

Given that,

$$\textcircled{1} P(n) = 1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1)$$

$$= \frac{n(n+1)(n+2)}{3}$$

a) L.H.S for $P(1)$

$$L.H.S = 1 \cdot 2 = 2$$

R.H.S for $P(1)$ means $n=1$

$$R.H.S = \frac{n(n+1)(n+2)}{3}$$

$$= \frac{1 \cdot 2 \cdot 3}{3} = 2$$

$$\therefore R.H.S = L.H.S = P(1)$$

Hence, $P(1)$ is true Proved.

c) what do you need to Prove in the inductive step?

\Rightarrow in the inductive step, we need to Prove:

if $P(k)$ is true for all value of k , then

$P(k+1)$ is also true.

Ans: 02

Let $P(n)$ be the statement that $2^n > n^2 + n$ when n is an integer greater than 4.

$\Rightarrow P(5)$ is true because $2^5 = 32 > 5^2 + 5 = 30$

inductive step:

Assume that $P(k)$ is true.

i.e., $2^k > k^2 + k$

We have to prove that $P(k+1)$ is true.

Now, $(k+1)^2 + (k+1)$

$$= k^2 + 2k + 1 + k + 1$$

$$= k^2 + k + 2k + 2$$

$$\leq k^2 + 2k + 2$$

$$< 2k + 2^k$$

$$= 2 \cdot 2^k$$

$$= 2^{k+1}$$

Therefore $(k+1)^2 + (k+1) < 2^{k+1}$

i.e. $2^{k+1} > (k+1)^2 + (k+1)$

Thus $P(k+1)$ is true

Ans: 08

How many numbers must be selected from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ to - - - - 9?

⇒ Given;

Numbers must be selected from the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ to guarantee that at least one pair of these numbers add up to 9.

with 2 elements Pairs which give sum as 9
 $= \{(1, 8), (2, 7), (3, 6)\}$

So Choosing 1 element from each group
 $= 4$ elements (in worst case) $\{1, 7, 3, 5\} \{2, 6, 4, 8\}$

Hence, there are 3 Pairs of numbers that add up to 9.