FE570 Zero Intelligence Model simulation

—— Using Poisson distribution

Team 8

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Content

Chapter I

Part I Santa Fe Model Review

Assumptions:

Simplest ZI model capturing the main features of the limit order markets.

- 1.The model contains two types of traders:

  (1) Impatient traders: They place market orders which arrive randomly like a Poisson process with rate μ shares/ unit time.

  (2) Patient traders: They place limit orders which arrive randomly like a Poisson process with rate α shares/ unit time.

- 2.All limit/market orders are of the same size σ(number of shares)
- 3.Buy and sell orders are equally probable.
- 4.Market and limit orders are independent of each other.

Part II Santa Fe Model simulation

Rules:

1.Limit buy orders arrive with equal probability at any price p$\leq a(t)$

2.Limit sell orders arrive with equal probability at any price p$\geq b(t)$

3.Market buys arrive as Poisson process with intensity μ and are executed

a(t)=best-ask

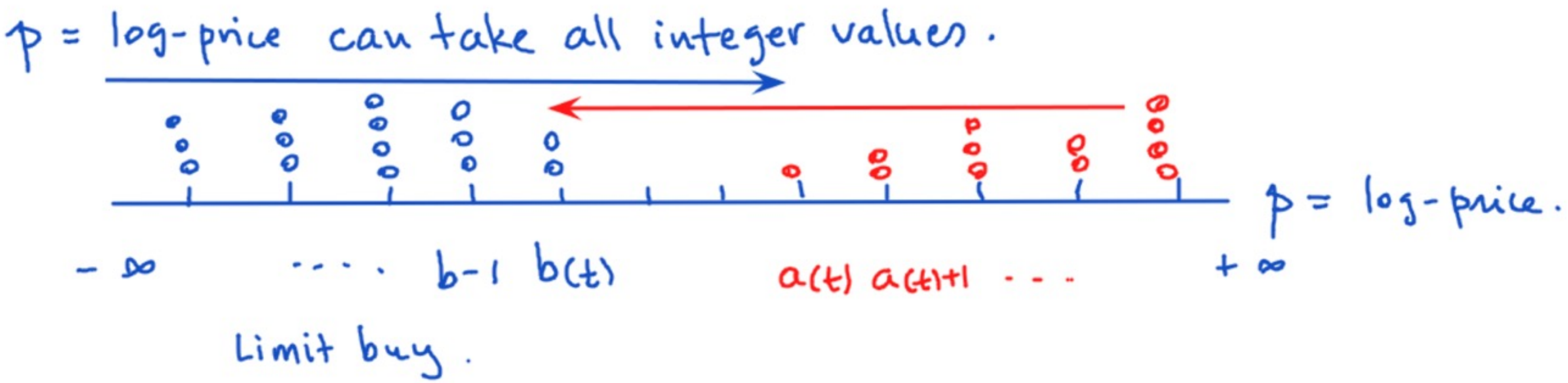
4.Market sells arrive as Poisson process with intensity μ and are executed

b(t)=best-bid

5.Limit orders are cancelled as Poisson process with intensity δ

Chapter I

Part II Santa Fe Model simulation



α—— limit orders

μ—— market orders

δ—— cancellation of limit orders

σ—— block size

μ/α—— ticks

μ/δ—— shares

1/δ—— time

Chapter II

Predictions

Prediction #1 Average spread

$$\hat{s} = \frac{\mu}{\alpha} f(\sigma \frac{\delta}{\mu})$$

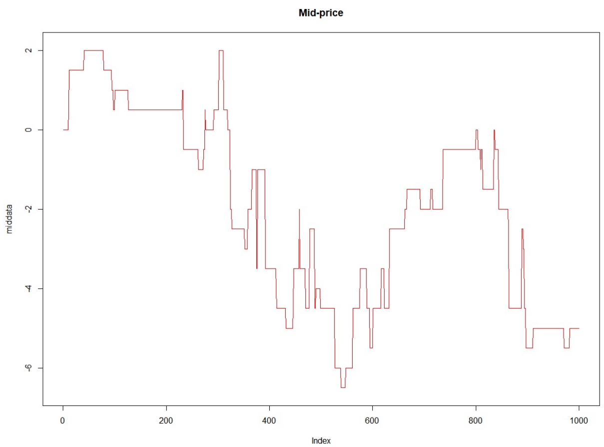
Prediction #2 The standard deviation

$$D \sim \left(\frac{\mu^{\frac{3}{4}} \delta^{\frac{1}{4}}}{\alpha^{\frac{1}{2}}} \right)$$

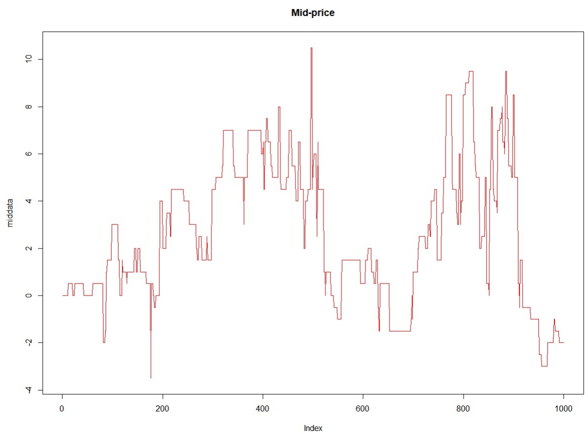
- The average spread increases with market orders μ or cancellations δ
- The average spread decreases as limit orders α increases
- The standard deviation of the mid-price and spread is expected to increase with μ and δ and decrease as α increases.

Chapter II

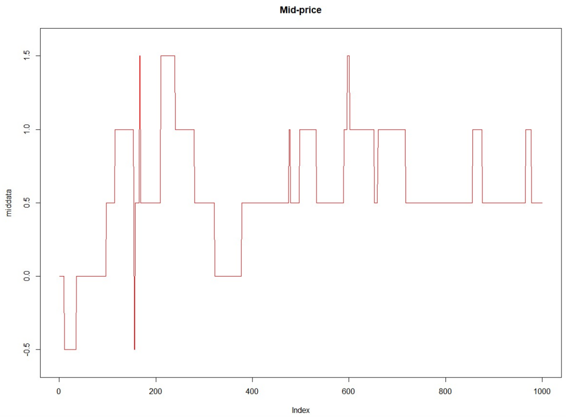
Mid-price



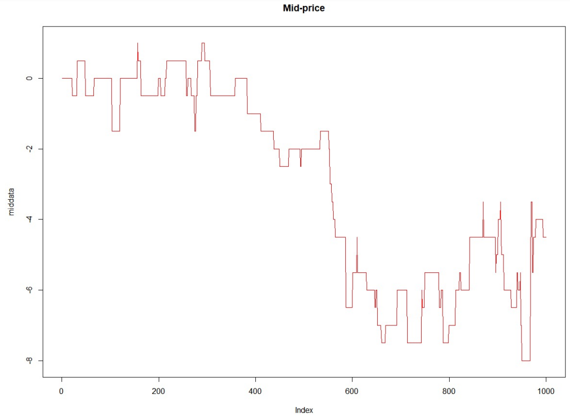
α=1, μ=10 ,δ=1/5



α=1, μ=20 ,δ=1/5



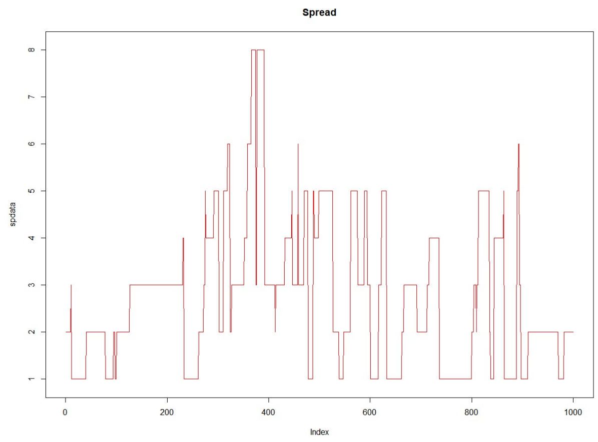
α=5, μ=10 ,δ=1/5



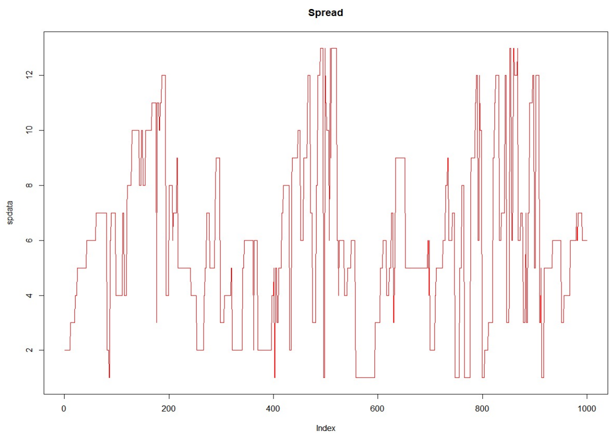
α=1, μ=10 ,δ=2/5

Chapter II

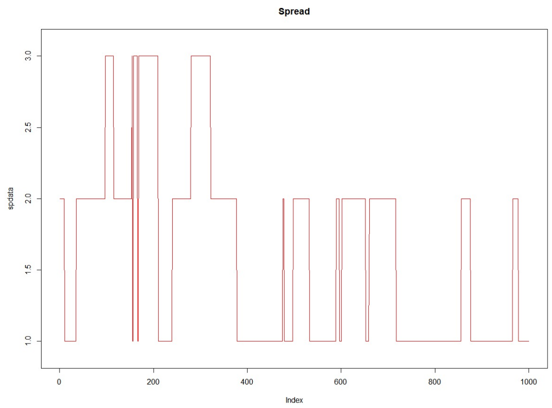
Spread



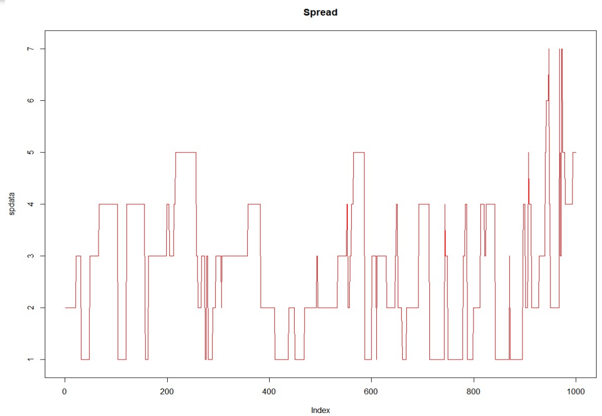
α=1, μ=10 ,δ=1/5



α=1, μ=20 ,δ=1/5



α=5, μ=10 ,δ=1/5



α=1, μ=10 ,δ=2/5

Chapter II

Results

Scenario	μ	α	δ	Comments	Spread	Mid-price
1	10	1	1/5	Base scenario	2.72 ± 1.58	-1.96 ± 2.38
2	20	1	1/5	High rate of market orders arrival	5.84 ± 3.13	2.64 ± 2.97
3	10	5	1/5	High rate of limit orders arrival	1.61 ± 0.68	0.58 ± 0.39
4	10	1	2/5	High rate of limit orders cancellations	2.64 ± 1.31	-2.95 ± 2.78

- The average spread increases with market orders μ
- The average spread decreases as limit orders α increases
- The standard deviation of the mid-price and spread is expected to increase with μ and decrease as α increases.

Chapter III

Part I : Depth and Slope

Prediction #1 Depth: the density of shares far away from the mid-price.

$$Depth \sim \frac{\alpha}{\delta}$$

Prediction #2 Slope: a measure of market order liquidity, the ratio of depth to spread.

$$Slope \sim \frac{\alpha^2}{\mu\delta}$$

Chapter III

Simulation vs Theoretical

Scenario	μ	α	δ	Comments	Slope	Slope $\left(\alpha^2/\mu\delta\right)$	Depth (Prediction)	Depth $\left(\alpha/\delta\right)$
1	10	1	1/5	Base scenario	0.495565	0.5	5.015938	5
2	20	1	1/5	High rate of market order arrival	0.3360431	0.25	4.392948	5
3	10	5	1/5	High rate of limit orders arrival	1.4501796	12.5	24.419772	25
4	10	1	2/5	High rate of limit orders cancellations	0.2078544	0.25	2.441263	2.5

- For depth:

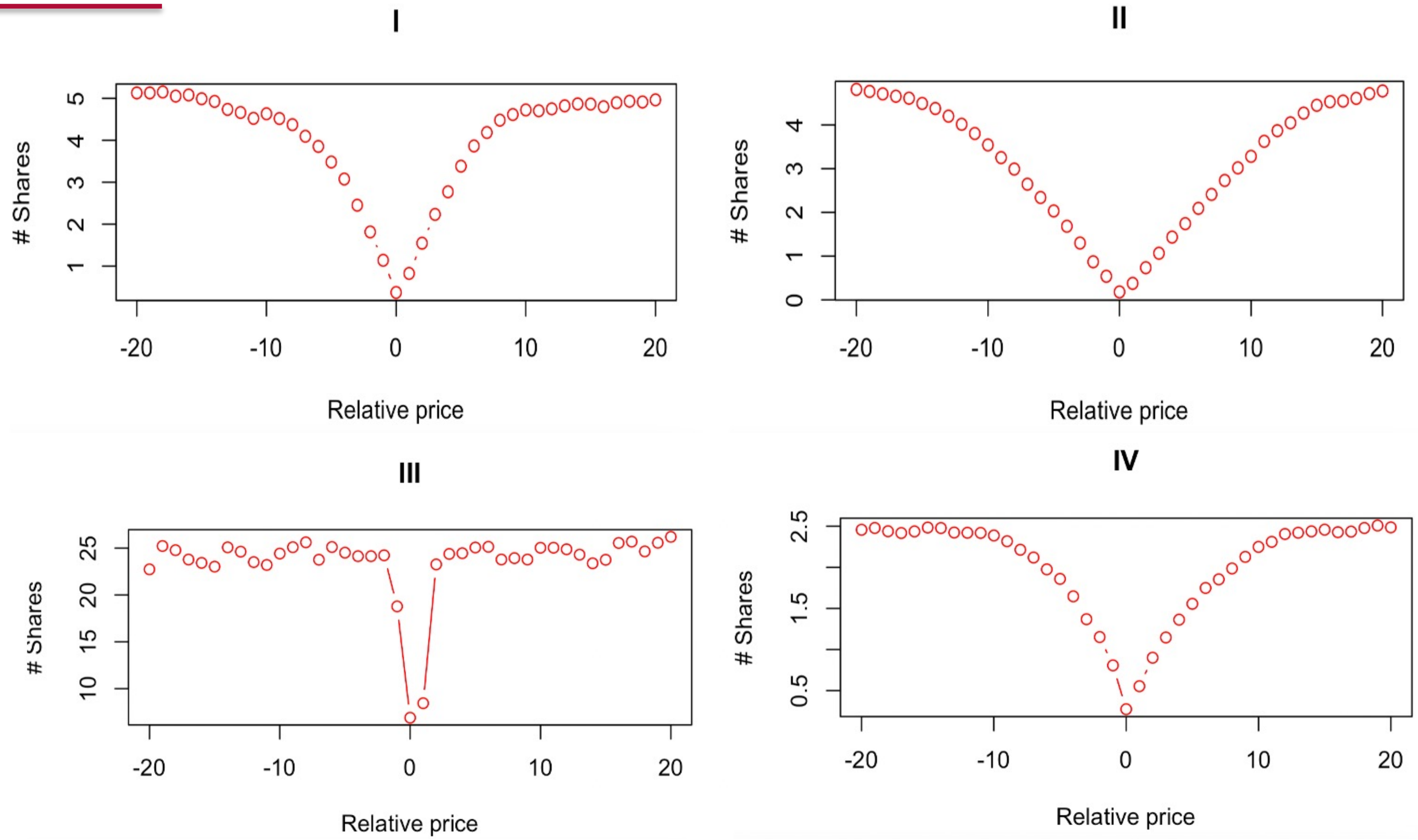
The simulation result of depth agree with the Santa Fe model prediction.

- For slope:

The simulation result for scenario 3 doesn’t match with the theoretical outcome. Because we use the relative price for regression.

Chapter III

Part II: Shape of LOB



Part III : Price impact

For $\epsilon \sim 0.01$, the Price impact:

$$\Delta p(w) \sim \sqrt{\frac{2w}{slope}}$$

In our simulation, the $\epsilon = \frac{\sigma\delta}{\mu}$ is small.

There is small accumulation of orders at best bid and ask, and near the midpoint price the depth profile increases nearly linear with price. As a result, as a crude approximation, the price impact increases roughly the square root of order size.

Chapter IV

Part I : Price dynamics

- Data: fake TAQ data (1000 events)
- Collect all market orders (number: 74)

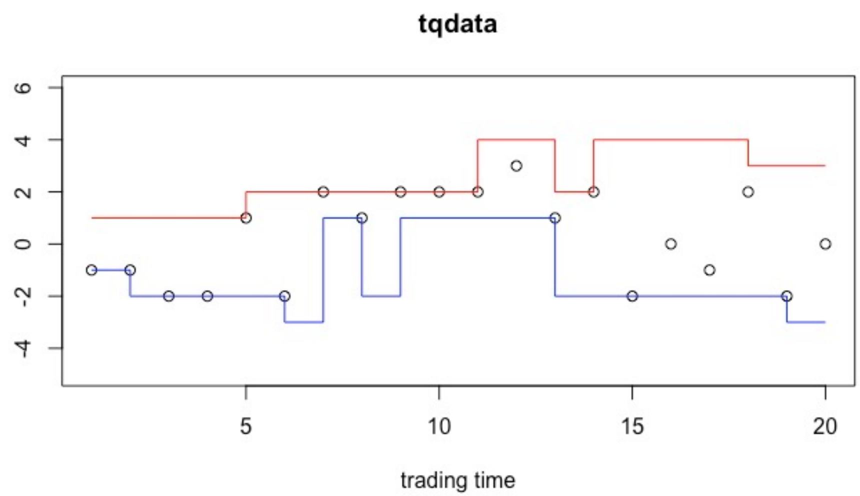
—— 37 market sells and 37 market buys

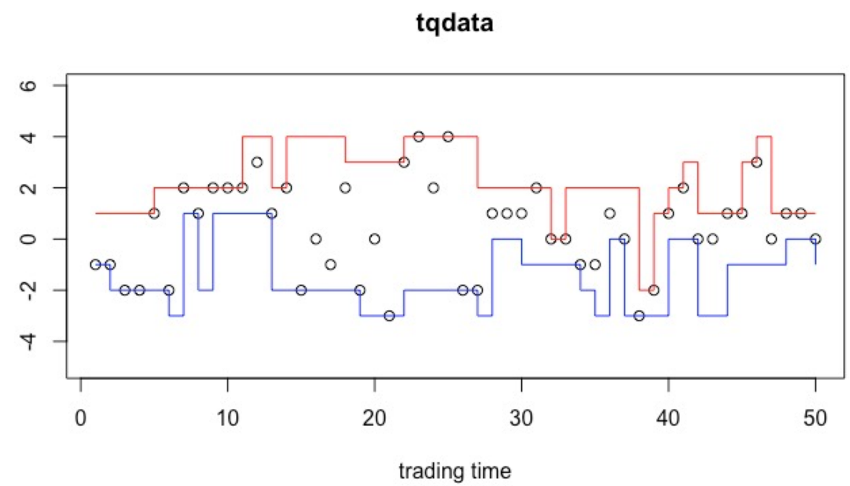
```
> tqdata_ms
    SYMBOL   EX BID BIDSIZ OFR OFRSIZ MODE PRICE DIR
4      YYY None  -1      5   1      5   20    -1  MS
46     YYY None  -1      5   2      5   20    -1  MS
68     YYY None  -1      5   0      5   20    -1  MS
82     YYY None  -2      5   0      5   20    -1  MS
92     YYY None  -2      5   0      5   20    -2  MS

> tqdata_mb
    SYMBOL   EX BID BIDSIZ OFR OFRSIZ MODE PRICE DIR
18     YYY None  -1      5   2      5   20     1  MB
21     YYY None  -1      5   2      5   20     2  MB
45     YYY None  -1      5   2      5   20     2  MB
103    YYY None  -2      5   2      5   20     0  MB
111    YYY None  -2      5   3      5   20     2  MB
```

Part I : Price dynamics

- Plot the first 20 and 50 events which described the changes in Best Bid and Best Ask price after each execution
- Red line: Best Ask

  Blue line: Best Bid

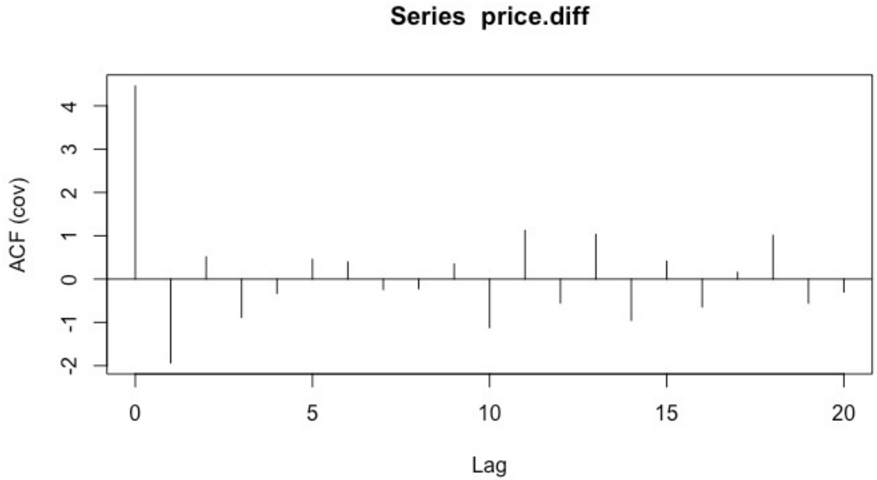


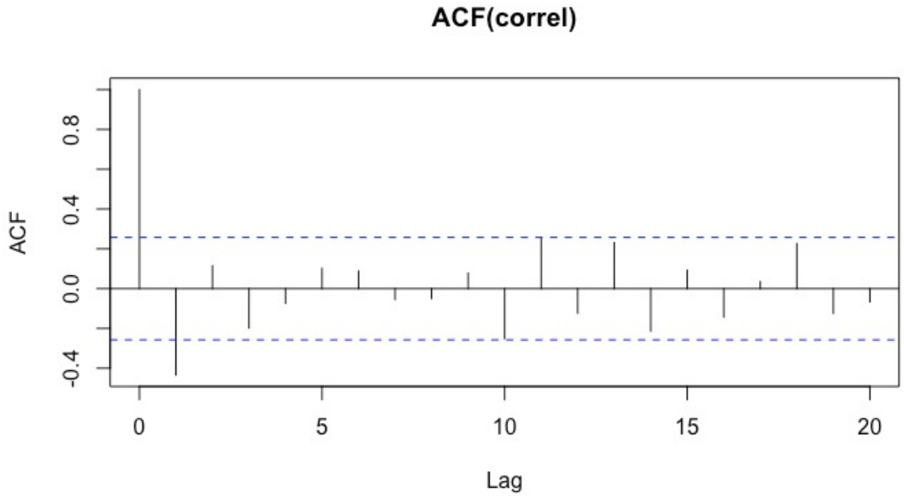


Part I : Price dynamics

❖ Then, let we see the covariance and autocorrelation

```
price <- as.numeric(tqdata$PRICE)
price.diff <- diff(price)
price.diff.acf <- acf(price.diff, lag.max=20, type="covariance")
price.diff.acfc <- acf(price.diff, lag.max=20, type="correlation",
                       main="ACF(correl)")
```

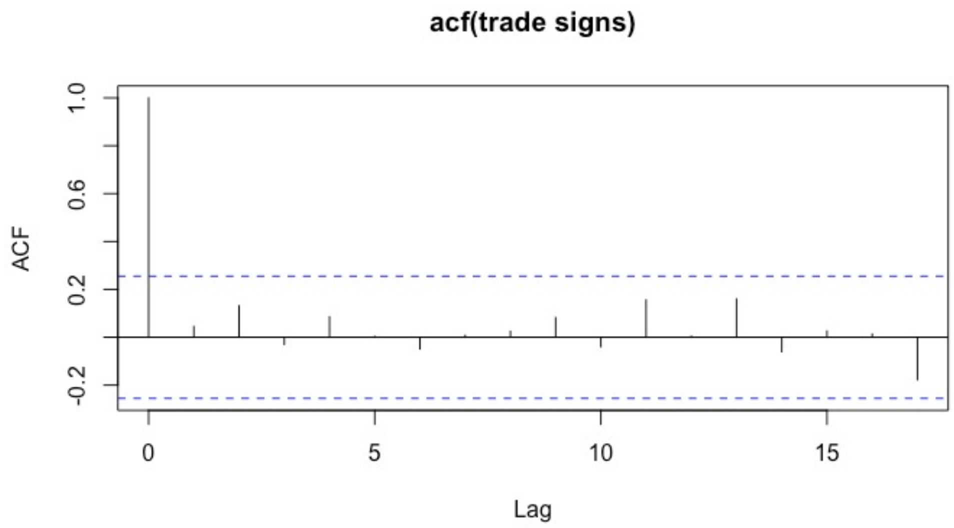




Part II : Autocorrelation of trade signs and compared with Roll Model

- trade signs are d_t, where $d_t = \begin{cases} 1, buy \\ -1, sell \end{cases}$

```
trade.dir <- coredata(tqdata$DIR[,1])[,1]
trade.sign.recorded <- ifelse(trade.dir=="MB",1,-1)

plot(trade.sign.recorded, type="l")

acf(trade.sign.recorded, main="acf(trade signs)")
```



Part II : Autocorrelation of trade signs and compared with Roll Model

❖ Roll Model Result

Scenario	μ	α	δ	Comments	c	σ_u	spread
1	10	1	1/5	Base scenario	1.39	0.77	2.78
2	20	1	1/5	High rate of market order arrival	2.07	3.06	4.14
3	10	5	1/5	High rate of limit orders arrival	0.67	0.17	1.35
4	10	1	2/5	High rate of limit orders cancellations	1.61	1.16	3.22

❖ Compared with ZI model

Scenario	μ	α	δ	Comments	Spread	Mid-price
1	10	1	1/5	Base scenario	2.72 ± 1.58	-1.96 ± 2.38
2	20	1	1/5	High rate of market orders arrival	5.84 ± 3.13	2.64 ± 2.97
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Chapter V

Improvements with relevant topic

If there is extra time one could explore further along different situations.

1. Change the assumptions with uniformed distributed arrival rate

   or Erlang distribution.

2. Add more simulation times.
3. Discuss the slope of LOB using absolute price.
4. Maybe use the real-world data to estimate the validity of Zero Intelligence Model.
5. Discuss the price impact of different size of arrival order

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