(QUIZ02)

① (a) $1\cdot2+2\cdot3+\cdots+n(n+1) = \frac{n(n+1)(n+2)}{3}$

for $P(1)$, $n=1$

$L.H.S = 1\cdot2 = 2$

$R.H.S = \frac{1(1+1)(1+2)}{3}$

$= 2$

$L.H.S = R.H.S \Rightarrow P(1)$ is true.

(b) Inductive Hypothesis is the induction Process where some statement or equation holds for some Particular value of n. We assume the Correctness of equation on n and Prove it for $n+1$.

(c) In the inductive step we have to Prove $P(n+1)$ based on the assumption that $P(n)$ is true. If this happens, the orginal $P(n)$ becomes for all valid n.

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(d) We assumed that $P(n)$ is true

$$1 \cdot 2 + 2 \cdot 3 + \text{------} + n(n+1)$$

$$= \frac{n(n+1)(n+2)}{3} \quad \text{----} \ (1)$$

Statement for $P(n+1)$ becomes:-

$$1 \cdot 2 + 2 \cdot 3 + \text{-------} + n(n+1) + (n+1)(n+2)$$

$$= \frac{(n+1)(n+2)(n+3)}{3} \quad \text{-----} \ (2)$$

Now, Put the value of (1) in (2) and observing the L.H.S:-

$$\text{L.H.S} = \frac{n(n+1)(n+2)}{3} + (n+1)(n+2)$$

$$= \frac{n(n+1)(n+2) + 3(n+1)(n+2)}{3}$$

$$= \frac{(n+1)(n+2)(n+3)}{3}$$

$= \text{R.H.S}$

Hence, $P(n+1)$ holds if $P(n)$ holds

$\therefore$ $P(n)$ is true for all valid n.

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Ans: 02

for $n=5$

$L.H.S = 2^5$
$= 32$

$R.H.S = 5^2+5$
$= 25+5$
$= 30$

So, $L.H.S > R.H.S$ for $n=5$

Let's assume. $L.H.S > R.H.S$

for $n=k$

So $2^k > k^2+k$ for $n=k+1$

$L.H.S = 2^{k+1} = 2\times 2^k > 2\times(k^2+k) = 2k^2+2k$

$R.H.S = (k+1)^2+(k+1)$

$= k^2+2k+1+k+1$

$= k^2+2k+k+2$

We know for $k>4$,

$k^2 > k+2$

So $k^2+2k+k+2 < 2k^2+2k = L.H.S$

So, $L.H.S > RHS$

for $n=k+1$. Hence Proved by induction.

Given,

$$f(m,n) = \begin{cases} 0 & \text{if } m \geq 1 \text{ and } n=0 \text{ ----- (1)} \\ 2 & \text{if } m \geq 1 \text{ and } n=1 \text{ ----- (2)} \\ 2n & \text{if } m=0 \text{ ------- (3)} \\ f(m-1, f(m, n-1)) & \text{if } m \geq 1 \text{ and } n \geq 2 \text{ --- (4)} \end{cases}$$

find

1) f(1,3) Here, m=1, n=3, so condition (4) satisfied

so, f(1,3) = f(0, f(1,2))

To find f(1,3) value, first we here to find f(1,2) and f(0, output of f(1,2))

2) Let find f(1,2).

f(1,2) = m=1, n=2 so condition (4) satisfies.

so, f(1,2) = f(0, f(1,1))

To find f(1,2) value, first we have to find value of f(1,1) and then f(0, output f(1,1))

3) Let find f(1,1) :-

f(1,1) = m=1, n=1, so condition (2) satiesfies

so, output of f(1,1) = 2

After finding the value of f(1,1) = 2. now move to step (2) again

$f(1,2) = f(0, f(1,1)) = f(0,2)$

$f(0,2) = 4$ (As $m=0$, so output is $2n$.
i.e. $2 \times 2 = 4$

$\boxed{\text{So, } f(1,2) = f(0,2) = 4}$

(4) Now, move to step (1)

i.e. $f(1,3) = f(0, f(1,2))$
$= f(0,4)$
$= 8$

So, $f(1,3) = 8$

Ans: 04

Given $F(n) = 1+2+3+ - - - - +n$.

Sum of 1st 'n' natural numbers $= \frac{n(n+1)}{2}$

So, replace L.H.S with $\frac{n(n+1)}{2}$.

$$\boxed{F(n) = \frac{n(n+1)}{2}}$$

$1\left(\frac{1+1}{2}\right)$ for $n=1$. $F(n) = 1$

$2\left(\frac{2+1}{2}\right)$ for $n=2$, $F(n) = 3 = 1+2$

$3\left(\frac{3+1}{2}\right)$ for $n=3$ $F(n) = 6 = 1+2+3$

$$\boxed{\therefore F(n) = \frac{n(n+1)}{2}}$$

Ans: 05

Given, $A \in S$

$\Rightarrow 0x \in S,\ x \in S$

$\Rightarrow x1 \in S,\ x \in S$

⊛ Initially, before performing any iterations, we only know about the string λ is an element of S.

(b) ~~First~~ second Iteration:

we use the recursive definition on $x=0$, $x=1$

$x=0$ and $x=1$

$\Rightarrow 0x = 00 \in S$

$\Rightarrow x_1 = 01 \in S$

$\Rightarrow 0x = 01 \in S$

$\Rightarrow x_1 = 11 \in S$

(a) First Iteration: We use the recursive definition on $z = \lambda$

$\Rightarrow 0x = 0\lambda = 0 \in S$

$\Rightarrow 1x = 1\lambda = 1 \in S$

(c) Third Iteration:- we use the recursive definition on $x = 00$. $x = 01$ & $x = 11$

$\Rightarrow 0x = 000 \in S$

$\Rightarrow x1 = 001 \in S$

$\Rightarrow 0x = 001 \in S$

$\Rightarrow x1 = 011 \in S$

$\Rightarrow 0x = 011 \in S$

$\Rightarrow x1 = 111 \in S$

Note: s = set of all Strings that have all 0's Preceding all 1's

$S = \{\lambda, 0, 1, 00, 01, 11, 000, 001, 011, 111\}$

Ans: 06 :-

$\Rightarrow$ Given Code snippet -

$t = 0$ —— $O(1)$

for $i = 1$ to $n \rightarrow O(n)$

for $j = 1$ to $n \rightarrow O(n \times n) = O(n^2)$

$t = t + i * j \rightarrow O(1)$

$\therefore O(1) + O(n^2) + O(1)$

$\Rightarrow O(n^2)$

Ans: 07

$$\sum_{j=1}^{n} j(j+1)$$

The formula for the sum of n terms for these series can be given as:-

$(n \times (n+1) \times (n+2))/3$

Derivation :-

nth term $= n(n+1) = n^2+n$

$t(n) = n \times (n+1) = n^2+n$

$\Rightarrow$ Sum of the series upto for first n terms:-

$S(n) = \sum t(n) = \sum (n^2+n)\ (n^2+n)$

$= \sum n^2 n^2 + \sum n$

$= (n \times (n+1) \times (2n+1))/6 + (n \times (n+1))/n\ (n \times (n+1) \times (2n+1))/6 + (n \times (n+1))/2$

$= ((n \times (n+1) \times (2n+1))/6\ ((n \times (n+1) \times (2n+1))/6$

$= (n \times (n+1) \times (n+2))/3$

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⇒ The highest term in this expression will be the order of n^3

$s(n) \leq cn^3$ for all $n >= n_0$

Hence, Big-O notation for the given expression $= O(n^3)$.