Calculate the number of steps and Big-O estimate for this function.

```
function do_it(A, B: matrices){

for i= 1 to m {
  for j= 1 to n {
      cij=0
  for q= 1 to k
       Cij=Cij+Aiq*Bqj
  }
 }
 return c
}
```

Ans :

Function do if (AB: matrices)

For i=1 to m { m loops : linear plus

For j=1 to n { n loops : linear plus

cij =0 → 1 step

for q=1 to k ⇒ k loops : linear plus.

cij=cij+Aiq*Bqj → 1 step.

}

}

return c → possibly 1 step.

}

$m(1+n(1+k(1+1)))+1$

$= m+m\{n(2k+1)\}+1$

$= m+2kmn+mn+1$

$m = n = k$

$= n+2n^3+n^2+1$

$= 2n^3+n^2+n+1$

$= 2n^3+n^3+n^3+n^3$

$= 5n^3$ (Ans)

$O(n^3)$

Find the big O of the function $f(n) = n^{2^n} + n^{n^2}$

Ans :

Given, $f(n) = n^{2^n} + n^{n^2}$

Let us assume that $g(n) = n^{2^n}$

when $n > 4$ we have the properties
$n^2 \le 2^n$

For convenience sake, we will choose
$k = 4$ and then use $x > 4$

$$|f(n)| = |n^{2^n} + n^{n^2}|$$
$$= n^{2^n} + n^{n^2} \le n^{2^n} + n^{2^n}$$
$$= 2n^{2^n}$$

[$n^2 \le 2^n$ as $n > 4$
combine like terms]

Thus we need to choose c to be at least 2.

Let us take $c = 2$

By the definition of the Big o notation
$f(n) = n^{2^n} + n^{n^2}$ is $O(n^{2^n})$ with $k = 4$ and
$c = 2$

Ans : $O(n^{2^n})$

Let $f(x) = 3x + 2$ and $g(x) = x^2$ be functions defined on the integers ($f : Z \rightarrow Z$, $g : Z \rightarrow Z$). Find the Big O estimate of gof.

Ans :

Q1 (A) $f(x) = 3x+2$

$g(x) = x^2$

$(g \circ f)(x) = g(f(x)) = g(3x+2) = (3x+2)^2$

$= 9x^2 + 12x + 4$

Thus we see for $x > 12$

$(g \circ f)(x) = 9x^2 + 12x + 4 < 10.x^2$

Thus $(g \circ f)(x) = O(x^2)$.

We know that $f(x) = O(g(x))$ if there exists constant $C > 0$ and $n_0 \in \mathbb{N}$ such that

$$f(n) \leq C.g(n) \ \forall\ n > n_0$$

This big O estimate of $g \circ f$ is $O(x^2)$

Prove by induction that: $1 + 4 + 7 + \cdots + (3n - 2) = n(3n - 1)/2$

Ans :

Q

→ $1+4+7+\cdots+(3n-2) = \dfrac{n(3n-1)}{2}$

Prove by Induction

For $n=1$

$$1 + = \frac{1(3\cdot1-1)}{2}$$

$$1 = \frac{2}{2}$$

Result true for $n=1$

Assume Result true for $n=k$

Prove that Result true for $n=k+1$

$$1+4+7+\cdots+(3K-2)+(3(k+1)-2$$

$$= [1+4+7+\cdots+(3k-2)] + [3(k+1)-2]$$

$$= \frac{k(3k-1)}{2} + (3(k+1)-2)$$

$$= \frac{3k^2-k}{2} + (3k+3-2)$$

$$= \frac{3k^2-k}{2} + (3k+1)$$

$$L\cdot H\cdot S = \frac{3k^2-k+6k+2}{2} = \boxed{\frac{3k^2+5k+2}{2}}$$

Now check R·H·S for $n=k+1$

$$R\cdot H\cdot S = \frac{(k+1)(3(k+1)-1)}{2}$$

$$= \frac{(k+1)(3k+3-1)}{2} = \frac{(k+1)(3k+2)}{2}$$

$$= \frac{3k^2+2k+3k+2}{2} = \frac{3k^2+5k+2}{2}$$

Use mathematical induction to show that $2^n > n^2 + n$ whenever n is an integer greater than 4.

Explanation Verified

Step 1

1 of 4

To show that $2^n > n^2 + n$ for all integers $n > 4$.

Step 2

2 of 4

Base case: For $n = 5$, $2^5 = 32$ and $5^2 + 5 = 30$. Hence $2^5 > 5^2 + 5$. Base case is verified.

Step 3

3 of 4

Inductive case: Suppose the result is true for $n = k$. Now,

$$\begin{aligned} 2^{k+1} &= 2 \cdot 2^k \\ &> 2 \cdot (k^2 + k) \text{...(Induction hypothesis)} \end{aligned}$$

Now,

$$\begin{aligned} (k+1)^2 + (k+1) &= k^2 + 2k + 1 + k + 1 \\ &= (k^2 + k + 2) + 2k \\ &\leq 2k^2 + 2k \text{...(Easy to see)} \\ &< 2^{k+1} \text{...(From earlier calculation.)} \end{aligned}$$

Hence $2^{k+1} > (k+1)^2 + (k+1)$. So by mathematical induction, the result is true for all integers greater than 4.

Result

4 of 4

$$2^n > n^2 + n$$

Give a recursive definition of $P_m(n) = m*n$, the product of the integer m and the nonnegative integer n.

Step 1 of 3

Let $P_m(n)$ be the product of the integer m and the non-negative integer n.

So $P_m(n) = m \cdot n$

We have to give the recursive definition of $P_m(n)$.

The recursive definition contains two parts.

Firstly $P_m(0)$ is specified.

$P_m(0) = 0$ $(\text{since } m \times 0 = 0, m \text{ is any integer})$ ... (1)

Comment

Step 2 of 3

Then the rule for finding $P_m(n+1)$ is

$$P_m(n+1) = m(n+1)$$
$$= m \cdot n + m$$

Given

$P_m(n+1) = P_m(n) + m$ for $n = 0,1,2,3,...$ ...(2)

Comment

Step 3 of 3

The two equations (1) and (2) uniquely define $P_m(n)$ for integer m and non-negative integer n.

Then the recursive definition of $P_m(n)$ is

$P_m(0) = 0$

$P_m(n+1) = P_m(n) + m$ for $n = 0,1,2,3,...$

Give a recursive definition of *F(n)* where *F(n)* =*1* + *2* + *3* + ... + *n*.

Q Given,

$$F(n) = 1+2+3+\cdots\cdots+n$$

Replace,

n by $(n+1)$

$$F(n+1) = 1+2+3+\cdots\cdots+n+(n+1)$$

$$F(n+1) = [1+2+3+\cdots\cdots+n]+(n+1)$$

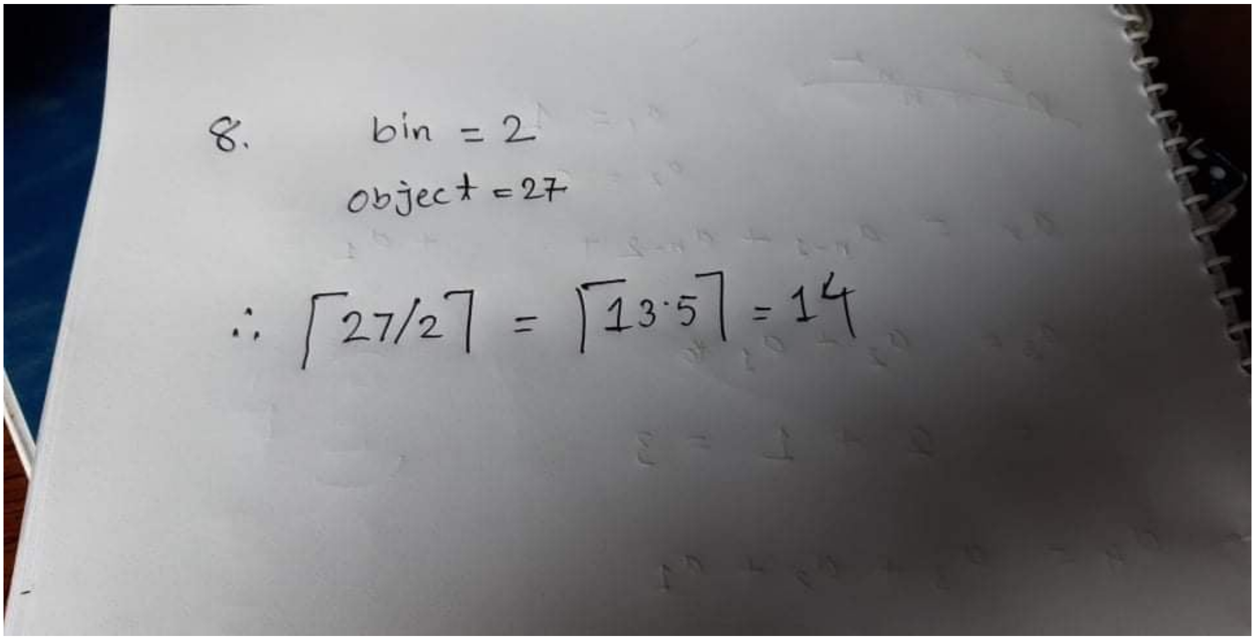
$$F(n+1) = F(n)+(n+1)$$

Hence, the recurrence relation is.

$$F(n+1) = F(n)+(n+1)$$

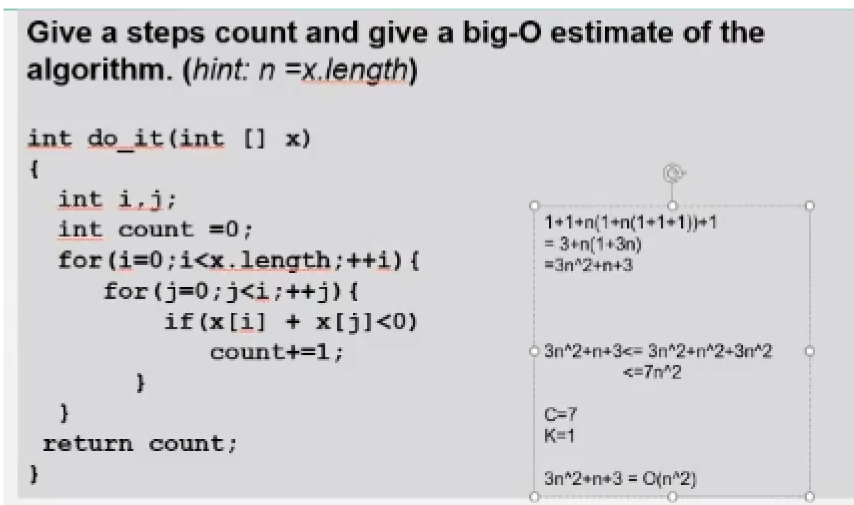
(Ans)

Suppose that there are 27 students in discrete mathematics class. Show that the class must have at least 14 male students or at least 14 female students.



8. bin = 2
object = 27

$\therefore \lceil 27/2 \rceil = \lceil 13.5 \rceil = 14$



Give a steps count and give a big-O estimate of the algorithm. (*hint: n =x.length*)

```
int do_it(int [] x)
{
  int i,j;
  int count =0;
  for(i=0;i<x.length;++i){
     for(j=0;j<i;++j){
         if(x[i] + x[j]<0)
            count+=1;
     }
  }
 return count;
}
```

1+1+n(1+n(1+1+1))+1
= 3+n(1+3n)
=3n^2+n+3

3n^2+n+3<= 3n^2+n^2+3n^2
<=7n^2

C=7
K=1

3n^2+n+3 = O(n^2)

3n^2 + n + 3 <= 3n^2 + n^2 + n^2 <= 5n^2
এইখানে **5 n^2 Hobe ..**
So, c=5

Prove that $1+2+---+n=\dfrac{n(n+1)}{2}$ by induction

EXAMPLE 1 Show that if n is a positive integer, then

$$1+2+\cdots+n=\frac{n(n+1)}{2}.$$

Extra Examples

Solution: Let $P(n)$ be the proposition that the sum of the first n positive integers, $1+2+\cdots n=\frac{n(n+1)}{2}$, is $n(n+1)/2$. We must do two things to prove that $P(n)$ is true for $n=1,2,3,\ldots.$ Namely, we must show that $P(1)$ is true and that the conditional statement $P(k)$ implies $P(k+1)$ is true for $k=1,2,3,\ldots.$

BASIS STEP: $P(1)$ is true, because $1=\dfrac{1(1+1)}{2}$. (The left-hand side of this equation is 1 because 1 is the sum of the first positive integer. The right-hand side is found by substituting 1 for n in $n(n+1)/2$.)

INDUCTIVE STEP: For the inductive hypothesis we assume that $P(k)$ holds for an arbitrary positive integer k. That is, we assume that

If you are rusty simplifying algebraic expressions, this is the time to do some reviewing!

$$1+2+\cdots+k=\frac{k(k+1)}{2}.$$

Under this assumption, it must be shown that $P(k+1)$ is true, namely, that

$$1+2+\cdots+k+(k+1)=\frac{(k+1)[(k+1)+1]}{2}=\frac{(k+1)(k+2)}{2}$$

is also true. When we add $k+1$ to both sides of the equation in $P(k)$, we obtain

$$\begin{aligned}1+2+\cdots+k+(k+1) &\overset{\text{IH}}{=} \frac{k(k+1)}{2}+(k+1)\\ &=\frac{k(k+1)+2(k+1)}{2}\\ &=\frac{(k+1)(k+2)}{2}.\end{aligned}$$

This last equation shows that $P(k+1)$ is true under the assumption that $P(k)$ is true. This completes the inductive step.

Prove by induction that $1+2+2^2+2^3+\ldots\ldots+2^n=2^{n+1}-1$ whenever *n* is a nonnegative integer.

EXAMPLE 3 Use mathematical induction to show that

$$1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$$

for all nonnegative integers n.

Solution: Let $P(n)$ be the proposition that $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$ for the integer n.

BASIS STEP: $P(0)$ is true because $2^0 = 1 = 2^1 - 1$. This completes the basis step.

INDUCTIVE STEP: For the inductive hypothesis, we assume that $P(k)$ is true for an arbitrary nonnegative integer k. That is, we assume that

$$1 + 2 + 2^2 + \cdots + 2^k = 2^{k+1} - 1.$$

To carry out the inductive step using this assumption, we must show that when we assume that $P(k)$ is true, then $P(k+1)$ is also true. That is, we must show that

$$1 + 2 + 2^2 + \cdots + 2^k + 2^{k+1} = 2^{(k+1)+1} - 1 = 2^{k+2} - 1$$

assuming the inductive hypothesis $P(k)$. Under the assumption of $P(k)$, we see that

$$\begin{aligned} 1 + 2 + 2^2 + \cdots + 2^k + 2^{k+1} &= (1 + 2 + 2^2 + \cdots + 2^k) + 2^{k+1} \\ &\overset{\text{IH}}{=} (2^{k+1} - 1) + 2^{k+1} \\ &= 2 \cdot 2^{k+1} - 1 \\ &= 2^{k+2} - 1. \end{aligned}$$

Note that we used the inductive hypothesis in the second equation in this string of equalities to replace $1 + 2 + 2^2 + \cdots + 2^k$ by $2^{k+1} - 1$. We have completed the inductive step.

Give a recursive definition of $S_m(n)=m+n$, the sum of the integer m and the nonnegative integer n.

The objective is to give a recursive definition of $s_m(n)$, the sum of the integer m and the nonnegative integer n;

Consider that the $s_m(n)$ is defined as,

$$S_m(n) = m + n$$

As n is the nonnegative integer, thus the first term is obtained when $n = 0$. So,

$$\begin{aligned} S_m(0) &= m + 0 \\ &= m \end{aligned}$$

For $n = n - 1$,

$$\begin{aligned} S_m(n-1) &= m + (n-1) \\ &= m + n - 1 \end{aligned}$$

Now, $s_m(n)$ can be written as,

$$\begin{aligned} S_m(n) &= m + n - 1 + 1 \\ &= S_m(n-1) + 1 \end{aligned}$$

Thus, the recursive definition of sum of integer and non-negative integer is as follows.

$$\boxed{S_m(n) = \begin{cases} m, & n = 0 \\ S_m(n-1) + 1, & n \neq 0 \end{cases}}$$

Find the value of a_4 if $a_1 = 1$, $a_2 = 2$, and $a_n = a_{n-1} + a_{n-2} + \cdots + a_1$

Given,

$a_1 = 1$; $a_2 = 2$

Again, $a_n = a_{n-1} + a_{n-2} + \cdots + a_1$

Now, $a_3 = a_2 + a_1$

$= 2 + 1$

$= 3$

and, $a_4 = a_3 + a_2 + a_1$

$= 3 + 2 + 1$

$= 6$

$\therefore$ The value of $a_4 = 6$

Show that $f(x) = 5x^2 + x + 1$ is $O(x^2)$ with suitable C and k.

2. **Show that $f(x) = 5x^2 + x + 1$ is $O(x^2)$ with suitable C and k.** **10**

Expert Answer

As per defination - Let f and g be real-valued functions. We say that f(x) is O(g(x)) if there are constants C and k such that :-

$|f(x)| \le C|g(x)|$ for all x>k.

So:-

$f(x)|= |5x^2 + x + 1|$

$\le |5x^2| + |x| + |1|$

$\le 5x^2 + x + 1$, for all x>0

$\le 5x^2 + x^2 + x^2$, for all x>1

$\le 7x^2$, for all x>1

Hence f(x) is $O(x^2)$ with C=7 and K=1

Observe that C= 7 and k= 1 from the definition of big-O.