





# 利息理论

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# 第一章 利息度量

## 1.1 累积函数

### 定义 1.1. 累积函数

时间零点的 1 元在时间  $t$  的累值, 记为  $a(t)$



性质 (1)  $a(0) = 1$

(2)  $a(t)$  通常是时间的增函数

(3) 当利息连续产生时,  $a(t)$  是时间的连续函数

## 1.2 贴现

### 定义 1.2. 贴现

$$v = \frac{1}{1+i} \quad d = iv$$



### 定义 1.3. 利息力

设可积函数连续可导, 则称

$$\delta_t = \frac{a'(t)}{a(t)} = [\ln a(t)]'$$

为时刻  $t$  的利息力



性质 累积函数:

$$a(t) = \exp\left(\int_0^t \delta_s ds\right)$$

贴现函数:

$$a^{-1}(t) = \exp\left(-\int_0^t \delta_s ds\right)$$

## 第二章 等额年金

### 2.1 符号一览

$$a_{\overline{n}|}, s_{\overline{n}|}$$

$$\ddot{a}_{\overline{n}|}$$

$$\ddot{s}_{\overline{n}|}$$

### 2.2 等额年金

定义 2.1. 年金的终值与现值

$$a_{\overline{n}|}, s_{\overline{n}|}$$

$$s_{\overline{n}|} = a_{\overline{n}|}(1+i)^n = \frac{(1+i)^n - 1}{i}$$



$$a_{\overline{n}|} = v + v^2 + v^3 + \cdots + v^n = \frac{1-v^n}{i}$$

$$\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d}$$

$$s_{\overline{n}|} = a_{\overline{n}|}(1+i)^n = \frac{(1+i)^n - 1}{i}$$

$$\ddot{s}_{\overline{n}|} = \ddot{a}_{\overline{n}|}(1+i)^n = \frac{(1+i)^n - 1}{d}$$

## 第三章 变额年金

### 3.1 符号一览

$(Ia)_{\overline{n}|}$ : 第一年支付 1 元

$(Ia)_{\overline{n}|}^{(m)}$ : 第一年支付 1 元, 以后每年支付增加 1 元, 每年支付  $m$  次

$(Ca)_{\overline{n}|}$ : 复递增年金

### 3.2 变额年金

#### 定义 3.1. 递增变额年金

$$(Ia)_{\overline{n}|} = v + 2v^2 + 3v^3 + \cdots + (n-1)v^{n-1} + nv^n$$



注  $(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$

$$(Is)_{\overline{n}|} = (1+i)^n (Ia)_{\overline{n}|}$$

### 3.3 复递增年金

$(Ca)_{\overline{n}|}$ : 复递增年金

#### 定义 3.2. 期末付复递增年金

$$(Ca)_{\overline{n}|} = \frac{(a)_{\overline{n}|}}{1+r} (r \neq i) \left( j = \frac{i-r}{1+r} \right)$$



注  $(Ca)_{\overline{n}|} = v + (1+r)v^2 + (1+r)^2v^3 + \cdots + (1+r)^{n-1}v^n$

## 第四章 收益率

### 4.1 收益率

### 4.2 基金的利息度量

#### 定义 4.1. Dollar-Weighted Return For a One-Year Period

Suppose the following information is known: (i) the balance in a fund at the start of the year is  $A$

(ii) for  $0 < t_1 < t_2 < \cdots < t_n < 1$ , the net deposit at time  $t_k$  is amount  $C_k$  (positive for a net deposit, negative for a net withdrawal), and

(iii) the balance in the fund at the end of the year is  $B$  Then the net amount of interest earned by the fund during the year is  $I = B - [A + \sum_{k=1}^n C_k]$ , and the dollar-weighted rate of return earned by the fund for the year is

$$\frac{I}{A + \sum_{k=1}^n C_k (1 - t_k)}$$



注  $(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$

#### 定义 4.2. Time-Weighted Return For a One-Year Period

Suppose the following information is known:

(i) the balance in a fund at the start of the year is  $A$

(ii) for  $0 < t_1 < t_2 < \cdots < t_n < 1$ , the net deposit at time  $t_k$  is amount  $C_k$  (positive for a net deposit, negative for a net withdrawal)

(iii) the value of the fund just before the net deposit at time  $t_k$  is  $F_k$ , and

(iv) the balance in the fund at the end of the year is  $B$  The time-weighted return rate earned by the fund for the year is

$$\left[ \frac{F_1}{A} \times \frac{F_2}{F_1 + C_1} \times \frac{F_3}{F_2 + C_2} \times \cdots \times \frac{F_k}{F_{k-1} + C_{k-1}} \times \frac{B}{F_k + C_k} \right] - 1$$



### 4.3 再投资

### 4.4 基金