}{T}\{-y(kT)+2y[(k-1)T]-y[(k-2)T]\} \end{array}\right\}$$

Thank you for your attention

VŠB TECHNICKÁ UNIVERZITA OSTRAVA

VSB TECHNICAL UNIVERSITY OF OSTRAVA



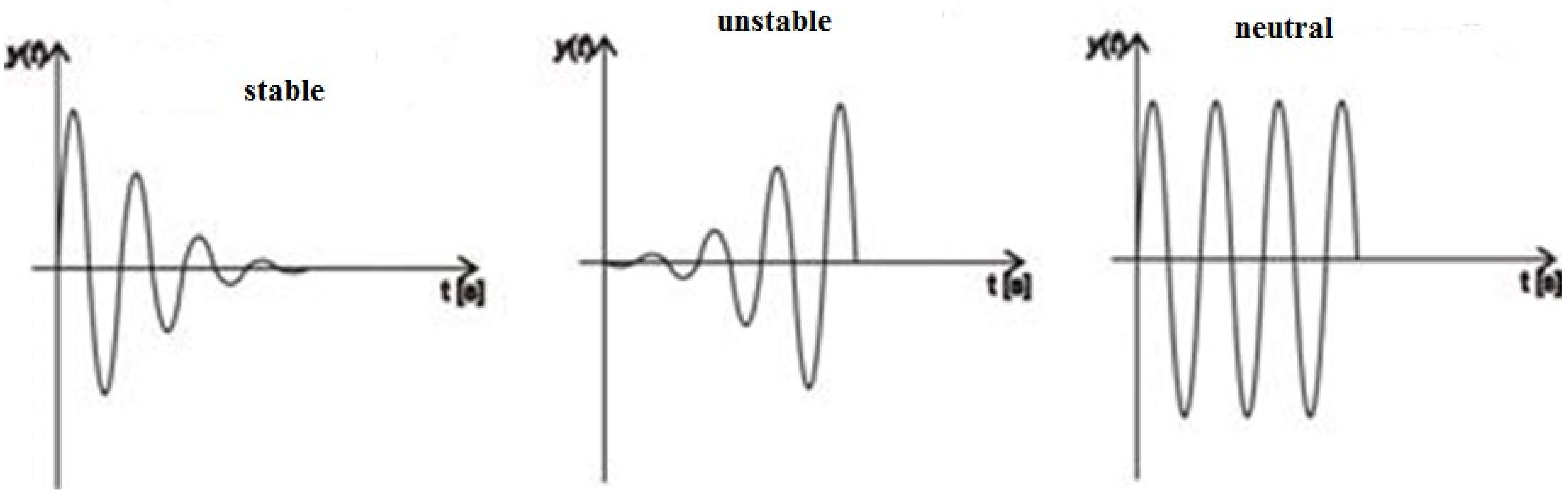
www.vsb.cz

Digital Control Systems Design
Stability

Renata Wagnerová

Stability

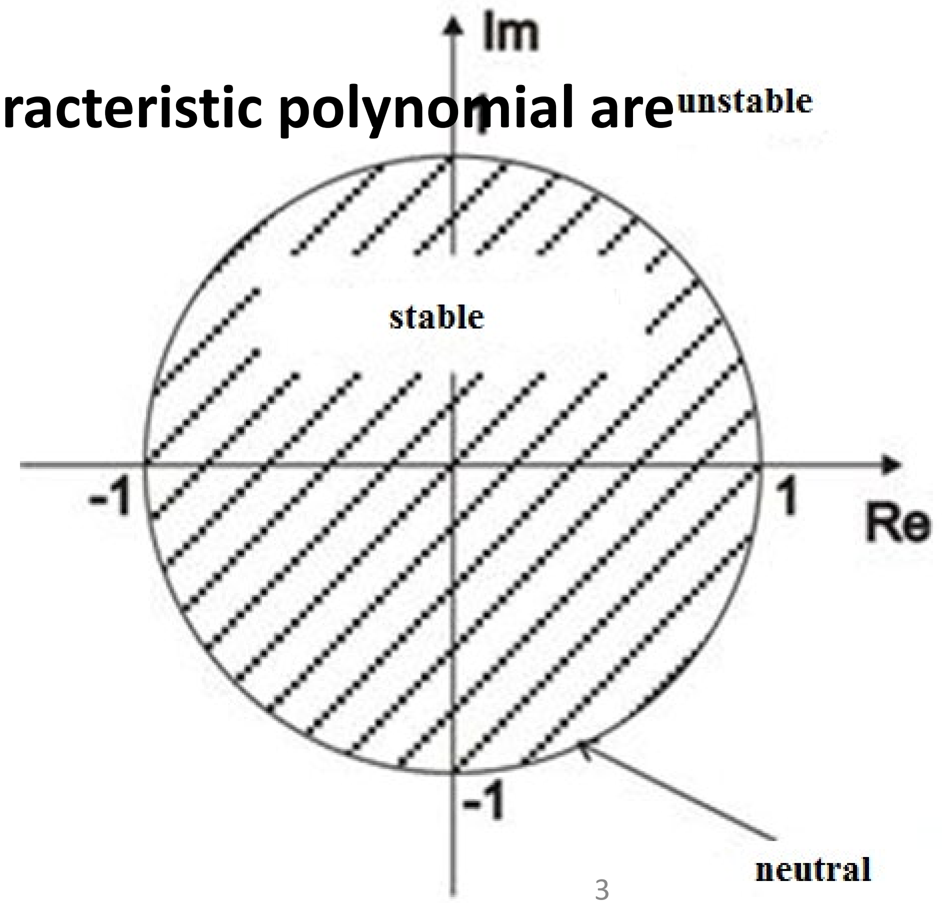
- System is stable when output of systems is moved back into steady state after input signals disappears.
- BIBO (BOUNDED INPUT BOUNDED OUTPUT) stability – system is stable if output signal is also bounded for bounded input.
- Possible results: stable, unstable and neutral (critically stable)



Stability

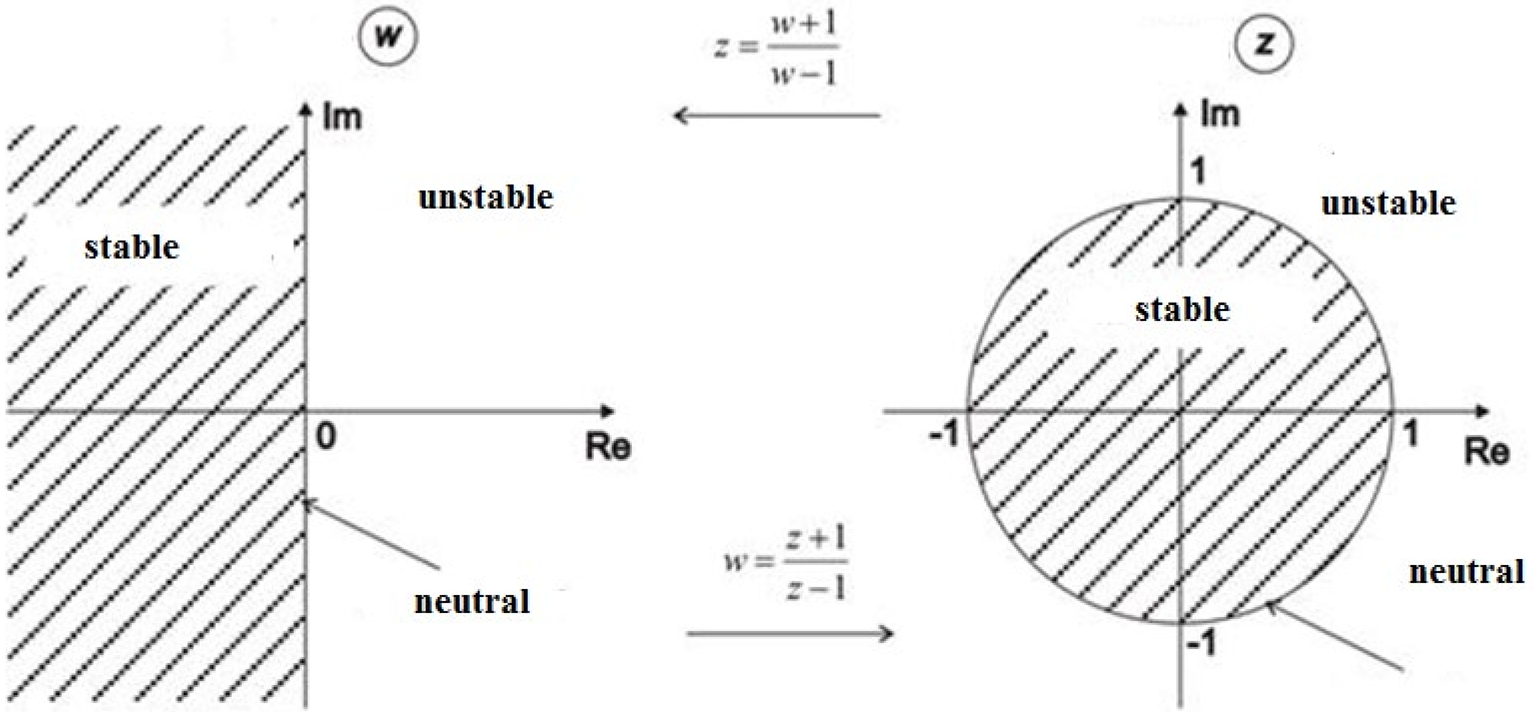
- **Characteristic polynomial**
- **Necessary and sufficient condition:**
  Discrete control system is stable if all roots of characteristic polynomial are located inside unit cycle.

$$\left|z_i\right| < 1 \quad i = 1,2,\ldots,n$$



Bilinear transform

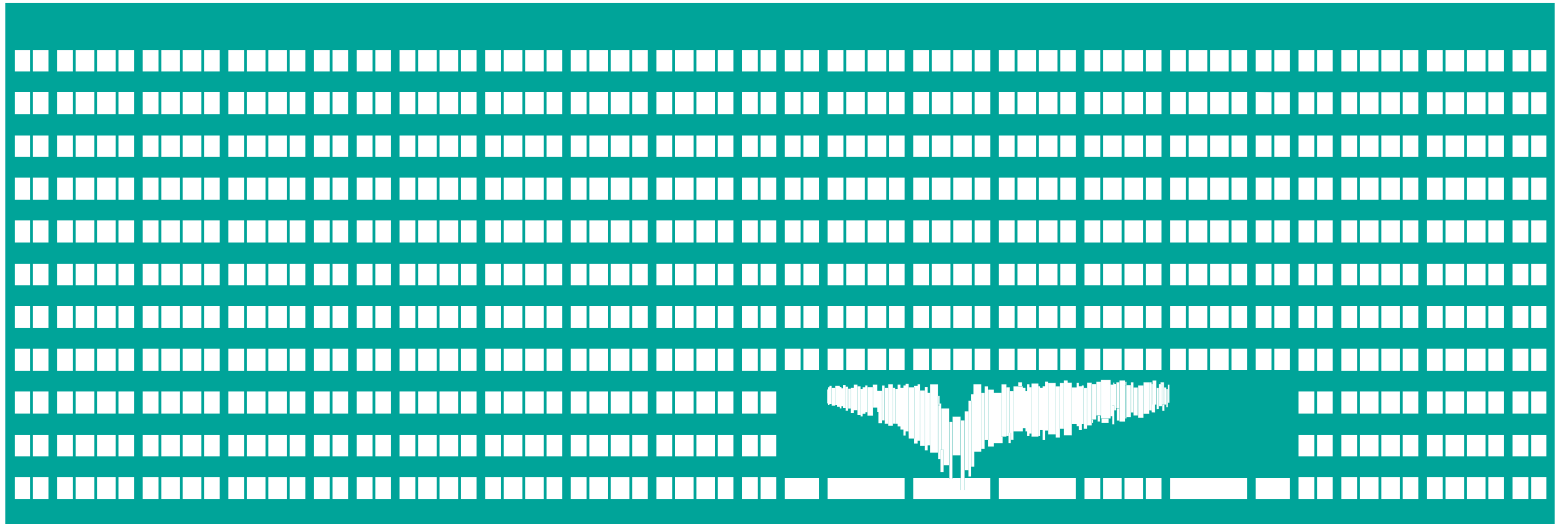
Transform the stability area of discrete systems into stability area of continuous systems



Thank you for your attention

VŠB TECHNICKÁ UNIVERZITA OSTRAVA

VSB TECHNICAL UNIVERSITY OF OSTRAVA



www.vsb.cz

VSB TECHNICAL UNIVERSITY OF OSTRAVA | FACULTY OF MECHANICAL ENGINEERING | DEPARTMENT OF CONTROL SYSTEMS AND INSTRUMENTATION

Digital Control Systems Design
Control Systems Performance

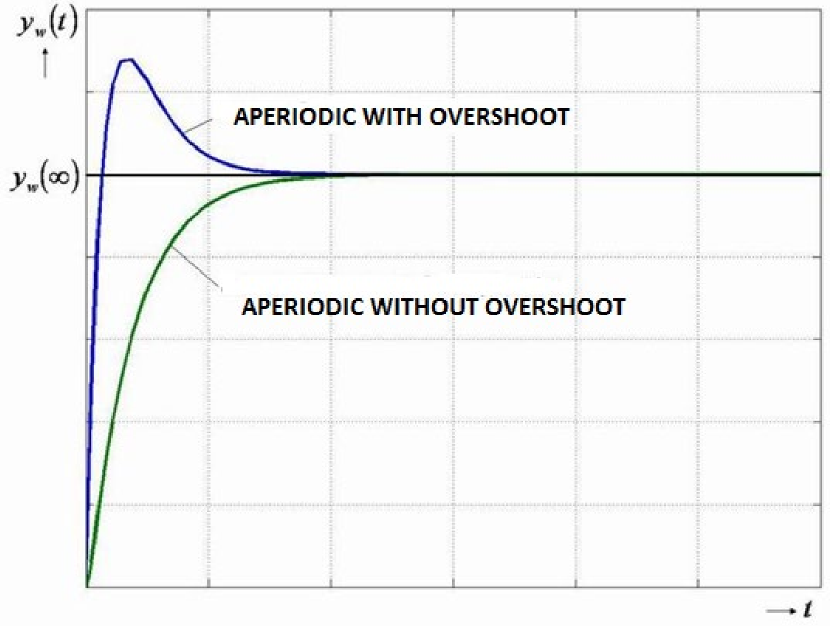
Renata Wagnerová

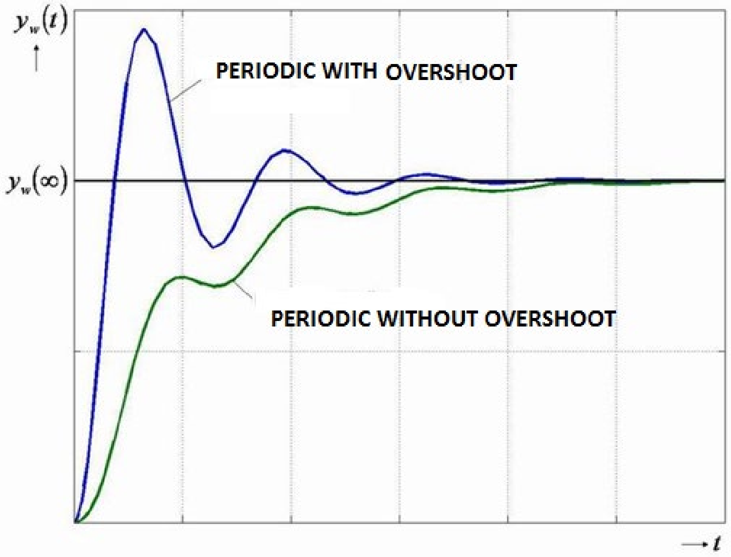
Quality of control systems
Time domain

It is necessary to know a step response of closed-loop control system.

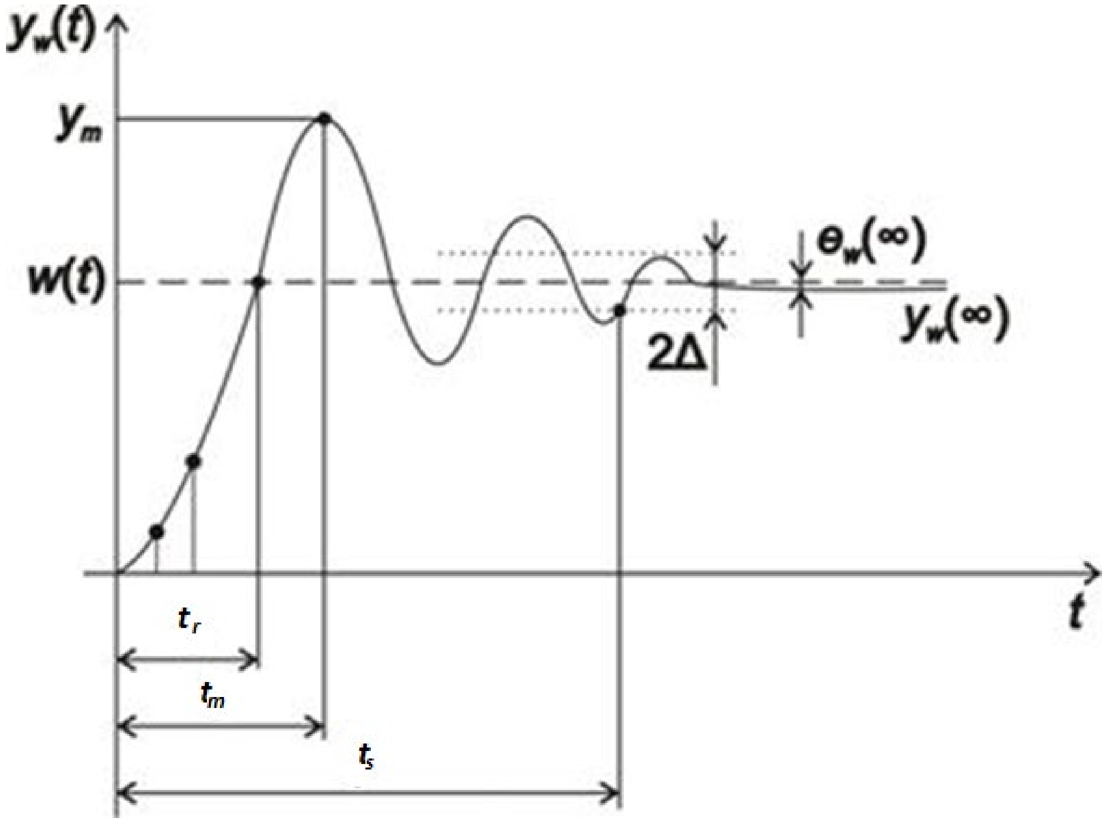
It can be divided into:

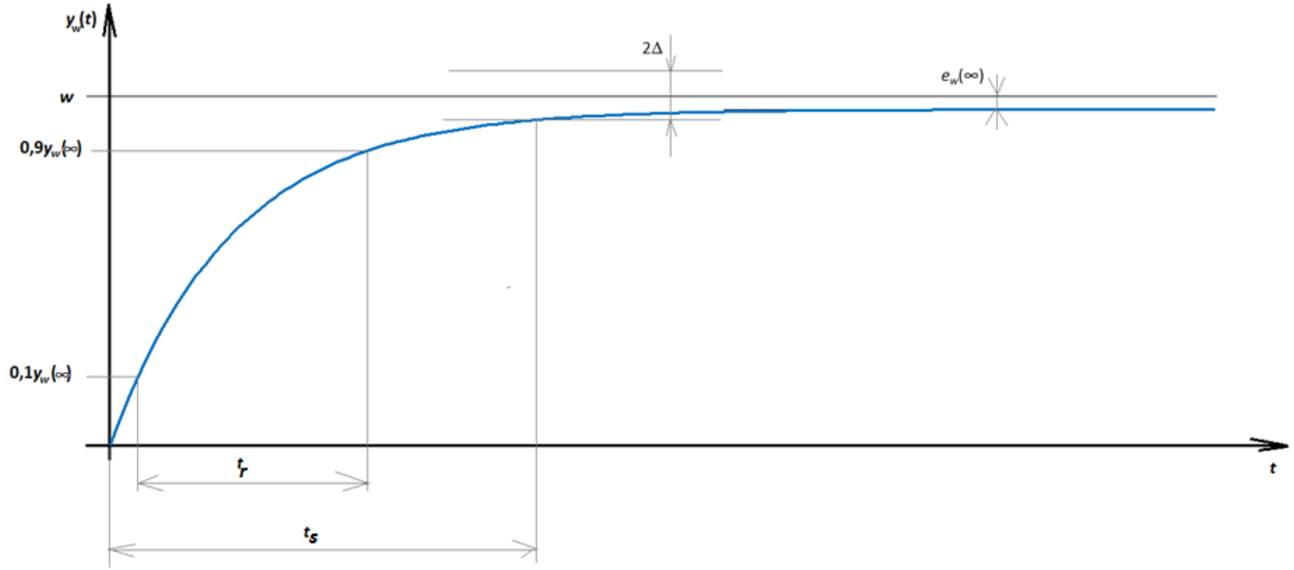
- Periodic
  - with overshoot
  - without overshoot
- Aperiodic –
  - with overshoot
  - without overshoot





Quality of control systems
Time domain





Performance criterias

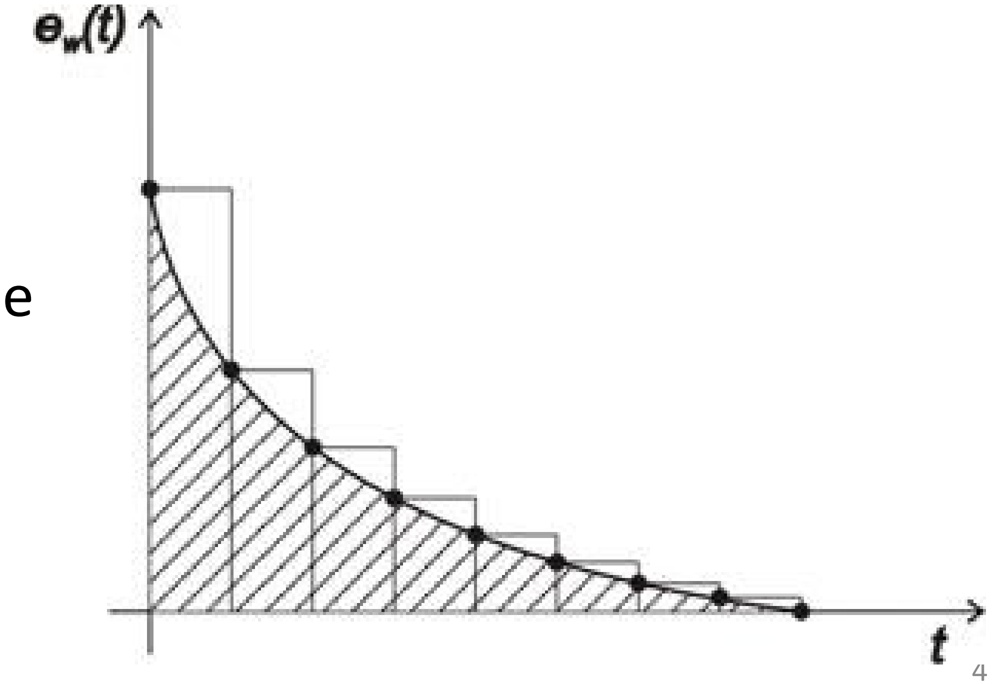
- Whether the aim is to improve the design of a system or to design a control system, the performance index must be chosen and measured.
- A system is considered an optimum control system when the system parameters are adjusted so that an extremum value (commonly minimum).

Linear error - IE

- Suitable only for processe

$$I_{IE} = \int_0^\infty e_w(t)dt$$

$$S_{IE} = T\sum_{i=0}^{\infty} e_w(iT)$$



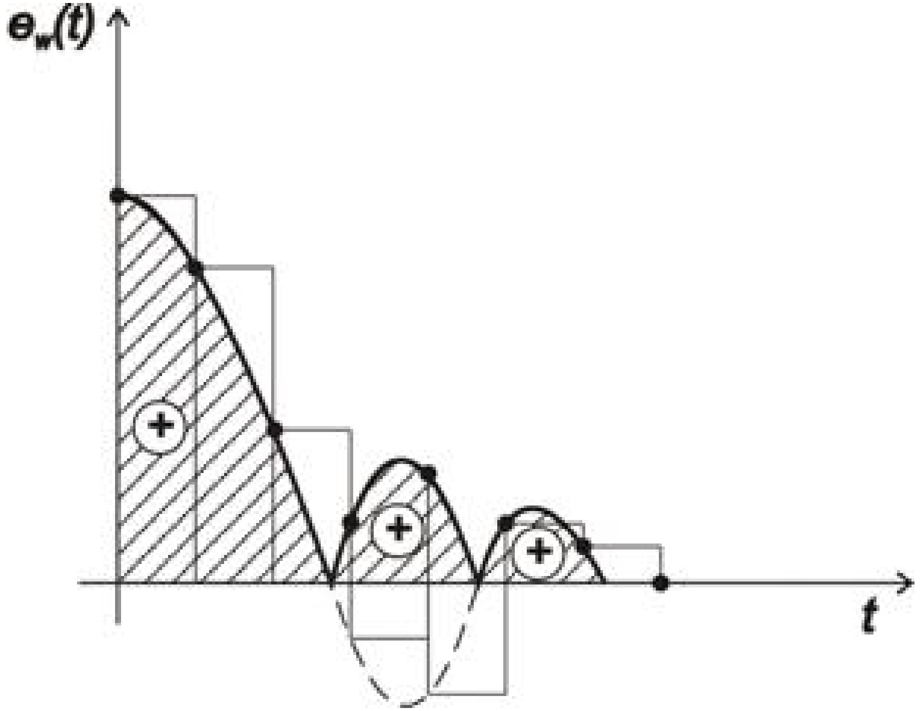
Performance criterias

Integral absolute error - IAE

-Suitable for processes with or without overshoot
-Not possible to solve analytically

$$I_{IAE} = \int_0^\infty |e_w(t)| dt$$

$$S_{IAE} = T \sum_{i=0}^{\infty} |e_w(iT)|$$



Performance criterias

Integral square error - ISE

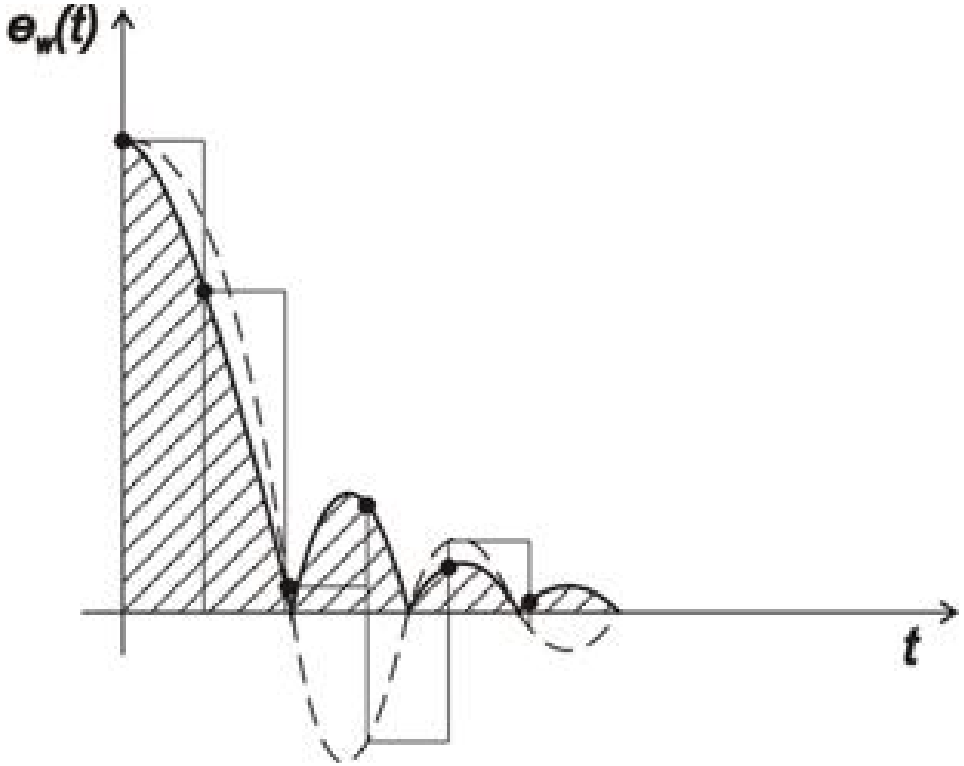
-Suitable for processes with or without overshoot

-It is possible to solve analytically

-Final course is with overshoot

$$I_{ISE} = \int_0^\infty e_w^{\ 2}(t)dt$$

$$S_{ISE} = T\sum_{i=0}^{\infty} e_w^{\ 2}(iT)$$



Performance criterias

ITAE

-To take into account also settling time
-Not possible to solve analytically

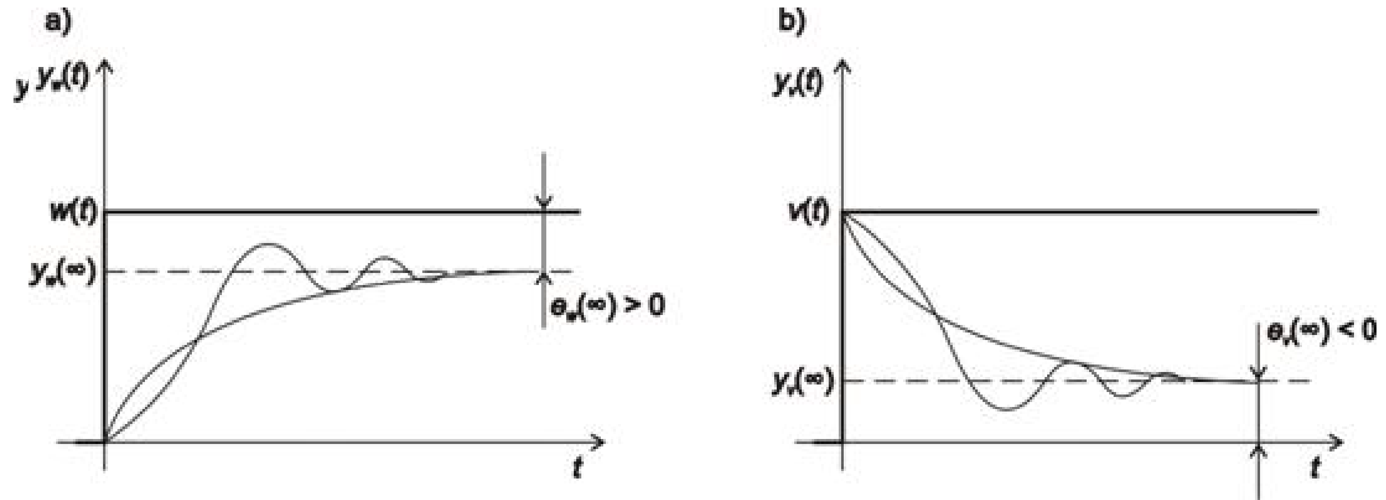
$$I_{ITAE} = \int_0^{\infty} |e_w(t)| t dt$$

$$S_{ITAE} = T \sum_{i=0}^{\infty} |e_w(iT)| iT$$

Steady state error

- Values depend on number of integral (summing) elements of closed-loop control systems and type of input signal
- Can have one of three values

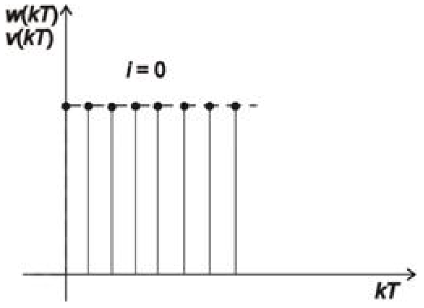
$$e(\infty) = 0 \qquad e(\infty) = \text{const.} \qquad e(\infty) = \infty$$



Steady state error

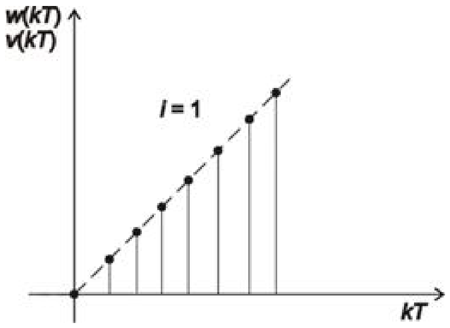
- Constant signal (i=0)

$$w(kT) = w_0 \eta(kT) \quad \rightarrow \quad W(z) = \frac{w_0 z}{z-1}$$



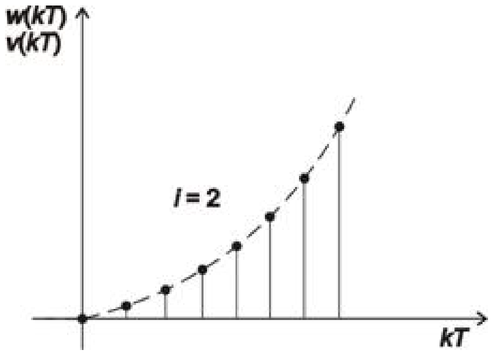
- Ramp signal (i=1)

$$w(kT) = w_1 kT \quad \rightarrow \quad W(z) = \frac{w_1 T z}{(z-1)^2}$$



- Parabolic signal(i=2)

$$w(kT) = w_2 (kT)^2 \quad \rightarrow \quad W(z) = \frac{w_2 T^2 z(z+1)}{(z-1)^3}$$



- General form of input

$$W(z) = \frac{w_i D_i(z)}{(z-1)^{i+1}}$$

$$D_0(z) = z$$
$$D_1(z) = Tz$$
$$D_2(z) = T^2 z(z+1)$$

Steady state error

Simplified solution

- It is necessary to determine q and k_0
- Then q and i are compared

$$G_0(z) = \frac{k_0 T^q}{(z-1)^q} G_1(z); \; \lim_{z \to 1} G_1(z) = 1$$

For $q > i$ $\quad e_w(\infty) = 0$

For $q = i$ $\quad e_w(\infty) = \text{const.}$

For $q < i$ $\quad e_w(\infty) = \infty$

For $q = 0$ $\quad e_w(\infty) = \dfrac{w_0}{1 + k_0}$

For $q = 1$ $\quad e_w(\infty) = \dfrac{w_1}{k_0}$

For $q = 2$ $\quad e_w(\infty) = \dfrac{2w_2}{k_0}$

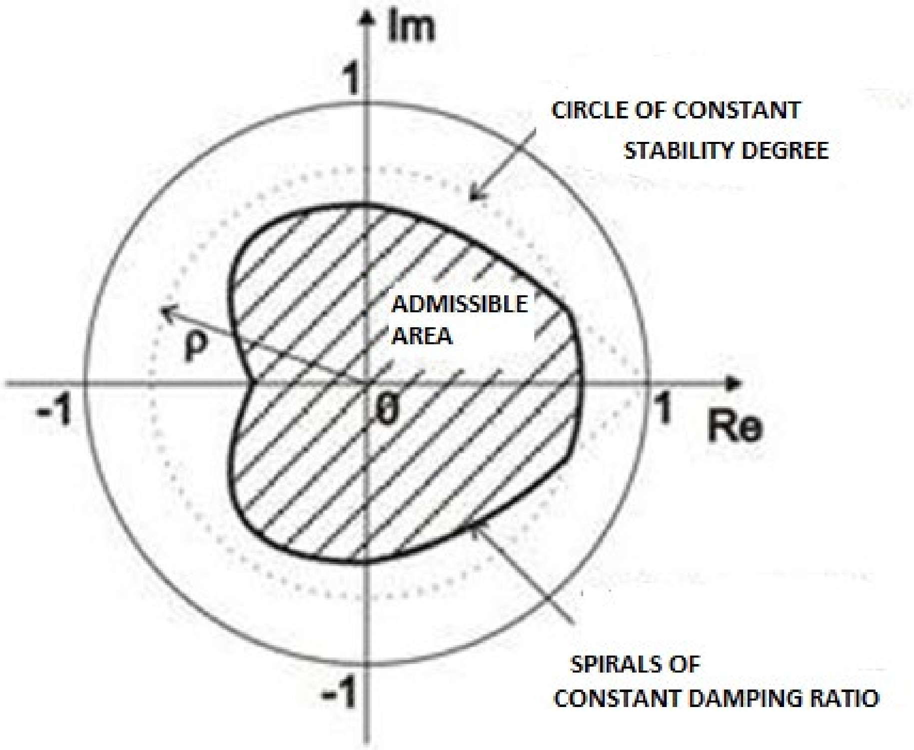
COMPLEX VARIABLE DOMAIN

- Placements of poles and zeros have impact on quality
- Necessary condition is stability
- Admissible area is described by settling time t_s and maximal relative overshoot κ
- Poles situated closed to admissible area are called **dominant**

Complex variable domain

Stability degree

$$\alpha_w \geq \left(\frac{1}{3} \div \frac{1}{5}\right) t_r$$



Damping ratio ξ_0

$$\varphi_w \leq \arccos \xi_0$$

$$\psi_w \geq \arcsin \xi_0$$

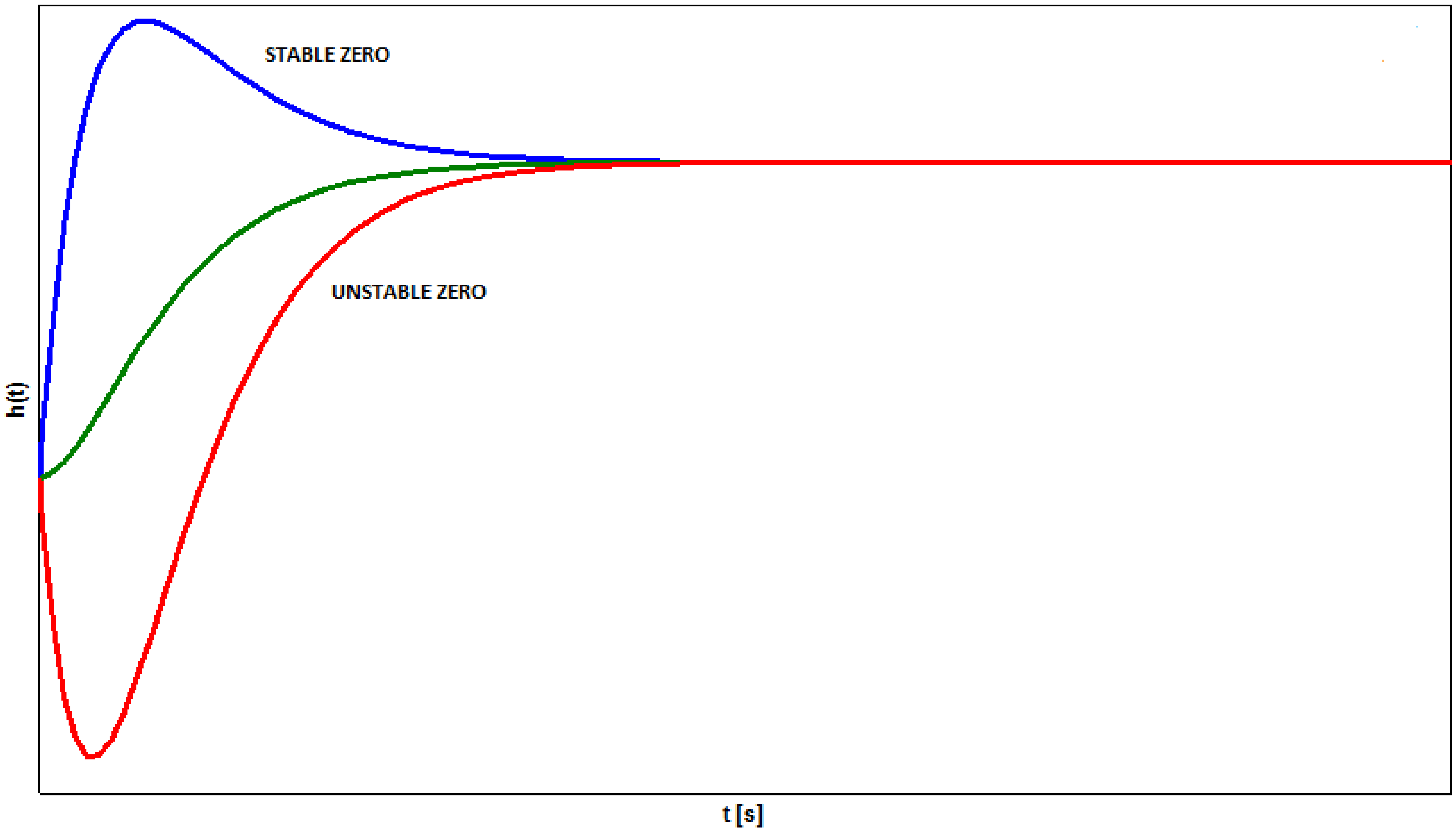
$$\rho \leq e^{-|\alpha_W| T}$$

$$\rho(\omega) \leq e^{-\frac{|\omega| T}{tg \psi_W}}$$

for $\quad -\frac{\pi}{2} \leq \omega \leq \frac{\pi}{2}$

COMPLEX VARIABLE DOMAIN

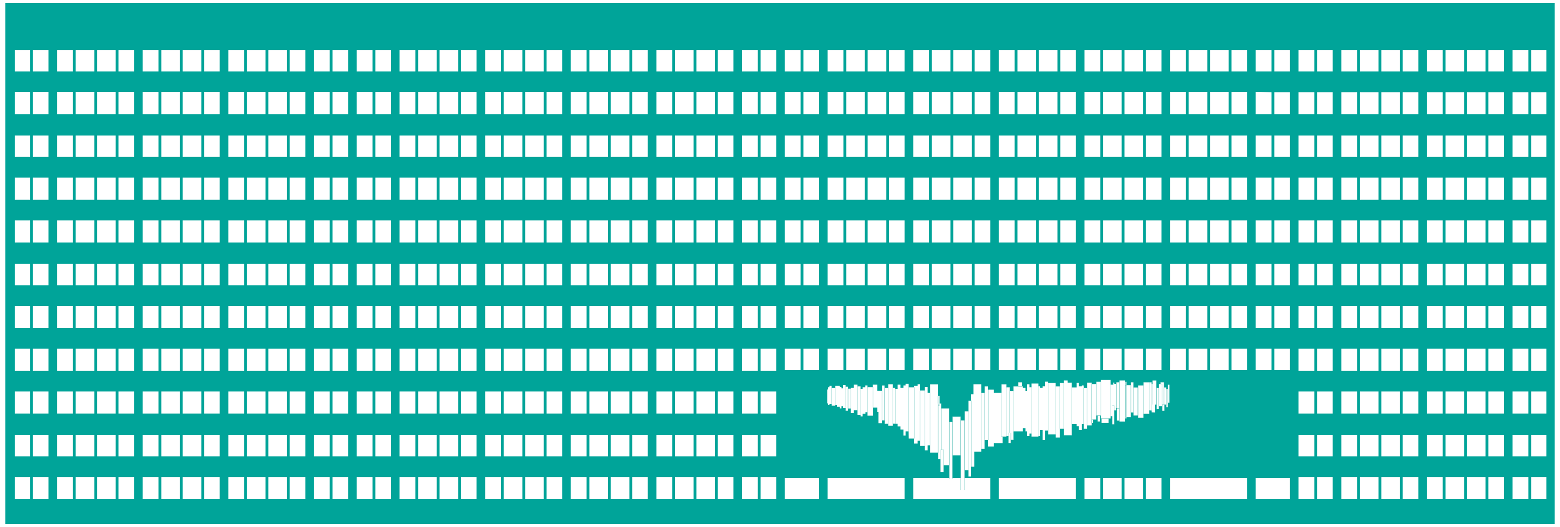
- Zeros have also impact into quality.
- **Stable zero** s_0=-1/T_1 – makes response faster, it causes overshoot.
- **Unstable zero** s_0=1/T_1 – makes response slower, it causes undershoot.


STABLE ZERO
UNSTABLE ZERO
h(t)
t [s]

Thank you for your attention

VŠB TECHNICKÁ UNIVERZITA OSTRAVA

VSB TECHNICAL UNIVERSITY OF OSTRAVA



www.vsb.cz

Digital Control Systems Design Synthesis

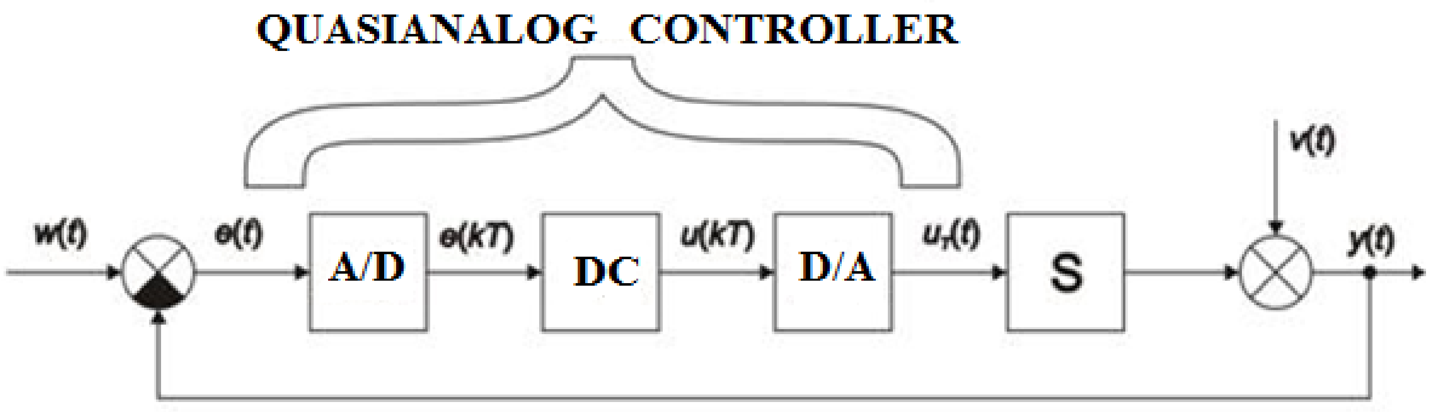
Renata Wagnerova

Digital Control Systems Synthesis

- **In case of low value of sampling period**

$$T << 0{,}25T_d$$

$$T << 0{,}17t_{0,95}$$



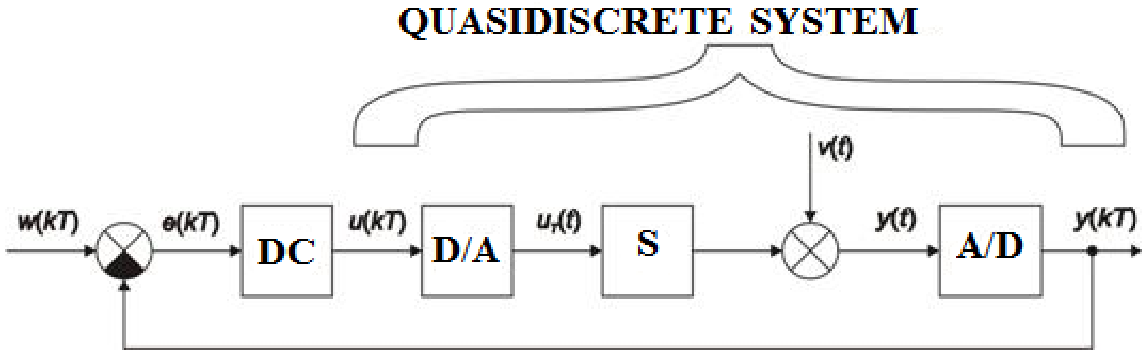
$$G_{CC}(s) = G_C(s)e^{-\frac{T}{2}s}$$

Digital Control Systems Synthesis

- **In case of middle value of sampling period**

$$T < 0{,}25T_d$$

$$T < 0{,}17t_{0{,}95}$$



$$G_S(z) = \frac{z-1}{z} Z\left\{ L^{-1}\left\{ \frac{G_S(s)}{s} \right\} \Big|_{t = kT} \right\}$$

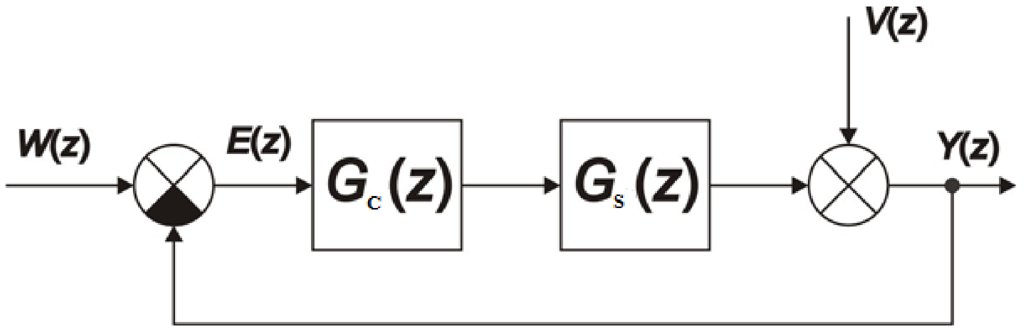
$$G'_S(s) = G_S(z)\Big|_{z = \frac{1+\frac{T}{2}s}{1-\frac{T}{2}s}}$$

- Continuous controller is chosen and tune according method for continuous systems

$$G_R(z) = G_R(s)\Big|_{s = \frac{2}{T}\frac{z-1}{z+1}}$$

Digital Control Systems Synthesis

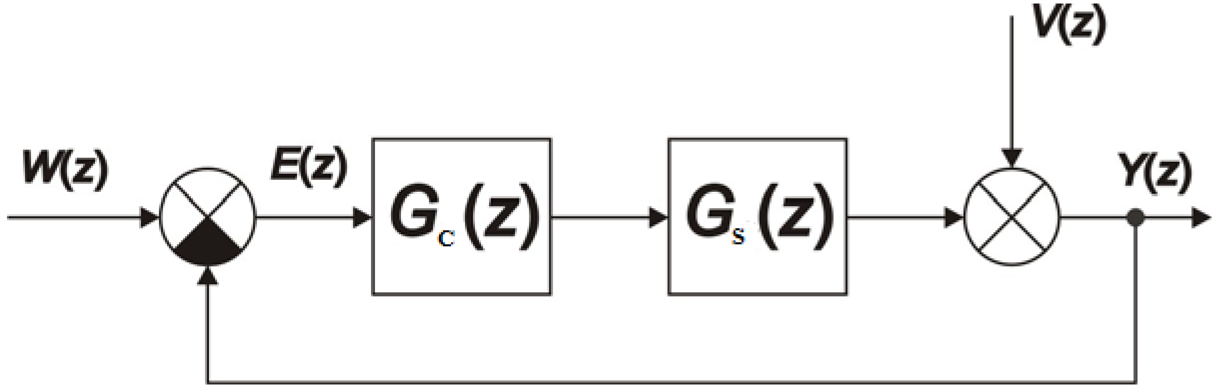
- **In case of high value of sampling period**



- Design and tuning are made in complex variable domain *z*, it is necessary to do discretization.
- Method for direct controller design is used.
- Controllers are not in standard form (like PID).

Method for direct digital controllers design

- Block scheme of closed-loop control systems.



- Equation for synthesis

$$G_C(z) = \frac{1}{G_S(z)} \frac{G_{wy}(z)}{1 - G_{wy}(z)}$$

Method for direct digital controllers design
Dead-beat controller

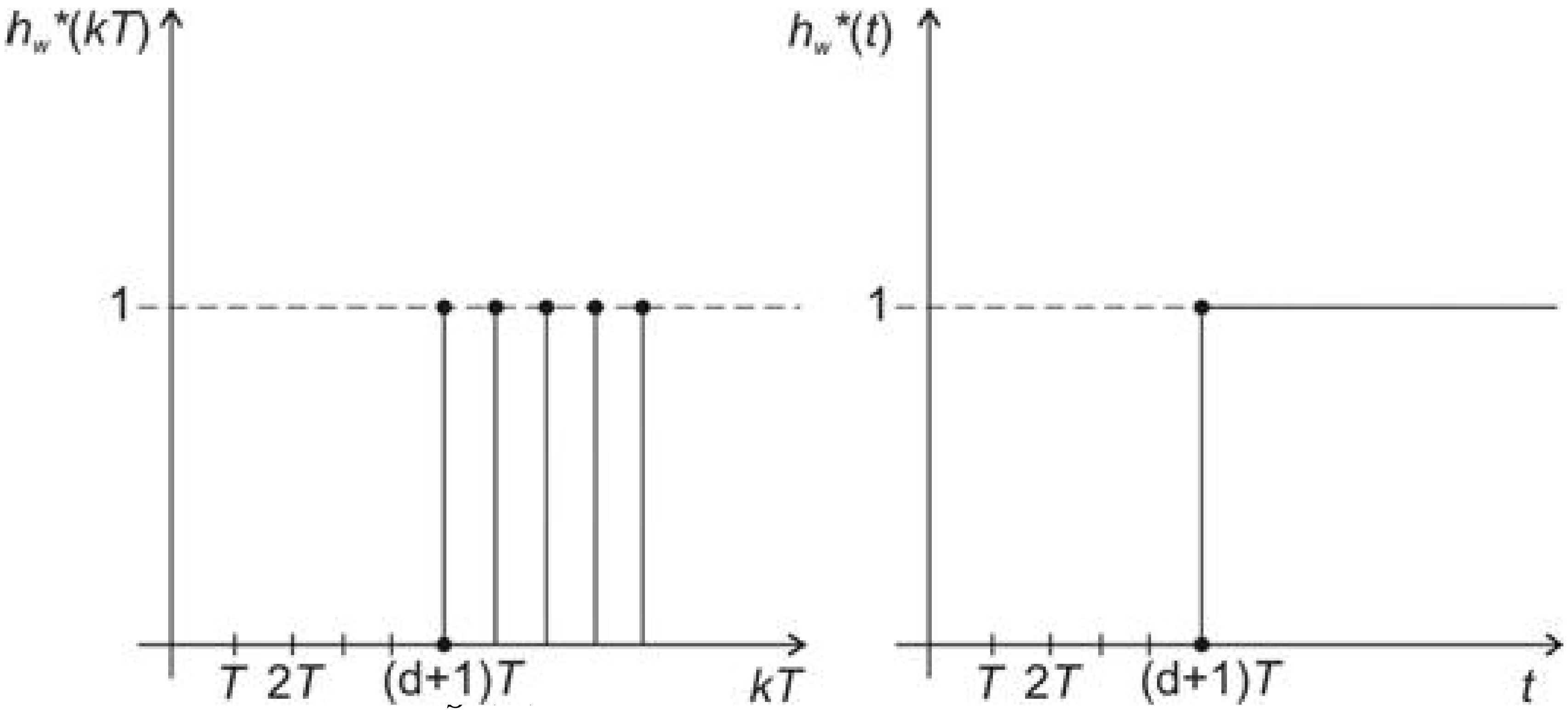
- In case of step input signal of reference variable $w(kT)$ controller enables to end transient part in $(d+1)T$ steps.
- Desired form of transfer function

$$G^*_{wy}(z) = z^{-(d+1)}$$

- Controller equation

$$G^*_C(z) = \frac{1}{G_S(z)} \frac{z^{-(d+1)}}{1 - z^{-(d+1)}}$$

Method for direct digital controllers design
Dead-beat controller



Method for direct digital controllers design
Dahlin controller

- Desired form of transfer function

$$G_{wy}^{*}(z)=\frac{1-c_w}{z-c_w}z^{-d}=\frac{1-c_w}{1-c_w z^{-1}}z^{-(d+1)},\ c_w=e^{\frac{T}{T_w}}$$

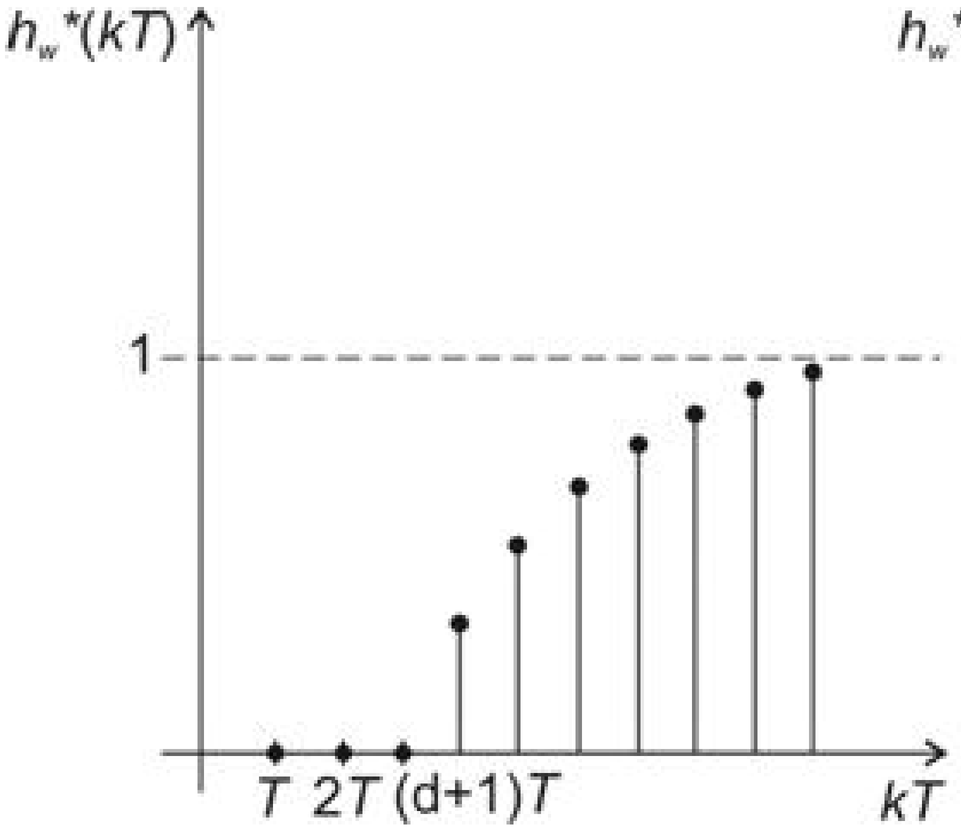
- It corresponds to the continuous system

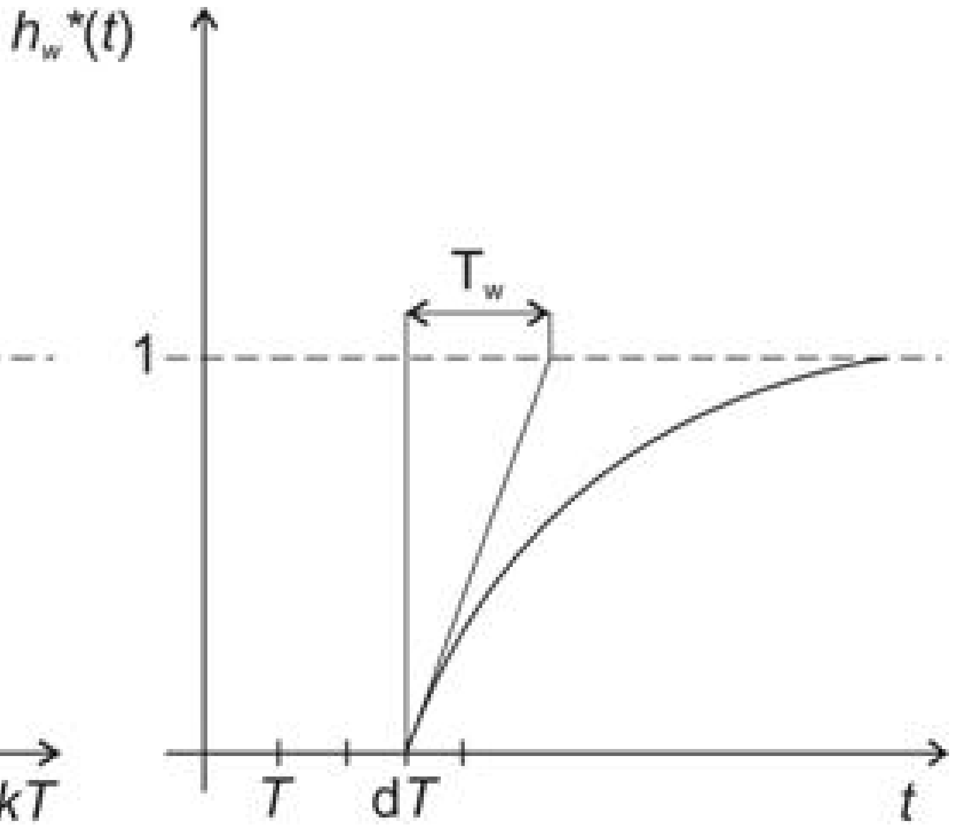
$$G_{wy}^{*}(s)=\frac{1}{T_w s+1}e^{-T_d s}$$

- Controller equation

$$G_C^{*}(z)=\frac{1}{G_S(z)}\frac{\frac{1-c_w}{1-c_w z^{-1}}z^{-(d+1)}}{1-\frac{1-c_w}{1-c_w z^{-1}}z^{-(d+1)}}$$

Method for direct digital controllers design
Dahlin controller





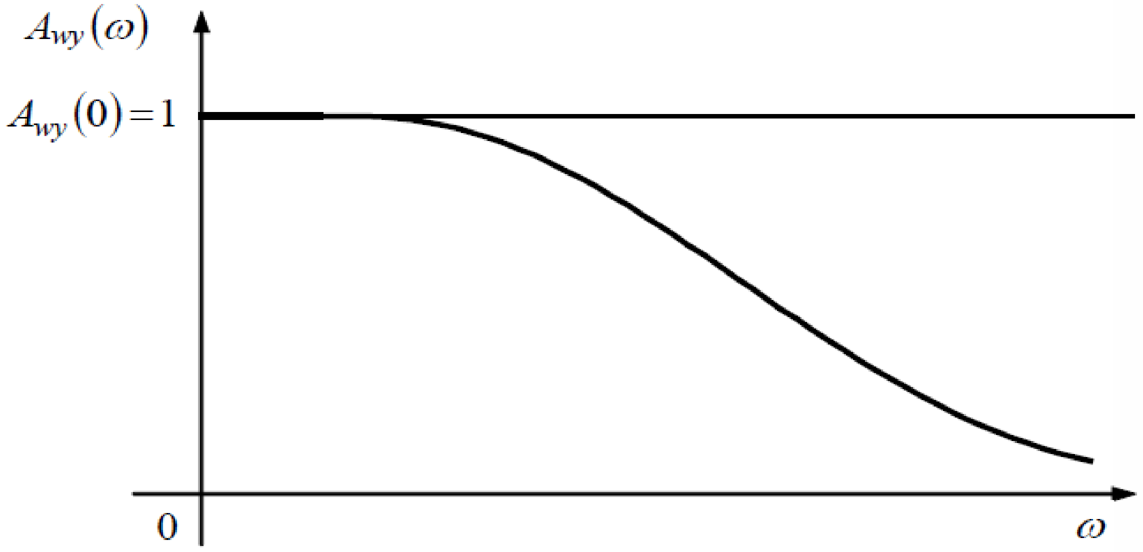
Example 2

- For the system described by transfer function it is necessary to design and verify digital controller so that the steady state error for constant set-point will be zero and relative overshoot will be up to 5%.
- Use all possible approaches.

$$G_S(s) = \frac{2}{5s + 1} e^{-6s}$$

Modulus Optimum Method

- It belongs among analytical tuning method
- It comes from desired condition for modulus of frequency control system transfer function
- It guarantees relative overshoot around 5%
- $q \leq 1$



Plant		Controller $<$ analog $T=0$, digital $T>0$			
		Type	K_P^*	T_I^*	T_D^*
1	$\frac{k_1}{T_1 s+1}$	I	–	$2k_1(T_1-0.5T)$	–
2	$\frac{k_1}{s(T_1 s+1)}$	P	$\frac{1}{2k_1T_1}$	–	–
3	$\frac{k_1}{(T_1 s+1)(T_2 s+1)}$ $T_1 \geq T_2$	PI	$\frac{T_I^*}{2k_1T_2}$	$T_1-0.5T$	–
4	$\frac{k_1}{s(T_1 s+1)(T_2 s+1)}$ $T_1 \geq T_2$	PD	$\frac{1}{2k_1(T_2+0.5T)}$	–	$T_1-0.5T$
5	$\frac{k_1}{(T_1 s+1)(T_2 s+1)(T_3 s+1)}$ $T_1 \geq T_2 \geq T_3$	PID	$\frac{T_I^*}{2k_1(T_3+0.5T)}$	T_1+T_2-T	$\frac{T_1T_2}{T_1+T_2}-\frac{T}{4}$

Example 3

- For the system described by transfer function it is necessary to design and verify digital controller so that the steady state error for constant set-point will be zero and relative overshoot will lower or equal to 5%.
- Use all possible approaches.

$$G_S(s) = \frac{2}{10s+1}$$

Thank you for your attention