Problem 1 – Random Tree

Assume that the strike price at maturity is 50.

American Call Option

For High Estimator of American Call option:

With the underlying security prices X_0, X_1's, X_2's given and the strike price = 50, the value of the American call option **at maturity or step = 2** is $V_2 = \max(X_2 - \text{Strike}, 0)$.

At step 1, the value of the American call option is:

$$V_1 = \max(\max(X_1 - \text{Strike}, 0), \text{Average of 3 successing } V_2)$$

"h_1" in the image below is $\max(X_1 - \text{Strike}, 0)$.

"(1/b)*sum(V_2)" in the image below is the Average of 3 successing V_2.

At step 0, the value of the American call option is:

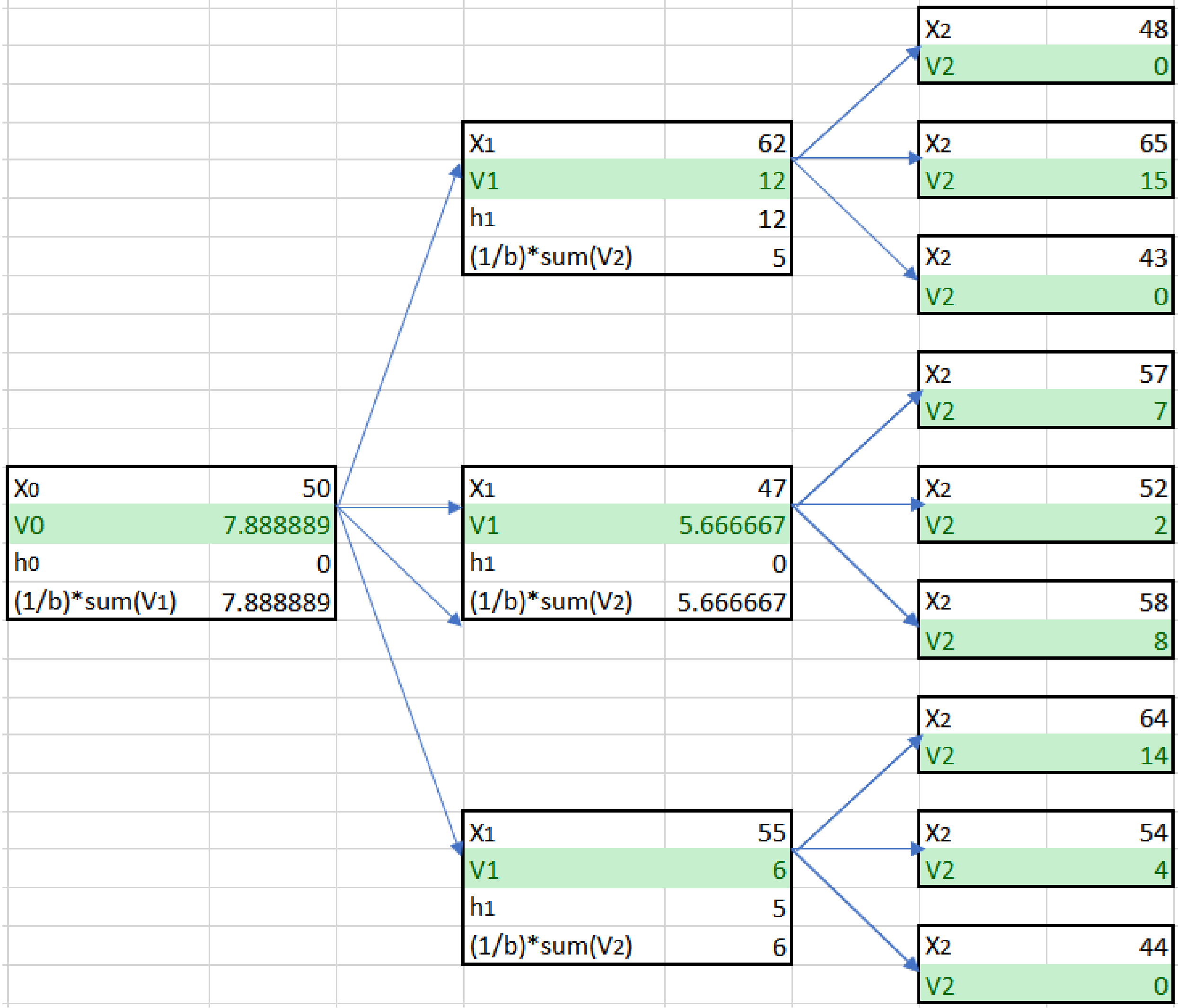
$$V_0 = \max(\max(X_0 - \text{Strike}, 0), \text{Average of 3 successing } V_1)$$

"h_0" in the image below is $\max(X_0 - \text{Strike}, 0)$.

"(1/b)*sum(V_1)" in the image below is the Average of 3 successing V_1.

Based on these formulas, the **high estimator of the American Call option is 7.8889**.

American Call - High Estimator



For Low Estimator of American Call option:

With the underlying security prices X_0, X_1's, X_2's given and the strike price = 50, the value of the American call option **at maturity or step = 2** is $V_2 = \max(X_2 - \text{Strike}, 0)$.

At step 1, the value of the American call option is:

$$V_1 = \text{mean}(\mathrm{U}_1, \mathrm{M}_1, \mathrm{L}_1)$$

"h_1" in the image below is $\max(X_1 - \text{Strike}, 0)$.

Let V_2^u, V_2^m, V_2^L be the V_2 values from the upper, middle, and lower successing nodes.

"U1" in the image below is $\left\{\begin{matrix} \mathrm{h}_1 & if\ \mathrm{h}_1 \geq \frac{1}{2}\left(V_2^m + V_2^l\right) \\ V_2^u & \text{Otherwise} \end{matrix}\right\}.$

"M1" in the image below is $\left\{\begin{matrix} \mathrm{h}_1 & if\ \mathrm{h}_1 \geq \frac{1}{2}\left(V_2^u + V_2^l\right) \\ V_2^m & \text{Otherwise} \end{matrix}\right\}.$

"L1" in the image below is $\left\{\begin{matrix} \mathrm{h}_1 & if\ \mathrm{h}_1 \geq \frac{1}{2}(V_2^u + V_2^m) \\ V_2^l & \text{Otherwise} \end{matrix}\right\}.$

At step 0, the value of the American call option is:

$$V_0 = \text{mean}(\mathrm{U}_0, \mathrm{M}_0, \mathrm{L}_0)$$

"h_0" in the image below is $\max(X_0 - \text{Strike}, 0)$.

Let V_1^u, V_1^m, V_1^L be the V_1 values from the upper, middle, and lower successing nodes.

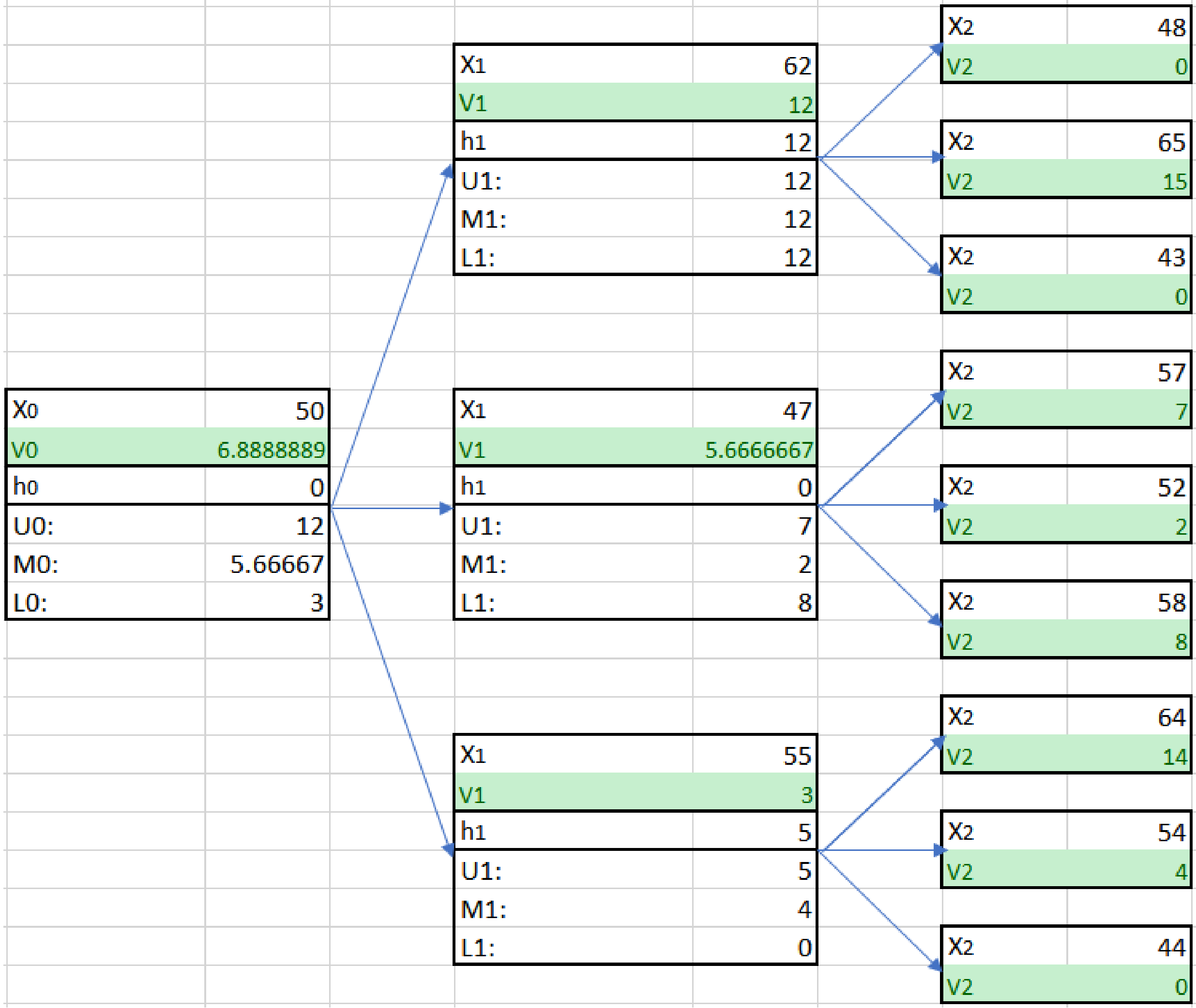
"U0" in the image below is $\left\{\begin{matrix} \mathrm{h}_0 & if\ \mathrm{h}_0 \geq \frac{1}{2}\left(V_1^m + V_1^l\right) \\ V_1^u & \text{Otherwise} \end{matrix}\right\}.$

"M0" in the image below is $\left\{\begin{matrix} \mathrm{h}_1 & if\ \mathrm{h}_0 \geq \frac{1}{2}\left(V_1^u + V_1^l\right) \\ V_1^m & \text{Otherwise} \end{matrix}\right\}.$

"L0" in the image below is $\left\{\begin{matrix} \mathrm{h}_1 & if\ \mathrm{h}_0 \geq \frac{1}{2}(V_1^u + V_1^m) \\ V_1^l & \text{Otherwise} \end{matrix}\right\}.$

Based on these formulas, the **low estimator of the American Call option is 6.8889**.

American Call - Low Estimator



American Put Option

For High Estimator of American Put option:

With the underlying security prices X_0, X_1's, X_2's given and the strike price = 50, the value of the American Put option **at maturity or step = 2** is $V_2 = \max(\text{Strike} - X_2, 0)$.

At step 1, the value of the American put option is:

$$V_1 = \max(\max(\text{Strike} - X_1, 0)\,, \text{Average of 3 successing } \mathrm{V}_2)$$

"h_1" in the image below is $\max(\text{Strike} - X_1, 0)$.

"(1/b)*sum(V_2)" in the image below is the Average of 3 successing V_2.

At step 0, the value of the American put option is:

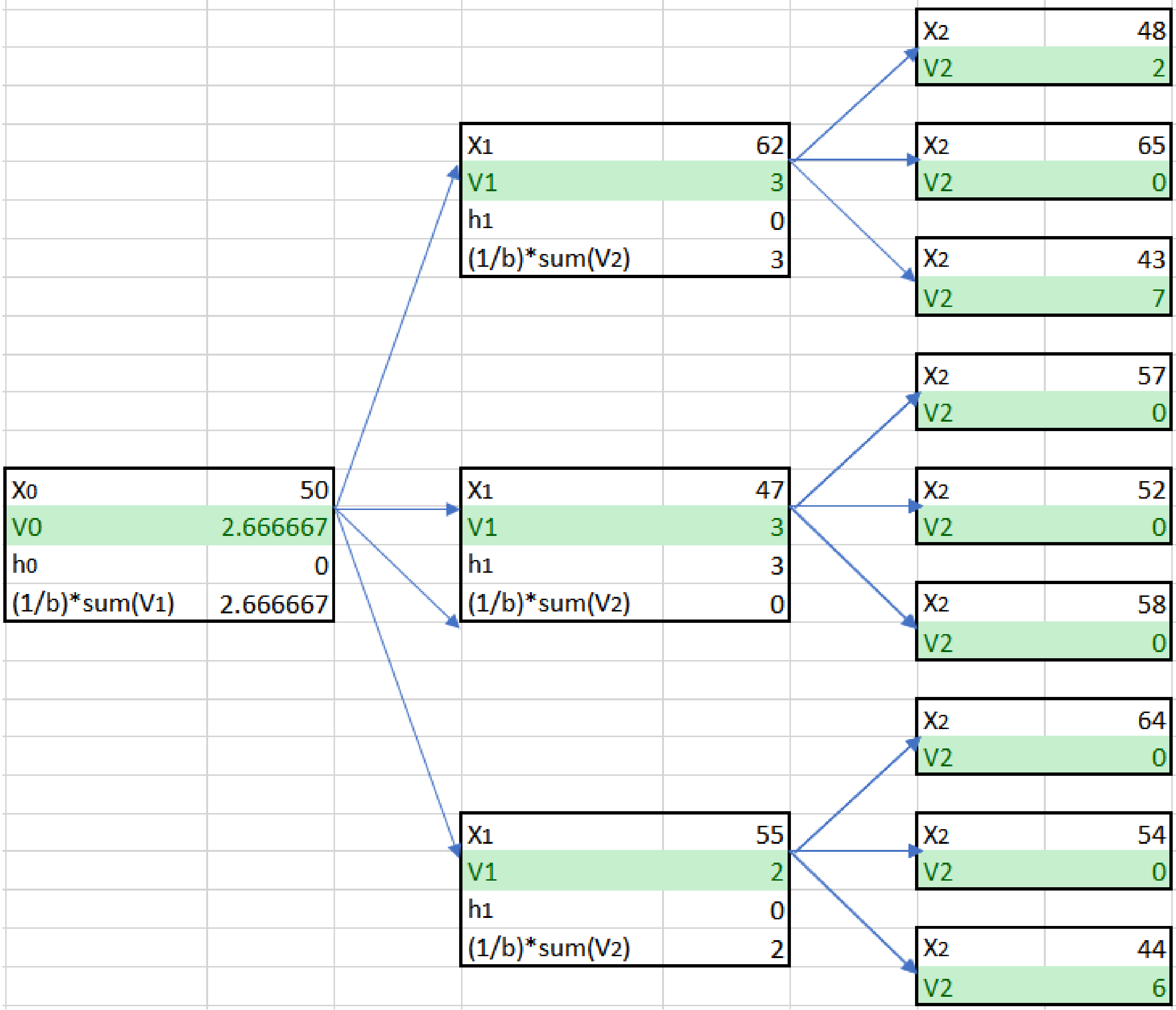
$$V_0 = \max(\max(\text{Strike} - X_0, 0)\,, \text{Average of 3 successing } \mathrm{V}_1)$$

"h_0" in the image below is $\max(\text{Strike} - X_0, 0)$.

"(1/b)*sum(V_1)" in the image below is the Average of 3 successing V_1.

Based on these formulas, the **high estimator of the American Put option is 2.6667**.

American Put - High Estimator



For Low Estimator of American Put option:

With the underlying security prices X_0, X_1's, X_2's given and the strike price = 50, the value of the American put option **at maturity or step = 2** is $V_2 = \max(\text{Strike} - X_2, 0)$.

At step 1, the value of the American put option is:

$$V_1 = \text{mean}(\mathrm{U}_1, \mathrm{M}_1, \mathrm{L}_1)$$

"h_1" in the image below is $\max(\text{Strike} - X_1, 0)$.

Let V_2^u, V_2^m, V_2^L be the V_2 values from the upper, middle, and lower successing nodes.

"U1" in the image below is $\left\{\begin{matrix} \mathrm{h}_1 & if\ \mathrm{h}_1 \geq \frac{1}{2}\left(V_2^m + V_2^l\right) \\ V_2^u & \text{Otherwise} \end{matrix}\right\}$.

"M1" in the image below is $\left\{\begin{matrix} \mathrm{h}_1 & if\ \mathrm{h}_1 \geq \frac{1}{2}\left(V_2^u + V_2^l\right) \\ V_2^m & \text{Otherwise} \end{matrix}\right\}$.

"L1" in the image below is $\left\{\begin{matrix} \mathrm{h}_1 & if\ \mathrm{h}_1 \geq \frac{1}{2}\left(V_2^u + V_2^m\right) \\ V_2^l & \text{Otherwise} \end{matrix}\right\}$.

At step 0, the value of the American put option, V_0, is:

$$V_0 = \text{mean}(\mathrm{U}_0, \mathrm{M}_0, \mathrm{L}_0)$$

"h_0" in the image below is $\max(\text{Strike} - X_0, 0)$.

Let V_1^u, V_1^m, V_1^L be the V_1 values from the upper, middle, and lower successing nodes.

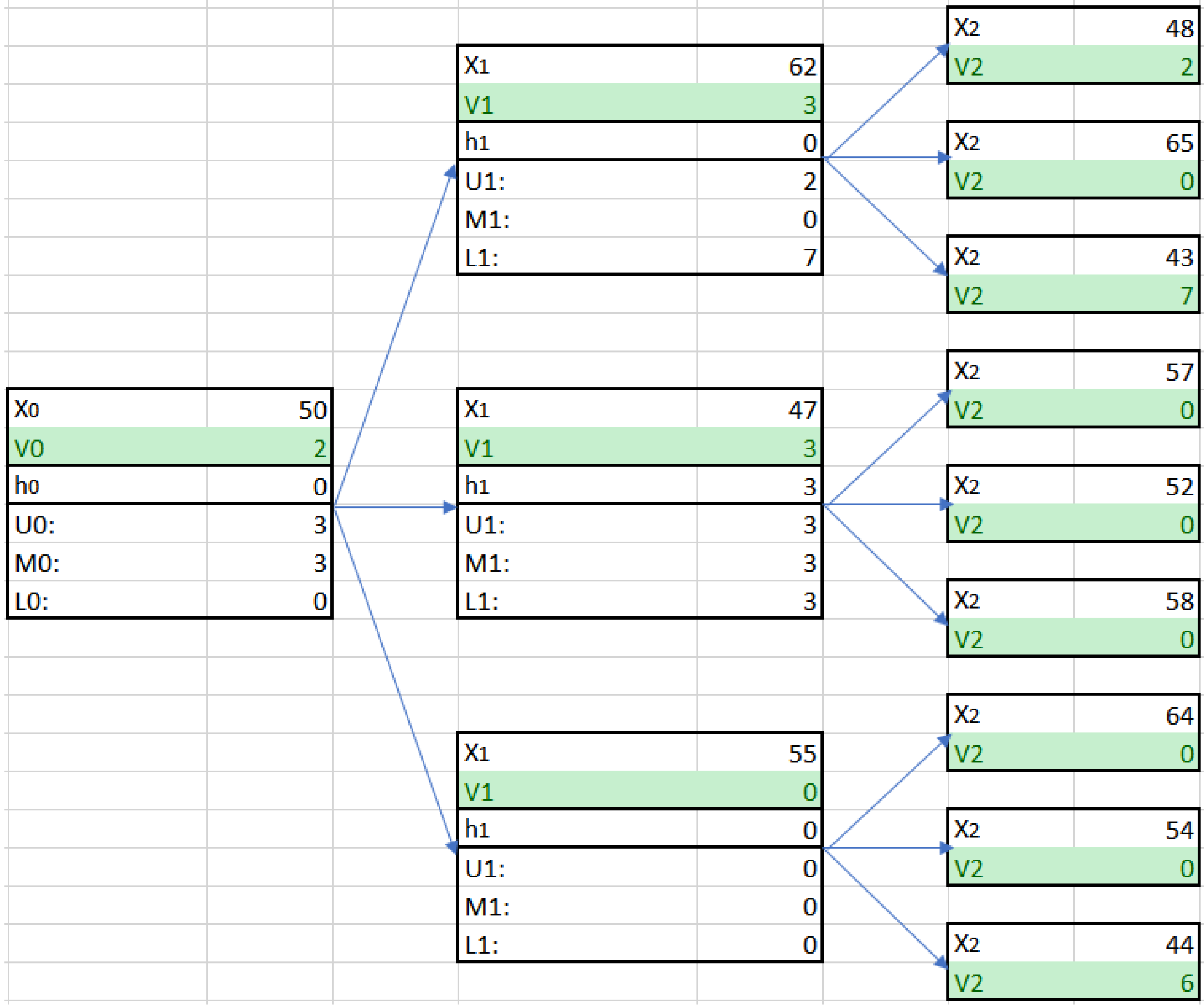
"U0" in the image below is $\left\{\begin{matrix} \mathrm{h}_0 & if\ \mathrm{h}_0 \geq \frac{1}{2}\left(V_1^m + V_1^l\right) \\ V_1^u & \text{Otherwise} \end{matrix}\right\}$.

"M0" in the image below is $\left\{\begin{matrix} \mathrm{h}_1 & if\ \mathrm{h}_0 \geq \frac{1}{2}\left(V_1^u + V_1^l\right) \\ V_1^m & \text{Otherwise} \end{matrix}\right\}$.

"L0" in the image below is $\left\{\begin{matrix} \mathrm{h}_1 & if\ \mathrm{h}_0 \geq \frac{1}{2}\left(V_1^u + V_1^m\right) \\ V_1^l & \text{Otherwise} \end{matrix}\right\}$.

Based on these formulas, the **low estimator of the American Put option is 2**.

American Put - Low Estimator



Problem 2 – Newton Ralphson

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lower volatility guess
0.2

high volatility guess
0.4

start volatility guess
0.23

Tolerance for volatility
0.0001

The target European Call option price is: 4.23


***Use bisection method:

Implied vol for the European Call option is:  0.262939

European Call Option price by implied volatility from Bisection method is: 4.23009


***Use Newton Raphson method:

Implied vol for the European Call option is:  0.262937

European Call Option price by implied volatility from Newton Raphson method is: 4.23005
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