航天飞行动力学
第三次作业 ——飞行方案设计

一、题目

1. 导弹参数：

* 导弹质量 $m_0 = 320kg$
* 发动机推力 $P = 2000N$
* 初始速度 $V_0 = 250m/s$
* 初始位置 $x_0 = 0m$
* 初始高度 $H_0 = 7000m$
* 初始弹道倾角 $\theta = 0°$
* 初始俯仰角 $\varphi_0 = 0°$
* 初始攻角 $\alpha_0 = 0°$
* 初始俯仰角速度 $\dot{\varphi}_0 = 0rad/\mathrm{s}$
* 初始速度 $V_0 = 250m/s$
* 参考长度 $S_{ref} = 0.45m^2$
* 参考面积 $L_{ref} = 2.5m$
* 升力系数 $C_y = 0.25\alpha + 0.05\delta_z$
* 阻力系数 $C_x = 0.2 + 0.005\alpha^2$
* 俯仰力矩系数 $m_z = -0.1\alpha + 0.024\delta_z$

2. 大气密度计算公式:

$$\begin{cases} \rho_0 = 1.2495\ kg/m^3 \\ T_0 = 288.15K \\ T = T_0 - 0.0065H \\ \rho = \rho_0 \left(\frac{T}{T_0}\right)^{4.25588} \end{cases} \tag{1}$$

3. 飞行方案：

(1) 当 $x < 9100m$ 时，采用瞬时平衡假设

$$\begin{cases} H^* = 2000 \times \cos(0.000314 \times 1.1 \times x) + 5000 \\ \delta_z = k_\varphi \times (H\ \text{-}H^*) + k_\varphi \times (H\ \text{-}H^*) \\ \delta_z = k_\varphi(H - H^*) + \dot{k}_\varphi H \\ m_s = 0.0kg/s \end{cases} \tag{2}$$

(2) 当 $24000m > x > 9100m$ 时，等高飞行方案，采用瞬时平衡假设。

$$\begin{cases} H^* = 3050m \\ \delta_z = k_\varphi(H - H^*) + \dot{k}_\varphi H \\ \delta_z = k_\varphi(H - H^*) + \dot{k}_\varphi H \\ m_s = 0.46kg/s \end{cases} \tag{3}$$

(3) 当 $x > 24000m\&\&y > 0$，目标位置为 $x_m = 30000m$, 采用比例导引法和瞬时平衡假设

$$\begin{cases} x_m = 30000m \\ m_z^\alpha \alpha + m_z^{\delta_z} \delta_z = 0 \\ m_s = 0.0kg/s \end{cases} \tag{4}$$

注：舵偏角约束 $|\delta_z| \leq 30°$

二、公式推导

1.$x < 24000m$ 的飞行方案：

基于“瞬时平衡”假设，将包含 20 个方程的导弹运动方程组简化为铅垂平面内的质心运动方程组。

$$\begin{cases} m\frac{\mathrm{d}V}{\mathrm{d}t} = P\cos\alpha_b - X_b - mg\sin\theta \\ mV\frac{\mathrm{d}\theta}{\mathrm{d}t} = P\sin\alpha_b + Y_b - mg\cos\theta \\ \frac{\mathrm{d}x}{\mathrm{d}t} = V\cos\theta \\ \frac{dy}{dt} = V\sin\theta \\ \frac{dm}{dt} = -m_s \\ \alpha_b = -\frac{m_z^{\delta_z}}{m_z^\alpha}\delta_{zb} \\ \delta_z = k_\varphi\left(H - H^*\right) + \dot{k}_\varphi\left(\dot{H} - \dot{H}^*\right) \\ H^* = 2000 \times \cos\left(0.000314 \times 1.1 \times x\right) + 5000 \end{cases} \tag{5}$$

2.$x > 24000m$ 的飞行方案：

(1) 末段第一种计算方法：

$$\begin{cases} r\frac{dq}{dt} = V_m \times \sin\eta - V_T\sin\eta_T \\ \tan q = \frac{y_T - y_m}{x_T - x_m} \\ \frac{d\theta^*}{dt} = k\frac{dq}{dt} \\ \theta^* - \theta_0 = k(q - q_0) \\ \theta_0, q_0? \\ \delta_z = k_\theta(\theta - \theta^*) + k_{\dot{\theta}}(\dot{\theta} - \dot{\theta}^*) \end{cases} \tag{6}$$

(2) 末段第二种计算方法：

只需要给出比例导引系数根据运动学方程

$$\begin{cases} r\frac{dq}{dt} = V_m \times \sin\eta \; - V_T \sin\eta_T \\ \tan q = \dfrac{y_T - y_m}{x_T - x_m} \\ \dfrac{dq}{dt} = \dfrac{-V_m \sin(\theta - q)}{r} \end{cases} \tag{7}$$

由比例导引法 $\dot{\theta}^* = k\dot{q}$, 可得动力学方程第二式

$$mV_m\dot{\theta}^* = P\sin\alpha + Y - mg\cos\theta \Rightarrow mV_mk\dot{q} = P\sin\alpha + Y - mg\cos\theta \tag{8}$$

由于攻角较小，进行线性化可得

$$mV_mk\dot{q} = P\alpha + Y^{\alpha}\alpha + Y^{\delta_z}\delta_z - mg\cos\theta \tag{9}$$

由于瞬时平衡 $m_z = 0$, 可得

$$-0.1\alpha + 0.024\delta_{\bar{z}} = 0 \Rightarrow \delta_{\bar{z}} = 0.1\alpha/0.024 \tag{10}$$

代入，可得

$$\alpha = \frac{mV_mk\dot{q} + mg\cos\theta}{P + Y^{\alpha} + Y^{\delta_z}(0.1/0.024)} \Rightarrow \frac{mV_mk\dot{q} + mg\cos\theta}{P + C_y^{\alpha}qL_{ref} + C_y^{\delta_z}qL_{ref}(0.1/0.024)} \tag{11}$$

最后得到弹道方程为

$$\begin{cases} \frac{dV}{dt} = \frac{P\cos\alpha - X}{m} - g\sin\theta \\ \alpha = \frac{mVk\dot{q} + mg\cos\theta}{P + C_y^{\alpha}qL_{ref} + C_y^{\delta_z}qL_{ref}(0.1/0.024)} \\ \frac{dx}{dt} = V\cos\theta \\ \frac{dy}{dt} = V\sin\theta \\ \dot{\theta}^* = k\dot{q} \\ \dot{\theta}^* = \dot{\theta} \\ \tan q = \frac{y_T - y_m}{x_T - x_m} \\ \frac{dq}{dt} = \frac{-V\sin(\theta - q)}{r} \\ \delta_z = 0.1\alpha/0.024 \end{cases} \tag{12}$$

推力 $\mathbf{P}$ 在弹体坐标系 $Ox_1y_1z_2$ 中的分量用 P_{x1}、P_{y1}、P_{z1} 表示，则根据题目信息可以得到

$$\begin{cases} \frac{P_{y1}}{P_{x1}} = \tan(-30°) \\ \frac{P_{y1}}{P_{z1}} = \tan 45° \\ P_{x1}^2 + P_{y1}^2 + P_{z1}^2 = P \end{cases} \tag{13}$$

带入 $\vec{P}=50000N$ 求得推力大小：

$$\begin{pmatrix}P_{x1}\\P_{y1}\\P_{z1}\end{pmatrix}=\begin{pmatrix}38730\\-22361\\-22361\end{pmatrix}\tag{14}$$

(1) 按照 $\psi\to\varphi\to\gamma$ 的顺序转换

从地面坐标系 $Axyz$ 转换为弹体坐标系 $Ox_1y_1z_2$ 的坐标变换矩阵为

$$L\left(\psi,\varphi,\gamma\right)=L_x\left(\gamma\right)L_z\left(\varphi\right)L_y\left(\psi\right)\tag{15}$$

其中：

$$L_y\left(\psi\right)=\begin{pmatrix}cos\psi&0&-sin\psi\\0&1&0\\sin\psi&0&cos\psi\end{pmatrix}\tag{16}$$

$$L_z\left(\varphi\right)=\begin{pmatrix}cos\varphi&sin\varphi&0\\-sin\varphi&cos\varphi&0\\0&0&1\end{pmatrix}\tag{17}$$

$$L_x\left(\gamma\right)=\begin{pmatrix}1&0&0\\0&cos\gamma&sin\gamma\\0&-sin\gamma&cos\gamma\end{pmatrix}\tag{18}$$

代入则有：

$$L\left(\psi,\varphi,\gamma\right)=\begin{pmatrix}cos\varphi cos\psi&\sin\varphi&-cos\varphi\sin\psi\\-\sin\varphi cos\psi cos\gamma+\sin\psi\sin\gamma&cos\varphi\cos\gamma&\sin\varphi\sin\psi\cos\gamma+cos\psi\sin\gamma\\\sin\varphi\cos\psi\sin\gamma+\sin\psi\cos\gamma&-cos\varphi\sin\gamma&-\sin\varphi\sin\psi\sin\gamma+cos\psi\cos\gamma\end{pmatrix}\tag{19}$$

同时，$L^{-1}\left(\psi,\varphi,\gamma\right)=L^T\left(\psi,\varphi,\gamma\right)$，所以

$$L^T\left(\psi,\varphi,\gamma\right)=\begin{pmatrix}\cos\varphi\cos\psi&-\sin\varphi\cos\psi\cos\gamma+\sin\psi\sin\gamma&\sin\varphi\cos\psi\sin\gamma+\sin\psi\cos\gamma\\\sin\varphi&\cos\varphi\cos\gamma&-\cos\varphi\sin\gamma\\-\cos\varphi\sin\psi&\sin\varphi\sin\psi\cos\gamma+\cos\psi\sin\gamma&-\sin\varphi\sin\psi\sin\gamma+\cos\psi\cos\gamma\end{pmatrix}\tag{20}$$

发动机推力矢量 $\vec{P}$ 在地面坐标系 $Axyz$ 的投影为

$$\begin{pmatrix}P_x\\P_y\\P_z\end{pmatrix}=L^T\left(\psi,\varphi,\gamma\right)\begin{pmatrix}P_{x1}\\P_{y1}\\P_{z1}\end{pmatrix}\tag{21}$$

(2) 按照 $\varphi\to\psi\to\gamma$ 的顺序转换

从地面坐标系 $Axyz$ 转换为弹体坐标系 $Ox_1y_1z_2$ 的坐标变换矩阵为

$$L(\varphi,\psi,\gamma) = L_x(\gamma)\,L_y(\psi)\,L_z(\varphi) \tag{22}$$

代入式 (4)、式 (5)、式 (6) 得到：

$$L(\varphi,\psi,\gamma) = \begin{pmatrix} \cos\varphi\cos\psi & \sin\varphi\cos\psi & -\sin\varphi \\ \sin\psi\cos\varphi\sin\gamma-\sin\varphi\cos\gamma & \sin\psi\sin\varphi\sin\gamma+\cos\varphi\cos\gamma & \cos\psi\sin\gamma \\ \sin\psi\cos\varphi\cos\gamma+\sin\varphi\cos\gamma & \sin\psi\sin\varphi\cos\gamma-\cos\varphi\sin\gamma & \cos\psi\cos\gamma \end{pmatrix} \tag{23}$$

同时，$L^{-1}(\psi,\varphi,\gamma) = L^T(\psi,\varphi,\gamma)$，所以

$$L^T(\psi,\varphi,\gamma) = \begin{pmatrix} \cos\varphi\cos\psi & -\sin\varphi\cos\psi\cos\gamma+\sin\psi\sin\gamma & \sin\varphi\cos\psi\sin\gamma+\sin\psi\cos\gamma \\ \sin\varphi & \cos\varphi\cos\gamma & -\cos\varphi\sin\gamma \\ -\cos\varphi\sin\psi & \sin\varphi\sin\psi\cos\gamma+\cos\psi\sin\gamma & -\sin\varphi\sin\psi\sin\gamma+\cos\psi\cos\gamma \end{pmatrix} \tag{24}$$

发动机推力矢量 $\vec{P}$ 在地面坐标系 $Axyz$ 的投影为

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = L^T(\psi,\varphi,\gamma)\begin{pmatrix} P_{x1} \\ P_{y1} \\ P_{z1} \end{pmatrix} \tag{25}$$

二、编程计算结果

(1) 按照 $\psi \to \varphi \to \gamma$ 的顺序转换计算结果

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = \begin{pmatrix} 2996 \\ -49152 \\ 8667 \end{pmatrix}(N) \tag{26}$$

(2) 按照 $\varphi \to \psi \to \gamma$ 的顺序转换

$$\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = L^T(\psi,\varphi,\gamma)\begin{pmatrix} 11180 \\ -47405 \\ -11305 \end{pmatrix}(N) \tag{27}$$

源代码 1: **main.py**

```python
import numpy as np
from matplotlib import pyplot as plt

# 展示高清图
from matplotlib_inline import backend_inline
backend_inline.set_matplotlib_formats('svg')

plt.rcParams['font.sans-serif'] = ['SimHei']
plt.rcParams['axes.unicode_minus'] = False

# 导弹参数
S_lef   =   0.45
L_ref   =   2.5

# 放大系数
K_phi = -0.6
K_phi_dot= 0.5 * K_phi
K_q = 3


# 仿真时间步
timestep = 0.001

# 导弹状态定义
class statu():
    __slot__=['Time','X','H','V','theta','mass','alpha','deltaz']

    # 位置
    # 速度
    # 欧拉角
    # 角加速度
    # 舵偏角

    # 初始化
    def __init__(self, Time, X=0, H=0, V=0, theta=0, mass=0):
        self.Time = Time
        self.X = X
        self.H = H
        self.V = V
        self.theta = theta
        self.mass = mass
```

```
        self.alpha = 0
        self.deltaz = 0

    # 显式Euler法，给定飞行高度
    def Euler(self, before, dmass):
        self.Time = before.Time + timestep

        self.X = before.X + before.V * np.cos(before.theta) * timestep
        self.H = before.H + before.V * np.sin(before.theta) * timestep

        self.deltaz = K_phi * (self.H - High_goal(self.X)) + K_phi_dot* (before.V * np.sin(before.theta) - High_goal_dot(self.X))

        if self.deltaz > 30:
            self.deltaz = 30
        elif self.deltaz < -30:
            self.deltaz = -30

        self.alpha =  0.24 *self.deltaz

        Y = (0.25 * self.alpha + 0.05* self.deltaz) * 0.5 * air(self.H) * before.V * before.V * S_lef

        X = (0.005 * self.alpha * self.alpha + 0.2) * 0.5 * air(self.H) * before.V * before.V * S_lef

        self.mass = before.mass - dmass * timestep
        if dmass == 0:
            P = 0
        else:
            P = 2000

        self.V = before.V + (P*np.cos(self.alpha*3.14159625/180) - X - self.mass*9.8*np.sin(before.theta)) /self.mass*timestep
        self.theta = before.theta + (P*np.sin(self.alpha*3.14159625/180) + Y - self.mass*9.8*np.cos(before.theta)) /self.mass/self.V*timestep

    # 比例导引法，给定目标位置
    def Euler2(self, before, Ym, Xm):
        self.Time = before.Time + timestep

        self.X = before.X + before.V * np.cos(before.theta) * timestep
        self.H = before.H + before.V * np.sin(before.theta) * timestep
```

```
        self.mass = before.mass
        r = np.sqrt((self.X - Xm)^2 + (self.H - Ym)^2)
        self.q = np.arctan((before.H - Ym)/(before.X - Xm))
        self.dq = -before.V * np.sin(before.theta - self.q)/ r * timestep

        self.theta = before.theta + K_q *self.dq * timestep

        P = 0

        self.alpha = (self.mass* before.V * K_q * self.dq + self.mass * 9.8 *np.cos(
            self.theta))/(P + )

        self.deltaz = self.alpha / 0.24

        if self.deltaz > 30:
            self.deltaz = 30
        if self.deltaz < -30:
            self.deltaz = -30


# 大气参数
def air (High):
    rho0 =1.2495
    T0  = 288.15
    Temp = T0 - 0.0065*High
    rho = rho0 * np.exp(4.25588*np.log(Temp / T0))
    return rho

# 飞行方案
def High_goal(X):
    if X <= 9100:
        return 2000 * np.cos(0.000314 * 1.1 * X) + 5000
    elif X <= 24000:
        return 3050
    else:
        return 0

def High_goal_dot(X):
    if X <= 9100:
        return -2000 * 0.000314 * np.sin(0.000314 * 1.1 * X)
    elif X <= 24000:
        return 0
    else:
```

```
        return 0


# 飞行初始状态
statu_n = [statu(0, 0, 7000, 250, 0, 320)]
statu_n[0].alpha = 0
statu_n[0].deltaz = 0

Time_goal = np.arange(0,200,timestep)
X_goal = np.arange(0,24000,10)
H_goal = [High_goal(i) for i in X_goal]
plt.plot(X_goal,H_goal, 'b--', alpha=0.5, linewidth=1, label='飞行方案高度')

while statu_n[-1].X < 9100:
    statu_n.append(statu(statu_n[-1].Time + timestep))
    statu_n[-1].Euler(statu_n[-2],0)
    #print(statu_n[-1].alpha)

while statu_n[-1].X <= 24000:
    statu_n.append(statu(statu_n[-1].Time + timestep))
    statu_n[-1].Euler(statu_n[-2],0.46)
    #print(statu_n[-1].alpha)

X_data = [n.X for n in statu_n]
H_data = [n.H for n in statu_n]
plt.plot(X_data,H_data, 'r-.', alpha=0.5, linewidth=1, label='实际飞行高度')

# 飞行高度绘图
plt.legend()  #显示上面的label
plt.xlabel('X') #x_label
plt.ylabel('H')#y_label
plt.ylim(0,8000)
plt.xlim(0,25100) #仅设置y轴坐标范围


plt.figure(2)
T_data = [n.Time for n in statu_n]
deltaz_data = [n.deltaz for n in statu_n]
plt.plot(T_data,deltaz_data, 'r-.', alpha=0.5, linewidth=1, label='舵偏角$\delta z$')
plt.legend()  #显示上面的label
plt.xlabel('Time(s)') #x_label
plt.ylabel('$\delta z$')#y_label
plt.ylim(-50,50)
```

```
plt.xlim(0,200)

"""
plt.figure(3)
M_data = [n.mass for n in statu_n]
plt.plot(T_data,M_data, 'r-.', alpha=0.5, linewidth=1, label='实际飞行速度V')
plt.legend()  #显示上面的label
plt.xlabel('Time(s)') #x_label
plt.ylabel('速度V')#y_label
plt.ylim(250,350)
plt.xlim(0,200)
"""
plt.figure(4)
V_data = [n.V for n in statu_n]
plt.plot(T_data,V_data, 'r-.', alpha=0.5, linewidth=1, label='实际飞行速度V')
plt.legend()  #显示上面的label
plt.xlabel('Time(s)') #x_label
plt.ylabel('速度V')#y_label
plt.ylim(100,250)
plt.xlim(0,200)

plt.show()
```

源代码 2: **work.tex**

```
\documentclass[UTF8]{ctexart}
\newcommand{\mycmdB}[1]{{\heiti #1}}
\renewcommand{\normalsize}{\fontsize{12}{12}\fangsong}
\usepackage{listings}
\usepackage{xcolor}
\usepackage{amsmath}
\usepackage{graphicx}
\usepackage{float}
\usepackage{indentfirst}
\usepackage{longtable}
\usepackage{fancyhdr}
\usepackage[a4paper, left=2.5cm, right=2.5cm, top=3cm, bottom=2cm]{geometry}
\usepackage{matlab-prettifier}
\usepackage{latexcolors}

\setlength{\parindent}{2em} %2em代表首行缩进2个字符

% 页眉页脚设置
\pagestyle{fancy}
\fancyhf{}
\lhead{2021300045}
\chead{李宗霖}
\rhead{第\thepage 页}

% 去除图注冒号
\usepackage{caption}
\captionsetup[table]{labelsep=space} % 表
\captionsetup[figure]{labelsep=space} % 图

\CTEXsetup[format={\Large\bfseries}]{section}

% 源代码引用
\renewcommand{\lstlistingname}{源代码}
\lstset{
    basicstyle          =   \zihao{5} \ttfamily,          % 基本代码风格
    keywordstyle        =   \bfseries,          % 关键字风格
    commentstyle        =   \ttfamily\itshape,  % 注释的风格，斜体
    stringstyle         =   \ttfamily,  % 字符串风格
    flexiblecolumns,                % 别问为什么，加上这个
    numbers             =   left,   % 行号的位置在左边
    showspaces          =   false,  % 是否显示空格，显示了有点
```

```
乱，所以不现实了
    numberstyle          =   \zihao{5}\ttfamily,    % 行号的样式，小五号，tt等宽字体
    showstringspaces     =   false,
    captionpos           =   t,      % 这段代码的名字所呈现的位置，t指的是top上面，b指下面
    frame                =   lrtb %lrtb,   % 显示边框
}
\lstset{
        language             = matlab,
    basicstyle          = \zihao{5}\ttfamily,          % 基本代码风格
        rulesepcolor         = \color{gray}, % 代码块边框颜色
        breaklines           = true, % 代码过长则换行
        numbers              = left, % 行号在左侧显示
        numberstyle          = \zihao{5}\ttfamily,  % 行号字体
        showspaces           = false, % 不显示空格
        columns              = fixed, % 字间距固定
        %morekeywords         = {as}, % 自加新的关键字(必须前后都是空格)
        %deletendkeywords     = {compile} % 删除内定关键字；删除错误标记的关键字用deletekeywords删！
}

\lstdefinestyle{Python}{
    language        =   Python, % 语言选Python
    basicstyle      =   \zihao{5}\ttfamily,
    numberstyle     =   \zihao{5}\ttfamily,
    keywordstyle    =   \color{blue},
    keywordstyle    =   [2] \color{teal},
    stringstyle     =   \color{magenta},
    commentstyle    =   \color{red}\ttfamily,
    breaklines      =   true,   % 自动换行，建议不要写太长的行
    columns         =   fixed,  % 如果不加这一句，字间距就不固定，很丑，必须加
    basewidth       =   0.5em,
}

\begin{document}

\begin{center}
    {\zihao{-2} \bf 航天飞行动力学}\\
    {\zihao{3} 第三次作业\ ——\ 飞行方案设计}

\end{center}
```

```

\section*{\zihao{-4} 一、题目}
\noindent {\heiti 1.导弹参数：}

\begin{itemize}
    \item[*] 导弹质量$m_0=320kg$
    \item[*] 发动机推力$P=2000N$
    \item[*] 初始速度$V_0=250m/s$
    \item[*] 初始位置$x_0=0m$
    \item[*] 初始高度$H_0=7000m$
    \item[*] 初始弹道倾角$\theta=0^{\circ}$
    \item[*] 初始俯仰角 $\varphi_0=0^\circ$
    \item[*] 初始攻角 $\alpha_0=0^\circ$
    \item[*] 初始俯仰角速度$\dot{\varphi}_0=0rad$/s
    \item[*] 初始速度$V_0=250m/s$
    \item[*] 参考长度$S_{ref}=0.45 m^{2}$
    \item[*] 参考面积$L_{ref}=2.5m$
    \item[*] 升力系数$C_{y}=0.25\alpha+0.05\delta_{z}$
    \item[*] 阻力系数$C_x=0.2+0.005\alpha^2$
    \item[*] 俯仰力矩系数$m_z=-0.1\alpha+0.024\delta_z$
\end{itemize}

\noindent {\heiti 2.大气密度计算公式:}
\begin{align}
    \begin{cases}
        \rho_0=1.2495\ kg/m^3 \\
        T_0=288.15   K        \\
        T=T_0-0.0065H         \\
        \rho=\rho_{0}\left( \frac{T}{T_{0}} \right)^{4.25588}
    \end{cases}
\end{align}

\noindent {\heiti 3.飞行方案：}

\begin{itemize}
    \item[(1)] 当$x<9100m$时，采用瞬时平衡假设
        \begin{align}
            \begin{cases}
                H^*=2000\times\cos(0.000314\times1.1\times x)+5000                     \\
                \delta_z=k_\varphi\times(H\text{ -}H^*)+k_\varphi\times(H\text{ -}H^*) \\
                \delta_z=k_\varphi(H-H^*)+\dot{k}_\varphi H                        \\
                m_{s}=0.0kg/s
```

```
            \end{cases}
        \end{align}
    \item[(2)] 当$24000m>x>9100m$时，等高飞行方案，采用瞬时平衡假设。
        \begin{align}
            \begin{cases}
                H^*=3050m                                    \\
                \delta_z=k_\varphi(H-H^*)+\dot{k}_\varphi H \\
                \delta_z=k_\varphi(H-H^*)+\dot{k}_\varphi H \\
                m_s=0.46kg/s
            \end{cases}
        \end{align}
    \item[(3)] 当$x>24000m\&\&y>0$，目标位置为$x_m=30000m$,采用比例导引法和瞬时平衡假设
        \begin{align}
            \begin{cases}
                x_m=30000 m                                 \\
                m_z^\alpha\alpha+m_z^{\delta_z}\delta_z=0 \\
                m_s=0.0kg/s
            \end{cases}
        \end{align}
\end{itemize}

注：舵偏角约束$\left|\delta_{z} \right|\leq 30^{\circ}$




\section*{\zihao{-4} 二、公式推导}


\noindent {\heiti 1.$x<24000m$的飞行方案：}

基于“瞬时平衡”假设，将包含20个方程的导弹运动方程组简化为铅垂平面内的质心运动方程组。

\begin{align}
    \begin{cases}
        m\frac{\mathrm{d}V}{\mathrm{d}t}=P\cos\alpha _{b}-X_{b}-mg\sin\theta\hfill                  \\
        mV\frac{\mathrm{d}\theta}{\mathrm{d}t}=P\sin\alpha _{b}+Y_{b}-mg\cos\theta\hfill            \\
        \frac{\mathrm{d}x}{\mathrm{d}t}=V\cos\theta \hfill                                          \\
```

```
        \frac{dy}{dt}=V\sin\theta \hfill                                    \\
        \frac{dm}{dt}=-m_{s} \hfill                                         \\
        \alpha_b=-\frac{m_{z}^{\delta_z}}{m_{z}^{\alpha}}\delta_{zb} \hfill              \\
        \delta_z = k_\varphi \left(H-H^{*}\right) +\dot{k}_{\varphi} \left(\dot{H} -\dot{H}^{*}\right) \hfill \\
        H^{*}=2000\times \cos\left(0.000314\times1.1\times x\right)+5000\hfill
    \end{cases}
\end{align}



\noindent {\heiti 2.$x>24000m$的飞行方案：}

\noindent {(1) 末段第一种计算方法：}


\begin{align}
    \begin{cases}
        r\frac{dq}{dt}=V_{m}\times\sin\eta-V_{T}\sin\eta_{T} \\
        \tan q=\frac{y_{T}-y_{m}}{x_{T}-x_{m}}               \\
        \frac{d\theta^*}{dt}=k\frac{dq}{dt}                  \\
        \theta^{*}-\theta_{0}=k(q-q_{0})                     \\
        \theta_{0},q_{0}?                                    \\
        \delta_{z}=k_{\theta}(\theta-\theta^{*})+k_{\dot{\theta}}(\dot{\theta}-\dot{\theta}^{*})
    \end{cases}
\end{align}

\noindent {(2) 末段第二种计算方法：}

只需要给出比例导引系数
根据运动学方程

\begin{align}
    \begin{cases}
        r\frac{dq}{dt}=V_{m}\times\sin\eta\:-V_{T}\sin\eta_{T}                        \\
        \begin{aligned}\tan q&=\frac{y_T-y_m}{x_T-x_m}\\
```

```
        \frac{dq}{dt}&=\frac{-V_m\sin(\theta-q)}r\end{aligned}
    \end{cases}
\end{align}

由比例导引法$\dot{\theta}^*=k\dot{q}$,可得动力学方程第二式
\begin{align}
    mV_m\dot{\theta}^*=P\sin\alpha+Y-mg\cos\theta\Rightarrow
       mV_mk\dot{q}=P\sin\alpha+Y-mg\cos\theta
\end{align}

由于攻角较小，进行线性化可得
\begin{align}
    mV_{m}k\dot{q}=P\alpha+Y^{\alpha}\alpha+Y^{\delta_{z}}\
       delta_{z}-mg\cos\theta
\end{align}

由于瞬时平衡$m_z=0$，可得
\begin{align}
    -0.1\alpha+0.024\delta_{\tilde{z}}=0\Rightarrow\delta_{\
       tilde{z}}=0.1\alpha/0.024
\end{align}

代入，可得
\begin{align}
    \alpha=\frac{mV_{m}k\dot{q}+mg\cos\theta}{P+Y^{\alpha}+Y^{\
       delta_{z}}(0.1/0.024)}\Rightarrow\frac{mV_{m}k\dot{q}+mg
       \cos\theta}{P+C_{y}^{\alpha}qL_{ref}+C_{y}^{\delta_{z}}
       qL_{ref}(0.1/0.024)}
\end{align}


最后得到弹道方程为

\begin{align}
    \begin{cases}
        \frac{dV}{dt}=\frac{P\cos\alpha-X}{m}-g\sin\theta  \\
        \alpha=\frac{mVk\dot{q}+mg\cos\theta}{P+C_{y}^{\alpha}
           qL_{ref}+C_{y}^{\delta_{z}}qL_{ref}(0.1/0.024)} \\
        \frac{dx}{dt}=V\cos\theta  \\
        {\frac{dy}{dt}}=V\sin\theta  \\
        \dot{\theta}^{*}=k\dot{q} \\
        \dot{\theta}^{*}=\dot{\theta} \\
        \tan q=\frac{y_{T}-y_{m}}{x_{T}-x_{m}} \\
        \frac{dq}{dt}=\frac{-V\sin(\theta-q)}{r} \\
        \delta_{z}=0.1\alpha/0.024
```

```latex
    \end{cases}
\end{align}

推力{\bf{P}}在弹体坐标系$Ox_{1}y_{1}z_{2}$中的分量用$P_{x1}$、$P_{y1}$、$P_{z1}$表示，则根据题目信息可以得到
\begin{align}
    \begin{cases}
        \frac{P_{y1}}{P_{x1}}=\tan{\left( - 30^\circ\right)} \\
        \frac{P_{y1}}{P_{z1}}=\tan{45^\circ}
        \\
        P_{x1}^2 + P_{y1}^2 + P_{z1}^2 = P
    \end{cases}
\end{align}

带入$\vec{P}=50000N$求得推力大小:
\begin{align}
    \begin{pmatrix}
        P_{x1} \\
        P_{y1} \\
        P_{z1}
    \end{pmatrix}
    =
    \begin{pmatrix}
        38730  \\
        -22361 \\
        -22361
    \end{pmatrix}
\end{align}

\noindent {\bf (1) 按照$\psi \rightarrow \varphi \rightarrow \gamma$的顺序转换}

从地面坐标系$Axyz$转换为弹体坐标系$Ox_{1}y_{1}z_{2}$的坐标变换矩阵为
\begin{align}
    L \left(\psi , \varphi , \gamma\right)
    =
    L_{x} \left(\gamma\right)
    L_{z} \left(\varphi\right)
    L_{y} \left(\psi\right)
\end{align}
其中:
\begin{align}
    L_{y} \left(\psi\right)
    =
```

```
    \begin{pmatrix}
        cos\psi & 0 & -sin\psi
        \\
        0       & 1 & 0
        \\
        sin\psi & 0 & cos\psi
    \end{pmatrix}
\end{align}

\begin{align}
    L_{z} \left(\varphi \right)
    =
    \begin{pmatrix}
        cos\varphi  & sin\varphi & 0
        \\
        -sin\varphi & cos\varphi & 0
        \\
        0           & 0          & 1
    \end{pmatrix}
\end{align}

\begin{align}
    L_{x} \left(\gamma\right)
    =
    \begin{pmatrix}
        1 & 0          & 0
        \\
        0 & cos\gamma  & sin\gamma
        \\
        0 & -sin\gamma & cos\gamma
    \end{pmatrix}
\end{align}

代入则有:
\begin{align}
    L \left(\psi , \varphi , \gamma\right)
    =
    \begin{pmatrix}
        cos\varphi cos\psi                               & \sin\varphi           & -cos\varphi\sin\psi
        \\
        -\sin\varphi cos\psi cos\gamma+\sin\psi\sin\gamma & cos\varphi\cos\gamma  & \sin\varphi\sin\psi\cos\gamma+cos\psi\sin\gamma
        \\
```

```
        \sin\varphi\cos\psi\sin\gamma+\sin\psi\cos\gamma  & -cos\varphi\sin\gamma & -\sin\varphi\sin\psi\sin\gamma+cos\psi\cos\gamma
    \end{pmatrix}
\end{align}

同时，$L^{-1} \left(\psi , \varphi , \gamma\right) = L^{T} \left(\psi , \varphi , \gamma\right)$，所以

\begin{align}
    L^{T} \left(\psi , \varphi , \gamma\right)
    =
    \begin{pmatrix}
        \cos\varphi\cos\psi  & -\sin\varphi\cos\psi\cos\gamma+\sin\psi\sin\gamma & \sin\varphi\cos\psi\sin\gamma+\sin\psi\cos\gamma
        \\
        \sin\varphi          & \cos\varphi\cos\gamma                         & -\cos\varphi\sin\gamma
        \\
        -\cos\varphi\sin\psi & \sin\varphi\sin\psi\cos\gamma+\cos\psi\sin\gamma  & -\sin\varphi\sin\psi\sin\gamma+\cos\psi\cos\gamma
    \end{pmatrix}
\end{align}

发动机推力矢量$\vec{P}$在地面坐标系$Axyz$的投影为

\begin{align}
    \begin{pmatrix}
        P_{x} \\
        P_{y} \\
        P_{z}
    \end{pmatrix}
    =
    L^{T} \left(\psi , \varphi , \gamma\right)
    \begin{pmatrix}
        P_{x1} \\
        P_{y1} \\
        P_{z1}
    \end{pmatrix}
\end{align}

\noindent {\bf (2) 按照$\varphi \rightarrow \psi \rightarrow \gamma$的顺序转换}
```

```
从地面坐标系$Axyz$转换为弹体坐标系$Ox_{1}y_{1}z_{2}$的坐标变换矩阵为
\begin{align}
    L \left(\varphi , \psi , \gamma\right)
    =
    L_{x} \left(\gamma\right)
    L_{y} \left(\psi\right)
    L_{z} \left(\varphi\right)
\end{align}

代入式$\left(4\right)$、式$\left(5\right)$、式$\left(6\right)$得到：
\begin{align}
    L \left(\varphi , \psi , \gamma\right)
    =
    \begin{pmatrix}
        \cos\varphi\cos\psi                            & \sin\varphi\cos\psi                            & -\sin\varphi
        \\
        \sin\psi\cos\varphi\sin\gamma-\sin\varphi\cos\gamma & \sin\psi\sin\varphi\sin\gamma+\cos\varphi\cos\gamma & \cos\psi\sin\gamma
        \\
        \sin\psi\cos\varphi\cos\gamma+\sin\varphi\cos\gamma & \sin\psi\sin\varphi\cos\gamma-\cos\varphi\sin\gamma & \cos\psi\cos\gamma
    \end{pmatrix}
\end{align}

同时，$L^{-1} \left(\psi , \varphi , \gamma\right) = L^{T} \left(\psi , \varphi , \gamma\right)$，所以

\begin{align}
    L^{T} \left(\psi , \varphi , \gamma\right)
    =
    \begin{pmatrix}
        \cos\varphi\cos\psi  & -\sin\varphi\cos\psi\cos\gamma+\sin\psi\sin\gamma & \sin\varphi\cos\psi\sin\gamma+\sin\psi\cos\gamma
        \\
        \sin\varphi          & \cos\varphi\cos\gamma                          & -\cos\varphi\sin\gamma
        \\
```

```
        -\cos\varphi\sin\psi & \sin\varphi\sin\psi\cos\gamma+\
           cos\psi\sin\gamma  & -\sin\varphi\sin\psi\sin\gamma
           +\cos\psi\cos\gamma
    \end{pmatrix}
\end{align}

发动机推力矢量$\vec{P}$在地面坐标系$Axyz$的投影为

\begin{align}
    \begin{pmatrix}
        P_{x} \\
        P_{y} \\
        P_{z}
    \end{pmatrix}
    =
    L^{T} \left(\psi , \varphi , \gamma\right)
    \begin{pmatrix}
        P_{x1} \\
        P_{y1} \\
        P_{z1}
    \end{pmatrix}
\end{align}

\section*{\zihao{-4} 二、编程计算结果}

\noindent {\bf (1) 按照$\psi \rightarrow \varphi \rightarrow \
   gamma$的顺序转换计算结果}
\begin{align}
    \begin{pmatrix}
        P_{x} \\
        P_{y} \\
        P_{z}
    \end{pmatrix}
    =
    \begin{pmatrix}
        2996   \\
        -49152 \\
        8667
    \end{pmatrix}
    \left(N\right)
\end{align}
\noindent {\bf (2) 按照$\varphi \rightarrow \psi \rightarrow \
   gamma$的顺序转换}
\begin{align}
    \begin{pmatrix}
```

```
        P_{x} \\
        P_{y} \\
        P_{z}
    \end{pmatrix}
    =
    L^{T} \left(\psi , \varphi , \gamma\right)
    \begin{pmatrix}
        11180  \\
        -47405 \\
        -11305
    \end{pmatrix}
    \left(N\right)
\end{align}


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