Spring 2022 **MATH 5543** **Homework Assignment #2** **Due 04/21/2022**

Note: For problems using MATLAB codes, please include all your subroutine files and driver files in the submission.

1. (30 pts) **(Coding Problem)** The goal of this problem is to solve the following boundary value problem on rectangular meshes using the 5-point finite difference stencil

$$\nabla^2 u = -(\pi^2 + 1)\sin(\pi x)\sin(y), \qquad \text{in } \Omega = [0,1] \times [0,\pi]$$
$$u = 0, \qquad \text{on } \partial\Omega.$$

Do the following tasks:

(a) Write two MATLAB subroutine files

A = fdm2Dmatrix(m, n, hx, hy)

F = fdm2Drhs(funF, funG, X, Y, m, hx, n, hy)

(b) Write a driver file to solve this problem with $m = 10, n = 30$. Plot your numerical solution. You may look at lecture notes Chapter 3, page 27-32.

(c) The true solution for this BVP is $u(x,y) = \sin(\pi x)\sin(y)$. Repeat your numerical tests on a sequence of meshes (i) $m = 10, n = 30$; (ii) $m = 20, n = 60$; (iii) $m = 40, n = 120$; (iv) $m = 80, n = 240$. Compute the error in L^∞ norm on each mesh:

$$\|E_h\|_\infty = \max\{|U - \tilde{U}|\}$$

where U is the finite difference solution and $\tilde{U}$ is the vector of the true solution evaluated at mesh nodes. Can you numerically confirm that the error decay in second order?

2. (30 pts) The goal of this problem is to solve the following initial value problem with some one-step and multistep methods:

$$u' = u^2 - 2u + 2, \qquad \text{for } t > 0.$$
$$u(0) = 1;$$

The true solution is $u(t) = tan(t) + 1$.

(a) Write a MATLAB subroutine for Trapezoidal method:

[t,U] = trapezoidal(fun,Dfun,a,b,U0,N)

The structure of this code is similar to backward Euler method on lecture notes Chapter 4, page 19.

(b) Write a MATLAB subroutine for BDF2 method (see lecture notes Chapter 4 page 25)

[t,U] = bdf2(fun,Dfun,a,b,U0,U1,N)

(c) Use these two methods to solve the above IVP with $N = 10$. Plot your numerical solution. (Hint: For BDF2 method, you can use some one-step method, such as Euler's method or Trapezoidal method to generate $U1$.)

(d) Solve the IVP with a sequence of time steps $h = 1/N$ with $N = 10, 20, 40, 80$. Record the error at $T = 1$. Can you numerically confirm that the global convergence order of these methods is 2?