Problem 1 – Random Tree

Assume that the strike price at maturity is 50.

American Call Option

For High Estimator of American Call option:

With the underlying security prices X_0 , X_1 's, X_2 's given and the strike price = 50, the value of the American call option at maturity or step = 2 is $V_2 = \max(X_2 - \text{Strike}, 0)$.

At step 1, the value of the American call option is:

$$V_1 = \max(\max(X_1 - \text{Strike}, 0), \text{Average of 3 successing } V_2)$$

"h₁" in the image below is $max(X_1 - Strike, 0)$.

" $(1/b)*sum(V_2)$ " in the image below is the Average of 3 successing V_2 .

At step 0, the value of the American call option is:

$$V_0 = \max(\max(X_0 - \text{Strike}, 0), \text{Average of 3 successing } V_1)$$

"h₀" in the image below is $max(X_0 - Strike, 0)$.

"(1/b)*sum (V_1) " in the image below is the Average of 3 successing V_1 .

Based on these formulas, the high estimator of the American Call option is 7.8889.

| | | American Call - High Estimator | | | | or |
|---------------|----------|--------------------------------|---------------|----------|----|-----|
| | | | | | X2 | 48 |
| | | | | | V2 | 0 |
| | | | X1 | 62 | X2 | 65 |
| | | | V1 | 12 | V2 | 15 |
| | | | h1 | 12 | | |
| | | | (1/b)*sum(V2) | 5 | X2 | 43 |
| | | | | | V2 | 0 |
| | | / | | | V- | F.7 |
| | | | | | X2 | 57 |
| | | | | | V2 | 7 |
| Xo | 50 | / . | X1 | 47 | X2 | 52 |
| V0 | 7.888889 | | V1 | 5.666667 | V2 | 2 |
| ho | 0 | | h1 | 0 | | |
| (1/b)*sum(V1) | 7.888889 | | (1/b)*sum(V2) | 5.666667 | X2 | 58 |
| | | | | | V2 | 8 |
| | | | | | | |
| | | | | | X2 | 64 |
| | | | | | V2 | 14 |
| | | | X1 | 55 | X2 | 54 |
| | | | V1 | 6 | V2 | 4 |
| | | | h1 | 5 | | |
| | | | (1/b)*sum(V2) | 6 | X2 | 44 |
| | | | | | V2 | 0 |

For Low Estimator of American Call option:

With the underlying security prices X_0 , X_1 's, X_2 's given and the strike price = 50, the value of the American call option at maturity or step = 2 is $V_2 = \max(X_2 - \text{Strike}, 0)$.

At step 1, the value of the American call option is:

$$V_1 = \text{mean}(U_1, M_1, L_1)$$

"h₁" in the image below is $max(X_1 - Strike, 0)$.

Let V_2^u , V_2^m , V_2^L be the V_2 values from the upper, middle, and lower successing nodes.

"U1" in the image below is
$$\begin{cases} h_1 & \text{if } h_1 \ge \frac{1}{2} \left(V_2^m + V_2^l \right) \\ V_2^u & \text{Otherwise} \end{cases}$$
.

"M1" in the image below is
$$\begin{cases} h_1 & \text{if } h_1 \ge \frac{1}{2} \left(V_2^u + V_2^l \right) \\ V_2^m & \text{Otherwise} \end{cases}.$$

"L1" in the image below is
$$\begin{cases} h_1 & \text{if } h_1 \ge \frac{1}{2}(V_2^u + V_2^m) \\ V_2^l & \text{Otherwise} \end{cases}$$
.

At step 0, the value of the American call option is:

$$V_0 = \text{mean}(U_0, M_0, L_0)$$

"h₀" in the image below is $max(X_0 - Strike, 0)$.

Let V_1^u , V_1^m , V_1^L be the V_1 values from the upper, middle, and lower successing nodes.

"U0" in the image below is
$$\begin{cases} h_0 & \text{if } h_0 \ge \frac{1}{2} \left(V_1^m + V_1^l \right) \\ V_1^u & \text{Otherwise} \end{cases}.$$

"M0" in the image below is
$$\begin{cases} h_1 & \text{if } h_0 \geq \frac{1}{2} \left(V_1^u + V_1^l\right) \\ V_1^m & \text{Otherwise} \end{cases}$$

"L0" in the image below is
$$\begin{cases} h_1 & \text{if } h_0 \ge \frac{1}{2}(V_1^u + V_1^m) \\ V_1^l & \text{Otherwise} \end{cases}$$
.

Based on these formulas, the low estimator of the American Call option is 6.8889.

| | | | America | • | | |
|------------|-----------|---|---------|-----------|----|----|
| | | | | | X2 | 48 |
| | | | X1 | 62 | V2 | |
| | | | V1 | 12 | | |
| | | | h1 | 12 | X2 | 65 |
| | | 1 | U1: | 12 | V2 | 15 |
| | | | M1: | 12 | | |
| | | | L1: | 12 | X2 | 43 |
| | | | | | V2 | 0 |
| | | | | | | |
| | | | | | X2 | 57 |
| Xo | 50 | | X1 | 47 | V2 | 7 |
| V 0 | 6.8888889 | | V1 | 5.6666667 | | |
| ho | 0 | / | h1 | 0 | X2 | 52 |
| U0: | 12 | | U1: | 7 | V2 | 2 |
| M0: | 5.66667 | | M1: | 2 | | |
| LO: | 3 | | L1: | 8 | X2 | 58 |
| | | | | | V2 | 8 |
| | | | | | | |
| | | | | | X2 | 64 |
| | | | X1 | 55 | V2 | 14 |
| | | | V1 | 3 | | |
| | | | h1 | 5 | X2 | 54 |
| | | | U1: | 5 | V2 | 4 |
| | | | M1: | 4 | | |
| | | | L1: | 0 | X2 | 44 |
| | | | | | V2 | 0 |

American Put Option

For High Estimator of American Put option:

With the underlying security prices X_0 , X_1 's, X_2 's given and the strike price = 50, the value of the American Put option at maturity or step = 2 is $V_2 = \max(\text{Strike} - X_2, 0)$.

At step 1, the value of the American put option is:

$$V_1 = \max(\max(\text{Strike} - X_1, 0), \text{Average of 3 successing } V_2)$$

"h₁" in the image below is $max(Strike - X_1, 0)$.

"(1/b)*sum (V_2) " in the image below is the Average of 3 successing V_2 .

At step 0, the value of the American put option is:

$$V_0 = \max(\max(\text{Strike} - X_0, 0), \text{Average of 3 successing } V_1)$$

" h_0 " in the image below is max(Strike – X_0 , 0).

" $(1/b)*sum(V_1)$ " in the image below is the Average of 3 successing V_1 .

Based on these formulas, the high estimator of the American Put option is 2.6667.

| | | | American Put | | | |
|---------------|----------|---------------|---------------|----|----|----|
| | | | | | X2 | 48 |
| | | | | | V2 | 2 |
| | | | X1 | 62 | X2 | 65 |
| | | 1 | V1 | 3 | V2 | 0 |
| | | | h1 | 0 | | |
| | | | (1/b)*sum(V2) | 3 | X2 | 43 |
| | | | | | V2 | 7 |
| | | | | | X2 | 57 |
| | | | | | V2 | 0 |
| | | | | | | |
| Xo | 50 | / . | X1 | 47 | X2 | 52 |
| V0 | 2.666667 | | V1 | 3 | V2 | 0 |
| ho | 0 | | h1 | 3 | | |
| (1/b)*sum(V1) | 2.666667 | | (1/b)*sum(V2) | 0 | X2 | 58 |
| | | | | | V2 | 0 |
| | | | | | | |
| | | | | | X2 | 64 |
| | | $\overline{}$ | | | V2 | 0 |
| | | | X1 | 55 | X2 | 54 |
| | | | V1 | 2 | V2 | 0 |
| | | | h1 | 0 | | |
| | | | (1/b)*sum(V2) | 2 | X2 | 44 |
| | | | | _ | V2 | 6 |

For Low Estimator of American Put option:

With the underlying security prices X_0 , X_1 's, X_2 's given and the strike price = 50, the value of the American put option at maturity or step = 2 is $V_2 = \max(S \text{trike} - X_2, 0)$.

At step 1, the value of the American put option is:

$$V_1 = \text{mean}(U_1, M_1, L_1)$$

" h_1 " in the image below is max(Strike – X_1 , 0).

Let V_2^u , V_2^m , V_2^L be the V_2 values from the upper, middle, and lower successing nodes.

"U1" in the image below is
$$\begin{cases} h_1 & if \ h_1 \ge \frac{1}{2} \left(V_2^m + V_2^l \right) \\ V_2^u & \text{Otherwise} \end{cases}.$$

"M1" in the image below is
$$\begin{cases} h_1 & \text{if } h_1 \ge \frac{1}{2} \left(V_2^u + V_2^l \right) \\ V_2^m & \text{Otherwise} \end{cases}.$$

"L1" in the image below is
$$\begin{cases} h_1 & \text{if } h_1 \geq \frac{1}{2}(V_2^u + V_2^m) \\ V_2^l & \text{Otherwise} \end{cases}.$$

At step 0, the value of the American put option, V_0 , is:

$$V_0 = \text{mean}(U_0, M_0, L_0)$$

"h₀" in the image below is $max(Strike - X_0, 0)$.

Let V_1^u , V_1^m , V_1^L be the V_1 values from the upper, middle, and lower successing nodes.

"U0" in the image below is
$$\begin{cases} h_0 & if \ h_0 \ge \frac{1}{2} (V_1^m + V_1^l) \\ V_1^u & \text{Otherwise} \end{cases}$$
.

"M0" in the image below is
$$\begin{cases} h_1 & \text{if } h_0 \ge \frac{1}{2} \left(V_1^u + V_1^l \right) \\ V_1^m & \text{Otherwise} \end{cases}$$
.

"L0" in the image below is
$$\begin{cases} h_1 & \text{if } h_0 \ge \frac{1}{2}(V_1^u + V_1^m) \\ V_1^l & \text{Otherwise} \end{cases}$$
.

Based on these formulas, the low estimator of the American Put option is 2.

| | | | American l | Put - Low Estin | nator | |
|-----|----|---|------------|-----------------|-------|----|
| | | | | | X2 | 48 |
| | | | X1 | 62 | V2 | 2 |
| | | | V1 | 3 | | |
| | | | h1 | 0 | X2 | 65 |
| | | 1 | U1: | 2 | V2 | 0 |
| | | | M1: | 0 | | |
| | | | L1: | 7 | X2 | 43 |
| | | | | | V2 | 7 |
| | | | | | | |
| | | | | | X2 | 57 |
| Xo | 50 | | X1 | 47 | V2 | 0 |
| V0 | 2 | | V1 | 3 | | |
| ho | 0 | / | h1 | 3 | X2 | 52 |
| U0: | 3 | | U1: | 3 | V2 | 0 |
| M0: | 3 | | M1: | 3 | | |
| LO: | 0 | | L1: | 3 | X2 | 58 |
| | | | | | V2 | 0 |
| | | | | | | |
| | | | | | X2 | 64 |
| | | | X1 | 55 | V2 | 0 |
| | | | V1 | 0 | | |
| | | | h1 | 0 | X2 | 54 |
| | | | U1: | 0 | V2 | 0 |
| | | | M1: | 0 | | |
| | | | L1: | 0 | X2 | 44 |
| | | | | | V2 | 6 |

Problem 2 – Newton Ralphson

```
lower volatility guess
0.2
high volatility guess
0.4
start volatility guess
0.23
Tolerance for volatility
0.0001
The target European Call option price is: 4.23
***Use bisection method:
Implied vol for the European Call option is: 0.262939
European Call Option price by implied volatility from Bisection method is: 4.23009
***Use Newton Raphson method:
Implied vol for the European Call option is: 0.262937
European Call Option price by implied volatility from Newton Raphson method is: 4.23005
```