



# American Options Hedging on AAPL

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FE620 Pricing and Hedging

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## Chapter I Problem setting and choice of underlying

For this project, a model is built to price three-month (3M) American options on AAPL. Apple Inc. (AAPL) stock was chosen due to the company's unique position in the technology sector and the group's interest in the company's offerings. The Black-Scholes model was implemented with binomial trees in R over the duration of January 05, 2022, to April 04, 2022.

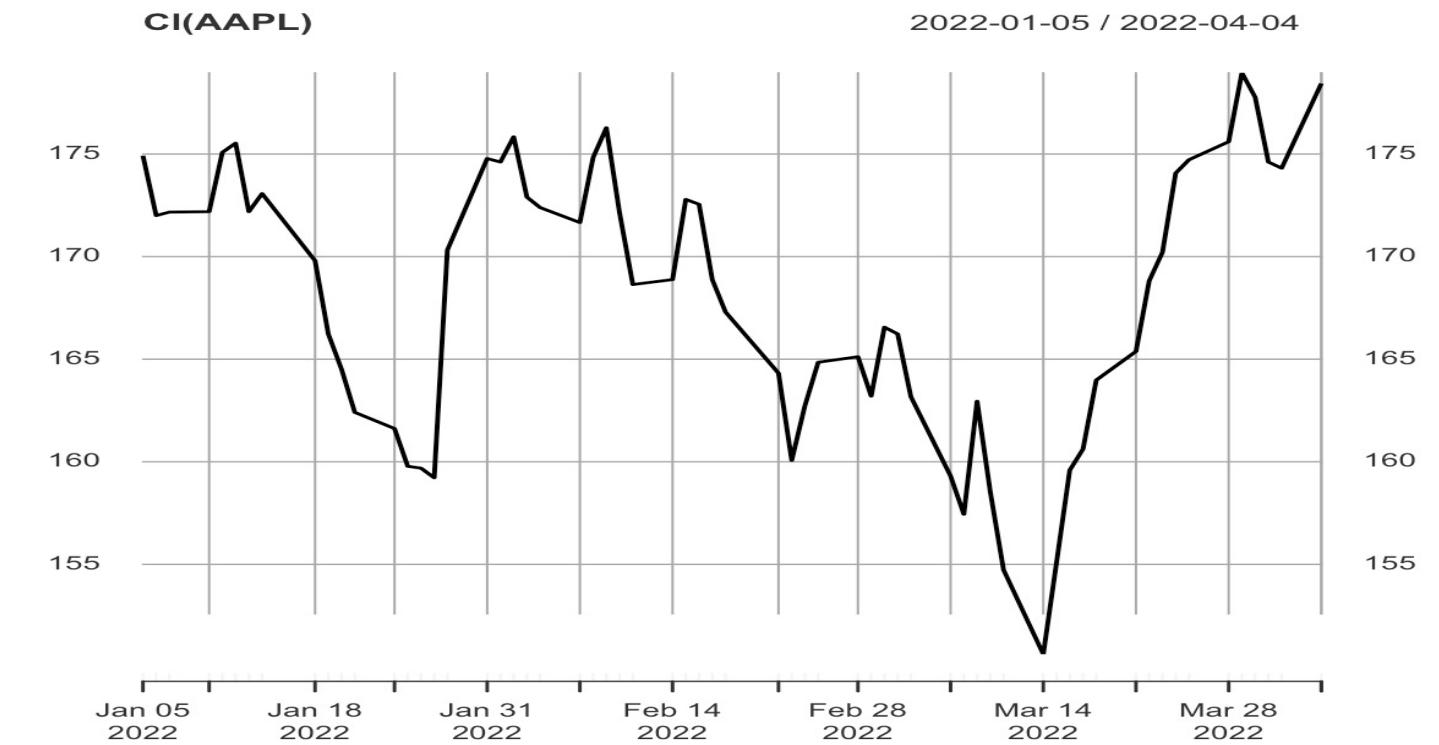
Pricing Black-Scholes model needed spot price, volatility, and risk-free rate.

The stock's spot price,  $S_0$ , was taken as the closing price on the first day of observation was found to be \$178.44.

Volatility is an estimate of the future variability for the asset underlying the options contract.

Risk-free rate use for the risk-free interest rate at the time of expiration for the option. The 3M Treasury Constant Maturity Rate on 2022-04-04 is found to be  $r=0.68\%$ .

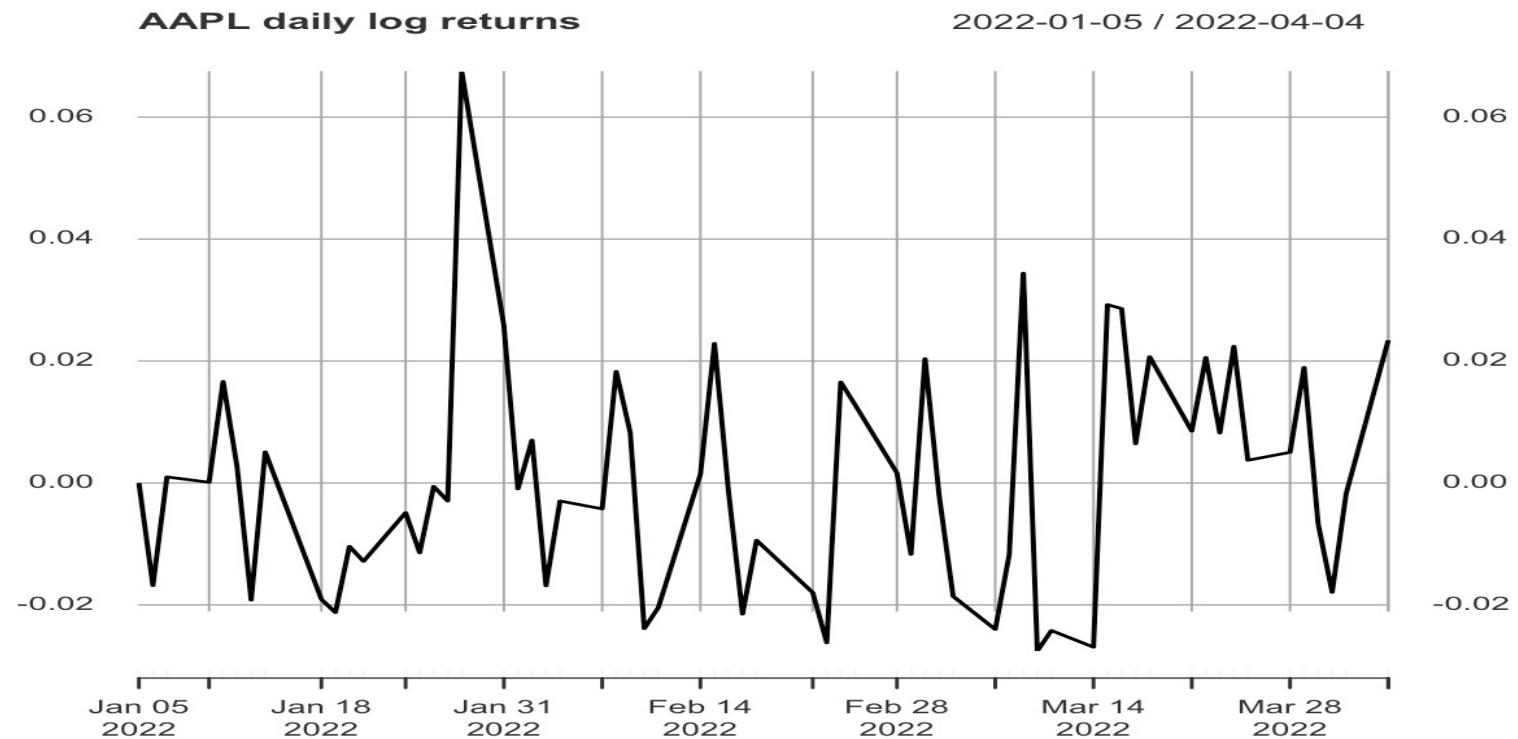
## Chapter II Market Data Analysis



**Figure 1** the daily closing prices of AAPL



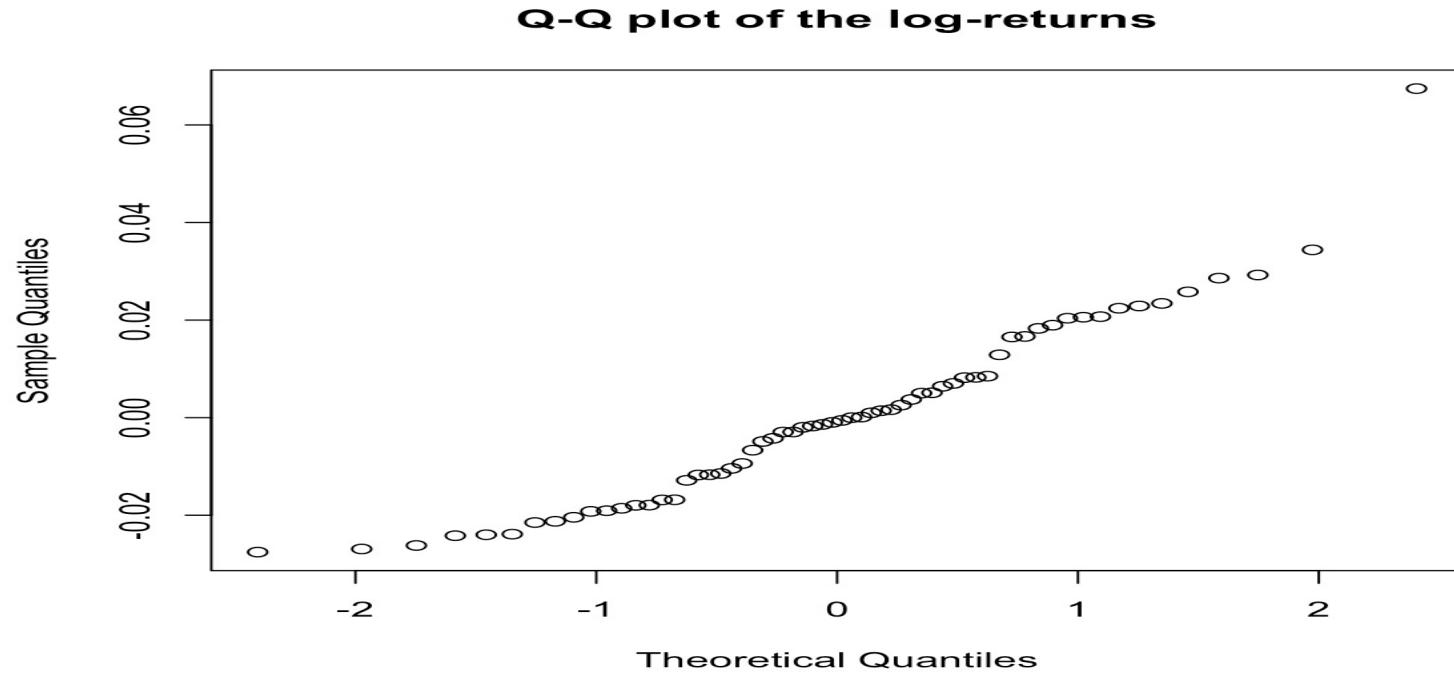
## Chapter II Market Data Analysis



**Figure 2 the daily log-returns**

## Chapter II Market Data Analysis

We observed a deviation of the tails from lognormality.



**Figure 3 the QQ plot of the log-returns**

## Chapter II Market Data Analysis

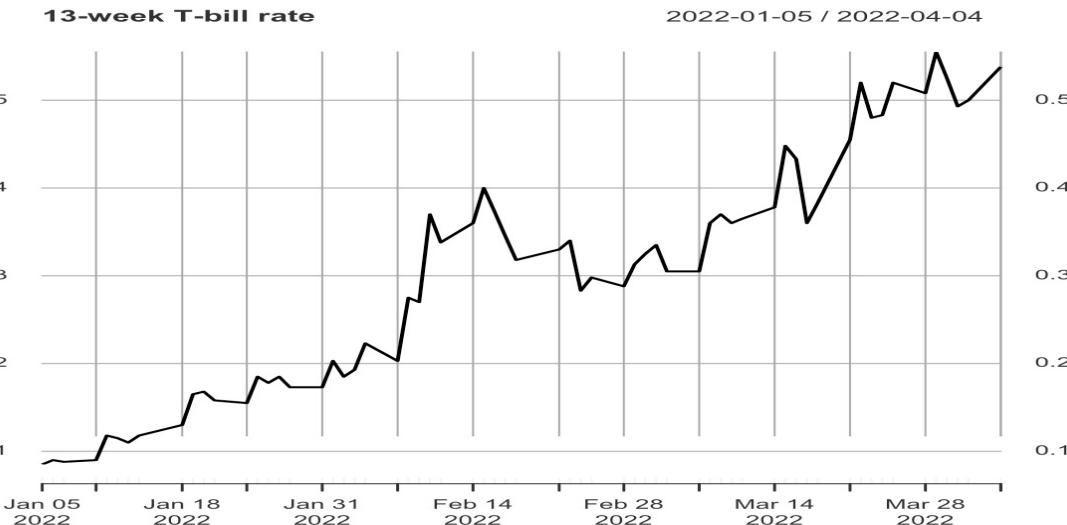
The volatility of the log returns is calculated by finding their standard deviation and annualized by multiplying the value by 252 (square root of the average number of trading days per year)

	<b>Volatility(%)</b>	<b>Standard error</b>
<b>1Y</b>	<b>24.10%</b>	<b>1.52%</b>
<b>9M</b>	<b>22.00%</b>	<b>1.39%</b>
<b>6M</b>	<b>20.96%</b>	<b>1.32%</b>
<b>3M</b>	<b>21.04%</b>	<b>1.33%</b>

**Figure 4: Volatility and Error**



## Chapter II Market Data Analysis



The 3M Treasury Bill Fixed Maturity Rate was chosen to represent the calculated interest rate. The 3-month Treasury bill rate on April 4, 2022, is found to be  $r = 0.68\%$



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# Chapter III American Options on AAPL



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Note: Given surface is more than 24hrs older than requested

APPLE INC 175.06 USD Bloomberg As of < 05-Apr-2022 > 16:15

1) Vol Table 2) 3D Surface 3) Term 4) Skew 5) Dividends 6) Prices

From 8 Apr 2022

Expiry	8 Apr 2022				14 Apr 2022				22 Apr 2022			
	Call		Put		Call		Put		Call		Put	
Put/Call	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask
157.5	16.45	18.9	0.01	0.03	17.2	18.4	0.17	0.2	17.15	18.65	0.33	0.43
160	14.75	15.45	0.02	0.03	15.15	16.25	0.24	0.26	14.95	15.8	0.48	0.53
162.5	11.95	13.2	0.04	0.06	12.3	13.3	0.33	0.38	12.8	13.8	0.67	0.72
165	9.8	10.8	0.09	0.1	10.4	10.95	0.48	0.52	10.4	11.5	0.93	0.98
167.5	6.55	9	0.16	0.18	7.9	8.65	0.73	0.8	8.65	9.35	1.29	1.36
170	5.3	5.65	0.36	0.4	6.2	6.4	1.15	1.2	6.7	7.3	1.81	1.88
172.5	3.3	3.5	0.8	0.85	4.3	4.5	1.74	1.83	5.05	5.3	2.52	2.62
175	1.72	1.78	1.65	1.71	2.75	2.83	2.68	2.75	3.55	3.75	3.45	3.55
177.5	0.71	0.75	3	3.25	1.61	1.66	3.95	4.1	2.42	2.45	4.7	4.85
180	0.25	0.27	4.75	5.45	0.86	0.88	5.65	5.85	1.47	1.52	5.65	7
182.5	0.09	0.1	6.95	8.35	0.43	0.46	7.65	8.2	0.89	0.96	7.7	8.55
185	0.04	0.05	9.45	10.9	0.22	0.25	9.5	10.9	0.51	0.58	9.85	10.85
187.5	0.02	0.03	11.95	13.45	0.13	0.14	11.9	13.15	0.3	0.36	11.35	13.8
190	0.01	14.1	16	0.08	0.09	14.45	15.9	0.2	0.22	14.25	16.25	
192.5	0.01	15.35	19.8	0.05	0.09	15.4	19.9	0.12	0.26	15.55	19.95	
195	0.01	18.75	21.4	0.04	0.08	19.25	21.05	0.08	0.11	18.95	21.2	
197.5	0.01	20.35	24.85	0.03	0.07	21.25	23.6	0.07	0.09	20.4	24.95	
200	0.01	24.15	26.1	0.02	0.04	23.6	26.15	0.05	0.08	23.95	26.15	
202.5	0.01	25.4	29.9									
205	0.01	28.8	31.25	0.01	0.03	29.15	31.75		0.06	28.9	31.15	
210	0.01	33.6	36		0.02	33.7	36.05	0.01	0.05	33.6	36.8	
215	0.01	38	42.35		0.02	39.2	41.15		0.05	39.1	41.85	
220	0.01	43	47.35		0.01	44.15	46.15		0.03	43.75	47.25	
225	0.01	47.85	52.35		0.01	49.2	51.1		0.04	48.1	51.95	
230	0.01	53	57.15		0.01	54.15	56.05		0.04	53	57.35	

Several options are traded on AAPL. As of 4-Apr-2022,

the first expiries are 8-Apr-2022, 14-Apr-2022, 22-Apr-

2022. These dates are the Fridays of each week.

The prices of these options for several strikes are

shown in the left, taken from Bloomberg.

We would like to price these American options in the

Black-Scholes model using a binomial tree approach.

# Basic Concepts for Pricing American Options

- *In each small interval of time the stock price is assumed to move up by a proportional amount  $u$  or to move down by a proportional amount  $d$*
- *We work back through the tree using risk-neutral valuation to calculate the value of the option at each node, testing for early exercise when appropriate*

# Difference between European options and American options(Calls) in R code

*European:*

```
option_tree[i, j] = ((1-q)*option_tree[i+1,j] + q*option_tree[i+1,j+1])/exp(r*delta_t)
```

*American:*

```
exercise.payoff<-if (type == "call") max(tree[i,j] -K, 0) else max(K -tree[i,j], 0)  
hold.payoff <-(q*option_tree[i+1,j+1] + (1-q)*option_tree[i+1,j])/exp(r*delta_t)  
option_tree[i,j] <-max(exercise.payoff, hold.payoff)
```

## Concerns

*Consider the put option with expiry 14-Apr-2022 and strike  $K = 177.5$  which is closest to the at-the-money point  $K = S_0$ . The binomial tree price with  $n = 9$  time steps (daily time steps) gives*  
 $P(K = 177.5; n = 9) = 3.467476$

How accurate is this?

*We recall that American call options on a stock which does not pay dividends have the same price as the European call options with the same maturity.*

The table below shows the binomial tree price of the American call option with strike  $K = 177.5$  for several values of  $n$ , the time steps of the tree. We observe that as  $n$  increases, the binomial tree prices approach the European option price, as expected.

n	9	30	50	80	100	BS
$C(K=177.5)$	4.72	4.644	4.642	4.638	4.636	4.642



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APPLE INC Equity | OVDV | Related Functions Menu

Click a volatility point for details

AAPL US Equity 90) Asset ▾ 91) Actions ▾ 92) Views ▾ 93) Settings ▾

APPLE INC 178.44 USD Bloomberg As of 04-Apr-2022 16:15

1) Vol Table 2) 3D Surface 3) Term 4) Skew 5) Dividends 6) Prices From 8 Apr 2022

Expiry	8 Apr 2022				14 Apr 2022				22 Apr 2022					
	Put/Call	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put	Call	Put	
Strike	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask	Bid	Ask
160	17.85	18.65	0.04	0.05	18.35	18.8	0.16	0.17	18.1	19	0.3	0.31		
162.5	15.4	17.05	0.05	0.07	14.9	17.4	0.19	0.21	16	17.1	0.39	0.4		
165	12.35	13.6	0.07	0.09	13.55	13.8	0.28	0.29	13.75	14.25	0.51	0.55		
167.5	10.65	11.55	0.11	0.12	10.9	11.45	0.4	0.42	11.45	12.3	0.74	0.76		
170	8.55	8.75	0.18	0.19	8.8	9.2	0.6	0.62	8.9	10	1.02	1.09		
172.5	6.2	6.35	0.34	0.35	6.8	6.95	0.91	0.95	6.4	7.6	1.46	1.53		
175	4.1	4.2	0.68	0.69	4.85	4.95	1.4	1.46	5.55	5.7	2.1	2.17		
177.5	2.31	2.36	1.38	1.41	3.2	3.25	2.21	2.27	3.9	4.1	2.94	3.05		
180	1.1	1.11	2.6	2.65	1.89	1.92	3.4	3.5	2.66	2.68	4.1	4.2		
182.5	0.42	0.44	4.35	4.6	1.04	1.07	5	5.15	1.65	1.73	5.6	5.8		
185	0.15	0.16	5.35	6.95	0.55	0.56	6.95	7.6	1.03	1.05	6.65	8.55		
187.5	0.05	0.06	8.3	9.35	0.28	0.3	9.15	9.55	0.62	0.63	9.05	10.8		
190	0.02	0.03	10.3	12.85	0.16	0.18	10.9	12.55	0.37	0.4	10.95	12.55		
192.5	0.01	0.02	12.75	15.7	0.1	0.12	12.8	16.05	0.22	0.26	12.9	16		
195	0.01	15.7	17.2	0.05	0.08	15.8	17.45	0.16	0.18	16	17.85			
197.5	0.02	17.75	21.5	0.04	0.06	17.95	20.3	0.11	0.13	17.8	21.25			
200	0.01	20.6	22.6	0.03	0.04	20.8	21.85	0.09	0.1	20.4	23.1			
202.5	0.01	22.75	26.5											
205	0.01	25.4	27.45	0.01	0.05	25.65	27.65	0.06	0.07	25.7	27.25			
210	0.01	30.3	32.5	0.01	0.02	30.6	32.55	0.03	0.05	30.65	33.05			
215	0.01	35.85	37.3	0.01	35.65	37.85	0.01	0.08	35.25	38.75				
220	0.01	40.25	42.45	0.02	40.85	42.4	0.03	0.05	40.25	43.75				
225	0.01	45.25	47.5	0.01	46.05	47.5	0.04	45.25	48.75					
230	0.01	50.25	52.5	0.01	50.8	52.35	0.01	0.04	50.25	53.45				
235	0.01	55.25	57.55	0.02	55.25	58.75	0.04	55.3	58.7					
240	0.01	60.25	64	0.01	60.35	63.45	0.03	60.25	63.75					

99) Legend

Suggested Functions HVG Track historical (realized) volatility ESG Environmental, Social & Governance

The left shows market prices of American options for AAPL in April of 2022 with various strikes and expiration dates. Some of the variation between the calculated and observed option prices can be attributed to the volatility smile effect which implies that market options can not be accurately calculated with a single volatility value.

kstrikes	callPrice
177.5	4.4731986
180.0	3.2723952
182.5	2.3088170
185.0	1.5909131
187.5	1.0571747
190.0	0.6759707
192.5	0.4149160
195.0	0.2438391
197.5	0.1401914

kstrikes	putPrice
177.5	3.500466
180.0	4.799794
182.5	6.336855
185.0	8.119489
187.5	10.086584
190.0	12.206563
192.5	14.447130
195.0	16.778192
197.5	19.176953



# Chapter IV The Greeks Analysis

*Implied volatilities (Vol table)  
of the American option prices on AAPL  
as of 05-Apr-2022 vs moneyness  $K/S_0$ .  
(Bloomberg)*

**Vega**  
*The amount call and put price will change every 1% change in implied volatility*  
*The only unknown.*

Click a volatility point for details											
AAPL US Equity		90) Asset		91) Actions		92) Views		93) Settings			
APPLE INC		175.06 USD		Bloomberg		Mid	As of	< 05-Apr-2022		16:15	▼
1) Vol Table	2) 3D Surface	3) Term	4) Skew	5) Dividends	6) Prices						
Moneyness	Listed	16) Edit				<input checked="" type="checkbox"/> Fwd			<input checked="" type="checkbox"/> Strikes		
Exp Date	ImpFwd	80.0%	90.0%	95.0%	97.5%	100.0%	102.5%	105.0%	110.0%	120.0%	
		140.048	157.554	166.307	170.684	175.06	179.437	183.813	192.566	210.072	
8 Apr 2022	175.07	70.43	48.28	35.26	30.67	27.15	26.29	29.26	39.67	51.51	
14 Apr 2022	175.10	57.21	39.34	31.19	27.91	25.21	23.66	23.97	29.80	41.71	
22 Apr 2022	175.14	45.50	33.44	28.16	25.83	23.78	22.39	22.12	24.77	32.26	
29 Apr 2022	175.17	45.68	35.91	31.94	30.14	28.53	27.24	26.40	26.67	32.66	
6 May 2022	174.94	44.07	35.08	31.28	29.61	28.16	26.99	26.19	26.07	30.83	
13 May 2022	174.97	42.57	34.34	30.86	29.32	27.93	26.77	25.92	25.43	28.87	
20 May 2022	175.00	41.53	33.77	30.54	29.13	27.89	26.85	26.06	25.36	27.74	
17 Jun 2022	175.13	38.64	32.70	30.21	29.10	28.11	27.26	26.57	25.67	25.95	
15 Jul 2022	175.30	36.98	31.89	29.80	28.90	28.09	27.39	26.79	25.90	25.54	
19 Aug 2022	175.53	36.83	32.69	30.99	30.22	29.52	28.86	28.27	27.31	26.46	
16 Sep 2022	175.78	35.99	32.25	30.73	30.05	29.42	28.84	28.30	27.38	26.35	
21 Oct 2022	176.17	35.02	31.82	30.49	29.89	29.32	28.80	28.32	27.49	26.48	
18 Nov 2022	176.39	35.06	32.07	30.83	30.27	29.75	29.27	28.82	28.04	27.02	
20 Jan 2023	177.20	33.89	31.36	30.32	29.86	29.43	29.03	28.66	28.01	27.06	
17 Mar 2023	177.85	33.69	31.40	30.47	30.05	29.66	29.30	28.97	28.39	27.52	
16 Jun 2023	179.18	33.28	31.39	30.62	30.27	29.95	29.66	29.39	28.91	28.17	
15 Sep 2023	180.64	32.82	31.27	30.63	30.33	30.06	29.80	29.55	29.11	28.38	
19 Jan 2024	182.66	32.29	31.01	30.47	30.22	29.99	29.76	29.55	29.16	28.49	
21 Jun 2024	185.10	31.93	31.01	30.60	30.41	30.22	30.04	29.86	29.53	28.91	
31 Dec 2024	187.99	31.86	31.06	30.70	30.53	30.36	30.20	30.05	29.75	29.21	
31 Dec 2025	193.04	31.84	31.10	30.78	30.63	30.49	30.35	30.22	29.98	29.55	
31 Dec 2026	197.74	31.78	31.11	30.81	30.68	30.55	30.42	30.30	30.08	29.68	
31 Dec 2027	202.36	31.77	31.18	30.92	30.79	30.67	30.56	30.45	30.24	29.86	
31 Dec 2028	207.01	31.60	31.15	30.91	30.80	30.60	30.50	30.40	30.20	29.92	

# Delta and Gamma

*Figure : Prices and Greeks of the American call and put options with maturity 14-Apr-2022 vs strike K obtained using the binomial tree method with n = 100 time steps.*

**Delta** is computed using central finite differences with step  $S_0 \rightarrow S_0 \pm 0.1$ .

$$\Delta = \frac{P(S_0 + 0.1) - P(S_0 - 0.1)}{0.2}$$

**Gamma** is computed on the same grid of points  $(S_0 - 0.1, S_0, S_0 + 0.1)$

$$\Gamma = \frac{P(S_0 + 0.1) - 2P(S_0) + P(S_0 - 0.1)}{0.1^2}$$

## callData

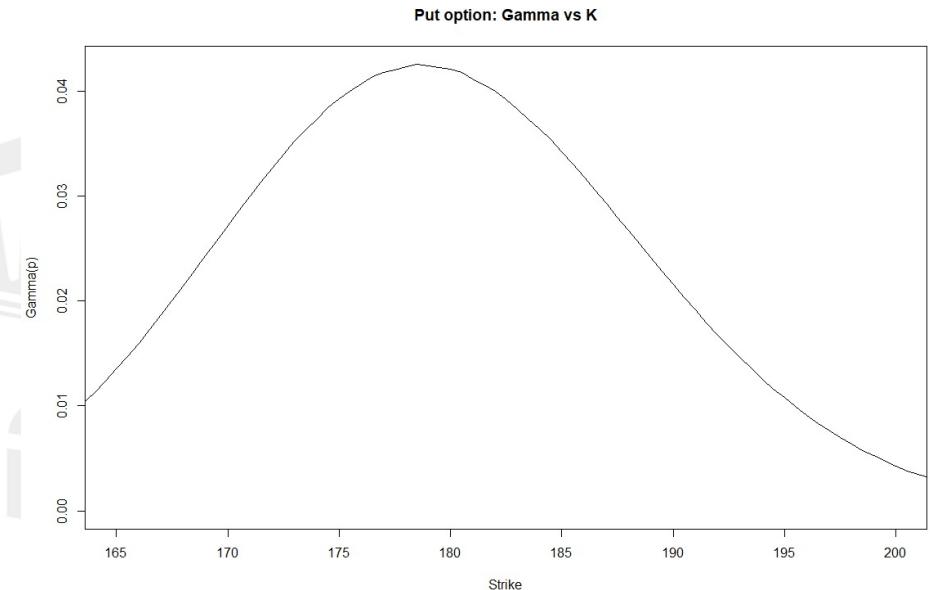
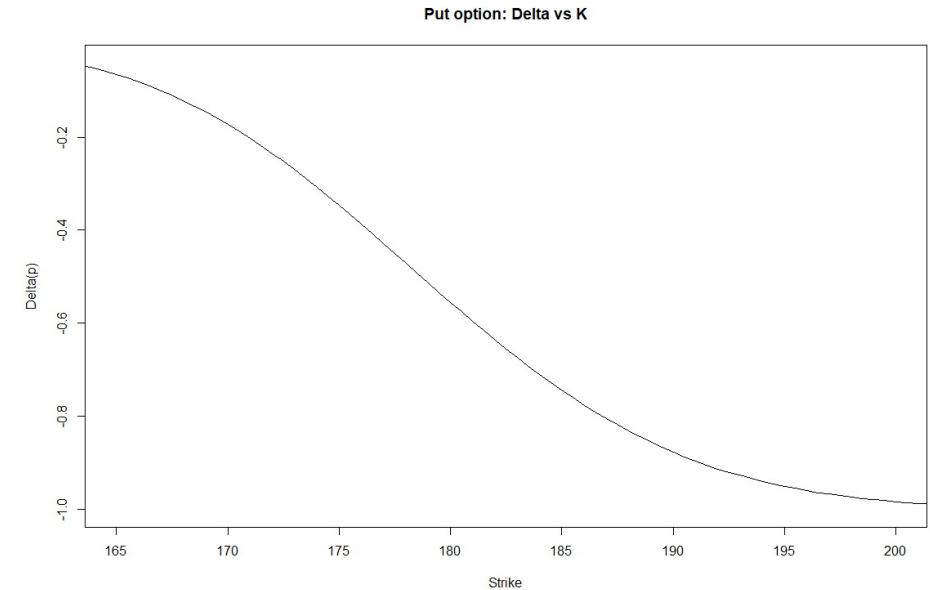
kStrikes	callPrice	callDelta	callGamma
162.5	16.152378	0.9585423	7.105427e-13
165.0	13.810873	0.9295696	1.556107e-01
167.5	11.586640	0.8710883	7.105427e-13
170.0	9.500974	0.8241318	3.552714e-13
172.5	7.586979	0.7585921	1.808657e-01
175.0	5.912638	0.6297407	1.065814e-12
177.5	4.473199	0.5522373	0.000000e+00
180.0	3.272395	0.4726871	4.440892e-13
182.5	2.308817	0.3438597	4.814109e-01

## putData

kStrikes	putPrice	putDelta	putGamma
162.5	0.1812065	-0.04145959	7.030658e-06
165.0	0.3392642	-0.07062732	1.586209e-01
167.5	0.6146617	-0.12891908	2.243196e-06
170.0	1.0286343	-0.17589540	7.355161e-05
172.5	1.6143766	-0.24209147	1.904306e-01
175.0	2.4399656	-0.37030451	6.236382e-05
177.5	3.5004661	-0.44785748	1.310261e-04
180.0	4.7997936	-0.52757796	5.043488e-04
182.5	6.3368545	-0.65727990	4.612555e-01

**Figure 3 : Put Option Delta vs Strike (K)**  
**Figure 4 : Put Option Gamma vs Strike (K)**

*The Greeks of American Options vs Strike price, delta levels out to -1 at the critical strike of 200, after which it will remain constant, and Gamma is discontinuous at a certain critical strike price K.*



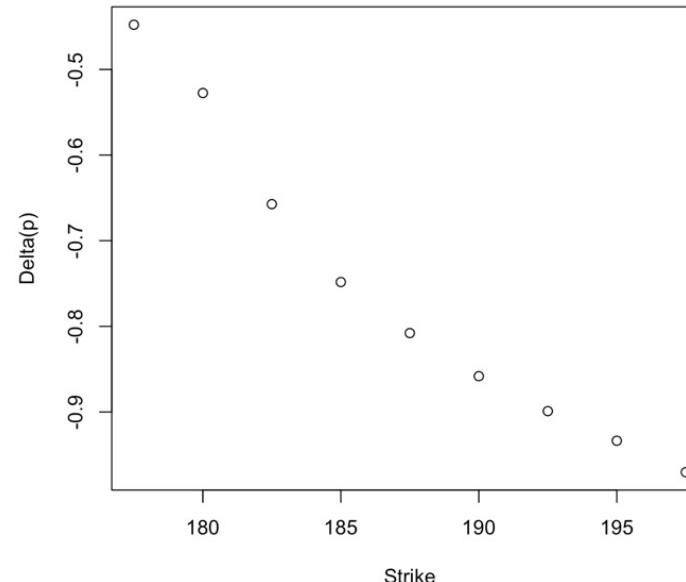


## Chapter V Hedging Exercise

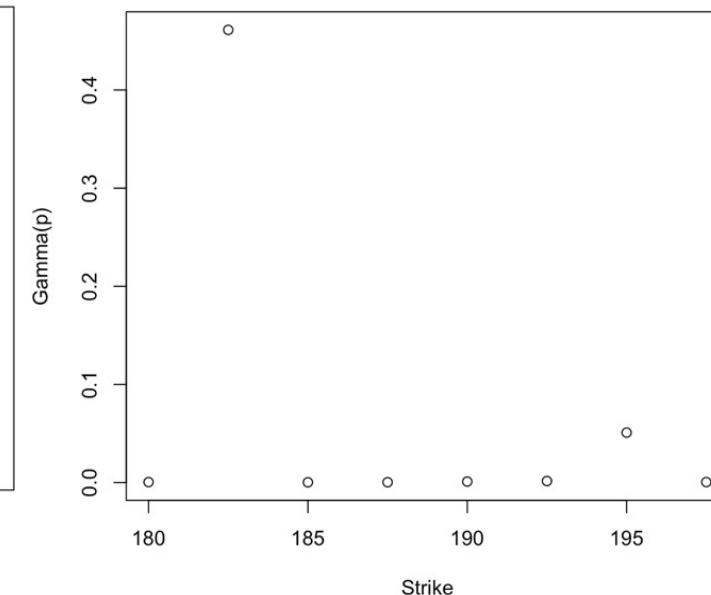
The closing prices of AAPL for the first 9 trading days of 2022-04-04 are shown below. For the hedging exercise, consider the put option with strike = 177.5 for each of the last 4 days of its existence:

4-Apr to 8-Apr.

Delta(put)



Gamma(put)



## Chapter V Hedging Exercise

> C1(AAPL)

	AAPL.Close
2022-04-04	178.44
2022-04-05	175.06
2022-04-06	171.83
2022-04-07	172.14
2022-04-08	170.09
2022-04-11	165.75
2022-04-12	167.66
2022-04-13	170.40
2022-04-14	165.29

The put option with strike  $K = 177.5$ , ends up on 8-Apr, when the stock price is 170.09.

Using the Delta values in the table, we construct a dynamically hedged portfolio: for one put option, we buy  $\Delta$  shares of stock. Denote the price of the option + stock hedged portfolio

## Chapter V Hedging Exercise

the value of the put price, put delta, daily PNL, hedge price

```
> putPrice
[1] 2.193213 3.682586 5.903238 5.424074 7.410000
> # 13.28089 17.82806 22.38536 39.58822 28.40000
> putDelta
[1] -0.4519412 -0.6850065 -0.9009339 -0.9548679
> # -0.4562717 -0.6145443 -0.7557467 -0.9893727
> dPrice # daily PnL of unhedged option
[1] 1.4893729 2.2206527 -0.4791649 1.9859264
> # 4.547176 4.557296 17.202862 -11.188219
> hedgedPrice # daily PnL of hedged option
[1] -0.038188163 0.008081577 -0.199875364 0.028447146
```

The daily price change of the hedged portfolio is :

The hedge is adjusted daily according to the Delta of the put option on each day, computed using the binomial tree model. We see that the hedged position has much smaller daily price volatility than the “naked” put option.

$$\Pi(t) - \Pi(t-1) = P(t) - P(t-1) + \Delta(t-1)(S(t) - S(t-1))$$

## Chapter V Hedging Exercise

sell one put option	1.252
sell one stock	178.44

Day	Maturity	$S_0$	Put Price	Delta	$P(t) - P(t-1)$	$\Pi(t) - \Pi(t - 1)$	share
4-Apr-22	4/252	178.44	2.193	-0.452	-	-	-0.452
5-Apr-22	3/252	175.06	3.683	-0.685	1.489	-0.038	-0.685
6-Apr-22	2/252	171.83	5.903	-0.901	2.221	0.008	-0.901
7-Apr-22	1/252	172.14	5.42	-0.955	-0.479	-0.2	-0.955
8-Apr-22	-	170.09	7.41	-1	1.986	0.028	-1

The  $P(t) - P(t-1)$  and  $\pi(t) - \pi(t - 1)$  columns in Table show the daily price changes of the unhedged put option, and of the Delta hedged position of option plus stock.

## Chapter V Hedging Exercise

Here is the hedging strategy:

Sell one share of put option and one share of stock in 4-Apr-22 and buy one share of put option in 8-Apr-22. With the hedging stocks' net income, you will finally get 5.487 dollars per share.

sell earn money	earn	stock total sell
80.65488	80.65488	0.452
119.9161	200.57098	1.137
154.81883	355.38981	2.038
164.3937	519.78351	2.993
170.09	689.87351	3.993

sell stock +	689.87351
buy stock -	679.16937
Net hedge	10.70414
Sell 1 put you get	2.193
Buy 1 put you cost	7.41
Final	5.48714

## Chapter VI Improvements with relevant topics

If there is extra time we could explore further along different directions:

1. Improve the tree pricing using the European option as control variate.
2. Use Bloomberg and R language to automatically price American options, and design arbitrage strategies, even multiple options trading strategies. We could calculate the Sharpe ratio and maximum drawdown.
3. Employ Least Square Monte Carlo Simulation Method to compute the American options.
4. Consider transact action cost and Corporate Risk.
5. Try to consider the extra dividend.



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